Electronic structure of a bulk (3D) in the magnetic field

\[ \frac{1}{2m^*} \left( \frac{\hbar}{c} \mathbf{A} - e^* \mathbf{A} \right)^2 \Psi(x, y, z) = E \Psi(x, y, z) \]

\[ \mathbf{A} = \nabla \times \mathbf{B} \]

\[ \mathbf{A} \rightarrow \tilde{\mathbf{A}} = \mathbf{A} + \nabla \Lambda, \quad \text{where} \quad \Lambda(x, t) \sim \text{scalar function} \]

\[ \mathbf{B} = \text{const} = (0, 0, B) \]

\[ \tilde{\mathbf{A}}(\vec{r}) = (-B_y, 0, 0) \]

\[ \tilde{\mathbf{A}}(\vec{r}) = \frac{1}{2} (-B_y, B_x, 0), \quad [\tilde{\mathbf{A}}(\vec{r}) = \frac{1}{2} \mathbf{B} \times \vec{r}] \]

\[ \tilde{\mathbf{A}}(\vec{r}) = (0, B_x, 0) \]

Electronic structure of a bulk (3D) in the magnetic field (cnt.)

\[ -\frac{\hbar^2}{2m^*} \left[ \left( \frac{\partial}{\partial x} \right)^2 + \left( \frac{\partial}{\partial y} \right)^2 + \left( \frac{\partial}{\partial z} \right)^2 \right] \Psi(x, y, z) = E \Psi(x, y, z) \]

\[ \Psi(x, y, z) = \exp[\imath(k_x x + k_z z)] g(y) \]

\[ \frac{d^2 g}{dy^2} + \frac{2m^*}{\hbar^2} \left[ E - \frac{\hbar}{2m^*} k_z^2 - \frac{1}{2m^*} (\hbar k_x - eB y)^2 \right] g(y) = 0 \]

\[ y_0 := \frac{\hbar k_x}{eB}, \quad \omega_c := \frac{eB}{m^*}, \quad \left\{ \omega_c := \frac{eB}{cm^*} \right\} \quad \varepsilon = E - \frac{\hbar^2}{2m^*} k_z^2 \]

\[ \frac{d^2 g}{dy^2} + \frac{2m^*}{\hbar^2} \left[ \varepsilon - \frac{m^*}{2} \omega_c^2 (y - y_0)^2 \right] g(y) = 0 \]
Electronic structure of a bulk (3D) in the magnetic field (cnt.)

\[ \frac{d^2 g}{dy^2} + \frac{2m*}{\hbar^2} \left[ \varepsilon - \frac{m*}{2} \omega_c^2 (y - y_0)^2 \right] g(y) = 0 \]

\[ \varepsilon_n = (n + 1/2) \hbar \omega_c \]

\[ g_n(y) = \left( \frac{2\sqrt{\pi} \alpha}{\Gamma(2n+1)} \right)^{1/2} H_n(\alpha(y - y_0)) \exp\left[ -\frac{1}{2} \alpha^2 (y - y_0)^2 \right] \]

\[ \alpha = \frac{m* \omega_c}{\hbar} = \frac{eB}{\hbar} = \frac{1}{l_m} \]

Electron density in a 2D system in the presence of a magnetic field:

\[ D(E) = \frac{1}{4\pi^2} \left( \frac{2m*}{\hbar^2} \right)^{3/2} \hbar \omega_c \sum_n \left( E - (n + 1/2) \hbar \omega_c \right)^{1/2} \]

Electronic structure of a bulk (3D) in the magnetic field (cnt.)

Degeneracy of the levels

The number of possible \( k_\alpha (\alpha = x, y, z) \) in the range \( \Delta k_\alpha \) is equal to

\[ \frac{L_\alpha}{2\pi} \frac{\Delta k_\alpha}{2\pi} \]

The orbit center \( y_0 \) must lie within the box

\[ -L_y/2 \leq y_0 \leq L_y/2 \]

\[ -L_x/eB \leq k_x \leq L_x/eB \]

It determines the range of allowed wavevectors

Number of states with certain \( k_z \)

Number of states in Landau level

Electronic structure of a bulk (3D) in the magnetic field (cnt.)

Quantization scheme for electrons

In the presence of the magnetic field, various \( k_x, k_y \) points condense into points on circles which represent constant energy surfaces with energies \( \frac{L_z}{2}, \frac{3L_z}{2}, \text{ etc.} \).
Electronic states in 2D Electron Gas (2DEG)
**2D structures in the magnetic field (3)**

- Natural unit of flux: \( \Phi_0 = \frac{hc}{e} = 4.136 \times 10^{-7} \text{ gauss cm}^2 \)

- Degeneracy of a level: \( N_L(B) = BS \frac{e}{hc} = \frac{\Phi}{\Phi_0} \)

**2D structures in the magnetic field (4)**

- Density of states with disorder:
  \[
  D(E) \sim \sum_n \delta(E - E_n)
  \]

- Confinement potential:
  \[
  U(y) = -A_y y
  \]

- Landau gauge: \( A_x = -By \quad A_y = 0 \)

