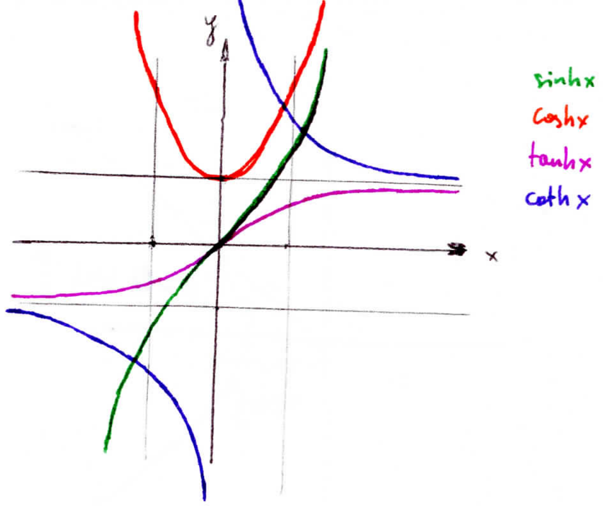
$\sinh x : \mathbb{R} \to \mathbb{R}$ i jest bijekcją

$\cosh x : \mathbb{R} \to [1;\infty[$ i jest surjekcją

$\tanh x : \mathbb{R} \to]-1;1[$ i jest bijekcją

$\coth x : \mathbb{R} \to \cancel{\mathbb{R}\setminus\{0\}}$ i jest bijekcją

$]-\infty;-1[\cup]1;\infty[$



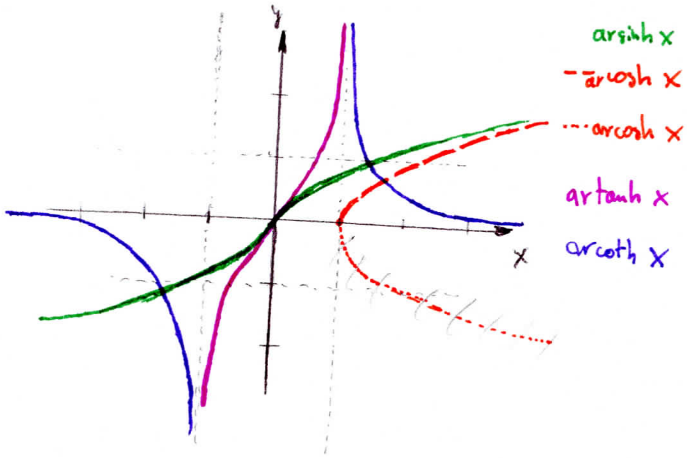
$\operatorname{arsinh} x : \mathbb{R} \to \mathbb{R}$

$\operatorname{arcosh} x$ $\begin{cases} \cancel{\operatorname{arcosh}_1 x \quad [1;\infty[\to [0;-\infty[} \\ \operatorname{arcosh}_2 x \quad [1;\infty[\to [0;\infty[\end{cases}$

$\operatorname{artanh} x :]-1;1[\to \mathbb{R}$

$\operatorname{arcoth} x :]-\infty;-1[\cup]1;\infty[\to \mathbb{R}$

$\sinh(a+b) =$



$$y = \sinh x = \frac{e^x - e^{-x}}{2} \qquad e^x = t, \quad t>0$$

$$2y = t - \frac{1}{t}$$

$$t^2 - 2yt - 1 = 0$$

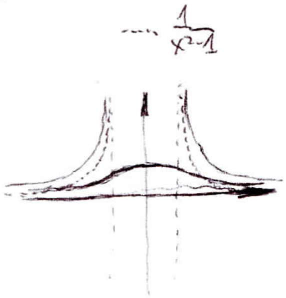
$$t = \frac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1} > 0$$

