Reduced Dynamics of General Relativity & Semiclassical Limit of Reduced Quantum Dynamics

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Plan of the Talk

Part I: Reduced Dynamics of GR

- Reduced phase space for GR by means of four massless Klein – Gordon scalar fields
- (Naive) LQC generalisation
- One massive scalar field & Dirac quantisation for diffeo [Domagala's talk]
- More general action involving scalar fields, Dirac quantisation techniques [Kuchar, Torre]
- Similarities and differences to BK dust Model

Part II: Semiclassical Limit

- Quantum dynamics generated by physical Hamiltonian
- How to compute semiclassical expectation values
- Idea of Semiclassical perturbation theory

- Part I: Reduced Dynamics of GR
  1. Choose reference matter fields
  2. Derive equivalent set of constraints adapted to reference fields
  3. Construct observables and physical Hamiltonian $\mathrm{H}_{\mathrm{phys}}$
- This can then be used as starting point for quantisation

Reference Matter fields

Dynamics in General Relativity

- Dynamics in GR at gauge invariant level: Relational formulation
- Introduction of reference fields maps GR into a true Hamiltonian system
- $\mathrm{H}_{\mathrm{can}} \approx 0$ whereas $\mathrm{H}_{\mathrm{phys}} \neq 0$
- Conceptual clearer way to make contact to sectors where dynamical effects of (quantum) gravity is negligible
- This allows to access directly the physical Hilbert space in the QT

Matter reference fields

- Complete quantum model for GR coupled to BK dust exist [K.G., T.Thiemann 0711.0119]
- Current LQC models use K.G. scalar field clock
- Also standard Higgs mechanism uses scalar fields, however with Mexican hat potential
- The reference fields can be used to reduce the constraints of GR classically (Goldstone bosons)
- Will BK mechanism also work?
- Deparametrisation: Generator of evolution is time indepedent
- How does the corresponding $\mathrm{H}_{\mathrm{phys}}$ look like for full GR?

Four K.G. scalar field

GR coupled to four scalar fields

- Action

$$S = S^g + S^m + S^{cl} = S^g + S^m + \int d^4X \sum_{J=0}^{3} \sqrt{|\det(g_{\mu\nu})|} g^{\mu\nu} \varphi^J_\mu \varphi^J_\nu$$

- Perform (3+1) split, then we have (p_A, q^A) and (π_J, φ^J)
- Constraints:

$$\begin{aligned} c^{tot} &= c^g + c^m + c^{\varphi^J} =: c + \sum_{J=0}^{3} \frac{\pi_J^2}{2\sqrt{\det(q)}} + \frac{\sqrt{\det(q)}}{2} \varphi^J_a \varphi^J_b q^{ab} \\ c^{tot}_a &= c^g_a + c^m_a + c^{\varphi^J}_a =: c_a + \sum_{J=0}^{3} \pi_J \varphi^J_{,a} \end{aligned}$$

- Solve constraints for clock momenta, then $\tilde{c}_J = \pi_J + h_J \quad J = 0, .., 3$

Solving constraints for the momenta

Reference system

- φ^{j} good reference system if $\det(\varphi^{\mathrm{J}}_{\mu}) \neq 0 \quad \mathrm{J}, \mu = 0, .., 3$
- Diffeo: $\varphi^{\mathrm{a}}_{\mathrm{j}} \varphi^{\mathrm{j}}_{\mathrm{b}} = \delta^{\mathrm{a}}_{\mathrm{b}}$ $\mathrm{j} = 1, 2, 3$

$$\pi_{\mathrm{j}} = \varphi^{\mathrm{j}}_{,\mathrm{a}} (\mathrm{c}_{\mathrm{a}} + \pi_{\mathrm{o}} \varphi^{0}_{,\mathrm{a}})$$

