Numerical simulations of Causal Dynamical Triangulations 2

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Outline

1. Numerical results in de Sitter phase
   - Semiclassical volume distribution
   - Minisuperspace model
   - Fluctuations around the semiclassical action
   - Geometry of 3d spatial slices
2. Universal behavior in de Sitter phase
3. Extending the effective action: more observables
4. Work in progress

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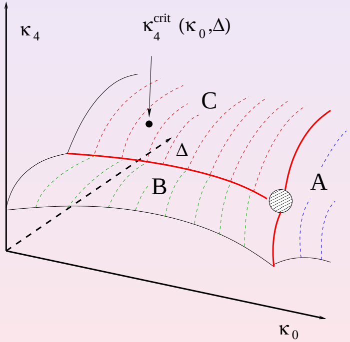
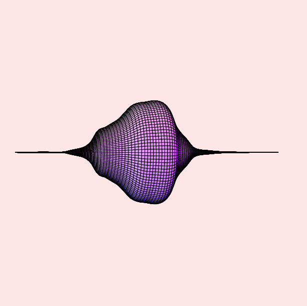
Approximate phase diagram of CDT



$\mathcal{Z}$ is defined for $\kappa_4 > \kappa_4^{crit}(\kappa_0, \Delta)$. Approaching a critical surface means taking an infinite volume limit. $\langle N_4 \rangle \sim 1/(\kappa_4 - \kappa_4^{crit})$.

Red lines - first order phase transitions. Perhaps a triple point.

Snapshot of a typical configuration in de Sitter phase

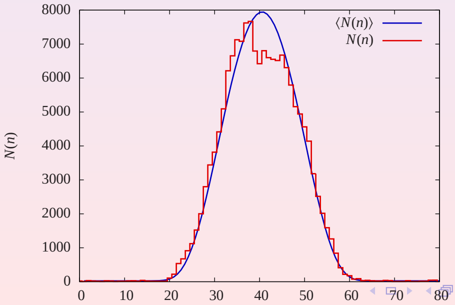


A typical configuration. Distribution of a spatial volume $N_3(t)$ as a function of discrete (imaginary) time t. Quantum fluctuation over a semiclassical background.

Configuration consists of a "stalk" of the cut-off size and a "blob". Center of the blob can shift. We fix the "center of mass" to be at zero time.

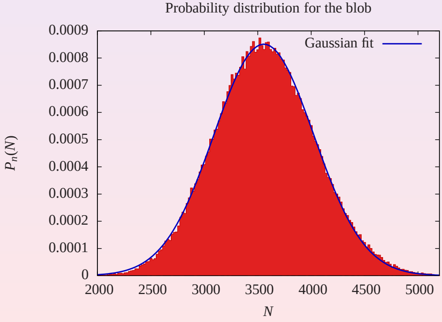
Approach to the semiclassical limit

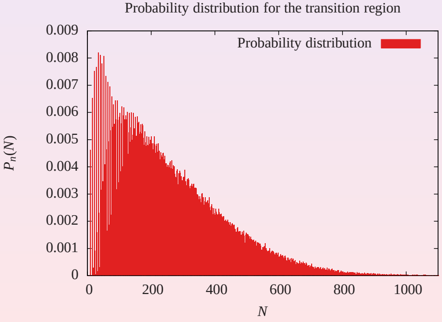
To obtain the semiclassical limit we average over configurations. On the plot we see the individual contribution (centered) and the limiting distribution (blue) obtained by averaging over many configurations with the same volume. Measurements at $\kappa_0 = 2.2$ and $\Delta = 0.6$



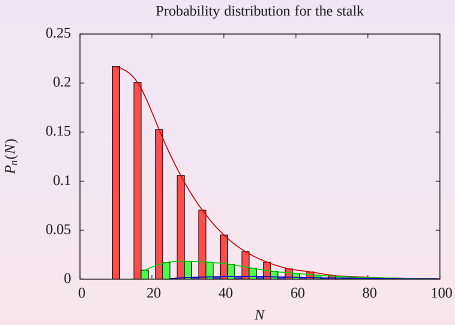
Approach to the semiclassical limit, contd.

Looking at the volume distribution at a fixed integer time we can estimate to what extent the distribution we see corresponds to a minimum of some effective action.





Approach to the semiclassical limit, contd.



The stalk is completely dominated by lattice artefacts.

Conclusion: Gaussian behaviour inside the blob. Artefacts in the transition region and in the stalk.

