The $\mathrm{U}(N)$ Structure of Loop Quantum Gravity

Etera Livine

Ecole Normale Supérieure de Lyon - CNRS

March 2010 in Zakopane

Open problems in Loop Quantum Gravity

An old idea with F. Girelli ,
then mostly based on work with L. Freidel ,
with more recent work with E. Borja , J. Diaz-Polo and I. Garay

Aim: Study deeper the Structure of $\mathrm{SU}(2)$ Intertwiners for Loop Quantum Gravity and how to glue them.

1. A $\mathrm{U}(N)$ Action on the Space of $\mathrm{SU}(2)$ Intertwiners:
   - ▶ defines area-preserving diffeomorphisms at the discrete level.
2. Counting Intertwiners and compare to Black Hole Entropy
3. The Intertwiner Space as a L^2 space
4. A Classical Dynamics for Intertwiner
5. Creation/Annihilation Operators and Coherent Intertwiners
6. New Operators for LQG Dynamics

A useful tool: Schwinger Representation for $\mathfrak{su}(2)$

The Object: the space of intertwiners with N legs i.e of $\mathrm{SU}(2)$-invariant states in the tensor product of irreps $V^{j_1} \otimes .. \otimes V^{j_N}$ for arbitrary values of spins $j_i \in \mathbb{N}/2$.

First Step: Write the spaces V^j as Hilbert spaces for a system of two harmonic oscillators at fixed total energy.

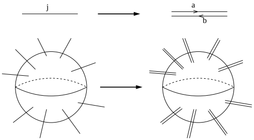
$$[a, a^\dagger] = [b, b^\dagger] = 1, \qquad [J_z, J_\pm] = \pm J_\pm,\ [J_+, J_-] = 2J_z,$$

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b) \qquad \Rightarrow \qquad \mathcal{C} = \vec{J}^2 = E(E+1),$$

$$J_+ = a^\dagger b,\ J_- = ab^\dagger, \qquad |j, m\rangle = |n_a, n_b\rangle_{OH},$$

$$E = \frac{1}{2}(a^\dagger a + b^\dagger b) \qquad j = \frac{1}{2}(n_a + n_b),\ m = \frac{1}{2}(n_a - n_b).$$

$\mathrm{SU}(2)$ Invariant Operators and the $\mathfrak{u}(N)$ Algebra



Second Step: Consider intertwiners with N legs and its dual boundary surface.
$\rightarrow 2 \times N$ oscillators a_i, b_i

We look for invariant observables , i.e operators that commute with global $\mathrm{SU}(2)$ transformations generated by $\vec{J} = \sum_{i=1}^{N} \vec{J}^{(i)}$.
$\rightarrow$ scalar product operators $\mathcal{O}_{ij} \equiv \vec{J}^{(i)} \cdot \vec{J}^{(j)}$, quartic in a, b's.

A problem: The commutators of the $\vec{J}^{(i)} \cdot \vec{J}^{(j)}$'s are cubic in the J's (volume) and generate an infinite tower of higher order operators. How to build semi-classical coherent states??

A solution: Build invariant operators which form a closed algebra?

The $\mathfrak{u}(N)$ Algebra as the Underlying Structure of the Intertwiner Space

Harmonic oscillators allow quadratic invariant operators, [Girelli, EL 05]

$$E_{ij} \equiv (a_i^\dagger a_j + b_i^\dagger b_j), \quad [E_{ij}, E_{kl}] = \delta_{jk} E_{il} - \delta_{il} E_{kj}.$$

The new operators E_{ij} form a closed $\mathfrak{u}(N)$ algebra !

$$E_i = E_{ii} = \text{ spin } 2j_i, \; E = \sum E_i = 2 \times \text{ total area.}$$

At fixed number N of legs, the $\mathfrak{u}(N)$ transformations change the individual spins j_i on each leg, but they still commute with the $\mathrm{U}(1)$ Casimir $E = 2\sum_i j_i$ which gives the (total) area.
Proposal: Identify $\mathrm{U}(N)$ as area-preserving diffeomorphisms on the discrete sphere defined as the space of $\mathrm{SU}(2)$ intertwiners with N legs. [Freidel, EL 09]

Here, the natural definition is Area $= \sum_i j_i$.

