What LQC tells us about the relation between LQG and Spin Foams

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Based on work done with Abhay Ashtekar, Miguel Campiglia and more recent work with William Nelson, Carlo Rovelli, Francesca Vidotto, Edward Wilson-Ewing

Introduction

- Major open question: Connection between Canonical LQG and Spin Foam models (SFM).
- The answer to this question will resolve many of the open questions of both.
- We use an exactly soluble model of Loop Quantum Cosmology (LQC) to probe this connection.
- We construct an expansion of LQC that is akin to the vertex expansion of SFM.
- Using this expansion we gain insight into many open questions.

SFM $\longrightarrow$ LQG

- Goal: Find SFM model that shares as many of the features of LQG as possible.
- EPRL: Connection to kinematics of LQG - Boundary states are SU(2) spin networks.
- Extension of EPRL to arbitrary graphs [Kaminski, Kisielowski, Lewandowski 09]
- Spin Foam models are defined on a fixed triangulation Δ : Continuum Limit? Refinement of single triangulation, sum over all triangulations, or...?
- Meaning of theory on one triangulation or a finite sum?
- Classical Limit $\longrightarrow$ $\cos S$ or e^{iS}? Relation to time orientation?
- Meaning of the GFT coupling constant λ?

LQG $\longrightarrow$ SFM

- Goal: Derive SFM from group averaged inner product.

$$([s_f],\,[s_i]) = \int \mathrm{d}\alpha \,\langle s_f|\, e^{i\alpha M}\, |s_i\rangle \quad \mathit{or} \quad \int \mathcal{D}N \,\langle s_f|\, e^{iC(N)}\, |s_i\rangle \tag{1}$$

- Original motivation for SFM - expansion of amplitude $\langle s_f|\, e^{iC(N)}\, |s_i\rangle$ expressed as sum over histories of spin networks [Reisenberger, Rovelli 97]
- Construction of SFM for 2+1 gravity - group average then expand [Noui, Perez 04]
- We seek a way to expand the physical inner product- allowing us to compute it perturbatively.
- This is non-trivial - Perturbatively computing something that should be giving distribution!

LQC as toy model

- We will study these issues using k=0 LQC with a massless scalar field [Ashtekar, Pawlowski, Singh 06].
- While far from the full theory, LQC provides a physically interesting yet technically simple arena to explore these issues.
- LQC has many of the key features of LQG (new representation, constrained, etc.) and shares many of its conceptual difficulties (problem of dynamics).
- This model is exactly soluble [Ashtekar, Corichi, Singh 08] allowing us to perform precise calculations. The calculations are not formal - they rely on just one assumption.

Expansion of LQC

- Want the 'transition amplitude' between basis vectors $|\nu, \phi\rangle$ in $\mathcal{H}_{\rm kin}$ which are the LQC analogs of spin networks that are used to specify the boundary states in SFMs

$$\langle \nu', \phi' \,|\, \nu, \phi\rangle = \delta_{\nu'\nu}\ \delta(\phi', \phi)\,. \tag{2}$$

- Given by the group averaged inner product between the physical states generated from the basis vectors

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = 2\int \mathrm{d}\alpha\, \langle \nu_f, \phi_f |\, e^{i\alpha C}\, |p_\phi|\, |\nu_i, \phi_i\rangle\,. \tag{3}$$

- Where the constraint is written in terms of Θ - a difference operator acting on $|\nu\rangle$

$$C = p_\phi^2 - \Theta \tag{4}$$

- We first obtain a sum over histories expansion of the amplitude below by closely following the standard Feynman construction for the gravitational part.

$$A(\nu_f, \phi_f; \nu_i, \phi_i; \alpha) = 2 \langle \nu_f, \phi_f | \, e^{i\alpha C} \, |p_\phi| \, |\nu_i, \phi_i\rangle \tag{5}$$

- By rearranging the sum over histories in terms of those paths whose volume is constant nearly everywhere, changing value only M times, the amplitude can be written as a sum over discrete histories

$$A(\nu_f, \phi_f; \nu_i, \phi_i; \alpha) = \sum_{M=0}^{\infty} \sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} \int \mathrm{d}p_\phi \; e^{i\alpha p_\phi^2} e^{ip_\phi \Delta\phi} \, |p_\phi| A(\nu_M, \ldots, \nu_0; \alpha) \tag{6}$$

- To get the ’transition amplitude’ we still need to carry out the group averaging.

$$([\nu_f, \phi_f], \; [\nu_i, \phi_i]) = \int \mathrm{d}\alpha \, A(\nu_f, \phi_f; \nu_i, \phi_i; \alpha) \tag{7}$$

- We have a well-defined expansion if we can carry out the group averaging integral for each term of the expansion seperately.
- Our assumption is that the integral over alpha commutes with the sum over M.

