Max-Planck-Institut
für Gravitationsphysik
(Albert-Einstein-Institut)

The new spin foam models

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Open Problems in Loop Quantum Gravity
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Plan of the talk

- Introduction
- Classical phase space
- The quantum 4-simplex
- Boundary states
- Spin foam amplitude
- Observables

Spin foams constitute an attempt to define a path integral for gravity.

$$\Psi(\text{3-geometries } h) = \sum_{\text{4-geometries } g} e^{i\frac{S(g)}{\hbar}}$$

The main assumption is a restriction to a fixed simplicial net.

Hope to define a dynamics for LQG.

Introduction

Two main approaches :

Spin foams

constrained BF theory

Reisenberger '94 &'97, Freidel and Krasnov '99...

quantum geometry

Barbieri '98, Barrett and Crane '98, Baez and Barrett '99...

Introduction

$$S_{EC\gamma} = \int \,^{\star}(e \wedge e)_{IJ} \wedge R(\omega)^{IJ} + \frac{s}{\gamma} \int (e \wedge e)_{IJ} \wedge R(\omega)^{IJ}$$

$$B = e \wedge e$$

$$S_P = \int (\star B + \frac{s}{\gamma} B)_{IJ} \wedge R(\omega)^{IJ} + \phi_{IJKL} B^{IJ} \wedge B^{KL}$$

discretize and constrain

$$Z_\Sigma = \sum_{\chi} \prod_{t} A_t \prod_{\tau} A_\tau \prod_{v} A_v$$

Introduction

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quantum geometry
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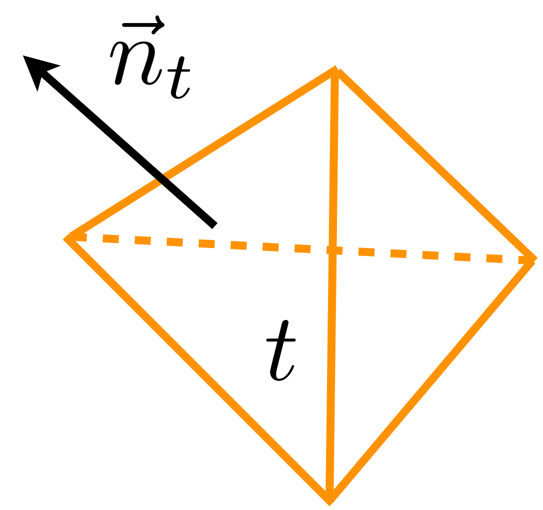
Introduction

Barbieri's quantum tetrahedron:

$$\sum_t \vec{n}_t = 0 \qquad |\vec{n}_t| = A_t$$



$$\vec{n}_t \rightsquigarrow \vec{L}$$

$$\mathcal{K}_3 = \otimes_t \mathcal{H}_{j_t} \xrightarrow{\text{closure}} \mathrm{Inv}\left(\otimes_t \mathcal{H}_{j_t}\right) \ni |i, j_t\rangle$$

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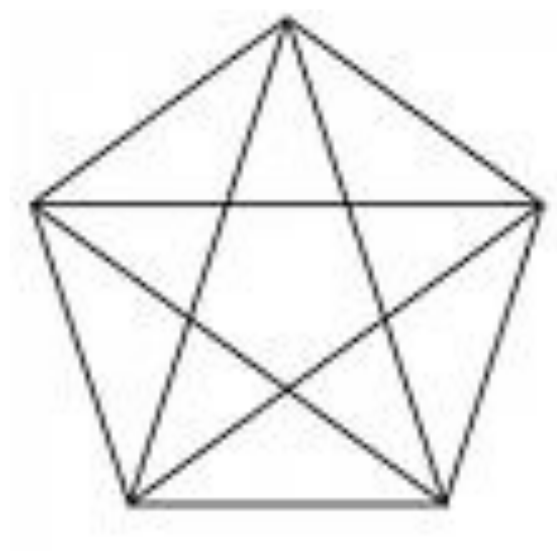
Classical phase space



Let us start with a single 4-simplex

general triangulations: Dittrich and Ryan '08 and talk by Ryan

The classical phase space associated to a 4-simplex can be described in a number of ways

Classical phase space

Barrett and Crane '98 :

Variables : $B_{ab}^{IJ} \leftrightarrow B_t^{IJ} \quad (a,b=1...5\,;\, I,J=0...3)$