Semiclassical volume distribution in de Sitter phase

The object of our analysis is the limiting semiclassical distribution of the observable $N_3(i)$ - volume of the spatial universe at (integer) time i. By construction

$$2\sum_i N_3(i) = N_4^{\{4,1\}}$$

If the space-time dimension is d we expect the semiclassical distribution of the spatial volume $N_3(i) = N_4^{(d-1)/d} P_{N_4}(\sigma)$ to be a universal function of the rescaled time σ

$$\sigma = i/N_4^{1/d}, \quad P_{N_4}(\sigma) = P(\sigma)$$

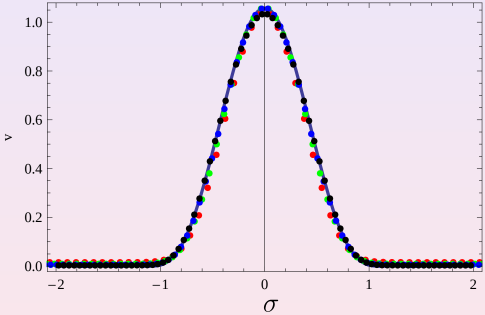
Scaling and its consequences: infinite volume limit

If indeed the semiclassical distribution would scale and if $d = 4$ it would mean that for a system size N_4

- The time extent of the blob is $\sim N_4^{1/d} = N_4^{1/4}$.
- The spatial size of the blob scales as $N_3 \sim N_4^{(d-1)/d} = N_4^{3/4}$.

This property will be fundamental to relate numerical results and physical properties of CDT in the continuum limit.
Notice that this scaling means that both the time and the space scale in a “canonical” way.

Universality of the volume distribution



Results of measurements of $P_{N_4}(\sigma)$ for $N_4 = 22.2$k, 45.5k, 91.1k, 181k and $d = 4$.

Measurements at $\kappa_0 = 2.2$ and $\Delta = 0.6$

The continuous line corresponds to $f(\sigma) = 1.0575 \cos^3(1.41\sigma)$ (4d sphere).

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Minisuperspace model

The limiting semiclassical distribution has a shape of a bounce. This is the effect of imaginary time. This shape can be obtained from the minisuperspace effective action of Hartle and Hawking (up to finite constants). Notice the opposite sign.

$$S^{eff} = \frac{1}{\Gamma} \int_0^T d\tau \left(\frac{1}{9N_3(\tau)} \left(\frac{dN_3(\tau)}{d\tau} \right)^2 + N_3^{1/3}(\tau) - \lambda^2 N_3(\tau) \right)$$

Here λ^2 is a Lagrange multiplier to enforce

$$\int_0^T d\tau N(\tau) = N_4$$

and Γ is the (dimensionless) effective Newton's constant.

Classical solution

In this example

$$N_3^{cl}(\tau) = \left(\frac{\cos(\lambda\tau)}{\lambda} \right)^3$$

and

$$\lambda^4 = \frac{4}{3N_4}$$

Restoring physical dimensions

Discretization of a theory always leads to a description of the theory in terms of dimensionless coupling constants and dimensionless objects.
We can replace the dimensionless objects by dimensionfull ones reintroducing the dimensionfull lengths a_t and a_s

$$V_4 = a_t a_s^3 N_4,\; V_3 = a_s^3 N_3,\; t = a_t \tau,\; G = a_s a_t \Gamma$$

and rewrite the effective action as

$$S^{eff} = \frac{1}{G}\int_0^{t_f} dt \left(\xi^2 \frac{1}{9V_3(t)} \left(\frac{dV_3(t)}{dt}\right)^2 + V_3^{1/3}(t) - \Lambda^2 V_3(t) \right)$$

with $\Lambda = \lambda/a_s,\; \xi = a_t/a_s$.

Size of the space-time

Parameter ξ is a finite number, representing the ratio between units of time and spatial length.
We can express a physical size of the studied system, using the Newton's constant as

$$V_4 = \frac{G^2}{\Gamma^2 \xi} N_4$$

Notice that parameters like Γ and ξ cannot be determined from the form of the semiclassical solution.
We also should really prove that the effective action has a form of the minisuperspace action.