- Reinsert π_{j} into $\mathrm{c}^{\mathrm{tot}}$:

$$\begin{aligned} 0 \quad = \quad & \pi_0^2 + \sum_j \varphi^a_j \varphi^b_j (c_b + \pi_0 \varphi^0_{,b})(c_a + \pi_0 \varphi^0_{,a}) + 2\sqrt{\det(q)} c \\ & + \det(q) \sum_J q^{ab} \varphi^J_{,a} \varphi^J_{,b} \end{aligned}$$

- quadratic equation for π_0

Solving constraints for the momenta

Equivalent set of constraints

- $$c^{tot} = \pi_0 + h(q, p, \varphi^J) \quad c_j^{tot} = \pi_j + h_j(q, p, \varphi^J)$$

$$\begin{aligned} h_j &= \varphi^j_{,a}(c_a + \pi_o \varphi^0_{,a}) \\ h &= -\frac{a}{2c} \pm \frac{1}{2c}\sqrt{a^2 - 4c^2 b} \end{aligned}$$

$$\begin{aligned} a &= \sum_j \varphi^0_b \varphi^b_j \varphi^a_j c_a, \quad c = 1 + \sum_j \varphi^0_a \varphi^0_b \varphi^a_j \varphi^b_j \\ b &= 2\sqrt{\det(q)}c + \sum_j \varphi^a_j \varphi^b_j c_a c_b + \det(q) q^{ab} \sum_J \varphi^J_{,a} \varphi^J_{,b} \end{aligned}$$

- We chose $\pm$ for π_0 such that $0 \leq \dot{\varphi}^0 = \frac{N\pi_0}{\sqrt{\det(q)}}$, $\pi_0 \geq 0$, hence $N \geq 0$

Comparison with BK dust model

BK dust model

- Recall four scalar fields $(\mathrm{P},\mathrm{T}),(\mathrm{P_j},\mathrm{S^j})\quad \mathrm{j}=1,..3$

$$c^{\mathrm{tot}} \quad = \quad \mathrm{P}+\mathrm{h}(\mathrm{q},\mathrm{p}) \quad c^{\mathrm{tot}}_{\mathrm{j}}=\mathrm{P_j}+\mathrm{h_j}(\mathrm{q},\mathrm{p},\mathrm{T},\mathrm{S^j})$$

- Here all dust field dependence cancels in h, mainly due to the fact that $\mathrm{P_j}=\mathrm{W_j}\mathrm{P}$
- Deparametrisation: time independent physical Hamiltonian $\mathrm{H_{phys}}$

Scalar field model

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$$c^{\mathrm{tot}} \quad = \quad \pi_0+\mathrm{h}(\mathrm{q},\mathrm{p},\varphi^0,\varphi^{\mathrm{j}}) \quad c^{\mathrm{tot}}_{\mathrm{j}}=\pi_{\mathrm{j}}+\mathrm{h_j}(\mathrm{q},\mathrm{p},\varphi^0,\varphi^{\mathrm{j}})$$

- Here clock fields occur in h and $\mathrm{h_j}$
- However, only in form of spatial derivative, this helps, different for instance for massless scalar fields

1.) Spatial diff'-invariant quantities

- Pull back from coord. manifold $\mathcal{M}$ to scalar field manifold $\mathcal{S}$

$$\mathrm{q_{ab}} \rightarrow \mathrm{q_{ij}} := \mathrm{q_{ab}}\varphi^{\mathrm{a}}_{\mathrm{i}}\varphi^{\mathrm{b}}_{\mathrm{j}}, \quad \mathrm{p^{ab}} \rightarrow \mathrm{p^{ij}} := \det(\partial\varphi^{\mathrm{j}}/\partial\mathrm{x^i})\mathrm{p^{ab}}\varphi^{\mathrm{i}}_{\mathrm{a}}\varphi^{\mathrm{j}}_{\mathrm{b}}$$