Constraints :

- Orientation:

$$B_{ab} = -B_{ba}$$

- Closure:

$$\sum_{b \neq a} B_{ab}^{IJ} = 0\ ,\ \forall a$$

- Diagonal simplicity:

$$(\star B_{ab})_{IJ} B_{ab}^{IJ} = 0\ ,\ \forall (ab)$$

- Cross simplicity:

$$(\star B_{ab})_{IJ} B_{ac}^{IJ} = 0\ ,\ \forall a, b \neq c$$

+ non-degeneracy conditions

Theorem 1. *(Barrett and Crane) Each geometric 4-simplex determines a set of bivectors satisfying the constraints above, and each set of bivectors satisfying these constraints determines a geometric 4-simplex unique up to parallel translation and inversion through the origin.*

Classical phase space

Let us consider some modifications :

Engle, R.P., Rovelli ’07

Constraints :

- Simplicity:

$$\forall a \ , \ \exists \ N_a \text{ s.t. } N_{a\,I} B_{ab}^{IJ} = 0 \ \forall b \neq a.$$

spacelike triangles in the Lorentzian case

talk by Conrady

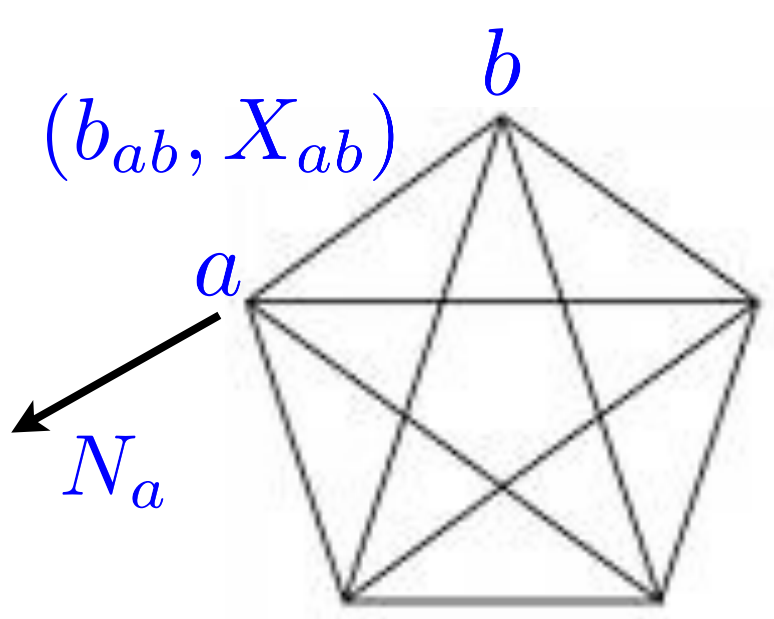
Variables :

$$X_a \mathcal{T} = N_a$$

$X_a \in G$ $(G\text{=SO}(4)$ or SO(3,1)$)$

$$X_{ab} = X_a^{-1} X_b$$

$$b_{ab} = X_a^{-1} \otimes X_a^{-1} \triangleright B_{ab}$$



Classical phase space

- Parallel transport:

$$b_{ab} = -\hat{X}_{ab} \otimes \hat{X}_{ab} \triangleright b_{ba} = -\hat{X}_{ab} b_{ba} \hat{X}_{ba}$$

- Closure:

$$\sum_{b \neq a} b_{ab}^{IJ} = 0 \ , \ \forall a$$

- Simplicity:

$$b_{ab}^{0i} = 0 \ , \ \forall i$$

- Non-degeneracy: The 3d and 4d geometries are non-degenerate;
- Flatness:

$$X_{ab} X_{bc} X_{ca} = 1 \ , \forall (abc) \Leftrightarrow \exists X_a \in \mathrm{SL}(2, \mathbb{C}) \ \text{s.t.} \ X_{ab} = X_a^{-1} X_b$$

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The quantum 4-simplex

Engle, Livine, R.P., Rovelli '08, Freidel and Krasnov '08

Group elements are quantized as multiplication operators and bivectors according to the following identification to left invariant vector fields:

$$J_t \rightsquigarrow \star b_t + \frac{s}{\gamma} b_t \quad \Longleftrightarrow \quad b_t \rightsquigarrow \frac{\gamma^2}{s\gamma^2 - 1}\left(\star J_t - \frac{s}{\gamma} J_t\right)$$

mimics the Immirzi parameter in the continuum theory, but in principle any choice of identification is possible