$$t = y + \sqrt{y^2+1}$$

$$e^x = y + \sqrt{y^2+1}$$

$$x = \ln\left(y + \sqrt{y^2+1}\right)$$

$$x = \operatorname{arsinh} y = \ln\left(y + \sqrt{y^2+1}\right)$$



$$\operatorname{arsinh} x = \ln\left(y + \sqrt{y^2+1}\right)$$

$$\operatorname{arcosh} x = \ln\left(y + \sqrt{y^2-1}\right)$$

~~ln(y ± √y²)~~

$$\operatorname{artanh} x = \ln\left(\sqrt{\frac{1+y}{1-y}}\right)$$

$$\operatorname{arcoth} x = \ln\left(\sqrt{\frac{1+y}{1-y}}\right)$$

$$\cancel{\left[(\operatorname{arsinh} x)'\right]' = \frac{1}{y+\sqrt{y^2+1}} \cdot \left(1 + \frac{2y}{2\sqrt{y^2+1}}\right) = \frac{\sqrt{y^2+1}+y}{\sqrt{y^2+1}\left(y+\sqrt{y^2+1}\right)} = \frac{1}{\sqrt{y^2+1}}}$$

$$\cancel{(\operatorname{arsinh} x)' = \sqrt{y^2+1} = \sqrt{\sinh^2 x + 1} = \cosh x}$$

$$(\operatorname{arsinh} x)' = \frac{1}{\sinh'(\operatorname{arsinh} x)} = \frac{1}{\cosh(\operatorname{arsinh} x)} = \frac{1}{\sqrt{1+\sinh^2(\operatorname{arsinh} x)}} = \frac{1}{\sqrt{1+x^2}}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$(\operatorname{arcosh} x)' = \frac{1}{\cosh' y} = \frac{1}{\sinh y} = \frac{1}{\sqrt{\cosh^2 y - 1}} = \frac{1}{\sqrt{x^2-1}}$$

$$\tanh' y = \frac{\sinh^2 y - \cosh^2 y}{\cosh^2 y} = \frac{-1}{\cosh^2 y} = \tanh^2 y - 1$$

$$\coth' y = \frac{\cosh^2 y - \sinh^2 y}{\sinh^2 y} = \frac{1}{\sinh^2 y} = \coth^2 y - 1$$

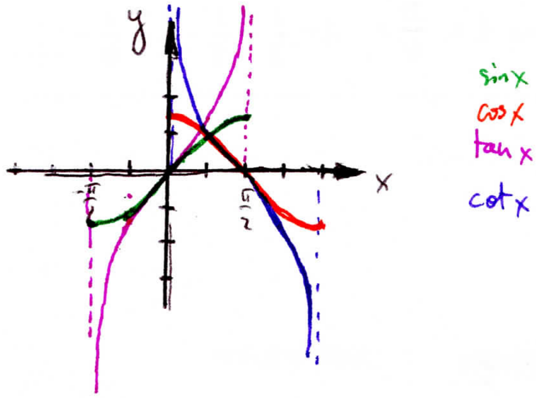
$$(\operatorname{artanh} x)' = \frac{1}{\tanh' y} = \frac{1}{\tanh^2 y - 1} = \frac{1}{x^2-1}$$

$$(\operatorname{arcoth} x)' = \frac{1}{\coth' y} = \frac{1}{\coth^2 y - 1} = \frac{1}{x^2-1}$$

$\sin x$
$\cos x$ } $x \in \mathbb{R}$
$\operatorname{tg} x$
$\operatorname{ctg} x$

ze względu na brak różnowartościowości dla $D = \mathbb{R}$ musimy obciąć dziedzinę naturalną do odcinka długości połowy ~~jednego~~ okresu

bijekcje
$$\begin{cases} \sin x : [-\frac{\pi}{2}; \frac{\pi}{2}] \to [-1, 1] \\ \cos x : [0; \pi] \to [-1, 1] \\ \tan x :]-\frac{\pi}{2}; \frac{\pi}{2}[\to]-\infty; \infty[\\ \operatorname{ctg} x :]0; \pi[\to]-\infty; \infty[\end{cases}$$



odwzorowania odwrotne
$$\begin{cases} \arcsin y : [-1, 1] \to [-\frac{\pi}{2}; \frac{\pi}{2}] & : \arcsin(\sin x) = x \\ \arccos y : [-1, 1] \to [0; \pi] & : \arccos(\cos x) = x \\ \arctan y :]-\infty; \infty[\to]-\frac{\pi}{2}; \frac{\pi}{2}[& : \arctan(\tan x) = x \\ \operatorname{arccot} y :]-\infty; \infty[\to]0; \pi[& ; \operatorname{arccot}(\cot x) = x \end{cases}$$

---

złożenie odwzorowań:

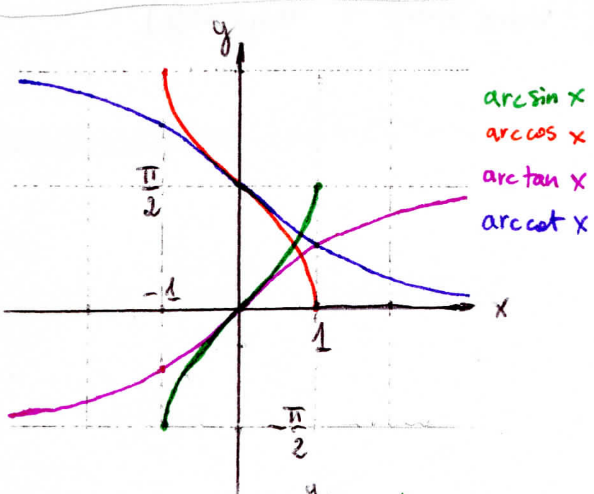
$\sin x : \mathbb{R} \to [-1; 1]$ surjekcja, ale nie iniekcja

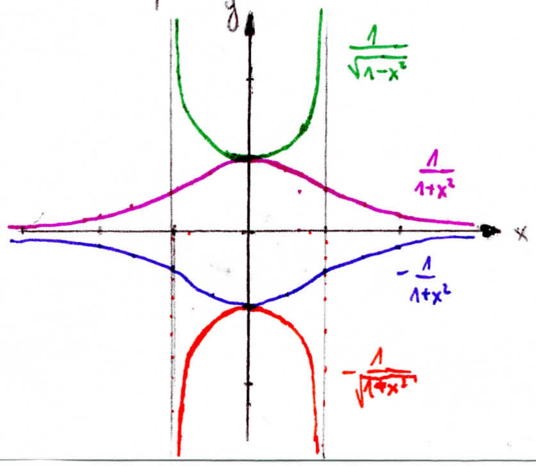
$\arcsin y : [-1; 1] \to [-\frac{\pi}{2}; \frac{\pi}{2}]$, bijekcja

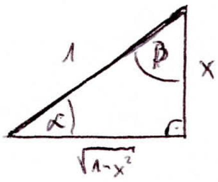
$\arcsin(\sin(x)) : \mathbb{R} \to [-1, 1]$, surjekcja : $x \mapsto x + k\pi$ tak, że $-\frac{\pi}{2} < x + k\pi < \frac{\pi}{2}$

z kolei

$\sin(\arcsin y) : [-1; 1] \to [-\frac{\pi}{2}; \frac{\pi}{2}]$, bijekcja; $y \mapsto y$







$\alpha = \arcsin(x/1) = \arcsin x$

$\beta = \arccos(x/1) = \arccos x$

$\beta = \frac{\pi}{2} - \alpha$

$$\boxed{\begin{aligned} &\arcsin x + \arccos x = \frac{\pi}{2} \\ &\arccos x = \frac{\pi}{2} - \arcsin x \end{aligned}}$$

$\boxed{\arcsin x = -\arcsin(-x) \quad \text{f. nieparzysta}}$

$\boxed{\arctan x = -\arctan(-x) \quad \text{f. nieparzysta}}$

$$\boxed{\begin{aligned} &\arctan x + \operatorname{arccot} x = \frac{\pi}{2} \\ &\operatorname{arccot} x = \frac{\pi}{2} - \operatorname{arctg} x \end{aligned}}$$

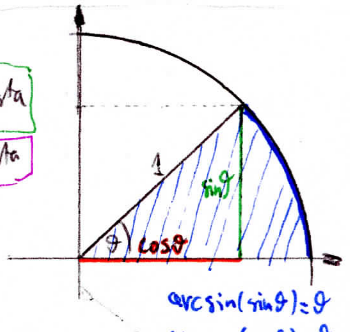
$$(\arcsin x)' = \frac{1}{\sin'(\arcsin x)} = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1 - \sin^2(\arcsin x)}} = \frac{1}{\sqrt{1-x^2}}$$

podobnie

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = \frac{-1}{1+x^2}$$



$\arcsin(\sin\vartheta) = \vartheta$

$\arccos(\cos\vartheta) = \vartheta$

$P = \frac{1}{2}\vartheta$