

Chair of Condensed Matter Physics Institute of Theoretical Physics Faculty of Physics, University of Warsaw

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Wykład

Modelowanie Nanostruktur

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AbsorptionReflectionDispersion	 These processes can be described as the action of a high-frequency macroscopic field on a solid the interaction between the elementary excitation of the solid and the quanta of the electromagnetic field – the photons
The macroscopic already contained Transverse, elec Material equation For nonmagnetic	interaction of electromagnetic fields with matter is d in the Maxwell equations tromagnetic waves \Longrightarrow LIGHT as $\vec{B} = \mu \vec{H}$ $\vec{D} = \varepsilon \vec{E}$ is solids $\mu = 1$
	stant C than contains all information about the





Absorption Coefficient & Measurement of Optical Constants • The intensity of light is proportional to $I \sim \vec{E} \times \vec{B} \sim \vec{E} ^2$					
$I(z) = I_0 e^{-\alpha z}$ with	$\alpha = \frac{2\omega\kappa}{c} = \frac{4\pi\kappa}{\lambda}$	Absorption coefficient			
• One can deduce that <i>I</i> satisfies the equation $\frac{dI}{dz} = -\alpha I$					
 Physical implification: Absorption coefficient is the amount of radiation energy absorbed by a unit volume of solid under unit radiation intensity (photon flux energy) in unit time 					
Empirical formula for semiconductors: $n^4 E_{Gan} = 77$ for $n \in [2.3 - 4.6]^{-1}$					
T. S. Moss, "Optical Properties of Semiconductors", Energy gap in eV Butterworths Scientific (1959)					
The optical constants are often determined by measuring					
i) Transmition ii) Reflection I_0 I_r $I_r = R_0$	Semiconductor $I_t = (1-R)I_0$ $n(\omega) \kappa(\omega)$				

Absorption Coefficient and Number of Absorbed Photons			
$\vec{E}(\vec{r},t) = -\nabla \phi(\vec{r},t) - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}(\vec{r},t)$	$\vec{B}(\vec{r},t) = \nabla \times \vec{A}(\vec{r},t) = curl\vec{A}(\vec{r},t)$		
Coulomb gauge: $\phi(\vec{r},t) = 0$	$div\vec{A}(\vec{r},t) = 0$		
$\vec{E} = \frac{i\omega}{c} A_0 \vec{e} \exp[i(\vec{q} \cdot \vec{r} - \omega t)] + c$	$\varepsilon.$		
$u = \frac{\varepsilon}{4\pi} \vec{E} \cdot \vec{E}^* = \frac{\varepsilon}{4\pi} \frac{\omega^2}{c^2} A_0^2$	$u\frac{c}{n} = \frac{\varepsilon}{n} \frac{1}{4\pi} \frac{\omega^2}{c} A_0^2 = n \frac{\omega^2}{c} \frac{1}{4\pi} A_0^2$		
$\boldsymbol{\alpha}(\boldsymbol{\omega}) = \frac{4\pi\hbar c}{n\omega A_0^2} W(\boldsymbol{\omega})$	$\varepsilon_2(\omega) = \frac{cn}{\omega} \alpha(\omega) = \frac{4\pi\hbar c^2}{\omega A_0^2} W(\omega)$		





Quantum mechanical calculation of absorption coefficient

$$\dot{a}_{m}(\vec{k}\,',t) = \frac{1}{i\hbar} \sum_{n\vec{k}} a_{n}(\vec{k},t) \exp\left(\frac{i}{\hbar} [E_{m}(\vec{k}\,') - E_{n}(\vec{k})]t\right) \langle m\vec{k}\,' | \hat{H}\,' | n\vec{k} \rangle$$
Let us assume: $a_{j}(\vec{k},0) = 1$ and for all other $a_{n}(\vec{k},0) = 0$
It means, at time $t = 0$ the electron is described by
 $\psi(0) = |j\vec{k}\rangle$ (is in state $|j\vec{k}\rangle$)
The probability $W(j,\vec{k},j',\vec{k}\,';\omega,t)$ that at time t it will be in state $|j'\vec{k}\,'\rangle$
is then equal to $|a_{j'}(\vec{k}\,',t)|^{2}$
In the first approximation
 $\dot{a}_{j'}^{(1)}(\vec{k}\,',t) = \frac{1}{i\hbar} \exp\left(\frac{i}{\hbar} [E_{j}(\vec{k}\,') - E_{j'}(\vec{k})]t\right) \langle j'\vec{k}\,' | \hat{H}\,' | j\vec{k}\rangle$
 $W(j,\vec{k},j',\vec{k}\,';\omega,t) = -\frac{1}{\hbar^{2}} \left| \int_{0}^{t} \exp\left(\frac{i}{\hbar} [E_{j}(\vec{k}\,') - E_{j'}(\vec{k})]t\right) \langle j'\vec{k}\,' | \hat{H}\,' | j\vec{k}\rangle dt' \right|^{2}$
 \circ Next we examine the matrix element $\langle j'\vec{k}\,' | \hat{H}\,' | j\vec{k}\rangle$











Optical properties of Quantum Structures

Specific features of optical processes originate from two basic physical peculiarities

- Spatial nonuniformity causes specific characteristics of the interaction of light with matter, including light propagation, absorption, etc..
- Electrons in quantum structures have energy spectra different from these of electrons in bulk materials.