The quantum 4-simplex

Let us look at the constraints one by one :

First the Flatness condition:

$$A(X_{ab}) = \int \prod_a dX_a \, \delta(X_a X_{ab} X_b^{-1})$$

this defines a function of ten copies of the group

To deal with the simplicity constraints, write the vertex amplitude in the dual representation:

$$\begin{aligned} A(j_{ab}, m_{ab}) &= \int \prod_{ab} dX_{ab} \, A(X_{ab}) \, D^{\chi_{ab}}_{j_{ab} m_{ab} \, j_{ba} m_{ba}}(X_{ab}) \\ &= \int \prod_a dX_a \, \langle j_{ab} m_{ab}; \chi_{ab} | \, X_a^{-1} X_b \, | \chi_{ab}; j_{ba} m_{ba} \rangle \end{aligned}$$

The quantum 4-simplex

$$L^2(G) = \bigoplus_\chi \bar{\mathcal{H}}_\chi \otimes \mathcal{H}_\chi$$

Euclidean

$$\mathcal{H}_{(j^+,j^-)}$$

$$\mathcal{H}_{(j^+,j^-)} = \bigoplus_{j=|j^+-j^-|}^{j=j^++j^-} \mathcal{H}_j$$

$$|(j^+,j^-);j,m\rangle$$

Lorentzian

$$\mathcal{H}_{(k,p)}$$

$$\mathcal{H}_{(k,p)} = \bigoplus_{j\geq|k|} \mathcal{H}_j$$

$$|(k,p);j,m\rangle$$

$$|\chi;j,m\rangle$$

$$K^i = J^{0i}\ ,\ L^i = \frac{1}{2}\epsilon^{0i}{}_{jk}J^{jk}$$

The quantum 4-simplex

Then the simplicity constraints: this is where the different models may differ one from each other

$$b_t^{0i} = 0 \Rightarrow L_t^i - \frac{s}{\gamma} K_t^i = 0$$

A number of ways to impose these constraints have been proposed.

They will impose some restrictions on the representations labeling the data on the boundary of the 4-simplex

The quantum 4-simplex

$$\vec{C}_t := \vec{L}_t - \frac{s}{\gamma}\vec{K}_t$$

Master constraint:

$$j^+ = \frac{1+\gamma}{|1-\gamma|}\, j^- \text{ and } j = \begin{cases} j^+ + j^- & \gamma < 1 \\ |j^+ - j^-| & \gamma > 1 \end{cases}$$

$$M_t = \sum \delta_{ij} C_t^i C_t^j$$

ordering

$$p = \gamma k \text{ and } j = |k|$$

Matrix elements:

Ding and Rovelli '09

$$\text{find } \mathcal{H} \text{ s.t. } \forall \psi, \phi \in \mathcal{H} \;\; \langle \psi | \vec{C}_t | \phi \rangle = 0$$

The quantum 4-simplex

$$\vec{C}_t := \vec{L}_t - \frac{s}{\gamma}\vec{K}_t$$

Expectation values:

Freidel and Krasnov ’08

$$\langle \vec{C}_t \rangle = 0$$

does not specify a Hilbert space!!

Expectation values + minimization of uncertainties

Conrady and Hnybida ’10 and talk by Conrady

The quantum 4-simplex

Closure and parallel transport:

Version I:

closure is automatically imposed by gauge invariance (remember Barbieri's quantum tetrahedron)

B_{ab} are quantized as left invariant vector fields and B_{ba} as right invariant vector fields on that copy of the group.

Version II:

Both constraints are recovered as critical point equations in the semiclassical analysis for the 4-simplex, and can be left free for the moment.

