A non–commutative flux representation for LQG

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based on work with Aristide Baratin, Bianca Dittrich and Daniele Oriti
(to appear)

'Open Problems in Loop Quantum Gravity'–workshop, Zakopane 01/03/2009

Motivation

LQG:

holonomies $h_e[A]$ $\longrightarrow$ 'multiplication operators'
fluxes E^i_{e*} $\longrightarrow$ 'derivative operators'

simplicial context:

interpretation: fluxes as elementary building blocks for 2d–surfaces

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- Notion of simplicial geometry from a full LQG context?
- Spinfoam dynamics from LQG?
- LQG $\rightarrow$ non–Abelian flux algebra $\rightarrow$ non–commutative geometry?

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- LQG $\rightarrow$ non–Abelian flux algebra $\rightarrow$ non–commutative geometry?

A flux representation for LQG:
holonomies $h_e[A]$ $\longrightarrow$ 'derivative operators'
fluxes E^i_{e*} $\longrightarrow$ 'multiplication operators'

Strategy: Start with LQG–Hilbert space $\mathcal{H}_{AL}$ $\rightarrow$ perform unitary transformation using 'group Fourier transform' methods

Outline of the talk

1. The LQG basis
2. Non–commutative Fourier transform techniques
3. LQG in the dual picture
4. Outlook

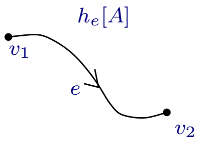
The LQG basis

[Ashtekar Isham '92; Baez '93; Ashtekar, Lewandowski '93; Marolf, Mourao '94; Ashtekar, Lewandowski '94; Ashtekar, Lewandowski, Marolf, Maurao, Thiemann '95]

Basic variables

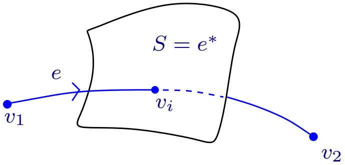
regularization of classical phase space [this version: Thiemann '00]

holonomies



$$h_e[A] = \mathcal{P} e^{\int_e A}$$

fluxes



$$E_e^i := \mathrm{Tr}\left[h_{(1,i)} \triangleright \int_{e*} dx^a \wedge dx^b \left[h_{(i,x)} \triangleright (*E^i)_{ab}\right](x)\right]$$

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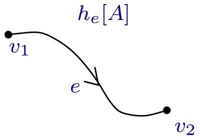

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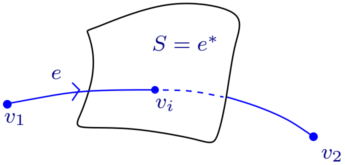


$$E^i_e := \mathrm{Tr}\left[h_{(1,i)} \triangleright \int_{e*} dx^a \wedge dx^b \left[h_{(i,x)} \triangleright (*E^i)_{ab} \right](x) \right]$$

phase space structure:

$h \in \mathrm{SU}(2)$, $E \in \mathfrak{su}(2)$ $\Rightarrow$ for each graph γ and dual graph γ^*: $\boxed{\mathcal{P}_\gamma = \otimes_{e\in\gamma} (T^*\mathrm{SU}(2))^{|\gamma|}}$

see also [Freidel, Speziale '10] for a derivation of $\mathcal{P}_\gamma$ from a simplicial point of view

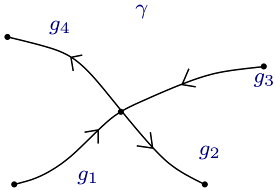
Quantization

space of cylindrical functions:
'functions of a finite number of holonomies'

$$\mathrm{Cyl}^\gamma = \{C^\gamma : \mathcal{A} \to \mathbb{C}; A \mapsto C^\gamma(A) \,|\, C^\gamma(A) := c(h_{e_1}(A), h_{e_2}(A), \dots h_{e_{|\gamma|}}(A))\}$$

$$\mathrm{Cyl} := \bigoplus_\gamma \mathrm{Cyl}^\gamma$$

example:



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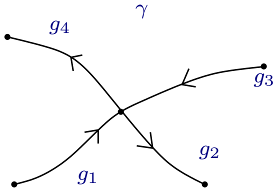

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for each graph:

Hilbert space: $\mathcal{H}_\gamma := L_2(\mathrm{SU}(2)^{|\gamma|}, d\mu_H)$

inner product: $\langle \phi \mid \psi \rangle_\gamma := \int \prod_{i=1}^{|\gamma|} dg_i \bar{\phi}(g_1, \ldots, g_{|\gamma|}) \psi(g_1, \ldots, g_{|\gamma|})$

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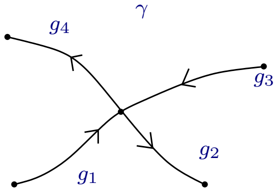

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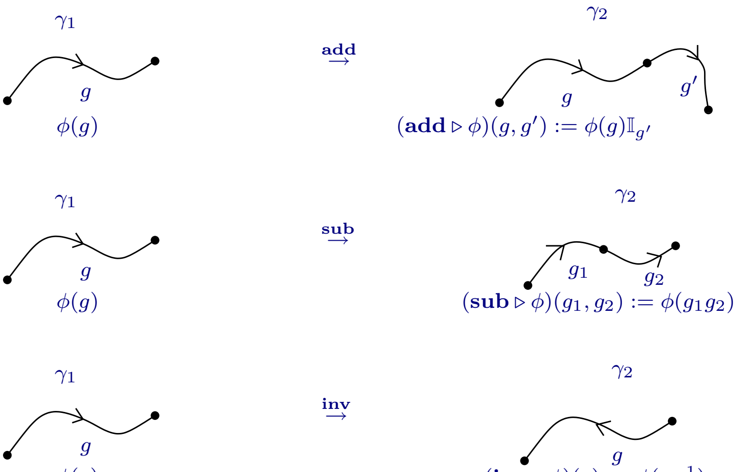
fundamental operators:

$(f_\gamma \triangleright \phi_\gamma)(g_i) := (f_\gamma \phi_\gamma)(g_i)$	$\rightarrow$ pointwise muliplication
$(E_e^i \triangleright \phi_\gamma)(g_i) := (X_e^i \triangleright \phi_\gamma)(g_i)$	$\rightarrow$ $(\mathrm{SU}(2)$–$)$ derivation

$\phi_\gamma \in \mathcal{H}_\gamma$, $f_\gamma \in \mathrm{Cyl}^\gamma$, X_e^i : left-, or right–invariant vector fields on the e–th copy of $\mathrm{SU}(2)$, depending on edge–orientation

Cylindrical consistency



$$\phi(g) \quad \xrightarrow{\textbf{add}} \quad (\textbf{add} \triangleright \phi)(g, g') := \phi(g)\mathbb{I}_{g'}$$

$$\phi(g) \quad \xrightarrow{\textbf{sub}} \quad (\textbf{sub} \triangleright \phi)(g_1, g_2) := \phi(g_1 g_2)$$

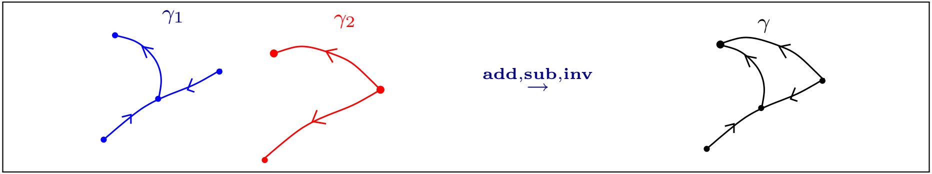
$$\phi(g) \quad \xrightarrow{\textbf{inv}} \quad (\textbf{inv} \triangleright \phi)(g) := \phi(g^{-1})$$

Ashtekar–Isham–Lewandowski Hilbert space

cylindrical consistency:

How to evaluate $\langle \phi_{\gamma_1} \mid \psi_{\gamma_2} \rangle$ for $\gamma_1 \neq \gamma_2$???

