On the use of evolutionary methods in metric theories of gravity VIII.

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Plans and Aims:

some of the arguments and techniques developed originally and applied so far exclusively only in the Lorentzian case do also apply to Riemannian spaces

5 The construction of binary initial data

- parabolic-hyperbolic system
- superposed Kerr-Schild
- initial-boundary value problem

  … no use of ad hoc boundary conditions in the strong field regime

  … there is an unprecedented full control of the ADM charges

Outline:

Motivations:

GW observations:

- inspiral and merger of binary black holes is of distinguished importance for the emerging field of gravitational wave astronomy
- non-linearities necessitate the use of accurate numerical approaches in determining the emitted waveforms
- precision of these simulations—in particular, their initializations—is of critical importance in enhancing the detection of gravitational wave signals

A new construction:

- I. Rácz: *Constraints as evolutionary systems*, Class. Quantum Grav. **33** 015014 (2016)
- I. Rácz: *A simple method of constructing binary black hole initial data*, Astronomy Reports **62** 953-958 (2018)
- I. Rácz: *On the ADM charges of multiple black holes* , arXiv:1608.02283

Intitialization:

The constraints:

- vacuum initial data: (h_{ij}, K_{ij}) on a 3-dimensional manifold Σ
  - evolution equations $\mathscr{L}_n h_{ij} = ...$ & $\mathscr{L}_n K_{ij} = ...$ (in analogy $\dot{\mathbf{x}} = \mathbf{v}$ & $\dot{\mathbf{v}} = \mathbf{f}$)

$$
\begin{aligned}
{}^{(3)}R + (K^e{}_e)^2 - K_{ef}K^{ef} &= 0 \\
D_e K^e{}_a - D_a K^e{}_e &= 0
\end{aligned}
$$

where D_a denotes the covariant derivative operator associated with h_{ab}

- it is an underdetermined system: 4 equations for 12 variables

The conformal (elliptic) method:

Lichnerowicz A (1944) and York J W (1972):

- replace

$$h_{ij} = \phi^4\, \widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\, h_{ij}\, K^l{}_l = \phi^{-2}\, \widetilde{K}_{ij}$$

- using these variables the constraints are put into the **semilinear elliptic system**

$$\widetilde{D}^l \widetilde{D}_l \phi - \tfrac{1}{8}\, \widetilde{R}\, \phi + \tfrac{1}{8}\, \widetilde{K}_{ij} \widetilde{K}^{ij}\, \phi^{-7} - \tfrac{1}{12}\, (K^l{}_l)^2\, \phi^5 = 0$$

where $\widetilde{D}_l$, $\widetilde{R}$, ... $\widetilde{h}_{ij}$

- $\widetilde{K}_{ij} = \widetilde{K}^{[L]}_{ij} + \widetilde{K}^{[TT]}_{ij}$, where $\widetilde{K}^{[L]}_{ij} = \widetilde{D}_i X_j + \widetilde{D}_j X_i - \tfrac{2}{3}\, \widetilde{h}_{ij} \widetilde{D}^l X_l$

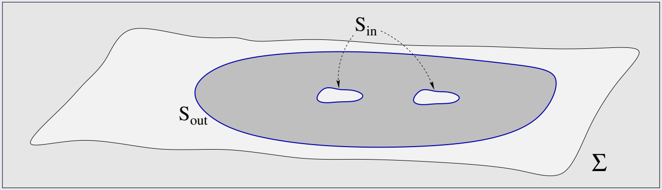
$$\widetilde{D}^l \widetilde{D}_l X_i + \tfrac{1}{3}\, \widetilde{D}_i (\widetilde{D}^l X_l) + \widetilde{R}_i{}^l X_l - \tfrac{2}{3}\, \phi^6 \widetilde{D}_i (K^l{}_l) = 0$$

- $$(h_{ij}, K_{ij}) \quad \longleftrightarrow \quad \left(\phi, \widetilde{h}_{ij}; K^l{}_l, X_i, \widetilde{K}^{[TT]}_{ij}\right)$$

The conformal method:

Impressive mathematical developments since 1944 but ...