Conrady and Freidel '08
Barrett, Dowdall, Fairbairn, Gomes and Hellmann '09
Barrett, Dowdall, Fairbairn, Hellmann and R.P. '09
talk by Hellmann

The quantum 4-simplex

Let us summarize:

$$A(X_{ab}) = \int \prod_a dX_a \, \delta(X_a X_{ab} X_b^{-1})$$

$$\Big\downarrow \text{Peter-Weyl}$$

$$\begin{aligned} A(j_{ab}, m_{ab}) &= \int \prod_{ab} dX_{ab} \, A(X_{ab}) \, D^{\chi_{ab}}_{j_{ab}m_{ab}\, j_{ba}m_{ba}}(X_{ab}) \\ &= \int \prod_a dX_a \, \langle j_{ab} m_{ab}; \chi_{ab} | \, X_a^{-1} X_b \, | \chi_{ab}; j_{ba} m_{ba} \rangle \end{aligned}$$

$$\Big\downarrow \text{simplicity constraints}$$

$$A(j_{ab}, m_{ab}) = \int \prod_a dX_a \, \langle j_{ab} m_{ab}; \chi_{ab}(j_{ab}) | \, X_a^{-1} X_b \, | \chi_{ab}(j_{ab}); j_{ab} m_{ba} \rangle$$

It defines an amplitude for a spin-net on the boundary   see talk by Rovelli

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Boundary states

For the moment the boundary state is a spin net with support on the dual graph to the boundary to a 4-simplex.

general graphs: Kaminski, Kisielowski and Lewandowski '09 and talk by Kaminski

Alternatively one can use coherent states to describe the boundary geometry.

Coherent tetrahedron: Livine and Speziale '07 and '08

Coherent states for any graph:

Immirzi '96; Freidel and Speziale '09; Bianchi, Magliaro and Perini '10;

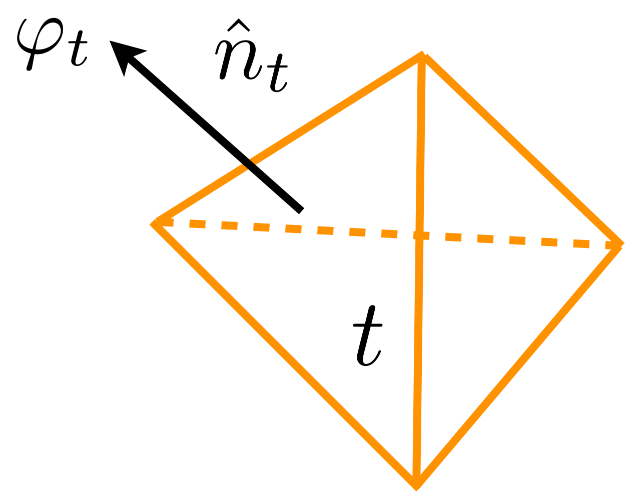
Barrett, Dowdall, Fairbairn, Gomes and Hellmann '09; Conrady and Freidel '09;

talk by Perini

Boundary states

The coherent tetrahedron:

$$\boxed{|\tau\rangle_{j_t,\hat{n}_t,\varphi_t} = \int_{\mathrm{SU}(2)} dg\, g \cdot \left(\otimes_{t\subset\tau} |j_t, \hat{n}_t, \varphi_t\rangle\right)}$$



$$|j,\hat{n},\varphi\rangle := g(\hat{n},\varphi) \triangleright |j,+j\rangle = e^{i\varphi j} g(\hat{n}) \triangleright |j,+j\rangle$$

Boosted tetrahedra:

$$|j_t,\hat{n}_t,\varphi_t\rangle \longrightarrow |\chi_t; j_t,\hat{n}_t,\varphi_t\rangle$$

Vertex amplitude:

$$A(j_{ab},\hat{n}_{ab},\varphi_{ab}) = \int \prod_a dX_a \, \langle j_{ab},\hat{n}_{ab},\varphi_{ab};\chi_{ab}|\, X_a^{-1} X_b \,|\chi_{ab}; j_{ba},\hat{n}_{ba},\varphi_{ba}\rangle$$

