Emergence of scalar matter from a spinfoam model

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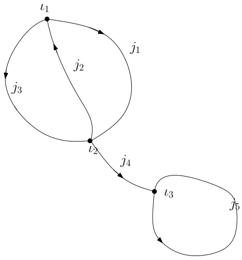
Outline

1. **Introduction and Motivation**
2. **A Model of 3D Euclidean Gravity Coupled with a Scalar Field**
3. **Emergence of Scalar Matter from the Spinfoam Model**
4. **Concluding Remark and Open Problems**

The Kinematical Framework of LQG

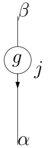
- ⋆ The internal gauge invariant Hilbert space $\mathcal{H}_G = L^2(\overline{\mathcal{A}/\mathcal{G}},\ d\mu^0)$
- ⋆ The gauge invariant spin network basis $T_{(\Gamma,\vec{\rho},\vec{\iota})}(\bar{A})$ in $\mathcal{H}_G$ is labeled by [Rovelli, Smolin, 1994; Baez 1994]

  - A finite graph Γ in the spatial manifold Σ
  - Associating an irreducible unitary representation of the gauge group to each edge e of Γ
  - Associating an intertwinor operator $\iota_v : \bigotimes_{e\in T(v)} V_{\rho_e} \mapsto \bigotimes_{e\in S(v)} V_{\rho_e}$ to each vertex v of Γ.

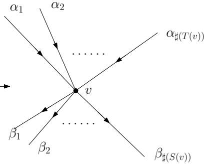


Graphic Calculus for Spin Networks

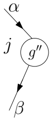
$R^j(g)^\alpha_\beta \longrightarrow$



$\iota^{\beta_1 \dots \beta_{\sharp(S(v))}}_{\alpha_1 \dots \alpha_{\sharp(T(v))}} \longrightarrow$

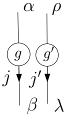


$R^j(g)^\alpha_\gamma R^j(g')^\gamma_\beta \longrightarrow$

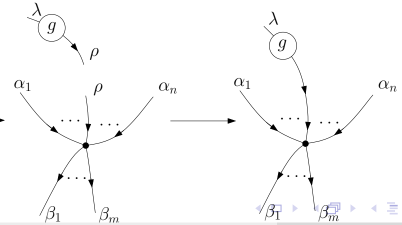


$g'' = gg'$

$R^j(g)^\alpha_\beta \otimes R^{j'}(g')^\rho_\lambda \longrightarrow$

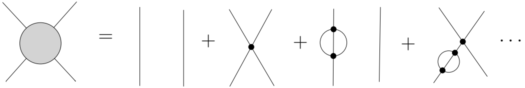


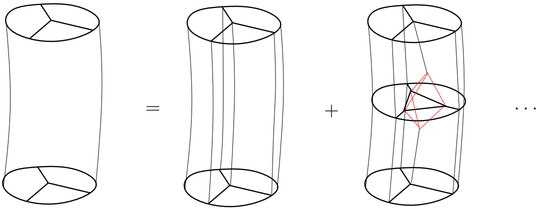
$R(g)^\lambda_\rho \iota^{\alpha_1 \dots \rho \dots \alpha_n}_{\beta_1 \dots \beta_m} \longrightarrow$



Spinfoams

- Spinfoam model as sum over histories of the evolutions of quantum states [Reisenberger, Rovelli, 1997].





Spinfoams

- Spinfoam model as a kind of lattice gravity [Ponzano, Regge, 1968]:

$$\mathcal{Z} = \int exp(\frac{\mathbf{i}}{\hbar}(Discrete\ \ Action))\mathcal{D}(Discrete\ \ Field\ \ History).$$

- Spinfoam model as a possible realization of the projector [Rovelli 1999]

$$P_{phy} = \int \mathcal{D}[N]\mathbf{e}^{-\mathbf{i}\int N\hat{C}_0}$$

  in canonical LQG.

