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$\boxed{\text{GROUP FIELD THEORY (GFT)}}$

GFT is a QFT on several opus (D: space-time dimension) of a group $G = SU(2)$ whose Feynman amplitudes

$$\int D\varphi \, e^{iS[\varphi]} = \sum_{\gamma} \frac{\mathcal{A}_\gamma}{S_\gamma} \qquad \varphi \; \underbrace{G \times \dots \times G}_{D} \to \mathbb{C}$$

γ Feynman graph $\Leftrightarrow$ 2 complex

$\mathcal{A}_\gamma$ = Spin foam amplitude

S_γ : Symmetry factor of the graph

Plan

Motivations
Feynman diagrams in QFT
Generalized matrix models
} Introductionary material

BF theory
EPRL/FK model
Open issues
} Construction of GFT

References

- C. Rovelli "Quantum gravity"
- T. Thieman "Modern Canonical gravity" 14.4.2.
- D. Oriti "The group field theory approach to QG: Some recent results" arxiv 0912.2441
- L. Freidel: "Group field theory: An overview" hep-th/0505016

I. Motivations

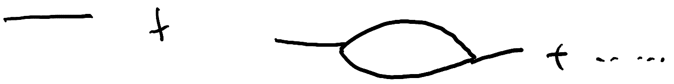
Covariant approach

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

non renormalisable as a QFT in $h_{\mu\nu}$

→ new physics at $\ell_P \simeq 10^{-33}$ cm (asymptotic safety)

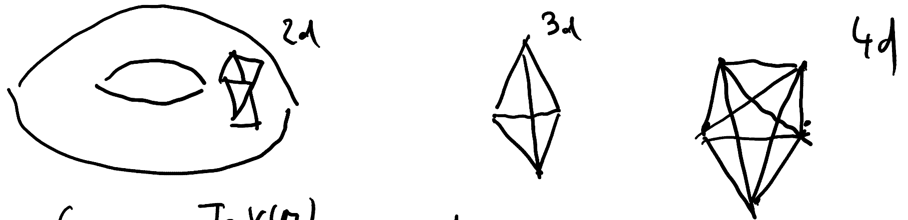
→ String theory (spin foam models or non commutative geometry)



perturbative expansion of a "generalized" QFT (string field Theory)

<u>Discrete approach</u>

$$\int \mathcal{D}g \, e^{iS_{EH}[g]} \xrightarrow{\text{discretize}} \sum_{T \text{ triangulations}} e^{iS_{Regge}[T]}$$



$\int \mathcal{D}M \, e^{-\mathrm{Tr}\, V(M)}$ $\qquad M_{ijk} \qquad M_{ijkl}$

M_{ij} matrix $\qquad$ tensors

<u>Canonical approach</u> (LQG)

$\mathcal{H}_P \quad L^2\left(SU(2)^E / SU(2)^V \right)$

fundamental role played by $SU(2)$

i, j, k, l → group elements

$\varphi(g_1, g_2, g_3, g_4)$ basic field

II. Feynman diagrams

<u>Definition</u>

:SrU7

$$\int [D\varphi]\, e^{iS[\varphi]}\, O(\varphi)$$

φ^a — $\varphi(x)$ QFT in position space

$\tilde{\varphi}(k)$ QFT in momentum space

$n_{ijkl\ldots}$ generalized matrix model

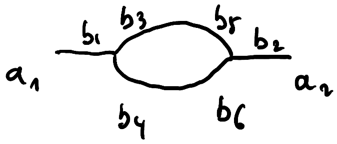
$$O(\varphi) = \varphi^{a_1} \ldots \varphi^{a_n}$$

$$S[\varphi] = \frac{1}{2}\sum_{b_1 b_2} \varphi^{b_1} (K^{-1})_{b_1 b_2} \varphi^{b_2} \quad \text{(kinetic)}$$

$$+ \sum_{n=1}^{\infty} \frac{\lambda_n}{n!} \sum_{b_1,\ldots,b_n} V_{b_1 \ldots b_n} \varphi^{b_1} \ldots \varphi^{b_n} \quad \text{(interaction)}$$

$$\int D\varphi\, O(\varphi)\, e^{iS[\varphi]} = \sum_{\substack{\gamma \\ \text{Feynman graphs}}} \frac{A_\gamma}{S_\gamma}$$

