(II). The relation $R$ is a closed subset of $B \times B$, i.e., $\forall b, b' \in B: (b, b') \in R$ if and only if for every neighborhood $G$ of zero in $B$ there exist $b_1, b_2 \in (b + G) \cap B$ and $b_2$ in $(b' + G) \cap B$ such that $b_2 b_2' = b_2 b_2'$.

This follows immediately from the well-known theorem of Alexander (cf. (a), (b), Theorems 1.2.3 and Exercise 3.3.1).

(iii). If $b b_2$, $b b_2'$ and $0 \in c_1$, then $(b(a)(b_2 + b_2 b_2') + b_2 b_2')$. In particular

$(b(a)(b_2 + b_2 b_2') + b_2 b_2')$.

Proof of (iv). Let $a, a'$ be elements of $B$ such that $\xi(a) = b, \xi(a') = b_2, \xi(a') = b_2, \xi(a') = b_2$.

Thus $\xi((a)(a) + a') = (a)(a) + a'$.

Hence, by (i).

$(a)(a) + a' = b_2 b_2 + b_2 b_2'$.

Proof of (iv). Let $b, b'$ be elements of $B$ and there exists $0 \in c_1$ such that $((b)(a) + b) b_2 b_2'$.

Proof of (iv). Let $b, b'$ be elements of $B$ and there exists $0 \in c_1$ such that $((b)(a) + b) b_2 b_2'$.

Since the equivalence algebras of $B$ are compact, we have $b b_2'$. Let $a, a'$ be elements of $B$ such...
Since \( t > 0 \) and \( 0 < s < t \), we have

\[
\begin{align*}
\frac{(t^2 - s^2) - 1}{2st} & > 0, \quad \frac{(t^2 - s^2) + 1}{2st} > 0, \\
\frac{(t^2 - s^2) - 1}{2st} & > 1.
\end{align*}
\]

Hence, by (IV), \( \beta \neq 0 \).

Let \( B \) be a neighbourhood of zero in \( B \). Since multiplication by scalars is continuous, there exists \( 0 < s < t \) such that \( B' \subseteq (b_1 + v) \cap B \) and \( B_1 \subseteq (b_2 + v) \cap B \).

Hence, by (II), we obtain \( b_1 B_2 \).

The converse implication is obvious.

(VIII). The sets \( B - B \) and \((B - B) \cap B\) are compact.

Proof of (VIII). The map \( g : B \times B \to B - B \) defined by \( g(b_1, b_2) = b_1 - b_2 \) for \((b_1, b_2) \in B \times B \) is a continuous surjection. The topology on \( B \times B \) is the product topology. Hence \( B - B \) is compact and \( g \) is a closed map. Therefore it is enough to prove that the set

\[
\mathcal{G} = g^{-1}((B - B) \cap B)
\]

is a closed subset of \( B \times B \). Let \((b_1, b_2) \) be an element of the closure of \( \mathcal{G} \). For every neighbourhood \( U \) of zero in \( B \), the set

\[
V = (b_1 + U) \cap (b_2 + U) \cap B
\]

is a neighbourhood of \((b_1, b_2) \) in \( B \times B \). Since there exists \((b_1, b_2) \) in \( \mathcal{G} \), but the condition \((b_1, b_2) \in \mathcal{G} \) is equivalent to \((b_1, b_2) \in \mathcal{G} \). Hence, by (II), \( b_1 \neq b_2 \), and consequently \((b_1 - b_2) \neq 0 \).

This completes the proof of (VIII).

(VIII). \((\beta_1, \beta_2)\) is a kernel pair in \( B \times B \).

Proof of (VIII). It is easy to see that the set \( \mathcal{G} = B - B \) is an absolutely convex, absorbing and compact in \( B \times B \).