- $\tilde{\mathrm{q}}_{\mathrm{ij}}(\sigma,\mathrm{t}) = \int_{\mathcal{M}} \mathrm{d^3x}|\det(\partial\varphi^{\mathrm{j}}(\mathrm{x})\ \partial\mathrm{x^i})|\delta(\varphi^{\mathrm{j}}(\mathrm{x}),\sigma^{\mathrm{j}})\mathrm{q_{ab}}(\mathrm{x})\varphi^{\mathrm{a}}_{\mathrm{i}}(\mathrm{x})\varphi^{\mathrm{b}}_{\mathrm{j}}(\mathrm{x})$
- local in σ but non – local in x
- $\varphi^{\mathrm{k}}_{\mathrm{a}} \longrightarrow \delta^{\mathrm{k}}_{\mathrm{j}}$ and $\varphi^0_{\mathrm{a}} \longrightarrow \tilde{\varphi}^0_{\mathrm{j}}$
- Hence $\tilde{\mathrm{h}} = \mathrm{h}(\tilde{\mathrm{q}},\tilde{\mathrm{p}},\tilde{\varphi}^0)$ and $\tilde{\mathrm{h}}_{\mathrm{j}} = \mathrm{h_j}(\tilde{\mathrm{q}},\tilde{\mathrm{p}},\tilde{\varphi}^0)$
- again only spatial derivative of the clock field φ^0 are involved

2.) Full Observables

- $\mathrm{Q}_{\mathrm{ij}}(\sigma,\tau)=\exp(\int_{\mathcal{S}}\mathrm{d}^3\sigma\beta\{\tilde{\mathrm{c}}^{\mathrm{tot}},.\})\cdot\tilde{\mathrm{q}}_{\mathrm{ij}}(\mathrm{t},\sigma)\Big|_{\beta=\tau-\varphi^0}$
- More complicated than in the dust case due to φ^0_{j} terms in $\tilde{\mathrm{h}}$
- Similar situation here: Physical Hamiltonian

$$\mathrm{H}_{\mathrm{phys}}=\int_{\mathcal{S}}\mathrm{d}^3\sigma\mathrm{H}(\sigma)\quad\text{with}$$

$$\mathrm{H}(\sigma)=\sqrt{-\left(2\sqrt{\det(\mathrm{Q})}\mathrm{C}+\mathrm{C}_{\mathrm{j}}\mathrm{C}_{\mathrm{k}}\delta^{\mathrm{jk}}+\det(\mathrm{Q})\mathrm{Q}^{\mathrm{ij}}\delta_{\mathrm{ij}}\right)}$$

Comparison of dynamics with BK dust model

BK dust model

- $$\mathrm{H}_{\mathrm{phys}} = \int_{\mathcal{S}} \mathrm{d}^3\sigma \mathrm{H}(\sigma) \quad \text{with} \quad \mathrm{H}(\sigma) = \sqrt{\mathrm{C}^2 - \mathrm{Q}^{\mathrm{ij}}\mathrm{C}_{\mathrm{i}}\mathrm{C}_{\mathrm{j}}}$$

Scalar field model

- $$\mathrm{H}_{\mathrm{phys}} = \int_{\mathcal{S}} \mathrm{d}^3\sigma \mathrm{H}(\sigma) \quad \text{with}$$
$$\mathrm{H}(\sigma) = \sqrt{-\left(2\sqrt{\det(\mathrm{Q})}\mathrm{C} + \mathrm{C}_{\mathrm{j}}\mathrm{C}_{\mathrm{k}}\delta^{\mathrm{jk}} + \det(\mathrm{Q})\mathrm{Q}^{\mathrm{ij}}\delta_{\mathrm{ij}}\right)}$$

Quantisation

Quantisation using LQG techniques

- Hamiltonian density

$$H(\sigma) = \sqrt{-\left(2\sqrt{\det(Q)}C + C_j C_k \delta^{jk} + \det(Q) Q^{ij}\delta_{ij}\right)}$$