Outline

Second order test of the effective action

If we expand the effective action S^{eff} around the classical solution $N_3^{cl}(\tau)$:

$$N_3(\tau) = N_3^{cl}(\tau) + \delta(\tau)$$

to second order in $\delta(\tau)$ we obtain:

$$S^{eff} = S^{eff}_{cl} + \frac{1}{9\Gamma}\int_0^T d\tau \delta(\tau) D(\tau) \delta(\tau) + \cdots$$

where $D(\tau)$ is the Sturm-Liouville operator

$$D(\tau) = -\frac{d}{d\tau}\frac{\xi^2}{N_3^{cl}(\tau)}\frac{d}{d\tau} - \frac{4}{N_3^{cl}(\tau)^{5/3}}$$

Gravitational constant Γ can be determined from the size of quantum fluctuations around the classical solution.

Correlation matrix of quantum fluctuations

For a sequence of volumes N_4 we perform measurement of the correlation function

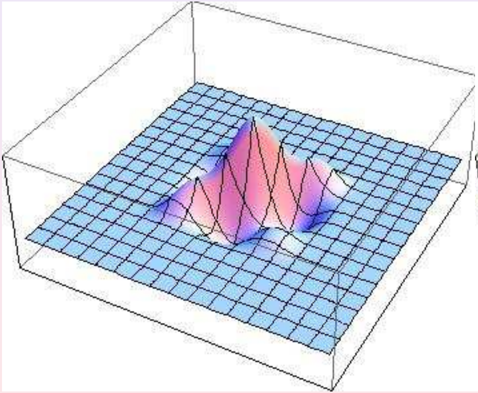
$$C(i,j) = \langle \delta(i)\delta(j) \rangle_{N_4}$$

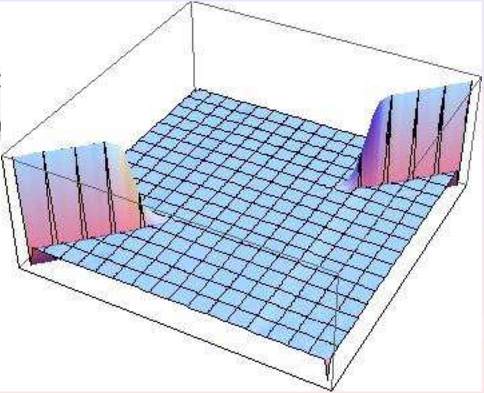
Using Gaussian approximation we have $C(i,j) \sim 9\Gamma D^{-1}(i,j)$. Inverting $C(i,j)$ gives (in principle) full information about $D(i,j)$ and Γ.

"Propagator" and "Inverse propagator"

Matrix $C(i,j)$

Matrix $D(i,j)$





Discretized effective action

The analysis of the semiclassical solution and fluctuations of three-volume shows that our observations can be interpreted as the effect of the existence of the effective action S_{eff}

$$S_{eff} = \frac{1}{\Gamma} \sum_t \Big(F\big(N_3(t), N_3(t+1)\big) + V\big(N_3(t)\big) \Big)$$

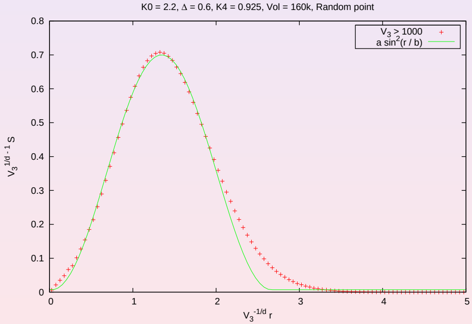
where

$$F(x,y) = \frac{(x-y)^2}{x+y}, \quad V(N_3(t)) = -\lambda_{eff} N_3(t) + \mu N_3(t)^{1/3} + \dots$$

S_{eff} is a discretization of the minisuperspace model.

Outline

Averaged geometry of spatial slices



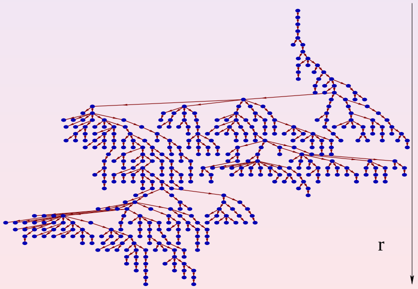
Minisuperspace model suggests that the geometry of 3d slices fluctuates around S^3. Is this true?
On this plot we see that for the averaged distribution the agreement with S^3 is fair.
Difference is in the tail, which can be understood (fluctuations of the radius).

What happens for individual realizations?