Let \( \mathcal{G} \) be the weak* topology of \( X \). Then \( B \times B \) is a Banach space and there exists a Banach space \( (X, \mathcal{G}) \) such that \( (B, \mathcal{G}) \) is isometrically isomorphic to the convex of \( (X, \mathcal{G}) \) (cf. Lemma 3.52). Hence, by the well-known Banach-Saks Theorem (cf. [5], IV.4.3), \( (B, \mathcal{G}) \) is a locally convex Hausdorff space. The topology \( \mathcal{G} \) and \( \mathcal{G} \) coincide with \( (X, \mathcal{G}) \) (and consequently on \( B \)). Since \( (X, \mathcal{G}) \) is compact in \( (X, \mathcal{G}) \), it is compact in \( (X, \mathcal{G}) \). Hence, by the well-known Banach-Saks Theorem (cf. [5], IV.4.3), \( X \) is \( (X, \mathcal{G}) \) - closed.

Let \( \mathcal{G} \) be the kernel topology on \( X \times X \) determined by \( g = (X, \mathcal{G}) \times (X, \mathcal{G}) \) is a locally convex Hausdorff space. Let

\[
\gamma : X \times X \to X / X
\]

be the quotient map. Then the map

\[
\beta : X \to \gamma(X)
\]

defined by \( \beta(x) = \gamma(x) \) for \( x \) in \( X \) is a convex-morphism.
It follows immediately from the definition of \( y \) that there exists a unique homomorphism (a \( \text{Comp}-\text{iso} \))
\[ h : W(B) \longrightarrow 0 \]
such that \( f = h g \). We shall show that \( (f_1, f_2) \) is a kernel pair of \( y \) in \( \text{Comp} \). Let \( (x_1, x_2) \)
be a parallel pair in \( \text{Comp} \) such that \( f_1 x_1 = f_2 x_2 \).

Hence \( f_1 x_1 = f_2 x_2 \) and therefore there exists a unique morphism \( u \) in \( \text{Comp} \) such that \( f_i u = f_i \) for \( i = 1, 2 \).

It is enough to prove that \( u \) is an affine map. Let \( \phi' \) be elements of \( B \) and let \( \phi \in (\alpha_1, \alpha_2) \). Then,
\[ f_1(\alpha_1(\phi')) + (1-\alpha_1(\phi')) f_2(\phi') = \phi \]
\[ = f_1(\phi') + (1-\alpha_1(\phi')) f_2(\phi') = \alpha_1(\phi') f_1(\phi') + (1-\alpha_1(\phi')) f_2(\phi') = f_1(u(\phi')) + (1-\alpha_1(\phi')) f_2(u(\phi')) \]
for \( i = 1, 2 \). Since \( (x_1, x_2) \) is a kernel pair,
\[ \phi(\phi') + (1-\alpha_1(\phi')) \phi(\phi') = \phi(\phi') + (1-\alpha_1(\phi')) \phi(\phi'). \]
This completes the proof of (VIII) and the proof of lemma 4.2.2.
(X, F) = \Theta(X,x,{\mathcal{V}}) = \left( S_{\lambda}(X,x), S_{\lambda} \right) \subseteq (X,x), S_{\lambda})

i.e., X = (X,x), F = S_{\lambda}. In other words, for any compact space X, and for any continuous map

\[ f : \mathcal{F}(X) \rightarrow X \]

satisfying the following conditions

(1) \[ f \left( \mathcal{F}(X) \right) = x \text{ for } x \text{ in } X \]

(2) \[ f \left( \mathcal{F}(X) \right) = f_{\lambda}(x) \]

there is the unique convex structure \( k \) on \( X \) such that

\( (X,x,\mathcal{V},k) \). is a compact convex set ( or is the given topology

on \( X \)), and \( f_{\lambda}(x) \) is the centroid of \( x \), for each \( x \) in

\( \mathcal{F}(X) \).

It is clear that one set

\[ \left\{ \sum_{i=1}^{n} a_i x_i : x_i = \lambda_i, \sum_{i=1}^{n} a_i = 1, \lambda_i \in \mathcal{F}(X) \right\} \]

is dense in \( f(\mathcal{F}(X)) \). Hence the condition (ii) is equivalent to

(3) \[ f \left( \sum_{i=1}^{n} a_i x_i \right) = f \left( \sum_{i=1}^{n} a_i \lambda_i \right) \]

Thus we get the following theorem:

4.2.5. Theorem. Let \( (X,x,\mathcal{V},k) \) be a compact space, and let

\[ f : \mathcal{F}(X) \rightarrow X \]

be a continuous map satisfying the conditions 4.2.4(i) and (iii). Then there is the unique convex structure \( k \) on \( X \)

such that \( (X,x,\mathcal{V},k) \) is a compact convex set and \( f(\lambda) \)

is the centroid of \( \lambda \), for each \( \lambda \) in \( \mathcal{F}(X) \).