- either "constancy" of ${K^l}_l$ or "smallness" of the TT part of $\widetilde{K}_{ij}$ is required
- it is highly implicit due to its elliptic character and the replacements $h_{ij} = \phi^4\,\widetilde{h}_{ij}$ and $K_{ij} = \frac{1}{3}\,\phi^4\,\widetilde{h}_{ij}\,{K^l}_l + \phi^{-2}\,\widetilde{K}_{ij}$ $\Longrightarrow$
  - no direct control on the physical parameters of the initial data specifications
- **boundary conditions:**
  - are known to influence solutions everywhere in their domains
  - the inner boundary conditions—they are applied with **excision** in the black hole interior—cannot simply be supported by intuition (**trumpet data** ...)
  - Bowen-York type initial data: $\widetilde{h}_{ij}$ is flat $\widetilde{h}_{ij} = \delta_{ij}$ and ${K^l}_l = 0$ ~~Kerr BH~~ non-negligible spurious gravitational wave content of yielded time evolutions



New variables by applying $2+1$ decompositions:

Splitting of the metric h_{ij}:

assume: $\Sigma \approx \mathbb{R} \times \mathscr{S}$

Σ is smoothly foliated by a one-parameter family of two-surfaces $\mathscr{S}_\rho$:
$\rho = const$ level surfaces of a smooth real function $\rho : \Sigma \to \mathbb{R}$ with $\partial_i \rho \neq 0$

$$\Longrightarrow \quad \widehat{n}_i = \widehat{N}\, \partial_i \rho \ \ldots\ \& \ldots\ h^{ij} \longrightarrow \widehat{n}^i = h^{ij}\widehat{n}_j \longrightarrow \widehat{\gamma}^i{}_j = \delta^i{}_j - \widehat{n}^i\widehat{n}_j$$

- choose ρ^i to be a vector field on Σ: the integral curves... & $\rho^i\partial_i\rho = 1$
- 'lapse' and 'shift' of ρ^i

$$\rho^i = \widehat{N}\,\widehat{n}^i + \widehat{N}^i\,, \quad \text{where} \quad \widehat{N} = \rho^j\widehat{n}_j \quad \text{and} \quad \widehat{N}^i = \widehat{\gamma}^i{}_j\,\rho^j$$

- induced metric, extrinsic curvature and acceleration of the $\mathscr{S}_\rho$ level surfaces:

$$\widehat{\gamma}_{ij} = \widehat{\gamma}^k{}_i\,\widehat{\gamma}^l{}_j\,h_{kl} \qquad \widehat{K}_{ij} = \tfrac{1}{2}\,\mathscr{L}_{\widehat{n}}\widehat{\gamma}_{ij} \qquad \dot{\widehat{n}}_i := \widehat{n}^e\nabla_e\widehat{n}_i = -\widehat{D}_i \ln \widehat{N}$$

- the metric h_{ij} can then be given as

$$h_{ij} = \widehat{\gamma}_{ij} + \widehat{n}_i\widehat{n}_j \quad \Longleftrightarrow \quad \{\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}\}$$

$2+1$ decompositions:

Splitting of the symmetric tensor field K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\, \widehat{n}_i \widehat{n}_j + [\widehat{n}_i\, \mathbf{k}_j + \widehat{n}_j\, \mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

  $$\boldsymbol{\kappa} = \widehat{n}^k \widehat{n}^l\, K_{kl}\,, \quad \mathbf{k}_i = \widehat{\gamma}^k{}_i \widehat{n}^l\, K_{kl} \quad \text{and} \quad \mathbf{K}_{ij} = \widehat{\gamma}^k{}_i \widehat{\gamma}^l{}_j\, K_{kl}$$

- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

  $$\mathbf{K}^l{}_l = \widehat{\gamma}^{kl}\, \mathbf{K}_{kl} \quad \text{and} \quad \mathring{\mathbf{K}}_{ij} = \mathbf{K}_{ij} - \tfrac{1}{2}\, \widehat{\gamma}_{ij} \mathbf{K}^l{}_l$$

