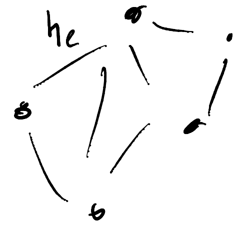
Lecture 2 :

1 marca 2011
07:11

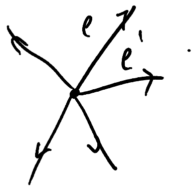
II DYNAMICS

$$\mathcal{H} = \lim_{\Gamma \to \infty} \mathcal{H}_\Gamma / \sim$$



$$\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$$

$$\psi(h_e)$$

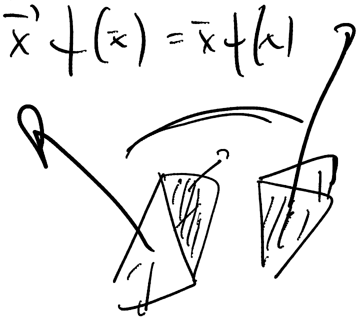


A $\quad G_{ee'} \qquad G_{ee'} = \vec{L}_e \cdot \vec{L}_{e'}$

A_e, $V_n \qquad \vec{R}_{e^{-1}} = \vec{L}_e \qquad \sum_{e \in n} \vec{L}_e = 0$

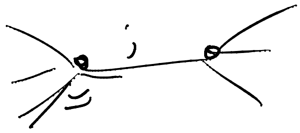
$|\Gamma, j_e, v_n\rangle \qquad |h_e\rangle$

$\hat{h}_e \psi(h_e) = h_e \psi(h_e) \qquad \hat{\vec{x}} \psi(x) = x \psi(x)$



$\vec{L}_e \longrightarrow$ fluxes

$h_e \longrightarrow$ holonomies



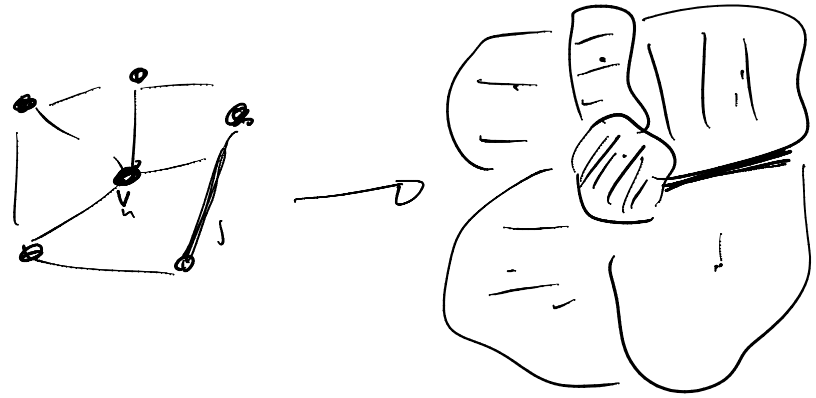
measurement

why this (maybe) in quantum gravity ?

(1) $\boxed{\text{CANONICAL QUANT of GR}}$

GRAV LATTICE QCD → QUANTUM GEOMETRY → QUANTA OF SPACE

(ii) beautiful



(iii) Dynamics → F.E.?

MATH

1. $SL(2,C)$ $\quad SO(3,1)$ $\quad$ Lorentz

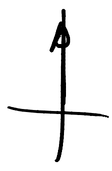
ψ^A $\quad g \in SL2C$ $\quad$ spinor $\quad A=0,1$

V^I $\quad \Lambda^I{}_J$ $\quad$ vector $\quad I,J,=0,1,2,3$

F_{IJ} $\quad J_{IJ}$ $\quad$ adjoint

choose a frame. $H \subset SL(2,C)$

$H \sim SU(2)$



vector $V^I = (v^0, \vec{v}) = (v^0, v^i)$ $\quad i=1,2,3$

ad, $\quad F_{IJ} \to \left(E_i = F_{0i} \quad , B_i = \frac{1}{2} \varepsilon_i^{\ jk} F_{jk} \right)$

$J_{IJ} \to K_i = J_{0i} \quad , L_i = \frac{1}{2} \varepsilon_{ijk} J_{jk}$

spinor. $\quad \underline{\underline{\mathbb{C}^2}} \qquad \mathcal{H}_{1/2} \qquad \langle \psi \phi \rangle = \bar{\psi}^A \phi^B \delta_{AB}$