Spinfoams

A spinfoam model is determined by

- The triangulation $\triangle$ and its dual $\triangle^*$ of spacetime manifold $\mathcal{M}$.
  The node, segment and triangle of $\triangle$ are denoted as n, s, t.
  The vertex, edge and face of $\triangle^*$ are denoted as v, e, f.
- The coloring of the 2-skeleton of $\triangle^*$:
  Each face f is colored by an irreducible unitary representation j_f of the gauge group, and each edge e is colored by an intertwinor operator ι_e.
- A weight $w(\triangle^*)$ is associated to each triangulation, and the amplitudes $A_v(j_f, \iota_e), A_e(j_f, \iota_e)$ and $A_f(j_f, \iota_e)$ are associated to each vertex, edge and face of $\triangle^*$.

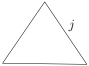
The partition functional reads

$$\mathcal{Z} = \sum_{\triangle^*} \mathbf{w}(\triangle^*) \sum_{j_f, \iota_e} \prod_{f \in \triangle^*} A_f(j_f, \iota_e) \prod_{e \in \triangle^*} A_e(j_f, \iota_e) \prod_{v \in \triangle^*} A_v(j_f, \iota_e).$$

Euclidean Spinfoams

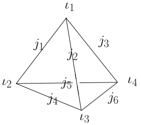
- The 2D spinfoam model [Livine, Perez, Rovelli, 2003]

$$\begin{aligned}\mathcal{Z} &= \sum_{\triangle} w(\triangle) \sum_{j} \prod_{f\in\triangle^*} \triangle_j \prod_{e\in\triangle^*} \frac{1}{\triangle_j} \prod_{v\in\triangle^*} \\ &= \sum_{\triangle} w(\triangle) \sum_{j} \triangle_j^{\chi(\mathcal{M})}.\end{aligned}$$



- The 3D spinfoam models

$$\mathcal{Z} = \sum_{\triangle} w(\triangle) \sum_{j_f} \prod_{f\in\triangle^*} \triangle_{j_f} \prod_{v\in\triangle^*}$$

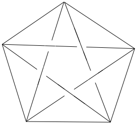


$\rightarrow$ *Regge Calculus*.

Euclidean Spinfoams

- The 4D spinfoam models [Barrett, Dowdall, Fairbairn, Gomes, Hellmann, 2009]

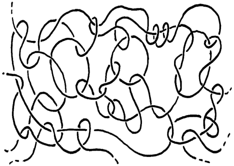
$$\mathcal{Z} = \sum_{\triangle} w(\triangle) \sum_{(j_f,\iota_e)} \prod_{f\in\triangle^*} \triangle_{j_f} \prod_{e} A_e \prod_{v\in\triangle^*}$$



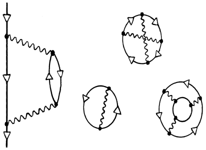
$\rightarrow$ *Regge Calculus*.

We can encode the dynamics of quantum geometry into the couplings of representations of the gauge group.

Matter Couplings in LQG



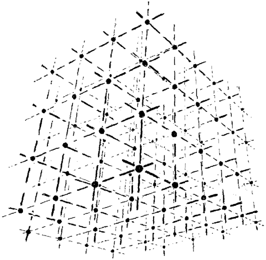
$\bigotimes$

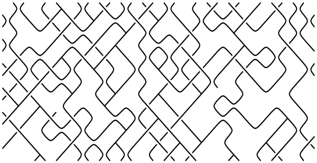


?

Planck Scale Quantum Geometry

Standard Model





Stringnet Condensation

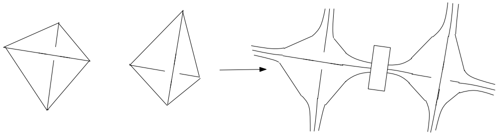
Recovery of discrete spacetime

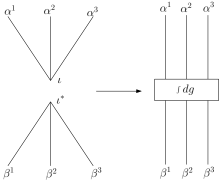
According to the formalism of spinfoam model,

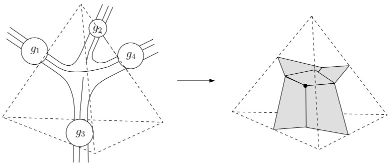
- the couplings of representations are the fundamental structure that lies behind the quantum dynamics of geometry.
- The n-simplex corresponding to the vertex amplitude is the building block in the recovery of the n-dimensional simplicial manifold from such couplings.