- In LQG representation finite diffeos $\hat{U}(\phi)$ are not weakly continuous
- Consequently, infinitesimal diffeos $\hat{C}_j$ cannot be implemented
- $H_{\rm phys}$ for scalar field clocks cannot be promoted to an operator in $\mathcal{H}_{\rm LQG}$
- Is this a problem? No
- Because One could also use Dirac quantisation for diffeos [Domagala's talk]
- However, it shows that they are classical clocks that are not allowed in QG if LQG representation is used

- Part II: Semiclassical Limit
  - Once we have an operator $\hat{\mathrm{H}}_{\mathrm{phys}}$ we would like to analyse its semiclassical limit, $\langle\psi, \hat{\mathrm{H}}_{\mathrm{phys}}\psi\rangle$
  - Details on coherent states [Bahr's talk]
  - $\mathrm{H}_{\mathrm{phys}}$ involves Volume operator
  - Semiclassical Computation not possible analytically, approximation techniques necessary
    $\longrightarrow$ Semiclassical perturbation theory
- Discuss semiclassical perturbation theory for the example of $\hat{\mathrm{H}}_{\mathrm{phys}}$ in the dust model

Operator for $\mathbf{H}_{\rm phys}$

- Physical Hamiltonian $\widehat{\mathbf{H}}_{\rm phys}$, $\tau_\mu = (1, \tau_{\rm j})$

$$\widehat{\mathbf{H}}_{\rm phys} = \frac{\hbar}{\ell_{\rm p}^4} \sum_{\rm v \in V(\alpha)} \sqrt{\left| \sum_{\mu=0}^{4} \eta^{\mu\mu} \left[\sum_{\rm a=1}^{3} \mathrm{Tr}\Big(\tau_\mu \mathbf{A}(\alpha_{\rm v}^{\rm a}) \mathbf{A}(\mathrm{e}_{\rm v}^{\rm a}) \big[\mathbf{A}(\mathrm{e}_{\rm v}^{\rm a})^{-1}, \mathbf{V}_{\rm v} \big] \Big) \right]^2 \right|}$$

- Volume operator:

$$\mathbf{V}_{\rm v} = \sqrt{\left| \epsilon_{\rm abc} \mathrm{Tr}\left(\mathbf{E}(\mathrm{S}_{\rm v}^{\rm a}) \mathbf{E}(\mathrm{S}_{\rm v}^{\rm b}) \mathbf{E}(\mathrm{S}_{\rm v}^{\rm c}) \right) \right|}$$

- Here two problems:
    - 1. Square root
    - 2. Volume operator

Problem 1. and 2.

- Discuss problem 2. related to Volume operator
- Problem 1. is then just a special application of the techniques developed for volume operator
- Problem 2. also occurs for Master constraint operator

Problem with SU(2) Calculations

- $\mathrm{H_{phys}}$ involves $\sqrt{|\hat{\mathrm{O}}_\mathrm{v}|}$
  We want to calculate
  $\langle\psi,\widehat{\mathbf{M}}_\mathrm{v}\,\psi\rangle \sim \langle\psi\,\mathrm{h}_\alpha\mathrm{h}[\mathrm{h}^{-1},\sqrt{\mathrm{V_v}}]\psi,\mathrm{h}_\alpha\mathrm{h}[\mathrm{h}^{-1},\sqrt{\mathrm{V_v}}]\psi\rangle$
  $\langle\psi,\hat{\mathrm{O}}_\mathrm{v}\psi\rangle \sim \langle\psi,\mathrm{h}_\alpha\mathrm{h}[\mathrm{h}^{-1},\mathrm{V_v}]\psi,\mathrm{h}_\alpha\mathrm{h}[\mathrm{h}^{-1},\mathrm{V_v}]\psi\rangle$
- of general form $\langle\psi,p_1(h)F_1(V_v)p_2(h)F_2(V_v)p_3(h)\psi\rangle$
- Volume operator

$$V_v := \ell_p^3\sqrt{|\frac{1}{48}\sum_{e_1\cap e_2\cap e_3=v}\epsilon_v(e_1,e_2,e_3)\epsilon^{ijk}E_i(e_1)E_j(e_2)E_k(e_3)|}$$

