Coupling Matter to Loop Quantum Gravity via the Spectral Triple

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Zakopane, Poland, 02.03.2010

Outline of talk

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Motivation

- Noncommutative geometry - Connes' work on the standard model of particle physics.
- Canonical quantum gravity / Loop quantum gravity.

Aim

- Intersection of noncommutative geometry and quantum gravity.

The Construction

- A spectral triple *over* a configuration space of connections.

Physical Interpretation

- Spaces of connections.
- The Poisson bracket of General Relativity.
- Semi-classical analysis: emergence of matter and the Dirac Hamiltonian.

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- **A Spectral Triple** is a collection (B, H, D):
  a $*$-algebra B represented as operators in the Hilbert space H; a self-adjoint, unbounded operator D, acting in H such that:
  1. The resolvent of D, $(1 + D^2)^{-1}$, is compact.
     (manageable spectrum)
  2. The commutator $[D, a]$ is bounded $\forall a \in B$.
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- First example: Riemannian geometry

$$(B = C^\infty(M), H = L^2(M, S), D = \not{D})$$

7 "axioms", Connes 2008: reconstruction theorem, complete equivalence.

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  - $B = C^\infty(M) \otimes B_F, \quad B_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

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    "almost commutative algebra"
  - The classical action of the standard model coupled to gravity emerges from a heat kernel expansion of the corresponding Dirac operator.
- The fact that the Standard Model coupled to gravity fits into the framework of NCG is a non-trivial result.

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Formulation of the classical standard model coupled to general relativity as a single gravitational theory. The standard model emerges from a modification of space-time geometry:

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Our goal

To construct a framework which combines noncommutative geometry with elements of quantum gravity/quantum field theory.

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  $\rightarrow$ noncommutative geometry;
  $\rightarrow$ LQG as a "top-down" program of unification.
- **Hope/Idea:** to look for a (semi-) classical limit where the algebra of loops descent to an almost commutative algebra (i.e. that some of the noncommutativity remains).

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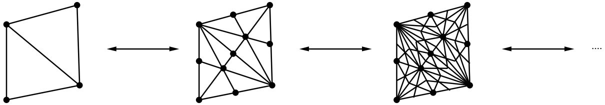

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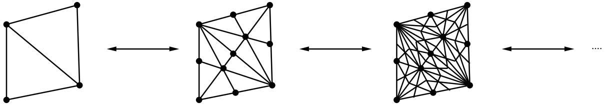

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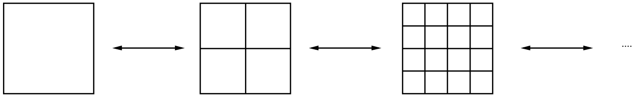

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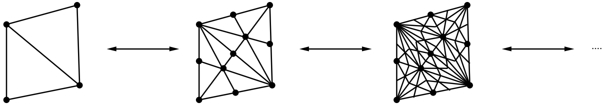

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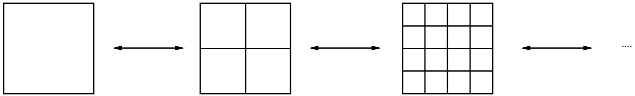

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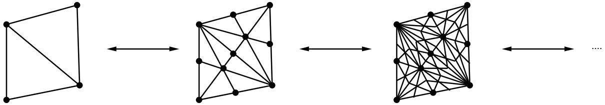

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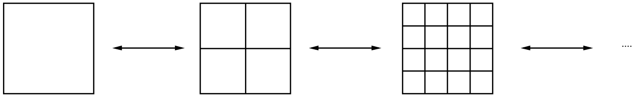

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- Semi-classical analysis indicate that cubic lattices are natural (end of talk).

The construction

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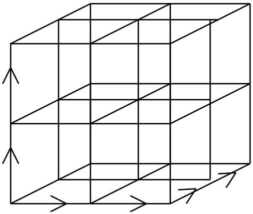
A single cubic lattice

- Let Γ be a finite 3-dim finite cubic lattice with edges $\{\epsilon_i\}$ and vertices $\{v_i\}$

$$\epsilon_j : \{0,1\} \rightarrow \{v_i\}$$



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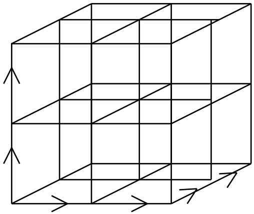

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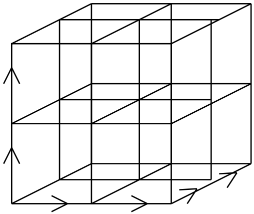

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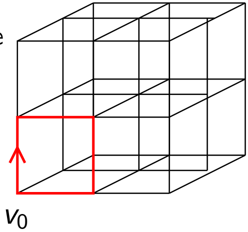
- The space $\mathcal{A}_\Gamma$ is a coarse-grained approximation of a configuration space of smooth connections, denoted by $\mathcal{A}$.