Follows from the Wick theorem



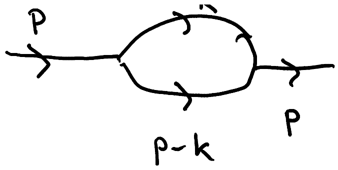
$$\sum_{b_1 b_2 \ldots b_6} K_{a_1 b_1} K_{b_2 a_2} K_{b_3 b_5} K_{b_4 b_6} \times (\lambda_3)^2 \, V_{b_1 b_3 b_4} V_{b_5 b_6 b_2}$$

K need not to be invertible

QFT in momentum space

$$K(p_1, p_2) = \delta(p_1, p_2) \frac{1}{p_1^2 + m^2} \quad \text{(Klein-Gordon propagator)}$$

$$V_n(p_1, \ldots, p_n) = \delta(p_1 + \ldots + p_n) \quad \text{(momentum conservation)}$$



$$\int \frac{d^D k}{(k^2+m^2)\,((k-p)^2+m^2)}$$

- I internal lines $\qquad E$ = external legs
  V vertices (n-valent)

$$\omega = D(I-V+1) - 2I$$

degree divergence
power of k in the integral

$$= D\left(\frac{nV-E}{2} - V + 1\right) - nV + E$$

$$nV = E + 2I \qquad = D - E\left(\frac{D}{2}-1\right) + V\left(\frac{nD}{2} - n - D\right)$$

$$I = \frac{nV-E}{2} \qquad \omega \geq 0 \iff \text{divergence}$$

$$\frac{nD}{2} - n - D = 0 \quad \text{theory is renormalizable}$$

$$n=4\ ,\ D=4$$

$$\omega = 4 - E$$

$\rightarrow$ divergences absorbed in the parameters λ
(mass, coupling constant)

regularization $\int dk \rightarrow \int_0^\Lambda dk \quad \Lambda$ = UV cut-off

$$\lambda, m \rightarrow \lambda(\Lambda), m(\Lambda)$$
renormalisation

we get a perturbatively finite result
(S. Weinberg "The Quantum theory of fields")

<u>Feynman graphs as 1D QG with matter</u>

$$S[x] = \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}} \qquad \text{relativistic particle}$$

$S[x, p]$

$$S[x,e] = \frac{1}{2}\Big[\underbrace{e^{-1} g_{\mu\nu} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}}_{\text{matter field}} + \underbrace{e m^2}_{\substack{\text{cosmological}\\ \text{constant}}} \Big]$$

e : einbein

$$\int De\, Dx\; e^{iS[x,e]} = \int_0^\infty d\alpha \int [Dx]\; e^{\frac{i}{2}\left(\alpha^{-1} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} g_{\mu\nu} + \alpha m^2\right)}$$

$$= \int_0^\infty d\alpha\, \frac{1}{\alpha^{D/2}}\, e^{i\frac{(x-y)^2}{2\alpha} + i\frac{\alpha m^2}{2}}$$

Zinn-Justin

"QFT and critical phenomena"

exercice (web)

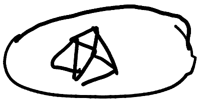
$$= \int d^Dk \int_0^\infty d\alpha\; e^{ik(x-y)}\, e^{i(k^2+m^2)\alpha}$$

$$= \int d^Dk\, \frac{e^{ik(x-y)}}{k^2+m^2}$$

II. Generalized matrix models

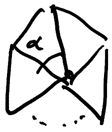
- Jan Ambjorn, Bergfinnur Durhuus, Thordur Jonsson "Quantum Geometry: a statistical field theory approach"
- R. de Pietri Carlo Petronio "Feynman diagrams of generalized matrix models and the associated manifold in dimension 4" gr-qc/0004045