$\rightarrow$ choose a bigger graph γ that contains γ_1, γ_2 and use $\mathbf{add}, \mathbf{sub}, \mathbf{inv}$ to evaluate scalar product on γ



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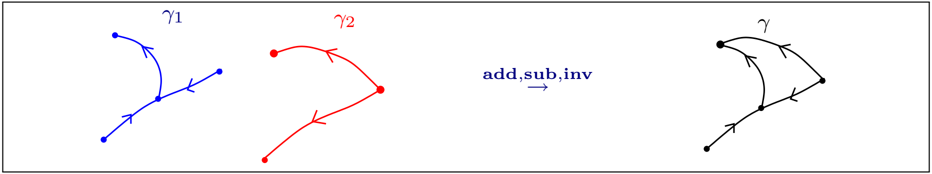

One has to ensure that $\langle \phi \mid \psi \rangle_\gamma$ does not depend on the choice of γ used to evaluate the inner product. [Ashtekar, Lewandowski '94].

$$L_2(\mathrm{SU}(2)^{|\gamma|}, d\mu_H) \overset{\text{projective limit}}{\longrightarrow} \mathcal{H}_{AL} := L_2(\bar{\mathcal{A}}, d\mu_{AL})$$

LQG is a true continuum theory! (see [Lewandowski, Okolow, Sahlmann, Thiemann '05] for uniqueness of $\mathcal{H}_{AL}$)

Non–commutative Fourier transformation techniques

[Freidel, Livine '05; Freidel, Majid '06; Joung, Mourad, Noui '08]

'map between functions on $\mathrm{SU}(2)$ and functions on $\mathfrak{su}(2)$ '

Plane Waves

define:

$$e : \mathrm{SU}(2) \times \mathfrak{su}(2) \to \mathbb{C}; \quad (g, x) \mapsto e_g(x) := e^{\mathrm{Tr}(x|g|)}$$

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- **symmetry:** $e_{-g}(x) = e_g(x)$, no distinction between upper and lower hemisphere of $\mathrm{SU}(2)$

Algebra and $\star$ –product

- **define** $\mathcal{C}(\mathbb{R}^3)$: linear span of $e_g(x)$ i.e. $\mathcal{C}(\mathbb{R}^3) \ni \hat{\phi}(x) = \int \phi(g) e_g(x)$

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- equip $\mathcal{C}(\mathbb{R}^3)$ with a $\star$–product (associative, non–commutative):

$$\star : \mathcal{C}(\mathbb{R}^3) \times \mathcal{C}(\mathbb{R}^3) \to \mathcal{C}(\mathbb{R}^3); \quad (e_g \star e_{g'})(x) := e_{gg'}(x)$$

extension to all $\mathcal{C}(\mathbb{R}^3)$ by linearity

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- $\mathcal{C}_\star(\mathbb{R}^3) := (\mathcal{C}(\mathbb{R}^3), \star)$ non–commutative algebra of functions over $\mathbb{R}^3$

Group Fourier transform

- **define** 'group Fourier transformation' as

$$\mathcal{F} : C(\mathrm{SU}(2)) \to \mathcal{C}_\star(\mathbb{R}^3); \quad f(g) \mapsto \hat{f}(x) := (\mathcal{F} \triangleright f)(x) := \int dg e_g(x) f(g)$$

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- effectively: restrictions to functions on $\mathrm{SO}(3)$

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- inverse Fourier transform:

$$\mathcal{F}^{-1} : \mathcal{C}_\star(\mathbb{R}^3) \to C^+(\mathrm{SU}(2)); \quad f(g) := (\mathcal{F}^{-1} \triangleright \hat{f})(g) := \int dx (e_{g^{-1}} \star \hat{f})(x)$$

Hilbert space and unitary map

- define inner product on $\mathcal{C}_\star(\mathbb{R}^3)$ as

$$\langle . \mid . \rangle_\star : \mathcal{C}_\star(\mathbb{R}^3) \times \mathcal{C}_\star(\mathbb{R}^3) \to \mathbb{C}; \quad (\hat{f}, \hat{f}') \mapsto \left\langle \hat{f} \mid \hat{f}' \right\rangle_\star := \int dx (\bar{\hat{f}} \star f')(x)$$

- Hilbert space: $\mathcal{H}_\star := \overline{\mathcal{C}_\star(\mathbb{R}^3)}$ wrt $\langle . \mid . \rangle_\star$

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- $\mathcal{F}$ is a unitary transformation between $\mathcal{H}_g := L_2(\mathrm{SU}(2)/\mathbb{Z}_2)$ and $\mathcal{H}_\star$.