The new variables:

- $$(h_{ij}, K_{ij}) \quad \Longleftrightarrow \quad (\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$$

- these variables retain the physically distinguished nature of h_{ij} and K_{ij}

The parabolic-hyperbolic form of the constraints:

An evolutionary system for the constrained fields $\widehat{N}, \mathbf{k}_i$ and $\mathbf{K}^l{}_l$:

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l \widehat{N})\,] - \widehat{N}^2(\widehat{D}^l \widehat{D}_l \widehat{N}) - \mathcal{A}\,\widehat{N} - \mathcal{B}\,\widehat{N}^3 = 0$$

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i - \tfrac{1}{2}\,\widehat{D}_i(\mathbf{K}^l{}_l) - \widehat{D}_i\boldsymbol{\kappa} + \widehat{D}^l\overset{\circ}{\mathbf{K}}_{li} + \widehat{N}\overset{\star}{K}\,\mathbf{k}_i + [\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,]\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\overset{\circ}{\mathbf{K}}_{li} = 0$$

$$\mathscr{L}_{\widehat{n}}(\mathbf{K}^l{}_l) - \widehat{D}^l\mathbf{k}_l - \widehat{N}\overset{\star}{K}\,[\,\boldsymbol{\kappa} - \tfrac{1}{2}\,(\mathbf{K}^l{}_l)\,] + \widehat{N}\,\overset{\circ}{\mathbf{K}}_{kl}\overset{\star}{K}{}^{kl} + 2\,\dot{\widehat{n}}^l\,\mathbf{k}_l = 0\,,$$

where $\widehat{D}_i$ denotes the covariant derivative operator associated with $\widehat{\gamma}_{ij}$

$$\overset{\star}{K} = \tfrac{1}{2}\,\widehat{\gamma}^{ij}\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_j\widehat{N}^j$$

$$\overset{\star}{K}_{ij} = \tfrac{1}{2}\mathscr{L}_\rho\widehat{\gamma}_{ij} - \widehat{D}_{(i}\widehat{N}_{j)}\,, \qquad \dot{\widehat{n}}_k = \widehat{n}^l D_l \widehat{n}_k = -\widehat{D}_k(\ln \widehat{N})$$

$$\mathcal{A} = (\partial_\rho \overset{\star}{K}) - \widehat{N}^l(\widehat{D}_l\overset{\star}{K}) + \tfrac{1}{2}[\,\overset{\star}{K}{}^2 + \overset{\star}{K}_{kl}\overset{\star}{K}{}^{kl}\,]$$

$$\mathcal{B} = -\tfrac{1}{2}\,\big[\widehat{R} + 2\,\boldsymbol{\kappa}\,(\mathbf{K}^l{}_l) + \tfrac{1}{2}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \overset{\circ}{\mathbf{K}}_{kl}\,\overset{\circ}{\mathbf{K}}{}^{kl}\,\big]$$

The parabolic-hyperbolic system:

The parabolic-hyperbolic system:

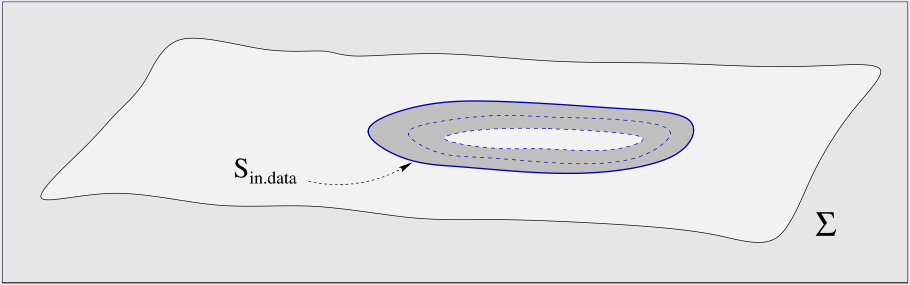
- no restriction applies to $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ $\Longrightarrow$ they are freely specifiable throughout Σ
- the parabolic equation is uniformly parabolic in those subregions of Σ, where $\overset{\star}{K}$ is either positive or negative
- $\overset{\star}{K}$ depends exclusively on the freely specifiable fields $\widehat{\gamma}_{ij}$ and $\widehat{N}^i$ $\Longrightarrow$ its sign can be tailored according to the desire of the investigated problem
- the combined evolutionary system is (locally) well-posed
  - if suitable initial values for the constrained fields $\widehat{N}, \mathbf{k}_i$ and $\mathbf{K}^l{}_l$ are given, on some level surface $\mathscr{S}_0$ in Σ, then, in the domain of dependence of $\mathscr{S}_0$, unique solution exists to the evolutionary system
  - the fields h_{ij} and K_{ij} that can be reconstructed from the freely specifiable and constrained variables do satisfy the Hamiltonian and momentum constraints

Solving the constraints:

- (h_{ij}, K_{ij}) represented by the variables $(\widehat{N}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i, \mathbf{K}^l{}_l, \mathring{\mathbf{K}}_{ij})$
- the constraints comprise a **parabolic-hyperbolic** system for $(\widehat{N}, \mathbf{k}_i, \mathbf{K}^l{}_l)$
  - with freely specifiable variables on Σ and on $\mathrm{S}_{\mathrm{in.data}}$:

$$(\widehat{N}|_{\mathrm{S_{in.data}}}, \widehat{N}^i, \widehat{\gamma}_{ij}; \boldsymbol{\kappa}, \mathbf{k}_i|_{\mathrm{S_{in.data}}}, \mathbf{K}^l{}_l|_{\mathrm{S_{in.data}}}, \mathring{\mathbf{K}}_{ij})$$

  - a fixed $(+/-)$ sign of $\overset{\star}{K} = \frac{1}{2}\,\widehat{\gamma}^{ij}\,\mathscr{L}_\rho \widehat{\gamma}_{ij} - \widehat{D}_j \widehat{N}^j$ can be guaranteed



The Kerr black hole:

In Kerr-Schild form:

- $$g_{\alpha\beta} = \eta_{\alpha\beta} + 2H\ell_\alpha\ell_\beta$$
- inertial coordinates (t, x, y, z) adapted to the Minkowski background $\eta_{\alpha\beta}$

$$H = \frac{r^3 M}{r^4 + a^2 z^2}$$

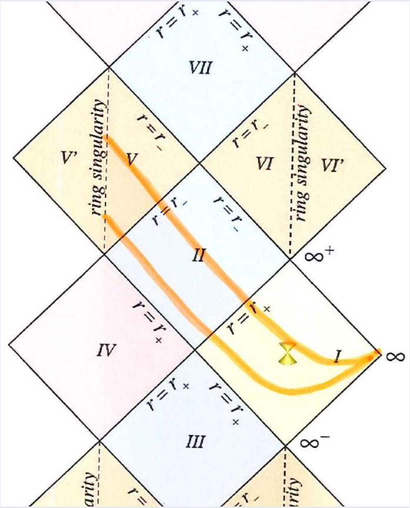
$$\ell_\alpha = \left(1, \frac{r\,x + a\,y}{r^2 + a^2}, \frac{r\,y - a\,x}{r^2 + a^2}, \frac{z}{r}\right)$$

- the Boyer-Lindquist radial coordinate r is related to the spatial part of the inertial coordinates as

$$r^4 - (x^2 + y^2 + z^2 - a^2)\, r^2 - a^2\, z^2 = 0 \qquad \frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1$$

- the $r = const$ surfaces are "ellipsoids"
  - degenerate to a disk $x^2 + y^2 \le a^2$ & $z = 0$ possessing the "ring singularity" (given as $x^2 + y^2 = a^2$ & $z = 0$) at its edge