$\underline{V}^I \Rightarrow v^0 \underline{1} + v^i \sigma_i$

R

→ all finite d. repres of SL2C are $\underline{\underline{\text{not}}}$ unitary

UNITARY repres. of $\underline{\text{SL2C}}$

$J_{IJ} \; : \; \vec{L}, \vec{K}$

$$\left\| \begin{array}{l} C_1 : L^2 - K^2 = p^2 - k^2 \\ C_2 : L \cdot K = 4pk \end{array} \right.$$

$$\left| \begin{array}{l} k = \text{half integer} \\ p = \text{real} \end{array} \right. \qquad \underline{\mathcal{H}_{pk}} = \bigoplus_{j=k}^{\infty} \mathcal{H}^j_{pk}$$

v^m

$$\underline{\underline{D^{pk}(g)^{jm}{}_{j'm'}}} \qquad |p, k, j, m, j', m'\rangle$$

$$\underline{\psi(g)} = \sum c_{pkjmj'm'} \, D^{pk}(g)^{jm}{}_{j'm'}$$

$$\boxed{P=\gamma j \qquad K=j} \qquad |\gamma j, j; jm, jm'\rangle$$

$$\mathcal{H}_\gamma \qquad\qquad \beta=\gamma$$

$$\psi(g) = \sum c_{jmm'}\, D^{\gamma j, j}(g)^{jm}{}_{jm'}$$

(i) $Y_\gamma : L_2[SU(2)] \longrightarrow \mathcal{H}_\gamma$

$Y_\gamma : |j,m,m'\rangle \longmapsto |\gamma j, j; jm, jm'\rangle$

(ii) $\boxed{\langle\psi|\vec{K}+\gamma\vec{L}|\phi\rangle = 0} \qquad \forall\, \phi,\psi \in \mathcal{H}_\gamma$

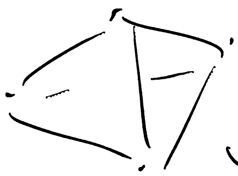
$$C = K^2 - L^2 = \gamma^2 L^2 - L^2 = (\gamma^2-1)\,L^2 = P^2 - K^2 = (\gamma^2-1)\,j^2$$

$$C = 4\vec{K}\cdot\vec{L} = -\gamma L^2 = 4PK = 4\gamma j^2$$

$L^2 \sim j(j+1) \to j^2 \qquad P=\gamma j \quad K=j$

$$\boxed{\vec{K}+\gamma\vec{L} = 0}$$

Spin networks



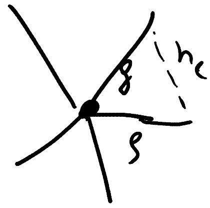
$\psi(h_e)$

$$(Y_\gamma\psi)(g_e)$$

$$P_{SL2C}\,\psi(g_e) = \int_{SL2C} dg_n\; \psi\left(g_{s(e)}\, g_e\, g^{-1}_{t(e)}\right)$$

$f_\gamma : P_{SL2C} \circ Y_\gamma \quad : \quad SU(2)$ s.n. $\to SL2C$ s.n.

$$(f_\gamma \psi)(\mathbb{1}) = \int dh_e\, \psi(h_e)\, A_\gamma(h_e)$$



$$A_\gamma(h_e) = \int_{(SL2C)^N} dg_n \prod_e K_e\left(h_e, g_{se}\, g^{-1}_{te}\right)$$

$$K_e(h,g) = \sum_j \int_{SU(2)} dk\; d_j\, \chi^j(hk)\, \chi^{\gamma j, j}(k,g)$$

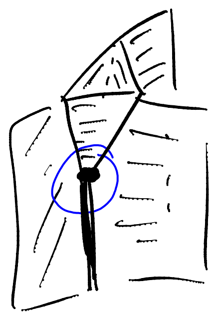
$\uparrow$ SU2 $\qquad$ $\uparrow$ SL2C

$$\chi^j(h) = Tr\left(D^j(h)\right)$$

II MATH $\qquad$ 2-complexes

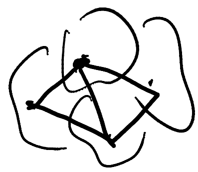
$$\mathcal{C} = (F, \mathcal{E}, V, \partial)$$

f, e, v



Γ_v

⊙ $\lim_{\mathcal{C}\to\infty} F_{\mathcal{C}}$



3d	4d
nodes $\leftrightarrow$ volumes	vertices $\leftrightarrow$ 4-volumes
links $\leftrightarrow$ surfaces	edges $\leftrightarrow$ 3-volumes
	faces $\leftrightarrow$ surfaces

⊙ boundary of a 2-complex

PHYSICS

$\mathcal{C} \qquad \partial\mathcal{C}=\Gamma$

$\mathcal{H}_\Gamma$

$$Z_\mathcal{C}(h_e)=\int_{SU(2)} dh_{vf}\,\prod_f \delta(h_f)\,\prod_v A_v(h_{vf})$$

$\int dh_e\, \overline{A_v(h_e)}\,\psi(h_e)$

$h_f=\prod_v h_{vf}$

$$\langle A_v|\psi\rangle \equiv (f_\gamma\psi)(\mathbb{1}) \qquad . \qquad f_\gamma : P_{SL2C}\circ Y_\gamma \qquad \vec{K}=-\gamma\vec{L}$$

$$Z(h_e)=\lim_{\mathcal{C}\to\infty} Z_\mathcal{C}(h_e)$$

$|\Gamma, j_e, v_n\rangle \qquad \psi_{\Gamma,j_e,v_n}(h_e)$

I STATES

II TRANS AMPL

III Really?

IV Extracting Physics

IV Extracting Physics