LQG in the dual picture

Space of cylindrical functions

standard LQG:

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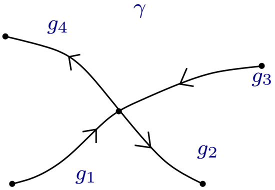
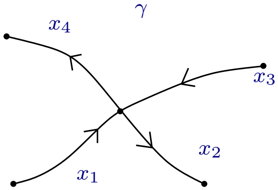
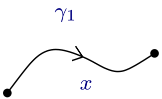




$$L_2([\mathrm{SU}(2)/\mathbb{Z}_2]^{|\gamma|}) \xrightarrow{\mathcal{F}} \mathcal{H}_\star^{|\gamma|}$$

$$\phi(g_1,\ldots,g_{|\gamma|}) \qquad \hat{\phi}(x_1,\ldots,x_{|\gamma|})$$

Cylindrical consistency I



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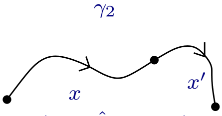

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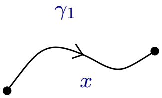

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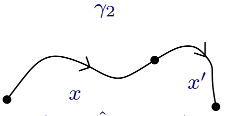

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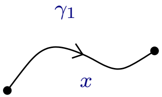

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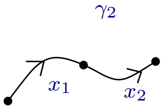

$(\textbf{sub} \triangleright \hat{\phi})(x_1, x_2) := (\delta_{x_1} \star \hat{\phi})(x_2)$

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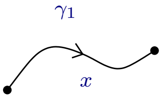

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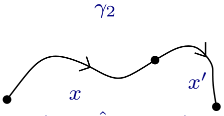

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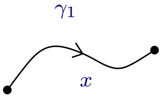

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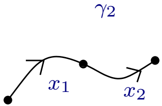

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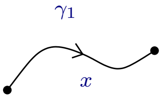
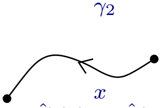


$\hat{\phi}(x)$

$\xrightarrow{\mathbf{inv}}$



$$(\mathbf{inv} \triangleright \hat{\phi})(x) := \hat{\phi}(-x)$$

Cylindrical consistency II

- associated with each graph we have a natural inner product

$$\left\langle \hat{\phi}_1 \mid \hat{\phi}_2 \right\rangle_{\star,\gamma} := \int \prod_{i=1}^{|\gamma|} dx_i (\bar{\hat{\phi}}_1 \star_{1,\dots,|\gamma|} \hat{\phi}_2)(x_1, \dots, x_{|\gamma|})$$

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- consider two functions $\hat{\phi}_{\gamma_1}(x_i), \hat{\psi}_{\gamma_2}(y_j)$ defined on possibly different graphs γ_1, γ_2
- choose any graph γ that contains γ_1, γ_2
- one can check that $\langle \phi_1 \mid \phi_2 \rangle_{\star,\gamma}$ does not depend on the choice of γ by
    1. either direct computation,
    2. or using unitarity of $\mathcal{F}$

$\Rightarrow$ Cylindrical consistency fulfilled, projective limit can be taken.