Recovery of discrete spacetime

- Gluing 3-cells:







- Can matter coupled to geometry be also recovered from the couplings of representations?

The classical theory

We consider the following system defined on a 3D manifold $\mathcal{M}$.

$$S[g,\phi]=\int_{\mathcal{M}}\sqrt{|\phi|}\sqrt{|g|}R[g],$$

where g_{ab} is the metric of Riemann signature and ϕ is the scalar field. This action is of physical interest [YM2001]:

- Its dynamics is "conformally" equivalent to that of

$$S[\tilde{g},\tilde{\phi}]=\int_{\mathcal{M}}\sqrt{|\tilde{g}|}(\tilde{R}[\tilde{g}]-\tilde{g}^{ab}\nabla_a\tilde{\phi}\nabla_b\tilde{\phi}).$$

- It describes the geometry of 4D static spacetime.

The classical theory

- The first order formalism in terms of (e, ω, ϕ):

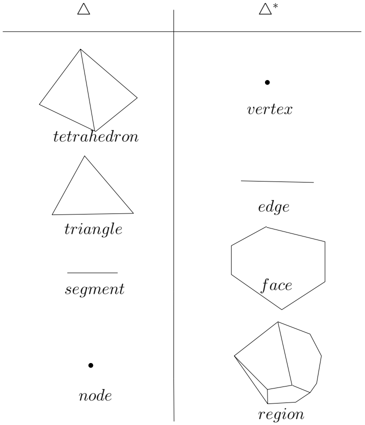
$$S[e, \omega, \phi] = \int_{\mathcal{M}} \sqrt{|\phi|} \epsilon_{IJK} e^I \wedge \Omega^{JK}(\omega)$$

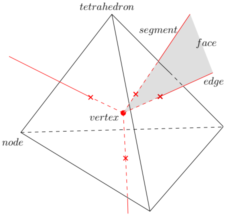
- The quantum physics should be determined by the partition functional

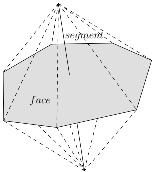
$$\mathcal{Z} = \int \mathcal{D}[\phi]\mathcal{D}[e]\mathcal{D}[\omega] \mathbf{e}^{-i \int_{\mathcal{M}} \sqrt{|\phi|} \epsilon_{IJK} e^I \wedge \Omega^{JK}}.$$

The triangulation of spacetime manifold

Introduce the triangulation $\triangle$ and its dual $\triangle^*$;
n, s, $t \in \triangle$ and v, e, $f \in \triangle^*$.





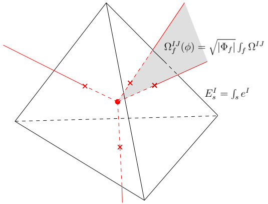


The regularization

- Smear the soldering form and the curvature term as

$$
\begin{aligned}
E_s^I &= \int_{s\in\triangle} e^I, \\
\Omega_f^{IJ}(\phi) &= \int_{f\in\triangle^*} \sqrt{|\phi|}\Omega^{IJ}.
\end{aligned}
$$

- The triangulation is assumed to be fine enough, so that on each face $\phi(x)$ can be treated as a constant Φ_f. Then $\Omega_f^{IJ}(\phi) = \sqrt{|\Phi_f|}\Omega_f^{IJ}$.