$t = const$ slices in Kerr spacetime:



the $\theta = \frac{\pi}{2}$ section

$$\Sigma \approx \mathbb{R}^3 \setminus \{\text{“ring singularity”}\}$$

Generic Kerr-Schild black holes:

- the Kerr-Schild metrics are form-invariant under Lorentz transformations
  - if a Lorentz transformation $x'^{\alpha} = \Lambda^{\alpha}{}_{\beta}\, x^{\beta}$ is performed
  - the metric retains its distinguished Kerr-Schild form

$$g'_{\alpha\beta} = \eta_{\alpha\beta} + 2H'\ell'_{\alpha}\ell'_{\beta}$$

  - where $H' = H'(x'^{\alpha})$ and $\ell'_{\beta} = \ell'_{\beta}(x'^{\varepsilon})$ are given as

$$H' = H\left([\Lambda^{\alpha}{}_{\beta}]^{-1}x'^{\beta}\right), \ \ell'_{\beta} = \Lambda^{\alpha}{}_{\beta}\,\ell_{\alpha}\left([\Lambda^{\varepsilon}{}_{\varphi}]^{-1}x'^{\varphi}\right)$$

- boosts and spatial rotations are special Lorentz transformations $\Longrightarrow$ it is straightforward to construct models of moving and rotating black holes with preferably oriented speed and spin

Superposed Kerr-Schild black holes:

A binary system will be approximated by:

- $$g_{\alpha\beta} = \eta_{\alpha\beta} + 2H^{[1]}\ell_\alpha{}^{[1]}\ell_\beta{}^{[1]} + 2H^{[2]}\ell_\alpha{}^{[2]}\ell_\beta{}^{[2]} \qquad (*)$$
  - $H^{[n]}$ and $\ell_\alpha{}^{[n]}$ correspond to the Kerr-Schild data for individual black holes
- (*) does not satisfy Einstein's equations
- good approximation close to the individual black holes
- Einstein tensor falls off at the rate $\mathcal{O}(|\vec{x}|^{-4})$, where $|\vec{x}| = \sqrt{x^2 + y^2 + z^2}$

Choice for the free data:

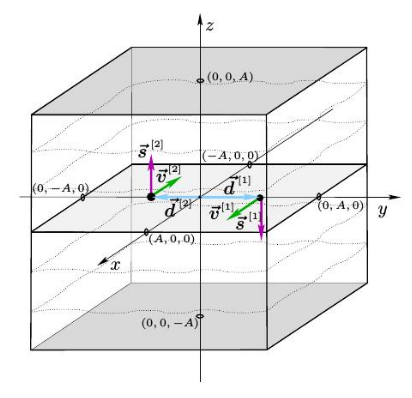
- $\widehat{N}^i, \widehat{\gamma}_{ij}, \boldsymbol{\kappa}$ and $\mathring{\mathbf{K}}_{ij}$ as if (*) solved the Einstein equations
- $\widehat{N}, \mathbf{K}^l{}_l$ and $\mathbf{k}_i$ on some level surface $\mathscr{S}_0$ in Σ deduced from (*) [only on $\mathscr{S}_0$!]
- **initial data surface**: the complementer of the two "ring" singularities in $t_{Mink} = 0$ hypersurface: $\Sigma \approx \mathbb{R}^3 \setminus \{ring_1 \text{ and } ring_2\}$

The initial-boundary value problem:

the $\mathscr{S}_\rho$ surfaces have tacitly been assumed to be compact without boundary:

- in numerical approaches Σ is chosen to be a large but bounded subset of $\mathbb{R}^3$
- the product structure $\Sigma \approx \mathbb{R} \times \mathscr{S}$ can be guaranteed by choosing the $\mathscr{S}_\rho$ leaves to be diffeomorphic to a closed disk in $\mathbb{R}^2$