We start from one (sub)simplex and move out (diffusion process) following nearest neighbors either using 3d geometry (tetrahedra at a fixed time t) or 4d geometry (4-simplices). The points on the plots correspond to the connected components.

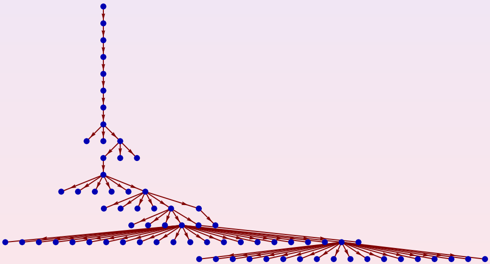
Fractal in 3d: definition of a radial distribution

For a particular realization of geometry the structure of the spatial slice is fractal. As a consequence we can only have spatial loops with a cut-off size.



Comparison in 4d: definition of a radial distribution

For the same realization 4d geometry is very regular. Only short-range fluctuations.

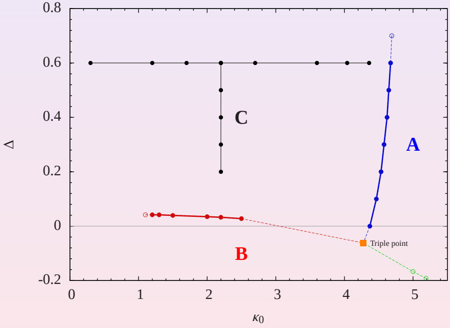


Interpretation

Geometry can be characterized by a Hassdorf dimension d_H and a spectral dimension d_S.

- In the 3d case we see that a tree with long "branches" is formed: a fractal. Although $d_H = 3$ we get $d_S \approx 1.5$.
- in 4d "branches" are very short. In this case $d_H = 4$ and at large scales $d_S = 4$. (On short scales we see $d_S \approx 2$).

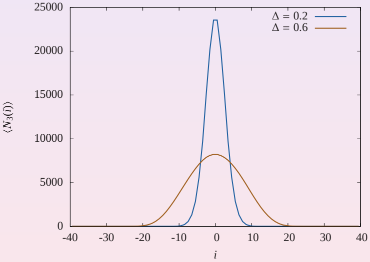
Phase diagram again

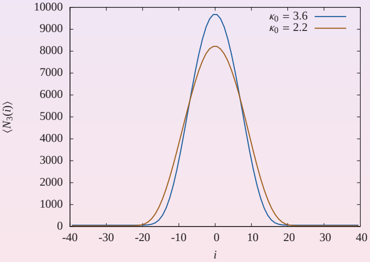


We may ask what happens when we move inside the de Sitter phase towards the phase transition. Black dots represent the positions where measurements were made. Red and blue dots represent approximate phase transition points.

Universality in the de Sitter phase

Changing values of κ_0 and Δ leads to a finite renormalization of the “effective” parameters Γ and ξ.

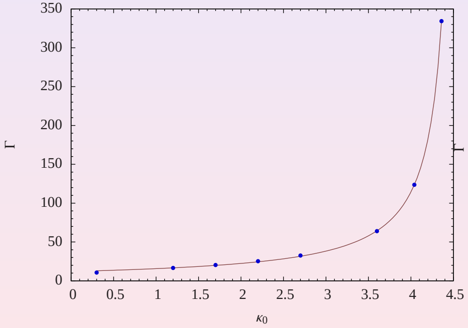




In the C phase volume distribution always represented by the same ($\cos^3$) curve, but the scale in time and the size of fluctuations are changing.

Effective gravitational constant Γ

$\Delta = 0.6$ $\kappa_0 = 2.2$



Conclusions

Approach to a critical line is accompanied by the increase of the effective gravitational constant. In physical units this can have an interpretation of ”stronger gravity” or ”larger cut-off” *a*. Recall that

$$V_4 = \frac{G^2}{\Gamma^2 \xi} N_4$$

and

$$G = a_s a_t \Gamma$$

New degrees of freedom

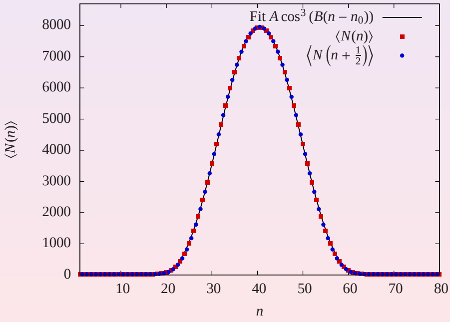
It was mentionned that the ”construction” of the space-time resembles the onion: There are consecutive layers of

- $\{1, 4\}$ and $\{4, 1\}$ simplices at integer discretized time.
- $\{3, 2\}$ and $\{2, 3\}$ layer which can be interpreted as half-integer time layer. This additional layer ”glues” together the spatial slices at integer t. It forms a closed 3d connected manifold.