The $\mathfrak{u}(N)$ Algebra from Harmonic Oscillators

Third Step: What's the action of $\mathrm{U}(N)$ transformations? Which $\mathrm{U}(N)$ representation(s)?

In fact, standard construction in mathematics !

▶ Take P sets of N harmonic oscillators $a_i^{(p)}$, $1 \leq i \leq N$, $1 \leq p \leq P$, then $E_{ij} = \sum_p a_i^{(p)\,\dagger} a_j^{(p)}$ form a $\mathfrak{u}(N)$ Lie algebra.

And explicitly. . .

Casimir Equation and Highest Weights

For **P=1**, we get quadratic relations on the $\mathfrak{u}(N)$ generators:

$$\forall i,\ \sum_j E_{ij}E_{ji} = E_i(E+N-1), \quad \sum_{i,j} E_{ij}E_{ji} = E(E+N-1).$$

Taking a highest weight vector v, s.t. $E_i\,v = l_i\,v$ with the weights $l_i \in \mathbb{N}$ and $E_{ij}\,v = 0$ for all $i<j$, with $l_1 \geq l_2 \geq .. \geq l_N \geq 0$, the Casimir equation implies that $l_2 = .. = l_N = 0$.

For **P=2**, we relate the Casimirs of $\mathrm{SU}(N)$, $\mathrm{U}(1)$ and $\mathrm{SU}(2)$:

$$\sum_{i,j} E_{ij}E_{ji} = E(\frac{E}{2}+N-2) + 2\vec{J}\cdot\vec{J}.$$

$$\Rightarrow \text{highest weight } [l_1, l_2, 0, 0, ..] \text{ with } \left\{ \begin{array}{c} l_1 - l_2 = 2\mathcal{J} \\ \vec{J}\cdot\vec{J} = \mathcal{J}(\mathcal{J}+1) \end{array} \right.$$

$\mathrm{SU}(2)$ Intertwiner Spaces as Representations of $\mathrm{U}(N)$

Intertwiners correspond to $\mathcal{J}=0$, thus $l_1 = l_2$. The representation is defined by the highest weight $[l, l, 0, ..]$ with $\mathrm{U}(1)$ Casimir is $E = 2l$, i.e Area$= l = \sum_i j_i$.

Meaning of the highest weight??

A bivalent intertwiner! ▶ A completely squeezed sphere.

The Young tableau is two lines of equal length l.

$$\text{hook formula} \Rightarrow \dim_N[l] = \frac{1}{l+1}\begin{pmatrix} N+l-1 \\ l \end{pmatrix}\begin{pmatrix} N+l-2 \\ l \end{pmatrix}.$$

- $\mathrm{U}(N)$ acts on the space of intertwiners at fixed total area l and number of punctures N, including trivial irreps $j_i = 0$.
- $\dim_N[l]$ gives the number of such intertwiners.

What's next?

What can we do this with framework?

1. Procedure works for groups $\mathrm{U}(M)$, $\mathrm{U}_q(M)$ and susy groups
2. $\mathrm{U}(N)$ useful for intertwiner counting and entropy calculations?
3. Geometric interpretation of $\mathrm{U}(N)$ transformations?
   - Build coherent states and link to coherent intertwiners
   - $N \to \infty$ limit of intertwiner space, embedding of $\mathrm{U}(N)$ in $\mathrm{U}(N+1)$ and projective limit and $\mathrm{U}(N)$ as diffeomorphisms in the continuum?
4. Actually useful for Loop Quantum Gravity??
5. Maybe useful for Spinfoams and Group Field Theory?

The Generating Functionals for the $\mathrm{U}(N)$ Dimensions. . .