- **Algebra:** A loop L is a finite sequence of edges $L = \{\epsilon_{i_1}, \epsilon_{i_2}, \ldots, \epsilon_{i_n}\}$ running in Γ (choose basepoint v_0). Discard trivial backtracking.



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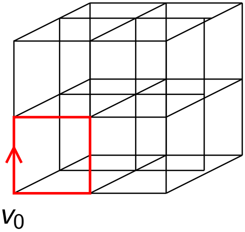

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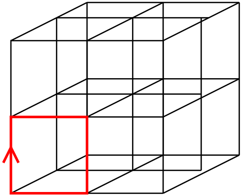

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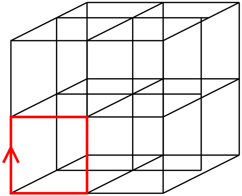

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- The algebra generated by based loops is a $\star$-algebra which we denote $\mathcal{B}_\Gamma$.

- **Hilbert space:** There is a 'natural' Hilbert space

$$\mathcal{H}_\Gamma = L^2(G^n, Cl(T^*G^n) \otimes M_l(\mathbb{C}))$$

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- Clifford bundle and matrix factor needed to accommodate a Dirac type operator and a representation of the algebra.
- The loop algebra $\mathcal{B}_\Gamma$ is represented on $\mathcal{H}_\Gamma$ by

$$f_L \cdot \psi(\nabla) = (1 \otimes \nabla(L)) \cdot \psi(\nabla) \, , \quad \psi \in \mathcal{H}_\Gamma$$

where the first factor acts on the Clifford-part of the Hilbert space and the second factor acts by matrix multiplication on the matrix part of the Hilbert space.

A family of lattices

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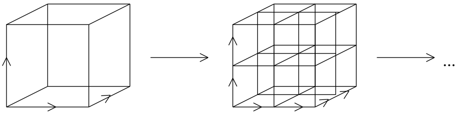
- Consider an infinite system of nested, 3-dimensional lattices

$$\Gamma_0 \longrightarrow \Gamma_1 \longrightarrow \Gamma_2 \longrightarrow \dots$$

with Γ_i a subdivision of Γ_{i-1}



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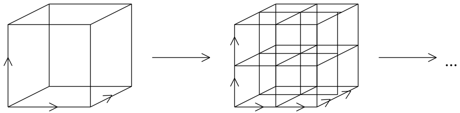

On the level of the associated manifolds $\mathcal{A}_{\Gamma_i}$ this gives rise to projections

$$\mathcal{A}_{\Gamma_0} \xleftarrow{P_{10}} \mathcal{A}_{\Gamma_1} \xleftarrow{P_{21}} \mathcal{A}_{\Gamma_2} \xleftarrow{P_{32}} \mathcal{A}_{\Gamma_3} \xleftarrow{P_{43}} \dots$$

- Consider next a corresponding system of spectral triples

$$(\mathcal{B}, \mathcal{H}, D)_{\Gamma_0} \leftrightarrow (\mathcal{B}, \mathcal{H}, D)_{\Gamma_1} \leftrightarrow (\mathcal{B}, \mathcal{H}, D)_{\Gamma_2} \leftrightarrow \ldots$$

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  with the requirement of compatible with the maps between graphs.
- This requirement restricts the choice of Dirac type operator.
- At the level of a graph, a compatible operator has the form

$$D = \sum_k a_k D_k$$

  where the sum runs over different copies of G and where

$$D_k(\xi) = \sum_i e_i \cdot d_{e_i}(\xi) \qquad \xi \in L^2(G, Cl(TG))$$

  where e_i are left-translated vectorfields. The a_k's are free parameters. The sum over copies of G is w.r.t. a change of variables.

The limit

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- Result: For a compact Lie-group G the triple $(\mathcal{B}, \mathcal{H}, D)_{\overline{\mathcal{A}}}$ is a semi-finite* spectral triple:
  - D's resolvent $(1 + D^2)^{-1}$ is compact (wrt. trace) and
  - the commutator $[D, b]$ is bounded

  provided the sequence $\{a_i\}$ approaches ∞.

**semi-finite: everything works up to a symmetry group with a trace (CAR algebra)* [Carey, Phillips, Sukochev].

What physical interpretation does this spectral triple construction have?