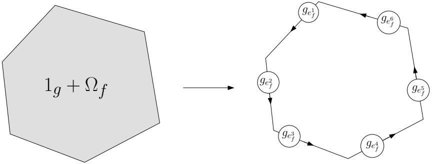


The regularization

- Also, the smeared curvature can be approximated by

$$\Omega_f = U_f - \mathbf{1}$$

where U_f is the holonomy around the face f,
$U_f = g_{e_f^1} \circ g_{e_f^2} \circ \ldots \circ g_{e_f^n}$



The spinfoam model

- By the regularization, the degrees of freedom left are (E^I_s, g_e, Φ_f).
- The discrete action becomes

$$S_\triangle = \sum_s tr(E_s(\sqrt{|\Phi_f|}\Omega_f))$$

- The partition functional now reads

$$\begin{aligned}\mathcal{Z} &= \sum_\triangle \mathbf{w}(\triangle) \int \prod_s dE^I_s \prod_e dg_e \prod_f d\Phi_f \, exp(-i \sum_s tr(E_s \mathbf{e}^{(\sqrt{|\Phi_f|}\Omega_f)})) \\ &= \sum_\triangle \mathbf{w}(\triangle) \int \prod_f d\Phi_f \prod_e dg_e \prod_f \delta(\mathbf{e}^{(\sqrt{|\Phi_f|}\Omega_f)})\end{aligned}$$

Discretization of the Scalar Field

- We assume that ϕ only take discrete values.
  Then the action can be rewritten as

$$S[e,\omega,\phi]=\int_{\mathcal{M}}\mathcal{C}\sqrt{\frac{|\phi|}{\mathcal{C}^2}}\epsilon_{IJK}e^I\wedge\Omega^{JK}(\omega)$$

  where $\sqrt{|\phi|/\mathcal{C}^2}$ take values in $\mathbb{Z}^+\cup\{0\}$, denoted as N, and $\mathcal{C}$ is a positive small constant.

- The discrete action becomes

$$S_\triangle=\sum_s \mathcal{C}tr(E_s(N_f\Omega_f))$$

Discretization of the Scalar Field

- The partition functional now reads

$$\mathcal{Z}^N_\triangle = \int \prod_s dE^I_s \prod_e dg_e \sum_{N_f} \rho(\overrightarrow{N}) \mathbf{e}^{-i\sum_s \mathcal{C}tr(E_s(N_f\Omega_f))},$$

where $\overrightarrow{N}$ denotes the new coloring for faces $\{N_{f_1}, N_{f_2}, \ldots\ldots\}$, and $\rho(\overrightarrow{N})$ is the weight that comes from the measure $d\Phi_f$.

Integrating out E_s, we have

$$\mathcal{Z}^N_\triangle = \int \prod_e dg_e \sum_{N_f} \rho(\overrightarrow{N}) \prod_f \delta(\mathbf{e}^{(N_f\Omega_f)})$$

$$\begin{aligned} \mathbf{e}^{(N_f\Omega_f)} &= \overbrace{\mathbf{e}^{\Omega_f}\mathbf{e}^{\Omega_f}\ldots\mathbf{e}^{\Omega_f}}^{N_f} \\ &= \overbrace{U_f \circ \ldots \circ U_f}^{N_f} \end{aligned}$$

Discretization of the Scalar Field

Hence we obtain

$$\mathcal{Z}^N_\triangle = \int \prod_e dg_e \sum_{N_f} \rho(\overrightarrow{N}) \prod_f \delta(\overbrace{U_f \circ \ldots \circ U_f}^{N_f})$$

$$= \int \prod_e dg_e \sum_{N_f} \rho(\overrightarrow{N}) \prod_f \sum_{j_f} \{\triangle_{j_f} \chi^{j_f} [\overbrace{(g_{e_f^1} \circ \ldots \circ g_{e_f^n}) \circ \ldots \circ (g_{e_f^1} \circ \ldots \circ g_{e_f^n})}^{N_f}$$

$$= \sum_{N_f} \sum_{j_f} \rho(\overrightarrow{N}) \int \prod_e dg_e \prod_f \{\triangle_{j_f} \delta_{\alpha_1}^{\alpha_{N_f n+1}} (R^{j_f}(g_{e_f^1})^{\alpha_1}_{\alpha_2} R^{j_f}(g_{e_f^2})^{\alpha_2}_{\alpha_3} \ldots R^{j_f}(g_{e_f^n})^{\alpha_n}_{\alpha_{n+1}}$$