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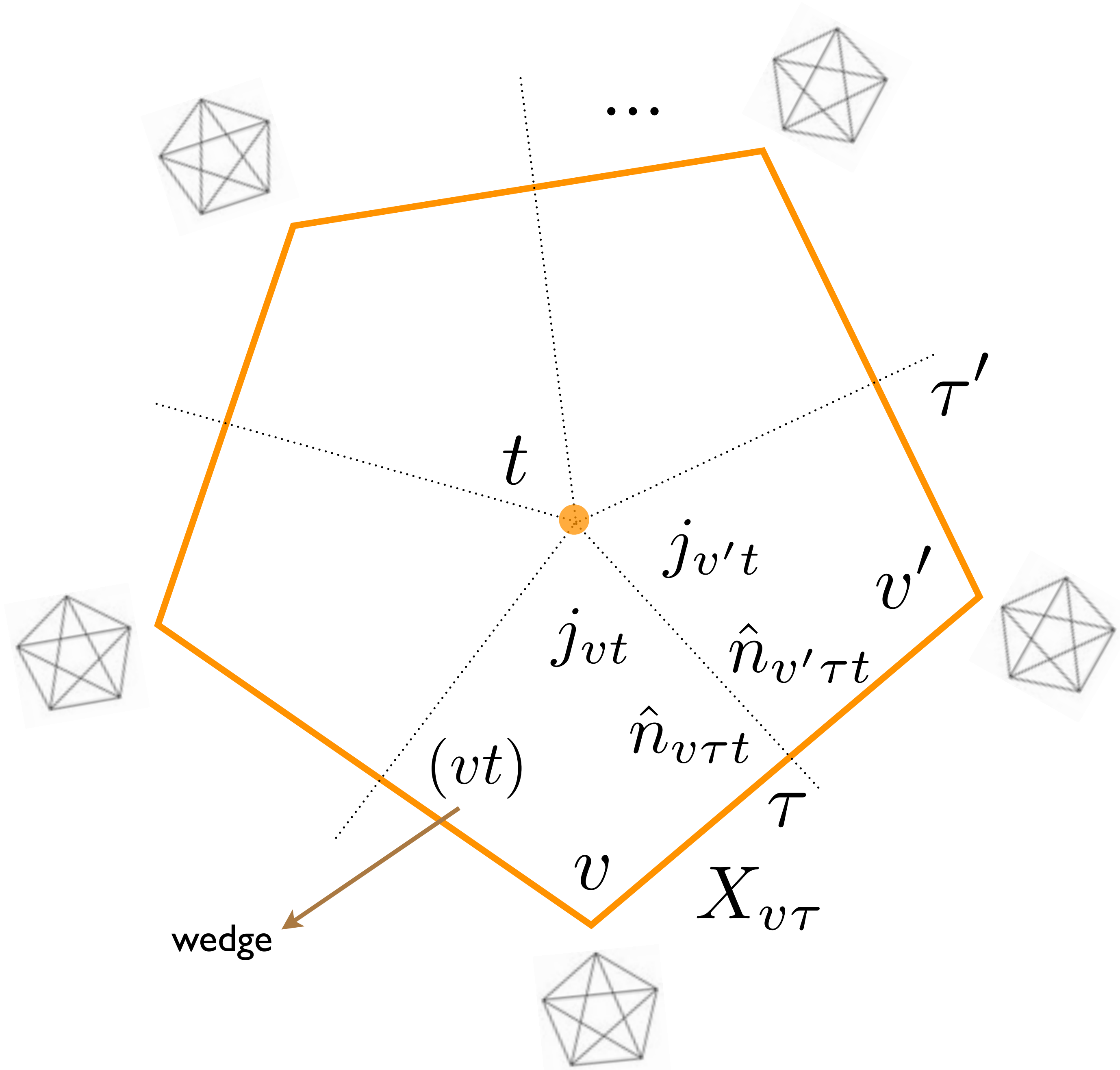
Livine and Speziale ’07
Freidel and Krasnov ’08
Conrady and Freidel ’08