For fixed N, we just established the equality:

$$\dim_N[l] = I^{(N)}[l] \equiv \sum_{j_1+..+j_N=l} \dim_0[j_1,..,j_N].$$

We can check this by introducing the generating functionals $F_N(t) = \sum_l t^{2l} \dim_N[l]$ and $\widetilde{F}_N(t) = \sum_l t^{2l} I^{(N)}[l]$. $\widetilde{F}_N$ can be computed directly as an integral over $\mathrm{SU}(2)$:

$$\widetilde{F}_N(t) = \int dg \left[\sum_j t^{2j}\chi_j(g)\right]^N = \frac{2}{\pi}\int_0^\pi d\theta \, \frac{\sin^2\theta}{(1-2t\cos\theta+t^2)^N}.$$

We check that both functional satisfy the same 2nd order (hypergeometric) diff eqn:

$$\Delta^{(N)} F_N = \Delta^{(N)} \widetilde{F}_N = 0, \quad \text{with}\ldots$$

. . . The Generating Functionals for the $\mathrm{U}(N)$ Dimensions

$$\Delta^{(N)} \equiv \frac{1}{4}(1-t^2)(t\partial_t^2+3\partial_t) - N(N-1)t - (N-1)t^2\partial_t.$$

We show this using two different methods:

- a recursion relation:
  $(l+1)(l+2)\,\dim_N[l+1] = (N+l)(N+l-1)\,\dim_N[l]$.
- a brute force calculation on the integral:

$$\Delta^{(N)}\,\widetilde{F}_N = \frac{N(1-t^2)}{\pi}\int_0^\pi d\theta\,\partial_\theta \frac{\sin^3\theta}{(1-2t\cos\theta+t^2)^{N+1}} = 0.$$

At the end of the day, $F_N(t)$ can be expressed in term of the first derivative of the Legendre polynomials.

The Big Generating Functional

Now, we introduce the full generating functional by also performing the sum over the number N of legs: $F(u,t) = \sum_{N,I} u^N t^{2I} \dim_N[I]$.
And we can compute explicitly from the integral representation:

$$F(u,t) = \frac{1}{2t^2}\left[t^2(u+2) - u^2 + u - \frac{u[t^2(t^2-2u-2)+(u-1)^2]}{\sqrt{((1+t)^2-u)((1-t)^2-u)}}\right].$$

For fixed $0 < u < 1$, the first pole is $t_c = 1-\sqrt{u}$, thus the asymptotics:

$$\log \sum_N u^N \dim_N[I] \underset{I\to\infty}{\sim} -2I\log(1-\sqrt{u}).$$

Black Hole Entropy?

$\dim_N[I]$ counts the number of $\mathrm{SU}(2)$ intertwiners, so it gives the black hole entropy? But...

- Standard LQG isolated horizon counts $\mathrm{U}(1)$ intertwiners
  - ▶ but see lots of recent works [e.g Engle,Noui,Perez 09]
- The total area is $\sum_i j_i$ instead of the standard $\sum_i \sqrt{j_i(j_i+1)}$
- We are counting many intertwiners which carry trivial legs !
- The dimension depends on the number of punctures N
  - ▶ but we can argue that N is given by the graph of the outside spin network state.
  - ▶ or we can simply sum over the number of punctures...

Removing Trivial Punctures

We count the irreps: $[j_1, .., j_N] \rightarrow \{(j, k_j)\}$ with $\left| \begin{array}{l} I = \sum_j jk_j \\ N = \sum_j k_j \end{array} \right.$.