2d Quantum gravity



triangulation

$$S_{Regge} = \sum_{\substack{\text{points}\\ v}} \Big(2\pi - \sum_{\substack{t \text{ triangles}\\ \text{attached}\\ \text{to } v}} \alpha_{v,t} \Big)$$



$$= 2\pi V - \sum_{v,t} \alpha_{v,t}$$

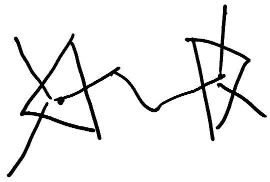
$$= 2\pi V - \pi F \quad \Big(\sum_v \alpha_{v,f} = \pi\Big) \text{ for fixed } f$$

$$= 2\pi (V - E + F) \qquad 2E = 3F$$

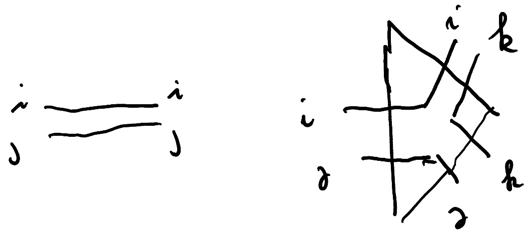
$$= 2\pi \chi$$

χ = Euler characteristics

$$\int Dn\, e^{iS[n]} = \sum_{\text{triangulation}} e^{i\chi}$$



n_{ij} $\quad n^*_{ji} = n_{ij}$



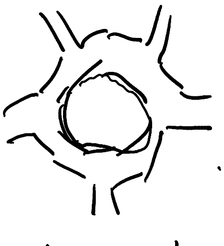
$$n_{ij} n_{jk} n_{ki}$$

$$S[n] = \frac{1}{2} \sum_{i,j} n_{ij} n^*_{ji} + \frac{\lambda}{3} \sum_{i,j,k} n_{ij} n_{jk} n_{ki}$$

$$= \frac{1}{2} \mathrm{Tr}\, n^2 + \frac{\lambda}{3} \mathrm{Tr}\, n^3$$

$$\int Dn\, e^{N \mathrm{Tr}\, S[n]} = \sum_\gamma N^{\chi(\gamma^*)}$$

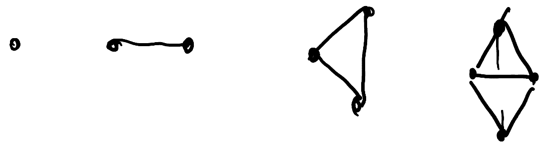
triangulation dual to γ



<u>Simplicial geometry</u>

n- Simplex is a convex span of $n+1$ points

$(x_1, \dots, x_{n+1})$



Orientation $(x_1, \dots, x_{n+1})$ up to the action of an even permutation

orientation reversed simplex

$$\overline{(x_1, \dots, x_n)} = (x_{\sigma(1)}, \dots, \sigma(n)) \quad \sigma \text{ odd}$$

$$(x_1, \dots, x_n) = (x_{\sigma(1)}, \dots, \sigma(n)) \quad \sigma \text{ even}$$

boundary operator

$$\partial(x_1, \dots, x_n) = (x_2, \dots x_n)\ \overline{(x_1, x_3, \dots x_n)}\ (x_1, x_2, x_4, \dots x_n) \dots$$

Generalized matrix models

Construct a field theory (generalized matrix model) whose perturbative expansion generates all ways to glue together D-simplices

analogous to matrix models M_{ij}

i = point D-2 simplex

ij = 1 simplex (D-1)

ijk = 2 simplex (D-2)