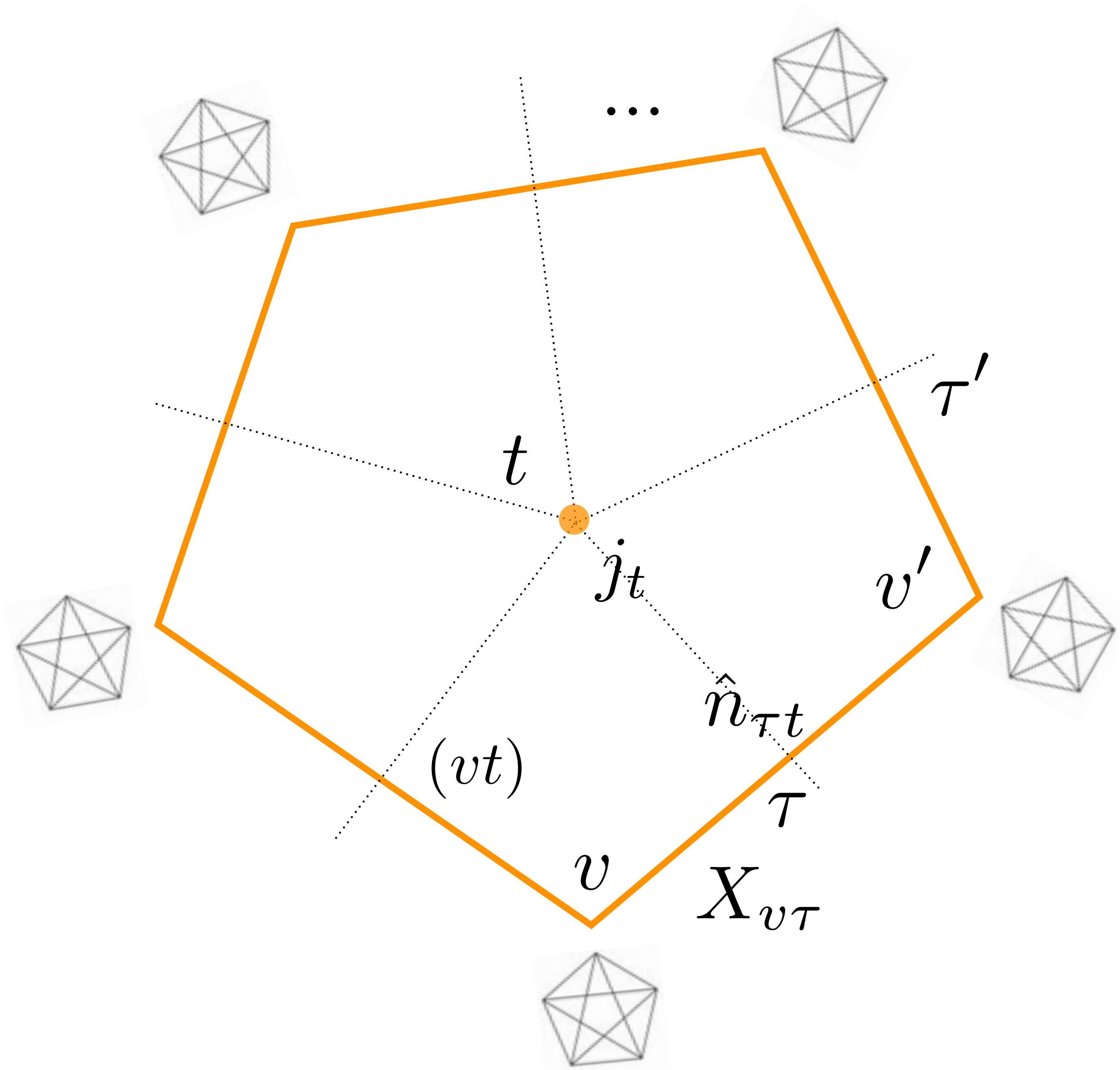
Spin foam amplitude

Σ with no boundary

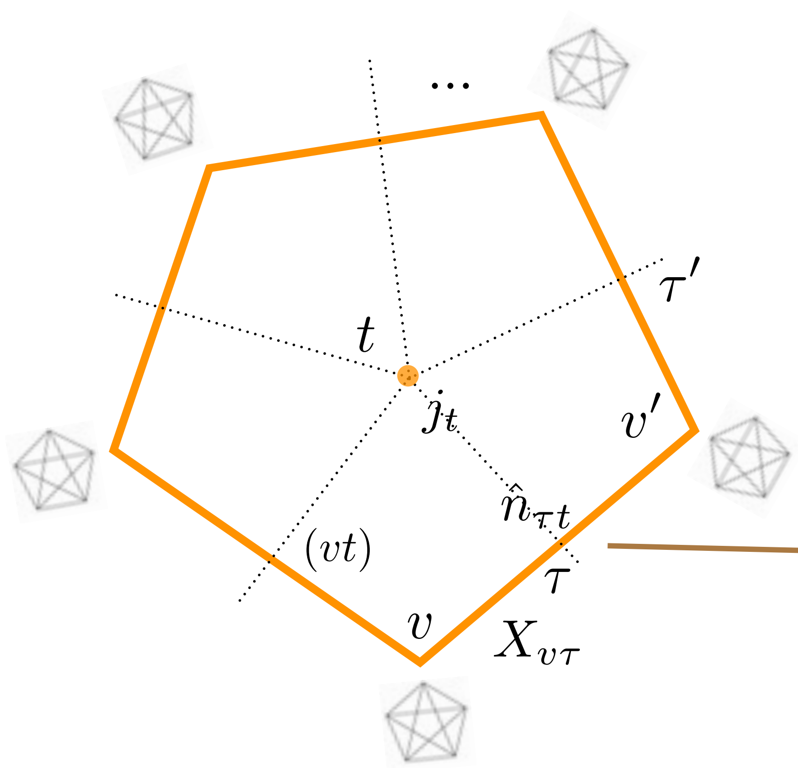


Spin foam amplitude

$\sum$ with no boundary



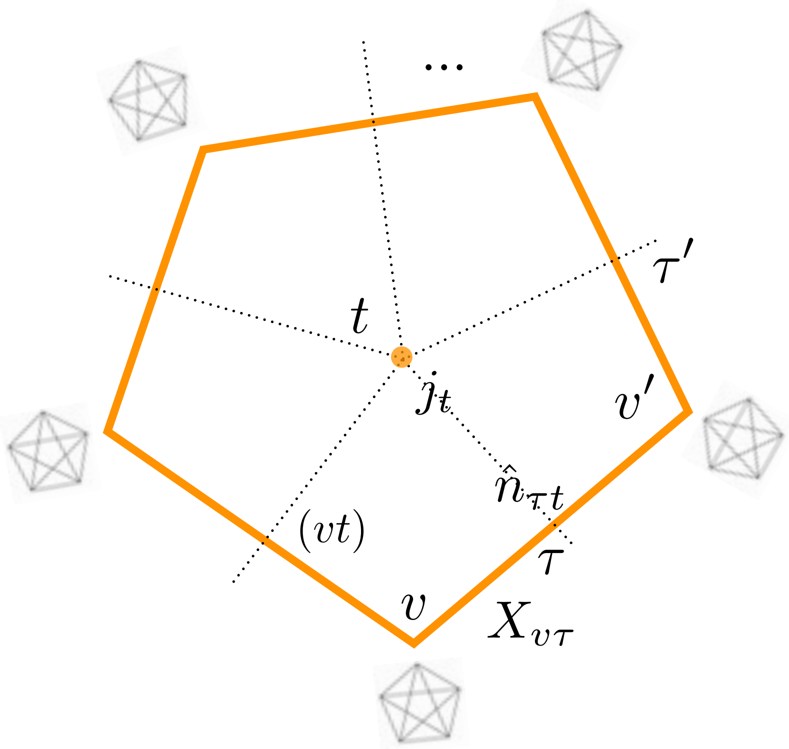
Spin foam amplitude



$$\mathbb{1}_{j_t} = d_{j_t} \int_{S^2} d^2\hat{n}_{\tau t}\, |j_t, \hat{n}_{\tau t}\rangle\langle j_t, \hat{n}_{\tau t}| = \sum_{m_{\tau t}=-j_t}^{+j_t} |j_t, m_{\tau t}\rangle\langle j_t, m_{\tau t}|$$

$$Z_\Sigma(j_t) = \int \prod_{v\tau} dX_{v\tau} \prod_{\tau t} d^2\hat{n}_{\tau t} \prod_t \prod_{v\supset t} \langle j_t, \hat{n}_{\tau t}; \chi_t | X_{v\tau}^{-1} X_{v\tau'} | \chi_t; j_t \hat{n}_{\tau' t}\rangle$$