$\dim_0[j_1, .., j_N]$ depends only the occurrence nbs k_j for $j > 0$.
Thus we separate trivial punctures, $K \equiv N - k_0$:

$$\Rightarrow \dim_N[I] = \sum_{K=0}^{N} \binom{N}{K} D_K[I],$$

$$D_K[I] = \sum_{\sum_{j\geq 1} k_j = K} \frac{K!}{\prod_j k_j!} \dim_0[\{k_j\}].$$

Final Counting

We want the number of intertwiners at fixed area without trivial punctures:

$$D[l] = \sum_K D_K[l], \qquad \blacktriangleright \text{ Sum is finite: } K \leq 2l$$

$D_K[l]$ is the binomial transform of $\dim_N[l]$:

$$D_K[l] = \sum_{N=0}^{K} (-1)^{K-N} \begin{pmatrix} K \\ N \end{pmatrix} \dim_N[l].$$

We introduce the generating functionals, which can be computed as integrals as before:

$$G_K(t) = \sum_l t^{2l} D_K[l], \; G(u,t) = \sum_{K,l} u^K t^{2l} D_K[l].$$

Final Result

The binomial transform has a simple generating functional !

$$G(u,t) = \frac{1}{1+u} F\left(\frac{u}{1+u}, t\right).$$

This gives $D[I]$ in term of the original dimensions $\dim_N[I]$:

$$\sum_I t^{2I} D[I] = G(1,t) = \frac{1}{2} F\left(\frac{1}{2}, t\right), \quad D[I] = \sum_N \frac{1}{2^{N+1}} \dim_N[I],$$

$$\Rightarrow S_\emptyset[I] = \ln D[I] \underset{I\to\infty}{\sim} I \ln \alpha - \frac{3}{2} \ln I,$$

with $\alpha = \frac{1}{\left(1-\sqrt{\frac{1}{2}}\right)^2} = 6 + 4\sqrt{2} \simeq 11.6568.$

What's next?

A few questions...

1. The $\mathrm{U}(N)$ structure holds too for $\mathrm{U}(1)$ punctures...
2. Writing a partition function?
   - Integral over $\mathrm{U}(N)$ as a sum over boundary 2-geometries?
   - Semi-classical limit for large N...
3. A realistic model for black hole?
   - Impossible with a single intertwiner, so can we do better?!

The Intertwiner Space as a L^2 Space...

The space of intertwiners with N legs can be represented as a space of L^2 functions:

$$\mathcal{H}_N = \bigoplus_{\{j_i\}} Inv[j_1 \otimes .. \otimes j_N] = \bigoplus_{J} R_N^J = L^2\left(Gr_{2,N}\right).$$

$Gr_{2,N}$ is a Grassmanian space:

$$Gr_{2,N} \equiv \frac{\mathrm{U}(N)}{\mathrm{U}(N-2) \times \mathrm{SU}(2)}$$

The subgroup $\mathrm{U}(N-2) \times \mathrm{SU}(2)$ stabilizes the highest weight vector: $\mathrm{U}(N-2)$ is generated by $E_{ij}, i,j \geq 3$ and $\mathrm{SU}(2)$ by $E_{12}, E_{21}, E_1 - E_2$.

...The Intertwiner Space as a L^2 Space

The space $L^2(Gr_{2,N})$ consists in functions on $\mathrm{U}(N)$ satisfying:

$$\forall G \in \mathrm{U}(N),\, \forall H \in \mathrm{U}(N-2) \times \mathrm{SU}(2), \quad f(GH) = f(G).$$

$Gr_{2,N} \sim P_N(A) \times U(1)^N$ can be interpreted as the space of polyhedra with N faces and only trivalent vertices for an arbitrary fixed total area A. Such polyhedra have $3(N-2)$ edges and $2(N-2)$ vertices, so dimensions match:

$$N^2 - (N-2)^2 - 3 = 4N - 7 = 3(N-2) - 1 + N.$$

This provides a geometric interpretation for intertwiners.