- $\mathrm{V_v} := \sqrt{|\mathrm{Q_v}|}$, then $V_v=(Q_v^2)^{\frac{1}{4}}$, $\sqrt{V_v}=(Q_v^2)^{\frac{1}{8}}$
  in general $F_I(V_v):=(Q_v^2)^{q_I}$
- Problem: We cannot calculate $\langle\psi(Q_v^2)^{q_I}\psi\rangle$ analytically

Naive Idea for approximation

- red not computable, green computable
- Define:

$$x_I := \frac{Q_v^2}{<\psi, Q_v\psi>^2} - 1$$

- The operator x_I is bounded from below, $x_I \geq -1$
- $<\psi, Q_v\psi>, <\psi, Q_v^n\psi>$ can be computed exactly [Winkler, Thiemann]
- Functions of volume operator

$$F_I(V_v) = (Q_v^2)^{q_I} = |<\psi\,,\,Q_v\psi>|^{2q_I}\ \ f_I(x_I),\quad f_I(x_I) = (1+x_I)^{q^I}$$

- Power expansion of $t \mapsto f(t) = (1+t)^q,\ -1 \leq t < \infty$

$$f(t) := 1 + \sum_{n=1}^{\infty} \left(\begin{array}{c} q \\ n \end{array}\right) t^n, \quad \left(\begin{array}{c} q \\ n \end{array}\right) = (-1)^{n+1} \frac{q(1-q)..(n-1+q)}{n!}$$

Expansion in terms of x_I operator

- Use the spectral theorem for operator valued function $f_I(x_I)$

$$
\begin{aligned}
f_I(x_I) &= \int_{-1}^{\infty} f_I(t) dE(t) = \int_{-1}^{\infty} [1 + \sum_{n=1}^{\infty} \left(\begin{array}{c} q \\ n \end{array} \right) t^n] \, dE(t) \\
&= [1 + \sum_{n=1}^{\infty} \left(\begin{array}{c} q \\ n \end{array} \right) x_I^n]
\end{aligned}
$$

where E is the projection valued measure associated with x_I.

- Coherent state matrix elements of x_I, x_I^n are computable [Winkler, Thiemann]
- Of course, the second equality is wrong if $t \notin (-1, 1)$!
- Naive idea false, must be substituted by a rigorous argument.

Rigorous Argument

- For each $k \geq 0$ there exists $0 < \beta_k < \infty$, such that

$$f_I^- := f_{2k+1}(t) - \beta_k t^{2k+2} \leq f(t) \leq f_{2k+1}(t) =: f_I^+$$

for $-1 \leq \mathrm{t} < \infty$ where $f_k(t)$ denotes the partial Taylor series of $f(t) = (1+t)^q$, $0 < q \leq \frac{1}{4}$ up to to order t^k.

- Polarisation identity

$$\Re(\langle\psi_1\,,\,f_I\,\psi_2\rangle) = \frac{1}{4}(\langle\underbrace{\psi_1+\psi_2}_{\psi_+}\,,\,f_I\,\psi_1+\psi_2\rangle - \langle\underbrace{\psi_1-\psi_2}_{\psi_-}\,,\,f_I\,\psi_1-\psi_2\rangle)$$

- Estimation ($f_I^\pm$ are computable!)

$$\frac{1}{4}\langle\psi_+\,,\,f_I^-\,\psi_+\rangle - \langle\psi_-\,,\,f_I^+\,\psi_-\rangle)$$
$$\leq \quad \Re(\langle\psi_1\,,\,f_I\,\psi_2\rangle) \leq \frac{1}{4}(\langle\psi_+\,,\,f_I^+\,\psi_+\rangle - \langle\psi_-\,,\,f_I^-\,\psi_-\,\rangle)$$