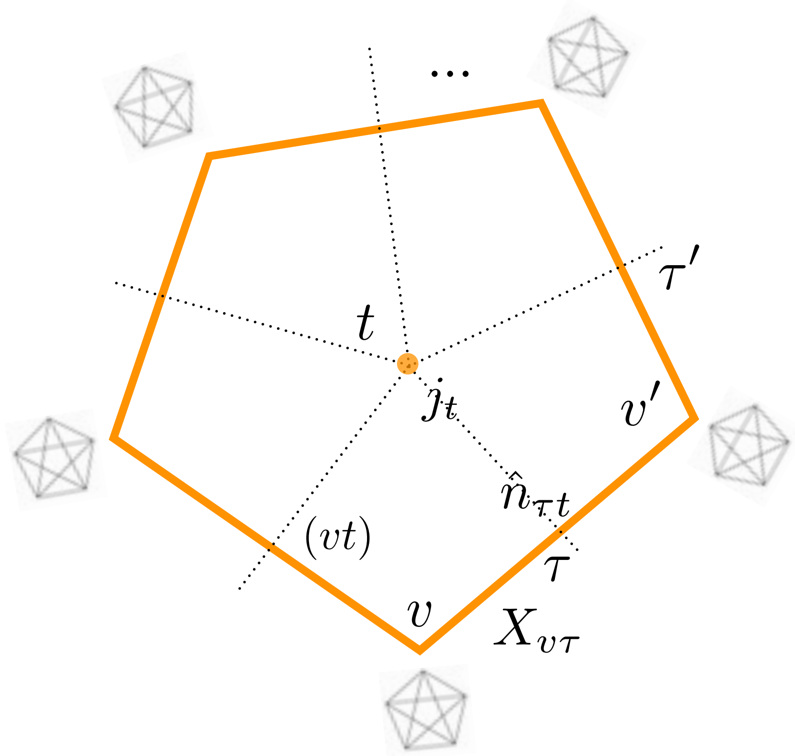
Spin foam amplitude



What about the sum over spins?

$$Z_\Sigma = \sum_{j_t} \mu(j_t) \int \prod_{v\tau} dX_{v\tau} \prod_{\tau t} d^2\hat{n}_{\tau t} \prod_t \prod_{v\supset t} \langle j_t, \hat{n}_{\tau t}; \chi_t | X_{v\tau}^{-1} X_{v\tau'} | \chi_t; j_t \hat{n}_{\tau' t} \rangle$$

Spin foam amplitude



What about the sum over spins?

talk by Kaminski

$$\mu(j_t) \sim j_t^{\alpha}$$

$$Z_\Sigma = \sum_{j_t} \mu(j_t) \int \prod_{v\tau} dX_{v\tau} \prod_{\tau t} d^2\hat{n}_{\tau t} \prod_t \prod_{v \supset t} \langle j_t, \hat{n}_{\tau t}; \chi_t | X_{v\tau}^{-1} X_{v\tau'} | \chi_t; j_t \hat{n}_{\tau' t} \rangle$$

Spin foam amplitude

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$$P_{vt} = \int \Omega_{z_{vt}} \, e^{j_t S_{vt}[X,n,z]}$$

$$Z_\Sigma = \sum_{j_t} \mu(j_t) \int \prod_{v\tau} dX_{v\tau} \prod_{\tau t} d^2\hat{n}_{\tau t} \prod_{vt} \Omega_{z_{vt}} \, e^{\sum_t j_t \sum_{v\supset t} S_{vt}}$$

inserting back the boundary

$$Z_\Sigma(\{\hat{n}_{\tau t}, \phi_t, j_t\}, t \subset \partial\Sigma) = \sum_{j_t} \mu(j_t) \int \prod_{v\tau} dX_{v\tau} \prod_{\tau t} d^2\hat{n}_{\tau t} \prod_{vt} \Omega_{z_{vt}} \, e^{\sum_t j_t \sum_{v\supset t} S_{vt}}$$

Spin foams constitute an attempt to define a path integral for gravity.

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The main assumption is a restriction to a fixed simplicial net.

Hope to define a dynamics for LQG.

Spin foam amplitude

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First, define it properly

Then, check if it has anything to do with gravity

Relation with Regge calculus: Asymptotic papers

Vector geometries : Barrett, Fairbairn and Hellmann '09

Degenerate configurations

Finally, what do we do with that?

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Observables

Area operator $\hat{A}_t^2 = \gamma^2 L_t^2$

Volume operator $\langle i|\hat{V}_\tau^2|j\rangle = \gamma^3 \langle i|\epsilon^{abc}\epsilon_{mnp} L_a^m L_b^n L_c^p|j\rangle$ Ding and Rovelli '09

Graviton propagator Bianchi, Magliaro and Perini '09 and talk by Rovelli

Conclusions

We are interested in defining and understanding a path integral for simplicial gravity

Let us summarize the steps in the construction

1) Classical phase space for a 4-simplex, in particular choice of boundary data!!

2) Quantize the 4-simplex

3) Glue simplices together

4) With the help of coherent states, exponentiate to get a path integral over classical histories

5) Do something with it!