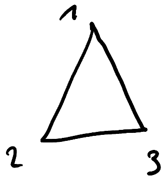
$$\underbrace{M_{\overbrace{i_1, i_2, \dots}^{D \text{ indices}}}}_{} \qquad i_1, \dots i_D : \text{D-2 Simplices}$$

D-1 Simplex

$$M_{i_{\sigma(1)}, i_{\sigma(2)} \dots} = \begin{cases} M_{i_1, i_2 \dots} & \sigma \text{ even} \\ M^*_{i_1, i_2, \dots} & \sigma \text{ odd} \end{cases} \quad \text{reality condition}$$

The interaction reproduces the gluing of D D-1 simplexes to give a D simplex

D=2



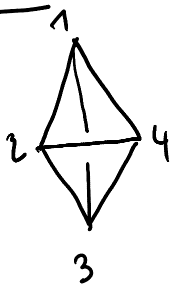
$\partial(123) = (23)\overline{(13)}(12)$

$M_{23} M^*_{13} M_{12} =$

$M_{12} M_{23} M_{31}$

(123 instead of i_1, i_2, i_3)

D=3



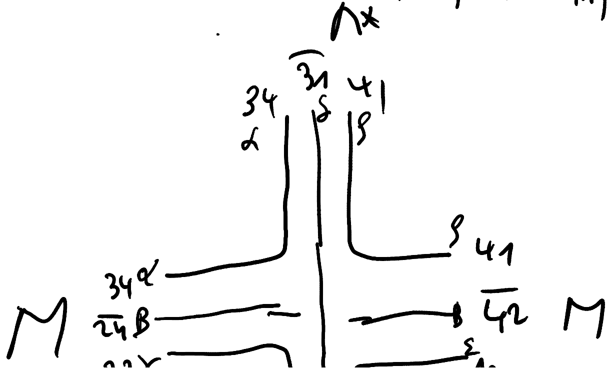
$\partial(1234) = (234)\overline{(134)}(124)\overline{(123)}$

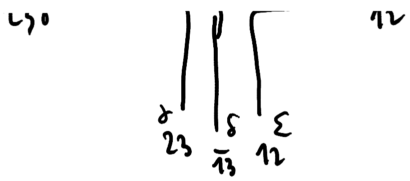
M represents a triangle
(D-1 simplex)

3→4→1   4→2→1

$M_{34,\overline{24},23}\ M^*_{34,\overline{14},13}\ M_{24,\overline{14},12}\ M^*_{23,\overline{13},12} =$

$M_{34,\overline{24},23}\ M^*_{23,\overline{13},12}\ M_{12,\overline{42},41}\ M^*_{41,\overline{31},34}$





M^*

$M_{\alpha,\beta,\gamma}$

$$S = \frac{1}{2}\sum_{\alpha,\beta,\gamma} |M_{\alpha\beta\gamma}|^2 + \frac{\lambda}{3}\sum_{\alpha\beta\gamma\delta\varrho\varepsilon} M_{\alpha\beta\gamma} M^*_{\gamma\delta\varepsilon} M_{\varepsilon\beta\varrho} M^*_{\varrho\delta\alpha}$$

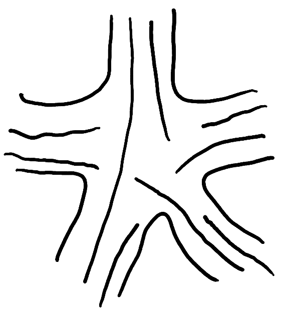
$$\int DM\, e^{iS[M]} = \sum_{T \text{ triangulation}} \frac{1}{S_T} \lambda^{\#\, D\text{ simplex}} N^{\#\, D-2\text{ simplex}}$$

$\alpha \in \{1, .., N\}$
β
$\vdots$

Triangulations dual to the Feynman graphs

Vertices D simplices (tetrahedra) glued along D-1 simplices (triangles)

D=4



Derive this vertex using the boundary of a 4-simplex