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \int \mathrm{d}\alpha \sum_{M=0}^{\infty} \sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} \ldots = \sum_{M=0}^{\infty} \sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} \int \mathrm{d}\alpha \ldots \tag{8}$$

- Surprisingly the integral converges for each discrete history.

$$A(\nu_M, \ldots, \nu_0; \phi_f, \phi_i) = \int \mathrm{d}\alpha \int \mathrm{d}p_\phi\ e^{i\alpha p_\phi^2} e^{ip_\phi \Delta\phi}\ |p_\phi| A(\nu_M, \ldots, \nu_0; \alpha) \tag{9}$$

SFM Vertex Expansion - Continuum Limit

- Arrived at an expansion akin to SFM vertex expansion

$$
\begin{aligned}
([\nu_f, \phi_f],\ [\nu_i, \phi_i]) &= \sum_{M=0}^{\infty} \Bigg[\sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} A(\nu_M, \ldots, \nu_0; \phi_f, \phi_i) \Bigg] \qquad (10) \\
&= \sum_{M=0} A[\Delta_M]
\end{aligned}
$$

- Each term M can related to a triangulation Δ_M - with sums over labellings of the dual triangulation. Each term then corresponds to the SFM amplitude on a fixed triangulation
- The full group averaged inner product is then obtained by summing over all such triangulations.
- This is a concrete realization of the expectation that the 'continuum limit' of SFM is given not by a refinement of a given triangulation but by a sum over all triangulations.

GFT

- An alternative derivation makes contact with GFT.
- Formally split the constraint into a 'free' and 'interaction' term introducing the coupling constant λ

$$C = (p_\phi^2 - D) - \lambda K \tag{11}$$

- Using textbook interaction picture perturbation theory we arrive at the same expansion (if $\lambda = 1$).

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0}^{\infty} \lambda^M \Bigg[\sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} A(\nu_M, \ldots, \nu_0; \phi_f, \phi_i) \Bigg] \tag{12}$$

- If GFT is more fundamental what is the meaning of the coupling constant λ and what happens as it flows under renormalization?

$\lambda \sim \Lambda$

- What is the physical meaning of $\lambda \neq 1$?
- Consider k=0 FRW with a cosmological constant Λ
- Same expansion can be carried out for this model.
- There is an isomorphism between the theory with $\lambda \neq 1$ and Λ and the theory with $\lambda = 1$ and $\tilde{\Lambda}$ where

$$\tilde{\Lambda} = \frac{\Lambda}{\lambda} + \frac{3}{2\gamma^2 \ell_o^2 \lambda} (\lambda - 1). \tag{13}$$

- Taking $\lambda \neq 1$ then corresponds to a shift in the value of the cosmological constant
- If we take $\tilde{\Lambda} = 0$ we find that taking $\lambda \neq 1$ is equivalent to changing the cosmological constant

$$\Lambda = \frac{3}{2\gamma^2 \ell_o^2} (1 - \lambda) \tag{14}$$

Matter is Good

- The addition of matter can lead to great simplifications.
- We are already familiar with this from LQC where the scalar field plays the role of a clock - making it easier to extract physics from the model.
- Here the scalar field makes each term of the expansion finite.
- Without the scalar field integrals over the parameter alpha are often divergent or are distributions for each path (example Bianchi I)

$$A[\Delta_M] = \sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} \int \mathrm{d}\alpha \int \mathrm{d}p_\phi \, e^{i\alpha p_\phi^2} e^{ip_\phi \Delta\phi} \, |p_\phi| A(\nu_M, \ldots, \nu_0; \alpha) \tag{15}$$

- Without matter it is necessary to regulate the group averaging procedure to obtain a well defined expansion. [Rovelli, Vidotto '10 AH, Rovelli, Vidotto, Wilson-Ewing wip]

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Semiclassical behavior and time orientation

- Relation between requiring positive frequency, semiclassical behavior e^{iS} or $\cos S$, and time orientation.
- If we restrict to positive frequency ($p_\phi > 0$) then the group averaged inner product is in general complex and semiclassically dominated by e^{iS}.
- If we reconstruct the classical metric from phase space trajectories there is a redundancy - the same geometries are contained in each part the phase space ($p_\phi > 0$ and $p_\phi < 0$) - but with different time orientations.
- Summing over both positive and negative frequency is the same then as summing over both time orientations and leads to $\cos S$ in the semiclassical limit.
- Realization of idea that we need to restrict to sums over a single time orientation.

Multiple Expansions

- There actually exist two distinct expansions.
- If one first carries out the group averaging procedure and then carries out the vertex expansion we arrive at a distinct expansion.
- While the two converge to the same result - term by term they look very different.
- This leads to an observation: In attempting to construct a SFM from LQG we may arrive at an expansion that looks quite different but actually gives the same physics.
- We may thus need to work carefully on both ends to ensure that the two match up!

Conclusions

- We have obtained a well defined Spin Foam like expansion of LQC - with one assumption.
- Gives insight into many open questions of both LQC, SFM, and the connection between them.
- Indicates that the continuum limit is given by a sum over all triangulations.
- The group field theory parameter may be physically related to the cosmological constant.
- Matter while not essential provides many technical simplifications.
- The reality of the physical inner product and obtaining $\cos S$ semiclassically can be traced to summing over both time orientations.
- There are multiple expansions that look very different term by term $\rightarrow$ may be non-trivial to compare construction from LQG to SFM.