Gluing Intertwiners into Spin Networks

Spin networks on a graph Γ are functions of E group elements and satisfying gauge invariance at each vertex:

$$\varphi \in L^2(\mathrm{SU}(2)^E/\mathrm{SU}(2)^V) \rightarrow \varphi(\{g_e\}) = \varphi(\{h_{s(e)}^{-1} g_e h_{t(e)}\}).$$

We can shift the degrees of freedom to the vertices:

$$\mathcal{H}_\Gamma = \bigoplus_{\{j_e\}} \bigotimes_{v\in\Gamma} \mathcal{H}_{j_1^v,\ldots,j_{N_v}^v} = L^2\left((\times_e \mathrm{U}_{(e)}(1))\backslash(\times_v Gr_{2,N_v})\right),$$

where $\mathrm{U}_{(e)}(1)$ generated by $E_e^{s(e)} - E_e^{t(e)}$ ensures the matching on the representations on the same edge e. The gauge invariance now reads on functions $f(K_v)$ with $K_v \in \mathrm{U}(N_v)$:

$$\left|\begin{array}{rcl} f(\{K_v\}) &=& f(\{K_v H_v\}) \\ f(\{K_{s(e)}, K_{t(e)}, K_v\}) &=& f(\{T_e K_{s(e)}, T_e K_{t(e)}, K_v\}). \end{array}\right.$$

A Dual Perspective on Spin Networks

Two interesting consequences:

- We can build operators focusing and acting on the intertwiner spaces (unlike holonomies).
- The intertwiner space is a L^2 space of wave-functions of a classical unitary matrix: we should be able to describe the intertwiner dynamics in term of an underlying matrix model .

  [Borja,Diaz,Garay,EL very soon]

- Spin network dynamics as a multi-matrix model?
  $\longrightarrow$ integrable structures?

A Classical System Behind Intertwiners

It all comes from a Hermitian matrix $M = M^\dagger$: $M_{ij} \to E_{ij}$

We impose quadratic constraints: [Borja,Diaz,Garay,EL in march]

$$M^2 = \frac{1}{2}(\mathrm{Tr} M)M \to \sum_j E_{ij}E_{jk} = E_{ik}(\frac{E}{2} + N - 2)$$

Implies that M has eigenvalues $[\lambda, \lambda, 0, .., 0]\dots$

$\dots$ and $\lambda = \frac{1}{2}\mathrm{Tr} M$ is the total boundary area .

A natural ansatz for a $\mathrm{U}(N)$-invariant action (external time):

$$S[M] = \frac{1}{2}\mathrm{Tr}(\partial_t M)^2 + \mathrm{Tr}\,\Lambda\left(M^2 - \frac{1}{2}(\mathrm{Tr} M)M\right) - V[M]$$

Trivial Dynamics for a Single Intertwiner

We solve the constraints $M = \lambda U^{-1}\Delta U$ with $\Delta = [1, 1, 0.., 0]$:

$$S[\lambda, U] = (\partial_t \lambda)^2 - V[\lambda] + \lambda^2 \mathrm{Tr}\, \Delta(\partial_t U)U^{-1}[\Delta, (\partial_t U)U^{-1}]$$

Classically, no dynamical coupling between the area λ and the surface deformations U! $\rightarrow$ Single intertwiner is trivial!

What to choose for a potential?

Quadratic in λ^2 or. . . a $\frac{1}{\lambda}$-potential?

What's next?

- Gluing Intertwiners and going to Multi-Matrix Models
- Role of Conformal Symmetry?

What are Coherent Intertwiners?

We want:

- A over complete basis of intertwiners which transforms consistently under $\mathrm{U}(N)$ transformations.
- Semi-classical intertwiners with peakedness properties.
- A relation with the standard coherent/holomorphic intertwiners. [EL,Speziale 07 Conrady,Freidel 09 Freidel,EL,Krasnov 09]

This will provide an explicit geometric interpretation of the $\mathrm{U}(N)$ transformations.

