Asymptotics of Spin Foams: What we learned

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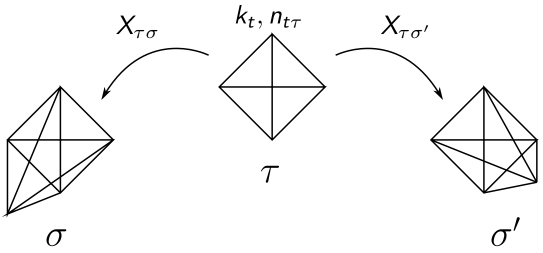
The Statesum

The statesum of EPRL, FK is given in terms of a triangulation of a manifold and can be written in terms of variables living on triangles t, tetrahedra τ and 4-simplices σ:

$$k_t \in \mathbb{N}/2 = \mathit{Irrep}(\mathrm{SU}(2))$$

$$n_{t\tau} \in S^2$$

$$X_{\tau\sigma} \in \mathrm{SL}(2, \mathbb{C}) \text{ or } \mathrm{Spin}(4)$$



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It is then given by:

$$Z(\mathcal{T}) = \sum_{k_t} \int_{n_{t\tau}} \int_{X_{\tau\sigma}} \prod_t f_2(k_t, n_{t\tau}) \prod_\tau f_3(k_t, n_{t\tau}) \prod_\sigma f_4(k_t, n_{t\tau}, X_{\tau\sigma})$$

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Heuristics

$X_{\tau\sigma}$ integration is local per 4-simplex. Heuristically: most spins in the sum are large, so approximately evaluate this integral using stationary phase ($S_4 = \ln f_4$ is linear in k):

$$\int_{X_{\tau\sigma}} f_4(\lambda k_t, n_{t\tau}, X_{\tau\sigma}) \sim \sum_{\delta S_4 = 0, \mathrm{Re}(S_4) = 0} \frac{e^{S_4(\lambda k_t, n_{t\tau}, X_{\tau\sigma})}}{\sqrt{detH}}$$

(same spirit as in Conrady, Freidel '08, works very well in the PR model as seen in Dowdall, Gomes, FH '09)

Geometry

$\delta S_4 = 0$ and $\mathrm{Re}(S_4) = 0$ (and $n_{t\tau}$ 3d non-degeneracy) imply:

- $b_{t\tau}$ from $(0, b_{t\tau}) = k_t X_{\tau\sigma}(0, n_{t\tau})$ is a constant $\mathfrak{su}(2)$ valued 2-form on σ (aka vector geometries), or
- $B_{t\tau}(\sigma_g) = \mu_\sigma \star k_t X_{\tau\sigma}(1,0,0,0) \wedge (0, n_{t\tau})$ are the bivectors of a geometric 4-simplex (σ_g),

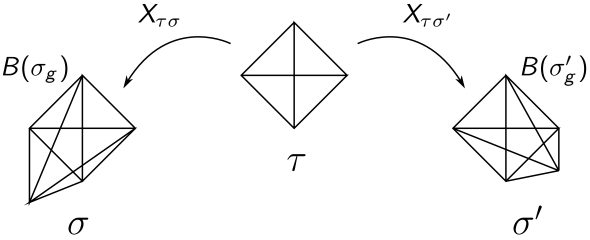
$$S^4 = \mu_\sigma S_{Regge}(\sigma_g).$$

The $\mathfrak{su}(2)$ sector is much larger than the geometric sector (dominant scaling contribution in BC (Barrett, Steele 02)).

The geometric sector can be distinguished in various ways: 4d non-degeneracy (Conrady, Freidel) gluing contraints (Dittrich, Ryan, Speziale), Regge likeness (Barrett, Dowdall, Fairbairn, Gomes, FH, Pereira)

Geometry gluing

The geometry of the 4-simplex σ_g agrees with the boundary geometry induced on the tetrahedra τ by $k_t n_{t\tau}$. Thus we get a consistent geometry throughout the manifold. X provide the connection between the frames in different 4-simplices.

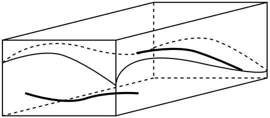


Two questions:

- Do the $\mathfrak{su}(2)$ solutions spoil the geometric sector?
- What is the meaning of μ?

$su(2)$ where are you?

The remaining k,n integrations should peak us on solutions to the variation of the action S_4 with respect to the solution space.



Do the $\mathfrak{su}(2)$ solutions induce spurious equation of motion? That is, is the geometric space a subspace of the $\mathfrak{su}(2)$ space?
Euclidean:
$(1 \pm \star)B_{t\tau}(\sigma_g)$ are constant $\mathfrak{su}(2)$ valued 2-form on σ. If we add an arbitrary constant $\mathfrak{su}(2)$ solution $b_{t\tau}$ to it we obtain good boundary data k,n but are out of the geometric sector (15 possible perturbations out of which only 10 are geometric) $\Rightarrow$ possibly spurious equations of motion for the geometric sector.

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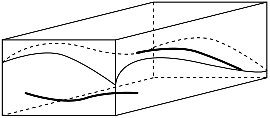

Do the $\mathfrak{su}(2)$ solutions induce spurious equation of motion? That is, is the geometric space a subspace of the $\mathfrak{su}(2)$ space?
Lorentzian:
The solutions require hyperbolic space to glue up, no way to perturb them into the $\mathfrak{su}(2)$ solutions. $\Rightarrow$ Regge equations of motion.

The meaning of μ

Bivectors reconstruct geometry and orientation. This orientation does not need to be constant throughout the manifold μ measures the orientation relative to a fiducial reference one, e.g. standard orientation in $\mathbb{R}^4$. Using $N_\tau = X_{\tau\sigma}(1,0,0,0)$.

$$B_{t\tau}(\sigma_g) = \star k_t \frac{N_\tau \wedge N_{\tau'}}{|N_\tau \wedge N_{\tau'}|} = \mu_\sigma \star k_t X_{\tau\sigma}(1,0,0,0) \wedge (0, n_{t\tau})$$

Then:

$$\frac{N_\tau \wedge N_{\tau'}}{|N_\tau \wedge N_{\tau'}|} = \mu_\sigma N_\tau \wedge X_{\tau\sigma}(0, n_{t\tau})$$

and thus $\mu X_{\tau\sigma}(0, n_{t\tau}) \cdot N_{\tau'} > 0$ and $\mu X_{\tau\sigma}(0, n_{t\tau})$ are the outward pointing triangle normals.

The meaning of μ (2)

Bivectors reconstruct geometry and orientation. This orientation does not need to be constant throughout the manifold μ measures the orientation relative to a fiducial reference one, e.g. standard orientation in $\mathbb{R}^4$. So what?

The action on the manifold is

$$\sum_{\sigma} \mu_{\sigma} S_{Regge}(\sigma_g).$$

These actions have very different solutions for different set of signs μ. In Ponzano Regge we get non-flat manifolds and divergences.

We need to interpret these solutions (anti space?) or make them go away (modify boundary state space?).

Conclusions

Caveats

- $n_{t\tau}$ required to be 3d non-degenerate.
- Uncertain status of inserting the asymptotics into the full statesum.
- No knowledge of the phase of the Hessian.
- Unclear if Regge likeness propagates from the boundary (maybe unlikely).

but if....

- Full geometric understanding of the non-metric solutions now available.
- Larger triangulations look vastly more feasible than in BC.
- Possibility of spurious EOMs in the Euclidean case, Lorentzian model looks better, maybe "good enough".
- Orientations problematic. Possibly indicates problem with the boundary state space (half the holomorphic states?).