Electrodynamics in Homogeneous Systems

In the simplest homogeneous case

$$\vec{E}(\vec{r},t) = \vec{e}E_0\cos(\vec{q}\cdot\vec{r}-\omega t) = \vec{e}E_0e^{-i(\vec{q}\cdot\vec{r}-\omega t)} + c.c$$

 The *energy of the wave* can be characterized by the density of the electromagnetic energy

$$w = \frac{1}{4\pi} \varepsilon_r \overline{E^2(t)} = \frac{1}{8\pi} \varepsilon_r E_0^2$$

The *intensity of the wave* = the energy flux through the unit area perpendicular to the wave vector *q*

$$I = \frac{1}{8\pi} \varepsilon_r E_0^2$$



















Light Absorption in Quantum Wells			
$R \sim \Gamma \sum_{i < i} \vec{e} \cdot \hat{\vec{p}} f > ^2 \delta(E_f - E_i - \hbar \omega) [f(E_i) - f(E_f)]$			
i, f Bulk: $ i\rangle = v\vec{k}\rangle$	$ f angle=ig cec{k} ight angle$		
QWs: $ i\rangle = v, n, \bar{k}\rangle$	$\left f \right\rangle = \left c, n, \vec{k}_{//c} \right\rangle$		
$\left i\right\rangle = \boldsymbol{\varPsi}_{\vec{v}\vec{k}_{//v}n}(\vec{r})$	$) = \frac{1}{\sqrt{S}} u_{\nu 0}(\vec{r}) e^{i\vec{k}_{//\nu} \cdot \vec{r}_{//}} \chi_{\nu n}(z)$		
$ f\rangle = \Psi_{c\vec{k}_{//c}n}(t)$	\vec{r}) = $\frac{1}{\sqrt{S}} u_{c0}(\vec{r}) e^{i\vec{k}_{//c}\cdot\vec{r}_{//}} \chi_{cn}(z)$		
$u_{v0}(\vec{r}) \& u_{c0}(\vec{r})$	- periodic parts of Bloch functions	s for $\vec{k}_0 = 0$	
$\vec{k}_{//v}$ & $\vec{k}_{//c}$	- two-dimensional (in-plane) wave vectors		
$\chi_{vn}(z)$ & $\chi_{cn}(z)$	 the envelope wave functions of the quantized transverse motion 		
S	- the area of the quantum well lay	yer	





Light Absorption in Quantum Wells				
• Using the matrix elements $\langle i \vec{e} \cdot \hat{\vec{p}} f \rangle$, one can calculate all characteristics, β, γ, α , for geometries presented previously.				
$\boldsymbol{\alpha} \sim \boldsymbol{\Gamma} \sum_{n,m} \sum_{\vec{k}_{//\nu}, \vec{k}_{//c}} p_{vc}^{bulk} ^2 \langle \boldsymbol{\chi}_{vn} \boldsymbol{\chi}_{cm} \rangle ^2 \boldsymbol{\delta}_{\vec{k}_{//\nu}, \vec{k}_{//c}} \times$				
$\times \boldsymbol{\delta}(E_{cm}(\vec{k}_{//c}) - E_{vn}(\vec{k}_{//v}) - \hbar\boldsymbol{\omega})[f(E_{vn}) - f(E_{cm})]$				
$\boldsymbol{\alpha} \sim \Gamma \sum_{n,m} p_{vc}^{bulk} ^2 \langle \boldsymbol{\chi}_{vn} \boldsymbol{\chi}_{cm} \rangle ^2 \int d^2 \vec{k}_{ll} \delta(E_{cm}(\vec{k}_{ll}) - E_{vn}(\vec{k}_{ll}) - \hbar \boldsymbol{\omega}) [f(E_{vn}) - f(E_{cm})]$				
• The energy-conservation law following from the δ function				
$E_{cm}(\vec{k}_{//}) - E_{vn}(\vec{k}_{//}) = \hbar \omega$				
shows that photo-transitions can involve different subbands from both the valence and the conduction bands.				













Intraband Transitions in Quantum Structures We start by recalling that *in an ideal bulk crystal, intraband photo-transitions are impossible* because of the energy- and the momentum-conservation laws E(k) - E(k') = ħω k - k' = q ≈ 0 Intraband photo-transitions in a bulk can be induced only by phonons, impurities, and other crystal imperfections. In contrast to bulk materials, intraband photo-transitions occur in semiconductor heterostructures.