Action of fundamental operators

Fluxes:

$$
\begin{aligned}
(R^i \triangleright e_g)(x) &:= \left[\frac{d}{dt} e_{[\exp(t\sigma^i)g]}(x)\right]_{t=0} \\
&= \left[\frac{d}{dt} e_{[\exp(t\tau^i)]}\right]_{t=0} \star e_g(x) \qquad a.e. \\
&= \mathrm{Tr}(x\tau^i) \star e_g(x) \\
&= x^i \star e_g(x) \\
\Rightarrow (E_e^i \triangleright \hat{\phi})(x_1, \ldots, x_{|\gamma|}) &:= (x_e^i \star_e \hat{\phi})(x_1, \ldots x_{|\gamma|})
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&= \quad \left[\frac{d}{dt} e_{[\exp(t\tau^i)]}\right]_{t=0} \star e_g(x) \qquad a.e. \\
&= \quad \mathrm{Tr}(x\tau^i) \star e_g(x) \\
&= \quad x^i \star e_g(x) \\
\Rightarrow (E^i_e \triangleright \hat{\phi})(x_1, \ldots, x_{|\gamma|}) \quad &:= \quad (x^i_e \star_e \hat{\phi})(x_1, \ldots x_{|\gamma|})
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Cylindrical functions:

each $f \in \mathrm{Cyl}$ can be written as (restricting to one edge here)
$f(g) = \int dx (e_g \star \hat{f})(x)$ $\Rightarrow$ enough to know the action of 'plane waves':

$$e_g(x) e_g(y) = e_g(x + y)$$

Action of fundamental operators

Fluxes:

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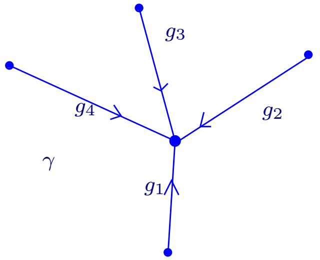
Fluxes act as $\star$–multiplication operators, cylindrical functions as generators of translation (in x –space)!

Geometrical interpretation

Gauge invariance

remark: construction not limited to 4–valent case!



$$\phi(g_1, g_2, g_3, g_4) = \int dg \phi(gg_1, gg_2, gg_3, gg_4)$$

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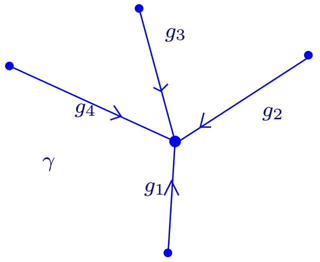

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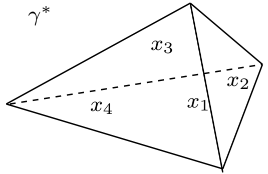

$\hat{\phi}(x_1, x_2, x_3, x_4) =$
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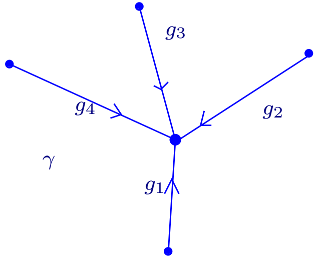

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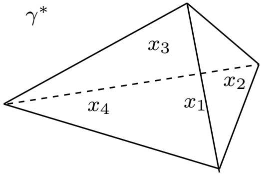

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$\Rightarrow$ Gauss constraint implies $\star$–closure in x–space. Natural interpretation: x^i are the oriented areas of faces $e^* \in \gamma^*$!
$\Rightarrow$ We can interpret $\hat{\phi}(x_i)$ as functions living on a dual cell–complex γ^*, flux operators provide information about (fuzzy) geometry.

Dual SNF and Semiclassical analysis

- what do 'dual SNF' look like?
  any function $\hat{\phi} \in \mathcal{H}_\star$ can be Peter–Weyl–decomposed into 'dual spin–network basis' as
  $\hat{\phi}(x) = \sum_{j,m,n} d_j \phi^j_{mn} \hat{D}^j_{mn}$

  $\hat{D}^j_{mn}(x) := \int d_g e_g(x) D^j_{mn}(g) = \frac{\mathcal{J}_{d_j}(|x|)}{|x|} D^j_{mn}(e^{-i\pi\vec{x}\cdot\vec{\sigma}})$ [Livine 08]
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- x–space provides a new possibility to define semiclassical (coherent?) states for LQG with a manifest (simplicial–) geometric interpretation (work in progress)

Outlook

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- expand $\star$–product in Planck–length, non–commutative geometry from LQG?