$$(R^{j_f}(g_{e_f^1})^{\alpha_{n+1}}_{\alpha_{n+2}} \ldots R^{j_f}(g_{e_f^n})^{\alpha_{2n}}_{\alpha_{2n+1}}) \ldots (R^{j_f}(g_{e_f^1})^{\alpha_{(N_f-1)n+1}}_{\alpha_{(N_f-1)n+2}} \ldots R^{j_f}(g_{e_f^n})^{\alpha_{N_f n}}_{\alpha_{N_f n+1}})$$

where $\triangle_{j_f}$ is the dimension of the representation j_f, and $\chi^{j_f}(g)$ is the character of the group element g in the representation j_f.

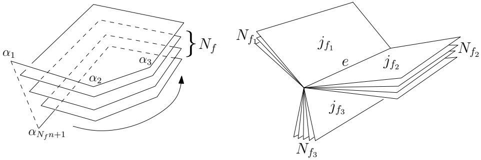
Discretization of the Scalar Field

For each edge, we have an integration of the following form

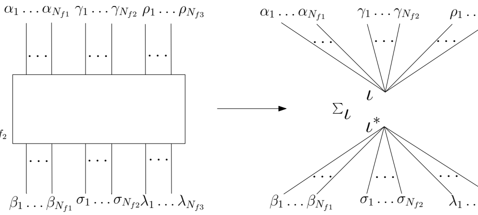
$$
\begin{aligned}
& \int dg_e \bigotimes_{i=1}^{N_{f_1}} R^{j_{f_1}}(g_e)^{\alpha_i}_{\beta_i} \bigotimes_{j=1}^{N_{f_2}} R^{j_{f_2}}(g_e)^{\gamma_j}_{\sigma_j} \bigotimes_{k=1}^{N_{f_3}} R^{j_{f_3}}(g_e)^{\rho_k}_{\lambda_k} \\
= & \sum_{\iota} \iota^{\alpha_1 \ldots \alpha_{N_{f_1}} \gamma_1 \ldots \gamma_{N_{f_2}} \rho_1 \ldots \rho_{N_{f_3}}} \iota^*_{\beta_1 \ldots \beta_{N_{f_1}} \sigma_1 \ldots \sigma_{N_{f_2}} \lambda_1 \ldots \lambda_{N_{f_3}}} \\
= & P_{inv}^{\otimes_{N_{f_1}} j_{f_1} \otimes_{N_{f_2}} j_{f_2} \otimes_{N_{f_3}} j_{f_3}} .
\end{aligned}
$$

Graphic Representation

For each face f we associate N_f copies of "virtual" faces which are all colored by the same representation j_f.



(a) (b)



The Vertex

According to the structure of $\triangle^*$, for each vertex we have integral

$$\int dg_{e^1}dg_{e^2}dg_{e^3}dg_{e^4}$$

$$\Big(\bigotimes_{i=1}^{N_{f_1}} R^{j_{f_1}}(g_{e^1})^{\alpha_i^1}_{\lambda_i^1} \bigotimes_{j=1}^{N_{f_2}} R^{j_{f_2}}(g_{e^1})^{\alpha_j^2}_{\lambda_j^2} \bigotimes_{k=1}^{N_{f_3}} R^{j_{f_3}}(g_{e^1})^{\alpha_k^3}_{\lambda_k^3}$$

$$\bigotimes_{l=1}^{N_{f_1}} R^{j_{f_1}}(g_{e^2})^{\lambda_l^1}_{\beta_l^1} \bigotimes_{m=1}^{N_{f_4}} R^{j_{f_4}}(g_{e^2})^{\alpha_m^4}_{\lambda_m^4} \bigotimes_{n=1}^{N_{f_5}} R^{j_{f_5}}(g_{e^2})^{\alpha_n^5}_{\lambda_n^5}$$

$$\bigotimes_{p=1}^{N_{f_5}} R^{j_{f_5}}(g_{e^3})^{\lambda_p^5}_{\beta_p^5} \bigotimes_{q=1}^{N_{f_2}} R^{j_{f_2}}(g_{e^3})^{\lambda_q^2}_{\beta_q^2} \bigotimes_{r=1}^{N_{f_6}} R^{j_{f_4}}(g_{e^3})^{\alpha_m^6}_{\lambda_m^6}$$