- Define
$$\bar{\mathrm{f}} := \frac{1}{2}(\mathrm{f}_+ + \mathrm{f}_-), \quad \Delta\mathrm{f} := \frac{1}{4}(\mathrm{f}_+ - \mathrm{f}_-)$$
- Δf is proportional to fluctuation of operator Q with $f := (Q^2)^q$
- Then one can show
$$\left|\Re\langle\psi_1\,,\,(f - \bar{f})\,\psi_2\rangle\right| \quad \leq \quad \langle\psi_+\,,\,\Delta f\,\psi_+\rangle - \langle\psi_-\,,\,\Delta f\,\psi_+\rangle$$
- Idea: Even if we cannot compute $\langle\psi, \mathrm{f}\psi\rangle$, $\bar{f}$ is good approximation provided matrix elements of Δf are small
- If necessary iterative use of above estimation, these terms are of higher order in $\hbar$

Application of estimation

- Consider $f := (Q^2)^q$

$$f = \lambda^q\big(1 + (Q^2 - \lambda)/\lambda)\big)^q = \lambda^q\big(1 + x\big)^q, \quad \lambda^q := \langle\psi Q^2 \psi\rangle$$

- Define (k=0) case:

$$f_+ := \lambda^q(1 + qx), \quad f_- := \lambda^q(1 + qx - (1 - q)x^2)$$

- Consider coherent state ψ, then

$$\langle\psi\, f_+\, \psi\rangle = \lambda^q \quad \text{and} \quad \langle\psi\, f_-\, \psi\rangle = \lambda^q(1-(1-q)[(1/\lambda^2)\langle\psi\, Q^4\, \psi\rangle - 1])$$

- Hence $\langle\psi, f\, \psi\rangle = \langle\psi\, , \, Q^2\, \psi\rangle^q + o(\hbar)$
- Means: If we are interested only in lowest order we can simply replace $\langle\psi\, , \, (Q^2)^q\, \psi\rangle$ by $\langle\psi\, , \, Q^2\, \psi\rangle^q$

Iteration for more complicated matrix elements

- Start with $\langle\psi, p_1(h) F_1(V) p_2(h) F_2(V) p_3(h)\,\psi\rangle$
- Use $F_1(V) \sim f_1$

$$\langle\psi, p_1(h) f_1 p_2(h) f_2 p_3(h)\,\psi\rangle$$

-

$$\mathrm{R} := \langle\psi, \mathrm{p}_1(\mathrm{h})\mathrm{f}_1\mathrm{p}_2(\mathrm{h})\mathrm{f}_2\mathrm{p}_3(\mathrm{h})\,\psi\rangle = \langle\psi_1\,,\,\mathrm{f}_1\,\psi_2\rangle \quad \psi_1 := \mathrm{p}_1\psi,\ \psi_2 := \mathrm{f}_2\mathrm{p}_3\psi$$

- Now apply lemma

$$\Big|\mathfrak{R}(\mathrm{R}) - \mathrm{R}_2\Big| \leq \mathrm{R}_3 + \mathrm{R}_4$$

- Now apply the lemma to $\mathrm{R}_2, \mathrm{R}_3$ and R_4 etc.
- Each iteration term will produce new terms $R_5, R_6, \ldots$ for which the order of $\hbar$ is increased for non – computable terms

- Start with $\langle\psi, \mathrm{p}_1(\mathrm{h})\mathrm{F}_1(\mathrm{V})\mathrm{p}_2(\mathrm{h})\mathrm{F}_2(\mathrm{V})\mathrm{p}_3(\mathrm{h})\psi\rangle$
- 1. Decide up to which order in $\hbar$ we want to calculate expectation value, assume $\hbar^k$
- 2. Check how many f are involved
- 3. Then number of iterations is known in order to ensure that non – computable terms of $o(\hbar^k)$ In general more than two f's

$$\langle\psi, p_1(h)F_1(V)p_2(h)F_2(V)p_3(h)\psi\rangle = R(f_1, f_2)$$

- We can show that (also for general case)

$$\begin{aligned} R(f_1, f_2) &= \langle\psi, p_1\bar{f}_1 p_2(h)\bar{f}_2 p_3(h)\psi\rangle + o(\hbar^k) \\ &= \langle\psi, p_1 f_1^+ p_2(h) f_2^+ p_3(h)\psi\rangle + o(\hbar^k) \end{aligned}$$