Moving in the C-phase we observe that the ratio of the numbers of $\{3, 2\}$ and $\{4, 1\}$ simplices is a function of the bare parameters of the theory.

Global volume distribution

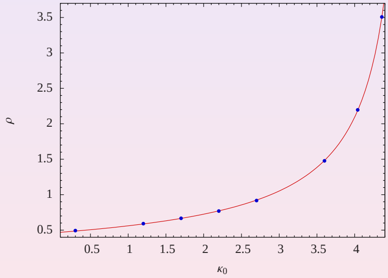
We may ask what is the role played by $\{3,2\}$ layers between the spatial slices. What is the volume of a $\{3,2\}$ simplex?

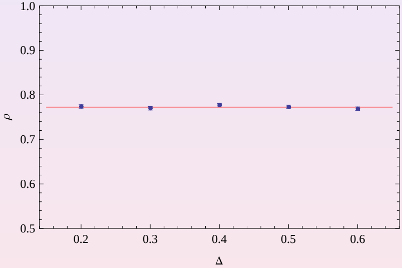


The plot presents a distribution of the $\{4,1\}$ volume at integer times and the $\{3,2\}$ volume scaled by a constant ρ and plotted at half-integer times.

$\kappa_0 = 2.2, \quad \Delta = 0.6$

Behavior of ρ (volume of a $\{3, 2\}$ simplex) in de Sitter phase





Surprising behavior: naively we expect dependence of ρ on the asymmetry parameter Δ. It shows that the relation between the bare and renormalized parameters is non-trivial.

New projects and beyond

There remain many questions for the simplest formulation (without matter):

- How to define semiclassically a system of coordinates which would make sense (fractal structures).
- If we accept that fractal structures are important, what is their dynamics?
- What other sensible observables can be measured (local effective action)?
- What are the cosmological consequences?

Other possible directions:

- Introducing matter is essential for a theory of gravity.
- How can we measure the semiclassical gravitational force?
- . . .

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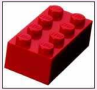
Outline

- Fractal dimensions

Problems of “Euclidean” DT

Dimension of space-time on large scales is not 4.

Naively if a building block has dimension d

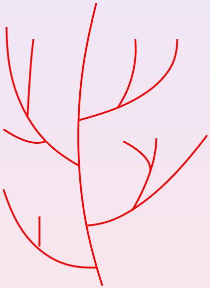


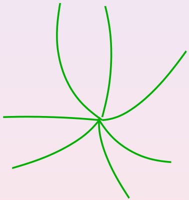
$\longrightarrow$

one expects that a final construction has the same dimension.

Problems of "Euclidean" DT cont'd.

In fact a "typical" structure of space time is different:

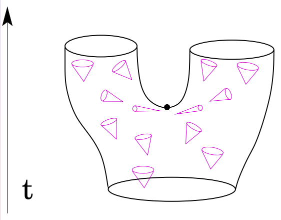




"Tree" $d_H = 2,\ d_S = 4/3$ "Bush" $d_H = \infty,\ d_S = ?$

Fractal dimensions d_H - Hausdorff, d_S - spectral dimension.

Problems of "Euclidean" DT cont'd.



Independently of how we define the "time" in DT, the quantum amplitude is always dominated by trajectories, for which the "spatial" universe splits into (infinity of) universes (baby-universes).

CDT requires that we restrict topologies to those, which do not admit such singularities:

Fractal dimensions

On random geometric structures we must define what we mean by the (averaged) dimension of a manifold. We use two definitions:

- Hausdorff dimension d_H. For a ball with a radius $r >> 1$ we measure the number of points inside the ball:

$$\langle N(r) \rangle \sim r^{d_H}$$

For a ball with a finite volume V we should have

$$\langle r \rangle_V \propto V^{1/d_H}$$

- Spectral dimension d_S. We define a diffusion process on a manifold in a pseudo-time σ. Return probability

$$\langle P(\sigma) \rangle \propto 1/\sigma^{d_S/2}$$