$$\bigotimes_{s=1}^{N_{f_6}} R^{j_{f_6}}(g_{e^4})^{\lambda_s^6}_{\beta_s^6} \bigotimes_{t=1}^{N_{f_4}} R^{j_{f_4}}(g_{e^4})^{\lambda_t^4}_{\beta_t^4} \bigotimes_{w=1}^{N_{f_3}} R^{j_{f_3}}(g_{e^4})^{\lambda_w^3}_{\beta_w^3}\Big).$$

The Vertex

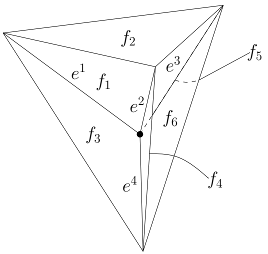
It equals to

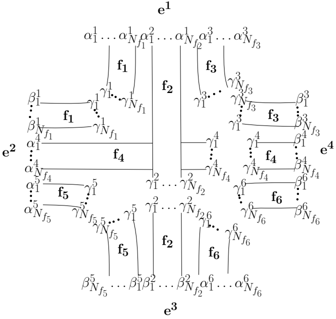
$$\sum_{\iota_{(1)}\iota_{(2)}\iota_{(3)}\iota_{(4)}} \Big(\iota_{(1)}^{\alpha^1_1\ldots\alpha^1_{N_{f1}}\alpha^2_1\ldots\alpha^2_{N_{f2}}\alpha^3_1\ldots\alpha^3_{N_{f3}}}\, \iota_{(2)}^{\alpha^4_1\ldots\alpha^4_{N_{f4}}\alpha^5_1\ldots\alpha^5_{N_{f5}}}{}_{\beta^1_1\ldots\beta^1_{N_{f1}}}$$

$$\iota^{*\alpha^6_1\ldots\alpha^6_{N_{f6}}}_{(3)}{}_{\beta^5_1\ldots\beta^5_{N_{f5}}\beta^2_1\ldots\beta^2_{N_{f2}}}\, \iota^{*}_{(4)\beta^6_1\ldots\beta^6_{N_{f6}}\beta^4_1\ldots\beta^4_{N_{f4}}\beta^3_1\ldots\beta^3_{N_{f3}}}\, \mathcal{A}_v(\iota_{(1)},\iota_{(2)},\iota_{(3)},\iota_{(4)})\Big)$$

The Vertex

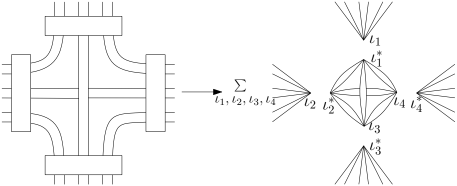
- How the indexes contract at the vertex can be illustrated as





The Vertex

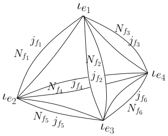
- The vertex integral gives rise to



Emergence of matter field in the spinfoam model

- The resulted amplitude A_v of the vertex reads

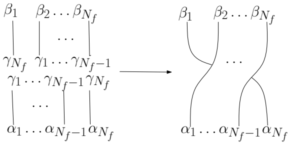
$$\mathcal{A}_v(\iota_{(1)}, \iota_{(2)}, \iota_{(3)}, \iota_{(4)}) =$$



$$= \iota^*_{(1)\lambda^1_1 \ldots \lambda^1_{N_{f1}} \lambda^2_1 \ldots \lambda^2_{N_{f2}} \lambda^3_1 \ldots \lambda^3_{N_{f3}}} \, \iota_{(2)}{}^{*\lambda^1_1 \ldots \lambda^1_{N_{f1}}}{}_{\lambda^4_1 \ldots \lambda^4_{N_{f4}} \lambda^5_1 \ldots \lambda^5_{N_{f5}}}$$
$$\iota_{(3)}{}^{\lambda^5_1 \ldots \lambda^5_{N_{f5}} \lambda^2_1 \ldots \lambda^2_{N_{f2}} \lambda^6_1 \ldots \lambda^6_{N_{f6}}} \, \iota_{(4)}{}^{\lambda^3_1 \ldots \lambda^3_{N_{f3}} \lambda^4_1 \ldots \lambda^4_{N_{f4}}}{}_{\lambda^6_1 \ldots \lambda^6_{N_{f6}}}$$

Emergence of matter field in the spinfoam model

- A crossing edge only causes a permutation of the indexes of the same brunch of faces and hence will not affect the calculation of amplitudes.