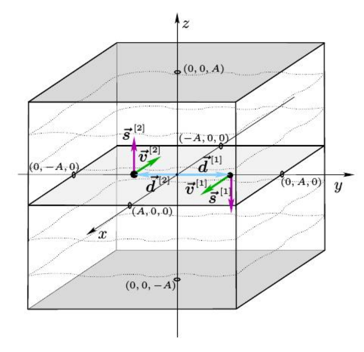
choose Σ to be a cube centered at the origin in $\mathbb{R}^3$:



- for large enough value of A ...
- boundary of Σ:

  six squares each with edges of size $2A$
- the black holes are assumed to be located on the $z = 0$ plane
- speeds are parallel, spins are orthogonal to the $z = 0$ plane
- foliation by $z = const$ level surfaces
- deduce $\overset{\star}{K}$ from (*)

The critical coefficient $\overset{\star}{K}$:



- the sign of $\overset{\star}{K}$ decides whether the parabolic-hyperbolic system evolves in the positive or negative ρ-direction

$$\overset{\star}{K}\,[\,(\partial_\rho \widehat{N}) - \widehat{N}^l(\widehat{D}_l \widehat{N})\,] = \widehat{N}^2(\widehat{D}^l \widehat{D}_l \widehat{N}) + \mathcal{A}\,\widehat{N} + \mathcal{B}\,\widehat{N}^3$$

  - it propagates aligned ρ^i for positive $\overset{\star}{K}$, while anti-aligned for negative $\overset{\star}{K}$
- restrict considerations to a binary BH system arranged as indicated on the figure

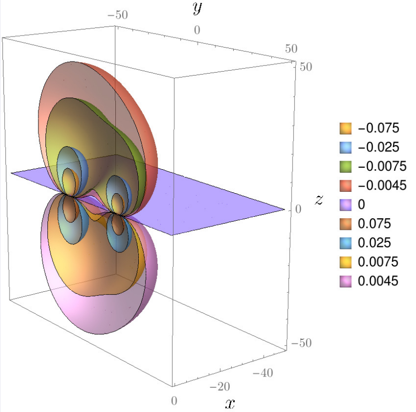
Splitting the boundary:

The princical coefficient $\overset{\star}{K}$:

- $\overset{\star}{K}$ can be given as the product of a strictly negative function and the z-coordinate
- $\overset{\star}{K}$ is positive below the $z = 0$ plane, while it is negative above that plane

$\overset{\star}{K} = const$ level surfaces:

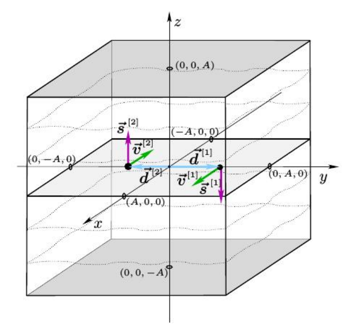
$$x < 0,\ 2A = 100$$
$$M^{[1]} = 1,\ \vec{d}^{\,[1]} = 20\,\vec{e}_y$$
$$\vec{v}^{\,[1]} = 0.5\,\vec{e}_x,\ a^{[1]} = 0.6$$
$$M^{[2]} = 2,\ \vec{d}^{\,[2]} = -10\,\vec{e}_y$$
$$\vec{v}^{\,[2]} = -0.25\,\vec{e}_x,\ a^{[2]} = -0.8$$



Solving the initial-boundary value problem:

The parabolic-hyperbolic system:

- $\overset{\star}{K}$ can be given as $\boxed{\overset{\star}{K} = -z \cdot \overset{+}{K}}$
- $\overset{\star}{K}$ is positive below the $z = 0$ plane, while it is negative above that plane
- solved by propagating, along the z-streamlines, initial values specified on the horizontal $z = \pm A$ squares
- boundary values are to be given on the four vertical sides of the cube



- $\widehat{N}, \mathbf{K}^l{}_l$ and $\mathbf{k}_i$ are developed on Σ^+ and Σ^- separately
  - global existence and matching of these solutions at their common Cauchy horizon—at the $z = 0$ plane—is of fundamental importance
- the auxiliary metric (*) possesses a $z \to -z$ reflection symmetry
- assume the existence of unique (at least) C^2 solutions (apart from singularities): proper matching at the "common Cauchy horizon" follows

