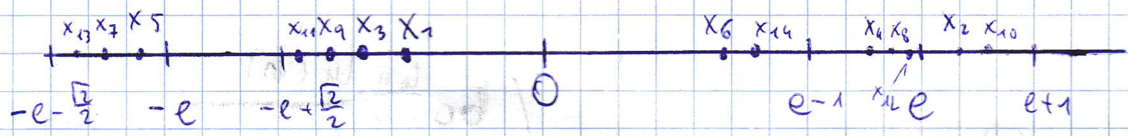
STANISŁAW ŻUKOWSKI
PRZEMYSŁAW SŁOTA

1

a) $a_n = \left(1+\frac{1}{n}\right)^n (-1)^n + \sin\left(\frac{n\pi}{4}\right)$

$$a_n = \begin{cases} \left(1+\frac{1}{n}\right)^n + \sin\left(\frac{n\pi}{4}\right) & \text{dla } n = 2k,\ k \in \mathbb{N} \\ -\left(1+\frac{1}{n}\right)^n + \sin\left(\frac{n\pi}{4}\right) & \text{dla } n = 2k+1,\ k \in \mathbb{N} \end{cases}$$



$$a_{2k+1} = -\left(1+\frac{1}{n}\right)^n + \sin\left(\frac{n\pi}{4}\right) \xrightarrow{n\to\infty} -e + \frac{\sqrt{2}}{2} \text{ lub } -e - \frac{\sqrt{2}}{2}$$

$$\liminf a_n = -e - \frac{\sqrt{2}}{2}$$

$$a_{2k} = \left(1+\frac{1}{n}\right)^n + \sin\left(\frac{n\pi}{4}\right) \xrightarrow{n\to\infty} e+1 \text{ lub } e \text{ lub } e-1$$

$$\limsup a_n = e+1$$

6)

$$a_n = \frac{\ln(n) - (1+\cos(n\pi))n}{\ln 2n}$$

$$1 + \cos(n\pi) = \begin{cases} 2 & \text{dla } n = 2k \\ 0 & \text{dla } n = 2k+1 \end{cases} \quad k \in \mathbb{N}$$

$$a_n = \begin{cases} \frac{\ln(n)}{\ln(2n)} & \text{dla } n = 2k+1 \\ \frac{\ln(n) - 2n}{\ln(2n)} & \text{dla } n = 2k \end{cases}$$

$$a_{2k+1} = \frac{\ln(n)}{\ln(2n)} \xrightarrow{n\to\infty} 1$$ (bo $\frac{\ln n}{\ln 2n} = \frac{\ln(n)}{\ln(2) + \ln(n)}$ przy $n \to \infty$ $\ln(2)$ zaniedbywalnie małe)

$$a_{2k} = \frac{\ln(n) - 2n}{\ln 2n} \xrightarrow{n\to\infty} -\infty$$ (logarytm rośnie wolniej niż liniowa)

$\liminf a_n = -\infty$ $\limsup = 1$