A new Set of $\mathrm{SU}(2)$-invariant Operators

We identify a new set of $\mathrm{SU}(2)$-invariant operators ! [Freidel,EL very soon]

$$F_{ij} \equiv a_i b_j - a_j b_i, \quad F_{ij} = -F_{ji}.$$

They are invariant under $\mathrm{SU}(2)$ but do not preserve the total area!!

$$[E, F_{ij}] = -2F_{ij}, \qquad [E, F_{ij}^\dagger] = +2F_{ij}^\dagger$$

$\Rightarrow$ annihilation/creation operators

With $E_\rho \equiv \sum_{ij} \rho^{ji} E_{ij}$, $F_\omega \equiv \frac{1}{2} \sum_{ij} \omega^{ji} F_{ij}$, the full algebra is:

$$[E_\alpha, E_\beta] = -E_{[\alpha,\beta]}, \qquad [E_\alpha, F_\omega] = -F_{(\alpha\omega+\omega\alpha^t)},$$
$$[F_{\omega_1}, F_{\omega_2}^\dagger] = E_{\omega_1\omega_2^\dagger}, \qquad [E_\alpha, F_\omega^\dagger] = F^\dagger_{(\alpha\omega+\omega\alpha^t)}.$$

Remember Spinors

We attach one spinor $|z_i\rangle = \begin{pmatrix} z_i^{(0)} \\ z_i^{(1)} \end{pmatrix}$ to each puncture/leg of the intertwiner. This defines a 3-vector n^i, the "normal vector" to the i^{th} patch, for each leg with normalization $|n_i| = \langle z_i|z_i\rangle$:

$$|z_i\rangle\langle z_i| = \frac{1}{2}\left(\langle z_i|z_i\rangle \mathrm{Id}_2 + \vec{n^i}\cdot\vec{\sigma}\right).$$

With $A(z)$ the "total area", the closure constraints reads:

$$\sum_i n^i = 0 \Leftrightarrow \sum_i |z_i\rangle\langle z_i| = A(z)\mathrm{Id}, \quad A(z) \equiv \frac{1}{2}\sum_i \langle z_i|z_i\rangle.$$

Defining Coherent Intertwiners

We define coherent states as: [Freidel,EL very soon]

$$|J, z_i\rangle \propto \frac{(F_\omega^\dagger)^J}{J!}|0\rangle, \quad \omega_{ij} = \langle z_i|\epsilon z_j\rangle = \overline{z_i^{(1)} z_j^{(0)}} - \overline{z_i^{(0)} z_j^{(1)}}.$$

These are states with total area $E = J$, which behave consistently under $\mathrm{U}(N)$ transformations :

$$U\,|J, z_i\rangle = |J, (u^{-1}z)_i\rangle, \quad \text{where} \quad U = \exp(E_\alpha),\ u = e^\alpha.$$

$\Rightarrow$ They cover the intertwiner space and provide a new resolution of the identity.

Semi-classical states?

We can compute the expectation value of the E_{kl} :

$$\frac{\langle J, z_i | E_{kl} | J, z_i \rangle}{\langle J, z_i | J, z_i \rangle} = J \frac{\langle z_k | z_l \rangle}{A(z)} = M_{kl}.$$

We check that M is a rank-two projector with eigenvalues $[J, J, 0, ..]$, which corresponds to our choice of $\mathrm{U}(N)$ irreps.

Then we can relate our new coherent states to the standard coherent intertwiners , which are known to be semi-classical:

$$|J, z_i\rangle \propto \sum_{\sum j_i = J} \frac{1}{\sqrt{(2j_1)!..(2j_N)!}} \int_{\mathrm{SU}(2)} dg\, g \triangleright \bigotimes_{i=1}^{N} |j_i, z_i\rangle$$

What's next?

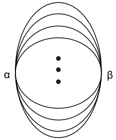
A couple of interesting questions:

- Understand the geometric action of the $\mathrm{U}(N)$ transformations acting on the z_i's
  ▶ Importance of the framing (phase of the spinors) !
- Apply these new $\mathrm{U}(N)$ coherent states to spinfoam models
  ▶ We insert the new resolution of the identity on the intertwiner space between 4-simplices.
  ▶ Improve the understanding of the symmetries of spinfoam amplitudes?