- Explicit expression for β is not needed
- We can reexpress $R(f_1, f_2)$ in terms of computable quantities f_1^+, f_2^+ + corrections of higher order than $\hbar^k$

Back to the start of part II

- We want to calculate $\langle\psi\, \hat{\mathrm{H}}_{\mathrm{phys}}\, \psi\rangle$ to zero order in $\hbar$
- $\langle\psi\, \hat{\mathrm{H}}_{\mathrm{phys}}\, \psi\rangle$ is of the form $\langle\psi\, \sqrt{|\hat{O}|}\, \psi\rangle$
- Problem 1.: Square root: Use Semiclassical perturbation theory for $\mathrm{q} = 1/4$
- Then $\langle\psi\, \sqrt{|\hat{\mathrm{O}}|}\, \psi\rangle = \langle\psi\, \hat{\mathrm{O}}\, \psi\rangle^{2\mathrm{q}} + \mathrm{o}(\hbar) = \sqrt{\langle\psi\, \hat{\mathrm{O}}\, \psi\rangle} + \mathrm{o}(\hbar)$
- Now $\langle\psi, \widehat{\mathbf{M}}_{\mathrm{v}}\, \psi\rangle \sim \langle \mathrm{h}_\alpha \mathrm{h}[\mathrm{h}^{-1}, \sqrt{\mathrm{V}_{\mathrm{v}}}]\psi, \mathrm{h}_\alpha \mathrm{h}[\mathrm{h}^{-1}, \sqrt{\mathrm{V}_{\mathrm{v}}}]\psi\rangle$
  $\langle\psi, \hat{\mathrm{O}}_{\mathrm{v}}\psi\rangle \sim \langle \mathrm{h}_\alpha \mathrm{h}[\mathrm{h}^{-1}, \mathrm{V}_{\mathrm{v}}]\psi, \mathrm{h}_\alpha \mathrm{h}[\mathrm{h}^{-1}, \mathrm{V}_{\mathrm{v}}]\psi\rangle$
- Hence for terms involved in $\hat{\mathrm{O}}$ apart from $\mathrm{q} = 1/2$ instead of $\mathrm{q} = 1/4$ similar calculation for $\widehat{\mathbf{M}}$
- For $\widehat{\mathbf{M}}$ we know already that semiclassical limit $(\hbar^0)$ is correct
- So $\langle\psi\, \hat{\mathrm{H}}_{\mathrm{phys}}\, \psi\rangle = \mathrm{H}_{\mathrm{phys}} + \mathrm{o}(\hbar)$

Summary & Conclusions

Part I: Reduced Dynamics of GR

- Reduced phase space for GR by means of four Klein – Gordon scalar fields
- Compared with dust model
- It turns out that GR & massless scalar fields cannot be quantised in $\mathcal{H}_{\mathrm{LQG}}$
- Requirements for choice of clocks wrt form how diffeos enter $\mathcal{H}_{\mathrm{phys}}$

Summary & Conclusions

Part II: Semiclassical Limit

- Discussed computational problems that occur for SU(2) case
- Introduced Semiclassical perturbation theory
- Allows to approximate non – computable terms by computable ones
- Error due to substitution is of higher order in $\hbar$
- Can be used to show that semiclassical limit of $\widehat{\mathbf{M}}$ and $\hat{\mathrm{H}}_{\mathrm{phys}}$ for dust model is correct
- Also interesting for quantum spacetimes in fulll theory (for LQC [Ashtekar, Kaminski, Lewandowski])
- Since $\langle\psi,\, \hat{\mathrm{N}}\,\psi\rangle$ and $\langle\psi,\, \hat{\mathrm{N}}^{\mathrm{a}}\,\psi\rangle$ will also involve similar terms