The ADM quantities as flux integrals:

- in the applied admissible coordinates the ADM mass, center of mass, linear and angular momenta are determined by the flux integrals

$$M^{ADM} = \frac{1}{16\pi} \oint_\infty [\,\partial_i h_{ij} - \partial_j h_{ii}\,] n^j \mathrm{d}S$$

$$M^{ADM} d_i = \frac{1}{16\pi} \oint_\infty \Big\{ x_i \,[\,\partial_k h_{kj} - \partial_j h_{kk}\,] - \Big[h_{kj}\, \delta^k{}_i - h_{kk}\, \delta_{ij} \Big] \Big\} n^j \mathrm{d}S$$

$$P_i^{ADM} = \frac{1}{8\pi} \oint_\infty \Big[K_{ij} - h_{kj}\, K^l{}_l \Big] n^j \mathrm{d}S$$

$$J_i^{ADM} = \frac{1}{8\pi} \oint_\infty \Big[K_{kj} - h_{kj}\, K^l{}_l \Big] Y_i^k n^j \mathrm{d}S$$

- the symbol $\oint_\infty$ is meant to denote limits of integrals over spheres while their radii tend to infinity
- n^i and $\mathrm{d}S$ denote the outward normal and the volume element of the individual spheres in the sequences
- the symbol $Y_i^k = \epsilon_i{}^{jk} x_j$ denote the components of the three rotational Killing vector fields, defined with respect to the applied admissible asymptotically Euclidean coordinates

Input parameters and global ADM charges:

- **Input parameters:** the rest masses $M^{[n]}$, displacements $\vec{d}^{\,[n]}$, speeds $\vec{v}^{\,[n]}$ and spins $a^{[n]}\vec{s}_\circ^{[n]}$ of the involved black holes
    - essentially the same as used in post-Newtonian description of binaries **!!!**

- **Global ADM charges:** in terms of the input parameters
    - though (*) does not satisfy Einstein's equations it is asymptotically flat
    - constructed by adding contributions of individual black hole metrics to a Minkowski background
    - the ADM quantities are linear in deviation from flat Euclidean space at infinity

$$\begin{aligned}
M^{ADM} &= \gamma^{[1]} M^{[1]} + \gamma^{[2]} M^{[2]} \\
M^{ADM} \vec{d}^{\,ADM} &= \gamma^{[1]} M^{[1]} \vec{d}^{\,[1]} + \gamma^{[2]} M^{[2]} \vec{d}^{\,[2]} \\
\vec{P}^{\,ADM} &= \gamma^{[1]} M^{[1]} \vec{v}^{\,[1]} + \gamma^{[2]} M^{[2]} \vec{v}^{\,[2]} \\
\vec{J}^{\,ADM} &= \gamma^{[1]} \left\{ M^{[1]} \vec{d}^{\,[1]} \times \vec{v}^{\,[1]} + M^{[1]} a^{[1]} \vec{s}_\circ^{[1]} \right\} \\
&\quad + \gamma^{[2]} \left\{ M^{[2]} \vec{d}^{\,[2]} \times \vec{v}^{\,[2]} + M^{[2]} a^{[2]} \vec{s}_\circ^{[2]} \right\}
\end{aligned}$$

Summary:

1. a new method to initialize time evolution of binary black hole systems by applying
   - a parabolic-hyperbolic formulation of constraint equations
   - superposing Kerr-Schild black holes
2. the parabolic-hyperbolic equations solved as an initial-boundary value problem
3. existence of unique (at least) C^2 solutions is guaranteed (apart from singularities)
4. construct initial data by integrating numerically the parabolic-hyperbolic form of the constraints: **!!!** paper is coming out soon
5. the input parameters—the rest masses, speeds, spins and displacements—are essentially the same as used in PN **!!!**
6. each of the ADM charges can be given in terms of the input parameters
7. no use of boundary conditions in the strong field regime (tidal deformations)

That is all for now...