- No analytic solutions for arbitrary confinement potential.
Confined 2D structures (effectively 1D) in the magnetic field (3)

\[ \Psi(x, y) = \frac{1}{\sqrt{L}} e^{ikx} \chi(y) \]

\[ [E_i + \frac{(\hbar k + eBy)^2}{2m^*} + \frac{\hat{p}_y^2}{2m^*} + U(y)]\chi(y) = E\chi(y) \]

A parabolic potential: \( U(y) = \frac{1}{2} m^* \omega_o^2 y^2 \)

\[ E(n, k) = E_i + (n + \frac{1}{2}) \hbar \omega_o + \frac{\hbar^2 k^2}{2m^*} \omega_o^2 \]

\[ \omega_{co} = \omega_o + \omega_c \quad \omega_c = \frac{|e|B}{cm^*} \]

Confined 2D structures (effectively 1D) in the magnetic field (3)

\[ E(n, k) = E_i + (n + \frac{1}{2}) \hbar \omega_{co} + \frac{\hbar^2 k^2}{2m^*} \omega_{co}^2 \]

Group velocity:

\[ v(n, k) = \frac{1}{\hbar} \frac{\partial E(n, k)}{\partial k} = \frac{\hbar k}{m^*} \frac{\omega_o^2}{\omega_{co}^2} \]

Electronic states in quantum dots

Fock-Darwin theory
Entirely confined systems

$L \approx \lambda = \frac{\hbar}{(2Em^*)^{1/2}}$

0D systems
quantum dots
quantum boxes
artificial atoms

Where physics of solids, atoms, nucleus, quantum chaos meet

Few body problem

Energy spectrum of 0D systems

always discreet levels (dimensional quantisation):

- $\varepsilon_n$, $\psi_n$, whose degeneracies, positions, and distribution depend on QD shape and disorder
- independently of disorder, spin and time reversal degeneracy if $B = 0$ and no spin interactions

Analogy to:
- nucleus
- finite elastic bodies (vibration modes)
- electromagnetic cavities

Fock-Darwin theory

Two-dimensional harmonic oscillator in the magnetic field

$L_z << L_x, L_y$ + 2D parabolic potential well
one-electron approximation

FIG. 1. Scanning electron micrograph of resist dots, with a 125 nm marker, together with a schematic sketch of the basic unit across the dots right at the InSb surface. The bright disks give an idea of the geometrical dot size. This monitor sample is shadowed with gold for contrast enhancement.
**Quantum Dot in Magnetic Field**

\[ H = \frac{(p_x - q_i A_t)^2}{2m^*} + \frac{1}{2} m^* \omega_0^2 r^2 + \frac{q^* \mu_B}{\hbar} B \cdot s_i \]

\[ \nabla \times A = B \quad \mu_B = |q|\hbar/2m_0 \]

**Single Particle Eigenstates**

\[ \psi_{n,m} \propto H_n(x) H_m(y) e^{-\frac{1}{2}(x^2 + y^2)} \]

Coulomb Gauge \( \Rightarrow \nabla \cdot A = 0 \)

\[ A = -\frac{1}{2} B (y\hat{j} - x\hat{j}) \]

**Fock-Darwin Eigenstates**

\[ H = -\frac{1}{2} \nabla^2 + \frac{1}{2} r^2 + \frac{1}{2} \omega_c (l_z + \gamma^s s_z) \]

\[ \psi_{n_L,l}(r, \theta) = C_{n_L,l} r^{|l|} e^{i\theta} L_{n_L}^{(|l|)}(r^2) \exp \left( -\frac{1}{2} r^2 \right) \]

\[ n_L \geq 0 \text{ and } |l| \leq n_L \quad C_{n_L,l}^2 = \frac{n_L!}{\pi (n_L + |l|)!} \]

\( L_{n_L}^{(|l|)} \) is an associated Laguerre polynomial

\( (n + 1) L_{n+1}^{(k)}(x) = (2n + k + 1 - x)L_{n}^{(k)}(x) - (n + k) L_{n-1}^{(k)}(x) \)

\[ L_0^{(k)}(x) = 1, \quad \text{and} \]

\[ L_1^{(k)}(x) = 1 - x + k \]

**Fock-Darwin Eigenstates**

\[ H = -\frac{1}{2} \nabla^2 + \frac{1}{2} r^2 + \frac{1}{2} \omega_c (l_z + \gamma^s s_z) \]

\[ \psi_{n_L,l}(r, \theta) = C_{n_L,l} r^{|l|} e^{i\theta} L_{n_L}^{(|l|)}(r^2) \exp \left( -\frac{1}{2} r^2 \right) \]

\[ E_{n_L,l} = 1 + 2n_L + |l| + \frac{1}{2} \omega_c (l + \gamma^s s) \]
Single Particle Energy Levels in Zero Magnetic Field

\[ E_{n_L,l} = 1 + 2n_L + |l| + \frac{1}{2}\omega_C (l + \gamma_s) \]

The single particle energy levels (without Zeeman Energy) as a function of magnetic field

Formation of Landau Levels for \( B \to \infty \) is clearly seen

Radial parts of the Fock-Darwin single particle states

Example of Quantum Transport:
Coherent Tunneling

Resonant Tunneling Diode (RTD)

Esaki, Chang, Tsu (IBM, 1974)
Fock-Darwin states

Tarucha et al., PRL'00