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- Take a cubulation of a 3-manifold Σ by the graphs $\{\Gamma_i\}_{i\in\mathbb{N}_+}$
- Denote by $\mathcal{A}$ the space of smooth G-connections. There is a natural map

$$\chi : \mathcal{A} \rightarrow \overline{\mathcal{A}}^{\square} \;, \quad \chi(\nabla)(\epsilon_i) = Hol(\nabla, \epsilon_i)$$

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- This result holds for many different systems of ordered graphs. Fx triangulations w. barycentric subdivisions.

D interacting with the algebra

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- Recall the Poisson bracket between loop and flux variables in LQG:

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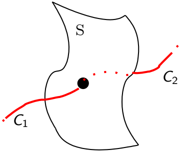

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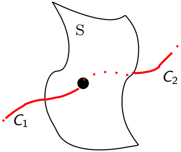

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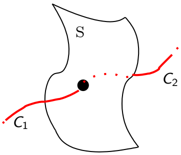

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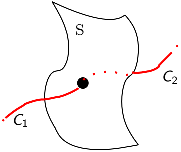

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- The left-invariant vector fields in D corresponds to infinitesimal flux-operators sitting at the vertices in the cubic graphs.
- In the continuum limit of repeated subdivisions the spectral triple contains information equivalent to a representation of the Poisson brackets of General Relativity:

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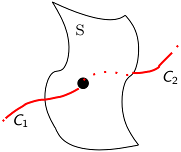

- The interaction between D and the algebra of loops reproduces the structure of this bracket.
- The left-invariant vector fields in D corresponds to infinitesimal flux-operators sitting at the vertices in the cubic graphs.
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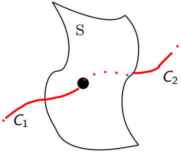

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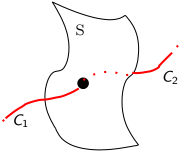

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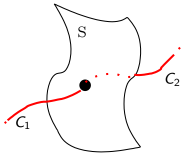

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  - the flux operators are stored in the Dirac type operator.
  - The classical loop- and flux-variables on a projective system of cubic lattices separates the Ashtekar variables.
- **Point:** the spectral triple construction captures information about the *kinematical* part of GR.

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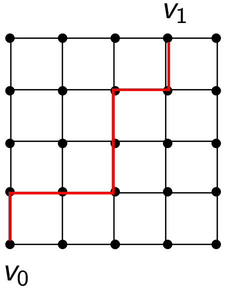
- **Aim**: to build invariance under choice of basepoint into the construction.
- Let $\mathcal{B}_v$ be the loop algebra based at the vertex v. The relationship between $\mathcal{B}_{v_0}$ and $\mathcal{B}_{v_1}$ is



$$\mathcal{B}_{v_0} = \mathcal{U}_p(v_0, v_1)\mathcal{B}_{v_1}\mathcal{U}_p^*(v_0, v_1)$$

where $p = \{\epsilon_{i_1}, \epsilon_{i_2}, \dots, \epsilon_{i_n}\}$ is a path from v_0 to v_i and $\mathcal{U}_p$ the corresponding parallel transport along p

$$\mathcal{U}_p(v_0, v_1) = g_{i_1} \cdot g_{i_2} \cdot \dots \cdot g_{i_n}$$

- Introduce the operators

$$\tilde{\mathcal{U}}_p = \tilde{\mathcal{U}}_{i_1} \tilde{\mathcal{U}}_{i_2} \cdot \ldots \cdot \tilde{\mathcal{U}}_{i_n}$$

with

$$\tilde{\mathcal{U}}_i = \mathbf{e}_i^a \, (1 \otimes \beta_i^a) \, (g_i \otimes 1)$$

associated to the path $p = \{\epsilon_{i_1}, \epsilon_{i_2}, \ldots, \epsilon_{i_n}\}$. β_i^a is an arbitrary matrix associated to the i'th edge and $a \otimes b$ refer to left and right actions.

- These operators are mutually orthogonal

$$\langle \tilde{\mathcal{U}}_p | \tilde{\mathcal{U}}_{p'} \rangle = \begin{cases} 1 & \text{if} \quad p = p' \\ 0 & \text{if} \quad p \neq p' \end{cases}$$

due to the elements of the Clifford algebra in $\tilde{\mathcal{U}}_i$.