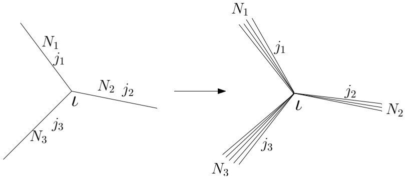


- Thus we arrive at the final form of the partition functional

$$\mathcal{Z}^N = \sum_{\triangle} \mathbf{w}(\triangle) \sum_{(j_f, N_f, \iota_e)} \rho(\overrightarrow{N}) \prod_{f \in \triangle^*} \triangle_{j_f} \prod_{v \in \triangle^*} \mathcal{A}_v(j_f, N_f, \iota_e).$$

Kinematics

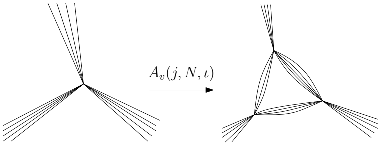
- The boundary states that encode the physical information on a hypersurface Σ which intersects with the spinfoams can be described as the "generalized" $SU(2)$ spin network states $\tilde{T}_{\gamma,N,j,\iota}$.
- These $\tilde{T}_{\gamma,N,j,\iota}$ can be casted into the pure $SU(2)$ spin network states, $T_{\gamma,N,j,\iota} \in \mathcal{H}_{kin}$, of 3D quantum gravity.



- Thus the information of the scalar field on Σ are totally encoded into the standard $SU(2)$ spin network states $T_{\gamma,N,j,\iota}$ or the states of quantum geometry.
  Hence the kinematics of this model has been casted into that of the 3D quantum geometry.

Dynamics

- The evolution of $T_{\gamma,N,j,\iota}$ is generated by the vertices and the transition amplitudes $\mathcal{A}_v(j_f, N_f, \iota_e)$.



- The new dynamics will reduce to that of Ponzano-Regge model, as one expects, if ϕ become a constant (or $N_f = 1$) for all faces. It is a natural extension of the dynamics of pure gravity by the new transition amplitudes coming also from the couplings of $SU(2)$ representations.
- As a generalization of Penrose's idea, both the kinematics and the dynamics of the scalar matter coupled with gravity are built into these couplings of representations.

Concluding Remark

- In the full quantum situation, the physics of this system can be cast into the modified dynamics of the pure quantum geometry, while in the low energy situation, the physics manifests itself as the dynamics of the geometry coupled with the scalar matter.
  Thus the scalar field can be viewed as a phenomenon emerged from the microscopic system of quantum spin foams.
- It is not difficult to generalize our result [Xu, YM, PRD80 (2009) 104024] to Lorentzian signature, provided there is such a valid spinfoam model for pure gravity.
- The distinct property of this model gives rise to a radical observation on the issue of unification of geometry and matter.
  Being the convergent point of distinct approaches to quantum gravity, spin foams might also be the convergent point of geometry and matter.

Open Problems

- Can we set up some relation between 4D Lorentzian spinfoam model (Conrady's talk) and our model by imposing certain symmetry condition at quantum level (Rovelli's talk) to represent static spacetimes?
- How to understand at quantum level the "conformal" relation between our model and the model of minimally coupled Klein-Gorden field?
- What is the relation between our spin foam model and the canonical loop quantization of the same classical model?
- Can matter fields with non-zero spins be somehow emerged from spinfoam models?
  (Comparing to the work in canonical approach: e.g., Smolin, Wan, 2007)

!Thanks!