This means that the conditions 4.2.4(i), (iii) give an

axiomatic characterization of the centroid or measure on a

compact space.

4.2.6. It is clear that the conditions 4.2.4(i), (iii) are equivalent to

(i) \[ f \left( \lambda_i \right) = x \text{ for } x \text{ in } X, \]

(ii) \[ f(\lambda) \in f(\mathcal{F}(X)) \text{ and } \lambda \in \mathcal{F}(X) \]

and

\[ \mu \lambda \in \mathcal{F}(X) \text{ then } f(\mu + \lambda) = f(\mu) \lambda + f(\lambda) \]

\[ f \left( \cdot \mu + \lambda \right) = f \left( \cdot \mu \right) + f(\lambda) \text{ (additivity).} \]
also satisfies these conditions and consequently, by Theorem 2.1, is non-singular. Thus, by Lemma 3.4, and 3.5, \( D \) is non-singular.

4.1. The case \( m = 3 \).

4.1.1. We shall now consider the canonical injection of a certain set \((x, a)\) into a pseudo-ordered vector space. Let \((x, a)\) be a coarser set. Let \( L \) be the subspace of the free vector space \( U(x) \) defined by

\[ L = \left\{ x_1, x_2, \ldots, x_k \in \mathbb{R}^n : x_1 \leq x_2 \leq \cdots \leq x_k \right\} \]

Let \( N(x, a) = U(x)/L \) and let \( \pi : U(x) \rightarrow N(x, a) \) be the quotient map. Then there is the unique injection \( \xi : E \rightarrow N(x, a) \) such that the diagram

\[ \begin{array}{ccc}
E & \xrightarrow{\xi} & N(x, a) \\
\downarrow \downarrow & & \downarrow \downarrow \\
U(x) & \xrightarrow{\pi} & \mathbb{R}^n \\
\end{array} \]

is commutative (cf. the proof of 4.1.10).

Let \( \xi : E \rightarrow \mathbb{R}^n \) be an affine map. Then \( \xi : E \rightarrow \mathbb{R}^n \) is affine and there exists the unique linear functional \( \xi^* \) on \( U(x) \) such that

\[ \xi^*(x) = \xi(x) \cdot a \quad \text{for all } x \in U(x) \]

This is the affine functional on \( U(x) \) defined by

\[ \xi^* \equiv \xi_0^* \]

In a vector-structure.

It is clear that the map

\[ \Phi : U(x) \rightarrow \mathbb{R}^n \]

defined by

\[ \Phi(x) = \sum_{s_{i_k}}^s x_{i_k} \]

is a norm in \( U(x) \). Consequently, the map

\[ \psi : N(x, a) \rightarrow \mathbb{R}^n \]

defined by

\[ \psi(y) = \sup \left\{ \sum_{s_{i_k}}^s x_{i_k} : x \in U(x) \right\} \]

is a pseudo-norm. It is easy to see that \( \psi \) is a norm for each \( x \in E \).

We shall show that for each \( x \in E \) in \( N(x, a) \)

\[ \psi(x) = \sup \left\{ \xi^* (x) : \xi \in \Xi \right\} \]
The inequality \( \sum (\phi_n(x,y) - \phi_m(x,y)) \leq \sum \phi_n(x,y) \leq 0 \) holds true for all \( x, y \). Let \( \phi_n(x,y) = x^n y^m \) for \( n, m \) such that \( n + m \leq 3 \). Then, we have:

\[
\sum (\phi_n(x,y) - \phi_m(x,y)) \leq \sum \phi_n(x,y) \leq 0
\]

where \( \phi_n(x,y) = x^n y^m \). In order to show that there is no collision in the map:

\[
\phi(x,y) = (x + 1, y + 1)
\]

we compute the changes in the coordinates:

\[
\phi(x,y) = (x + 1, y + 1)
\]

and show that they are bounded.