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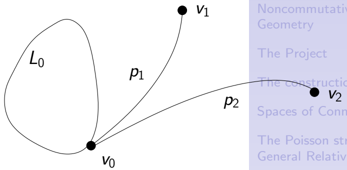

- Let L_0 be a loop based at v_0 and L_1 and L_2 the corresponding loops based at v_1 and v_2:

$$L_1 = \mathcal{U}^*_{p_1}(v_0, v_1) L_0\, \mathcal{U}_{p_1}(v_0, v_1)\ , \quad L_2 = \mathcal{U}^*_{p_2}(v_0, v_2) L_0\, \mathcal{U}_{p_2}(v_0, v_2)$$

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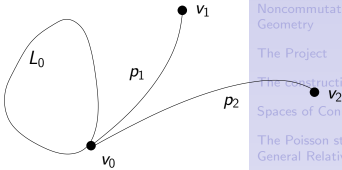

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Then

$$\langle \tilde{\mathcal{U}}_{p_1} \psi(v_1) + \tilde{\mathcal{U}}_{p_2} \psi(v_2) | L_0 | \tilde{\mathcal{U}}_{p_1} \psi(v_1) + \tilde{\mathcal{U}}_{p_2} \psi(v_2) \rangle = \langle \psi(v_1) | L_1 | \psi(v_1) \rangle + \langle \psi(v_2) | L_2 | \psi(v_2) \rangle \,,$$

where $\psi(v_i)$ is a matrix factor associated to v_i (will become a spinor field).

- This shows that the sum

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- **Important**: in $\xi_k(\psi)$ the sum runs over vertices in $\Gamma_k \backslash \Gamma_{k-1}$.

Special Semi-Classical States

- Pick a point (A, E) in phase-space (Ashtekar variables). Coherent states $\phi^t_k(E, A)$ on $\mathcal{A}_{\Gamma_k}$ are given by $(t \sim l_P^2)$

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- These states are well defined and normalizable also in the limit $k \to \infty$. (separable Hilbert space)
- Let ψ be a spinor field. Consider now the state

$$\Psi_k^t(\psi, E, A) = \xi_k(\psi)\Phi_k^t(A, E)$$

- This is a "natural" sequence of states $\{\Psi_k^t\}$ assigned to each level of subdivision of lattices.

The Dirac Hamiltonian

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provided we set $a_n = 2^{3n}$ and write $\beta^a_i = N(x)\gamma^a + N^a(x)\gamma^0$. Also, $g_i \simeq 1 + A_i$; $\nabla_i = \partial_i + A_i$ and $E^m_a = \sqrt{g}e^m_a$.

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- Here: $\psi(x) \in M_2(\mathbb{C}) \oplus M_2(\mathbb{C})$ and γ^μ acts from the right and A_i from the left.
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- We would like to take this limit of the *entire* construction to obtain an action of the diffeomorphism group.
- Thus, we should consider sequences of states $\{\psi_n(\mathcal{A}_{\Gamma_n})\}$ with certain continuity conditions.

A candidate for a partition function

- The trace of heat-kernel resembles a partition function

$$Tr \exp(-s(D)^2) \sim \int_{\overline{\mathcal{A}}} [d\nabla] \exp\left(-s(D)^2\right)$$

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- Thus, a key motivation for a spectral triple construction is it ensures a finite partition function.

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- We hope to find the Hamiltonian through the square of D.
- It is possible to write down an expression which gives the Hamiltonian in the classical limit. The operator

$$\sum_v 2^{3n}\left(\frac{8}{7}\right)^2 Tr(M(v)\sigma^a\sigma^b d_{\mathbf{e}^i_a} d_{\mathbf{e}^j_b} L_k)\epsilon^{ijk}$$

where L_i, $i \in \{1,2,3\}$, are loops in a plaquet and v is a vertex in $\Gamma_n \backslash \Gamma_{n-1}$, will descent to the Hamilton

$$\int N E^i_a E^j_b F^c_{ij} \epsilon^{ab}_{\ \ c} + N^a E^m_a E^n_b F^b_{mn}$$

in the semi-classical limit given by the states Φ^t_n, with

$$M(v) = N(v)1 + \mathrm{i}N^a(v)\sigma^a \ .$$

Connes Distance Formula

- Given a spectral triple $(\mathcal{B}, \mathcal{H}, D)$ over a manifold $\mathcal{M}$ the distance formula reads

$$d(\xi_x, \xi_y) = \sup_{b \in \mathcal{B}} \left\{ |\xi_x(b) - \xi_y(b)| \Big| |[D, b]| \leq 1 \right\}$$

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- **Question:** What about Connes distance formula for the spectral triple $(\mathcal{B}, \mathcal{H}, D)$ based on the algebra of loops?
- **Answer:** A distance between field configurations.
- The spectral triple construction is a metric structure on a configuration space of connections. This idea goes back to Feynman, Singer, Atiyah ...

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"A striking aspect of this approach to geometry of $\bar{\mathcal{A}}/\mathcal{G}$ is that its general spirit is the same as that of non-commutative geometry and quantum groups: even though there is no underlying differential manifold, geometrical notions can be developed by exploiting the properties of the *algebra* of functions."

- Ashtekar, Lewandowski, 1996