A simple model for LQG dynamics

Goal: Find a simple setting to define the dynamics of quantum Black Holes in LQG...

The 2-vertex "in-out" model



We consider the simplest class of graphs for LQG with 2 vertices in & out linked with N edges. [Borja,Diaz,Garay,EL next month]

$\rightarrow$ generalizes the dipolar model by Rovelli & Vidotto for cosmology with some inhomogeneities

Setting: Total boundary area
$+\ \mathrm{U}(N)$ defining the shape of α + $\mathrm{U}(N)$ for β

We have two independent intertwiner spaces related by the constraints of having a single $\mathrm{SU}(2)$ spin per link. We define the $\mathfrak{u}(N)$ boundary deformation algebra: $e_{ij} = E^{\alpha}_{ij} - E^{\beta}_{ji}$.

The matching conditions are the diagonal operators $e_i = E^{\alpha}_i - E^{\beta}_i$

An ansatz for dynamics...

Two simple operators that respect the matching conditions: $E^{\alpha}_{ij}E^{\beta}_{ij}$ shifts the spin j_i by $+\frac{1}{2}$ and j_j by $-\frac{1}{2}$ while $F^{\alpha}_{ij}F^{\beta}_{ij}$ decreases both spins j_i, j_j.

We introduce $f = \sum_{ij} F^{\alpha}_{ij}F^{\beta}_{ij}$ and $g = \sum_{ij} E^{\alpha}_{ij}E^{\beta}_{ij}$.

Both operators are invariant under $\mathrm{U}(N)$ boundary deformations: $[e_{ij}, f] = [e_{ij}, g] = 0$.

A simple ansatz for a Hamiltonian: $H \equiv \lambda g + (\sigma f + \bar{\sigma} f^{\dagger})$.

This generalizes the action of $\frac{1}{2}$-holonomy operators of the dynamics of BF theory.

... An ansatz for dynamics

Some simple remarks:

- Meaning of $f^\dagger$? Act with $(f^\dagger)^J$ on vacuum:

$$\mathrm{Tr}_\beta \left[(f^\dagger)^J |0,0\rangle\langle 0,0| f^J \right] = \text{totally mixed state for } \alpha$$

  $\Rightarrow$ $(f^\dagger)^J$ is a "black hole" creation operator.

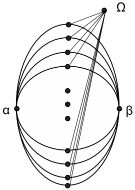
- E and F operators allow simple spin shifts and do not lead to superpositions of spins like holonomy operators.
- Identifying LQG's diffeo and Hamiltonian constraints?
  - invariant under which boundary deformations?
- A simple (cosmological) model to test LQG's dynamics !
- But very limited: no room for evaporation, ...

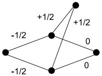
Going beyond the simplest two-vertex graph

A 3+N graph : N vertices on the boundary,
α In, β Out and Ω Other or "Radiation".

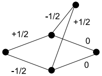
Ω allows to break $\alpha \leftrightarrow \beta$ matching conditions.

$\rightarrow$ Write dynamics in term of E, F operators?





"Evaporation" $\quad F^{\alpha}_{ij} E^{a_i}_{\gamma\alpha} E^{a_j}_{\gamma\alpha} F^{\gamma\dagger}_{ij}$



"Rotation" $\quad E^{\alpha}_{ij} E^{a_i}_{\alpha\gamma} E^{a_j}_{\gamma\alpha} E^{\gamma\dagger}_{ij}$

Both pieces of holonomy operator $\chi_{\frac{1}{2}}(\alpha a_i \beta a_j)\chi_{\frac{1}{2}}(\gamma a_i \beta a_j)$, but don't act on β

Then... study dynamics explicitly!! acting on coherent states?... maybe some more physical phenomena ...?

Outlook

Final Goal: Reformulate the dynamics on any graph in term of E, F operators and understand the “diffeomorphisms” acting at the discrete quantum level on spin network states in term of $\mathrm{U}(N)$ transformations.