Let \( \phi(x,y) = (x + 1, y + 1) \) be the map, and let \( z \) be a point in \( [0,1] \times [0,1] \). Then, we have:

\[
\phi(z) = (z + 1, z + 1)
\]

which implies that the changes are bounded.

In conclusion, the map is collision-free since there is no collision in the map.
It is clear that the topology on $X(A)$ induced by
the norm $\| x \|$ is the Mackey topology $\tau_X(X(A), \mathcal{A}_0(A))$.

Let $\mathcal{A}_0(A)$ be a locally convex Hausdorff space
$(\mathcal{A}_0(A), ||\cdot||)$. There exists a linear map $f : X(A) \to I$ and
$0 \neq y \in I$ such that $\psi(x) = f(x) + y$ for $x \in X(A)$. Let $x_0 \neq y_0$ be elements of $X(A)$ such
that $\psi(x_0) \neq \psi(y_0)$. Then there exists a continuous linear functional $g$ on $I$ such that $\psi(x_0) \neq \psi(y_0)$.

Consequently, $\mathcal{A}_0(A)$ is a locally convex Hausdorff space.

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Consequently, $\mathcal{A}_0(A)$ is a locally convex Hausdorff space.
4.3.4. Let $B$ be a bounded convex set. Then $B$ is a bounded convex subset of a locally convex linear space $(X, \mu)$ and $\text{Conv} B$ is the same convex set with the topology induced by the linear topology $\mathcal{T}(X^*')$, where $X^*$ is the vector space of all linear functions on $X$ which are bounded on $B$. Hence the norm of the adjunction

$$\mathcal{S}_B : \text{Conv}(X^*) \to \text{Conv}(X^*)$$

is defined by $\mathcal{S}_B(y) = y$ for $y \in B$. Moreover $\mathcal{S}_B : B \to B$.

Let $T = (\pi, \psi, \mu)$ be the adjunction determined by the adjunction $(\mathcal{S}_B, \mathcal{S}_B, \psi)$. Then

$$\mu^2 = \mathcal{S}_B \circ \mathcal{S}_B = I_{\text{Conv} B}$$

for each $x$ in Conv$^B$.

Let $(X, \mu)$ be a $T$-algebra. Then $\mathcal{S}_B(X, \mu)$ is an injection and consequently, by definition of $\mathcal{S}_B(X, \mu) = \mathcal{S}_B(X, \mu)$. Consequently, $\mathcal{S}_B(X, \mu) = \mathcal{S}_B(X, \mu)$. It is easy to see that $\mathcal{S}_B(X, \mu)$ is an injection if and only if $\mathcal{S}_B(X, \mu)$ separates $X$. Hence $(X, \mu)$ is a $T$-algebra if and only if $\mathcal{S}_B(X, \mu)$ separates $X$ and $\mathcal{S}_B(X, \mu)$ is continuous. Consequently the category Conv$^T$ may be...
Proof. The functor \( G_{10} \) is the composition of the functors \( G_2 \) and \( G_3 \). The functors \( G_2 \), \( G_3 \) satisfy the conditions \( 3.1(1), 3.1(2) \) of \( 3.1 \). Consequently, by \( 3.2 \), we obtain \( G_{10} \) is associative.

4.11. The case: \( \text{Compaaks} \xrightarrow{\alpha} \text{Compaaks} \xrightarrow{\beta} \text{Compaaks} \) (Compaaks).

The functor \( G_{11} \) is the composition of the functors \( G_2 \) and \( G_3 \). Hence, \( G_{11} \) is a left adjoint of \( G_{11} \).

4.11.1. Theorem: The forgetful functor

\[ \phi_{11} : \text{Compaaks} \longrightarrow \text{Compaaks} \] is monadic.

Proof. Similar as 4.10.2.

4.12. The case: \( \text{Compaaks} \xrightarrow{\alpha} \text{Res} \xrightarrow{\beta} \text{Res} \) (Res).

The functor \( G_{12} \) is the composition of the functors \( G_2 \) and \( G_3 \). Hence, \( G_{12} \) is a left adjoint of \( G_{12} \).

4.12.1. Theorem: The forgetful functor

\[ \phi_{12} : \text{Compaaks} \longrightarrow \text{Res} \] is monadic.