Introduction to Loop Quantum Gravity

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A broad perspective on the challenges, structure and successes of loop quantum gravity.

Addressed to Young Researchers: From Beginning Students to Senior Post-docs.

Organization:

1. Historical & Conceptual Setting
2. Structure of Loop Quantum Gravity
3. Outlook: Challenges and Opportunities

1. Historical and Conceptual Setting

Einstein's resistance to accept quantum mechanics as a fundamental theory is well known. However, he had a deep respect for quantum mechanics and was the first to raise the problem of unifying general relativity with quantum theory.

"Nevertheless, due to the inner-atomic movement of electrons, atoms would have to radiate not only electro-magnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in Nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."

(Albert Einstein, Preussische Akademie Sitzungsberichte, 1916)

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⋆ No experimental data with direct ramifications on quantum Gravity.
⋆ But then this should be a theorist's haven! Why isn't there a plethora of theories?

• In general relativity, gravity is coded in space-time geometry. Most spectacular predictions —e.g., the Big-Bang, Black Holes & Gravitational Waves— emerge from this encoding. Suggests: Geometry itself must become quantum mechanical. How do you do physics without a space-time continuum in the background?

• Several approaches: Causal sets, twistors, Non Commutative Geometry, AdS/CFT conjecture of string theory. Loop Quantum Gravity grew out of the Hamiltonian approach pioneered by Bergmann, Dirac, and developed by Wheeler, DeWitt and others.

Contrasting LQG with String theory

Because there are no direct experimental checks, approaches are driven by intellectual prejudices about what the core issues are and what will "take care of itself" once the core issues are resolved.

Particle Physics: 'Unification' Central: Extend Perturbative, flat space QFTs; Gravity just another force.

- Higher derivative theories; • Supergravity
- String theory incarnations:

⋆ Perturbative strings; ⋆ M theory
⋆ Matrix Models; ⋆ AdS/CFT Correspondence.

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• Particle Physics: 'Unification' Central: Extend Perturbative, flat space QFTs; Gravity just another force.
• Higher derivative theories; • Supergravity
• String theory incarnations:
⋆ Perturbative strings; ⋆ M theory/F theory;
⋆ Matrix Models; ⋆ AdS/CFT Correspondence.

• General Relativity: 'Background independence' Central: LQG
⋆ Quantum Geometry (Hamiltonian theory used for cosmology & BHs),
⋆ Spin-foams (Path integrals used to bridge low energy physics.)

• Issues:
• Unification: Ideas proposed in LQG but strong limitations;
Recall however, QCD versus Grand Unified Theories.
• Background Independence: Progress through AdS/CFT; but a 'small corner' of QG; Physics beyond singularities and S-matrices?

A. Ashtekar: LQG: Four Recent Advances and a dozen FAQs;
arXiv:0705.2222

• Recent Thrusts: Quite distinct.

2. Loop Quantum Gravity: Quantum Geometry

- Geometry: Physical entity, as real as tables and chairs.

Riemann 1854: Göttingen Address; Einstein 1915: General Relativity

- Matter has constituents. GEOMETRY??

'Atoms of Geometry'? Why then does the continuum picture work so well? Are there physical processes which convert Quanta of Geometry to Quanta of Matter and vice versa?

Towards Quantum Geometry: Paradigm Shift

- A Paradigm shift to address these issues

"The major question for anyone doing research in this field is: Of which mathematical type *are the variables . . . which permit the expression of physical properties of space. . . Only after that, which* equations *are satisfied by these variables?" Albert Einstein (1946); Autobiographical Notes.*

- Choice in General Relativity: Metric, $g_{\mu\nu}$. Directly determines Riemannian geometry; Geometrodynamics.

In all other interactions, by contrast, the basic variable is a Connection, i.e., a matrix valued vector potential A_a^i;
Gauge theories: Connection-dynamics

- Key new idea: 'Kinematic unification.' Cast GR also as a theory of connections. Import into GR techniques from gauge theories.

In Gravity: basic phase space variables are holonomies of the spin connection along 1-d curves and fluxes of conjugate electric fields E_i^a across 2-surfaces. E_i^a interpreted as *orthonormal frames/triads*. They determine the physical, curved geometry. Structure group: Rotations of triads SO(3) or, in presence of spinors, its double covering SU(2).

Uniqueness of Canonical Quantization?

- von Neumann's uniqueness theorem:

There is a unique IRR of the Weyl operators $\hat{U}(\lambda), \hat{V}(\mu)$ by 1-parameter unitary groups on a Hilbert space satisfying:

i) $\hat{U}(\lambda)\,\hat{V}(\mu) = e^{i\lambda\mu}\,\hat{V}(\mu)\,\hat{U}(\lambda)$; and ii) Weak continuity in λ, μ.

This is the standard Schrödinger representation: $\mathcal{H} = L^2(\mathbb{R}, \mathrm{d}x)$;

$\hat{x}\Psi(x) = x\Psi(x)$; $\hat{p}\Psi(x) = -i\hbar \mathrm{d}\Psi(x)/\mathrm{d}x$, and $U(\lambda) = e^{i\lambda\hat{x}}$, $V(\mu) = e^{i\mu\hat{p}}$

- Strategy for more general systems: Consider the analog $\mathfrak{a}$ of the Weyl algebra. Look for cyclic representations. The 'VEVs' i.e. expectation values in the cyclic state determine the representation through an explicit (GNS) construction. If the VEVs are invariant under a group, the group is unitarily implemented in the representation.

- Uniqueness does not hold for systems with an infinite number of degrees of freedom even after imposing additional symmetry requirements such as Poincaré invariance.

Uniqueness of LQG Kinematics!

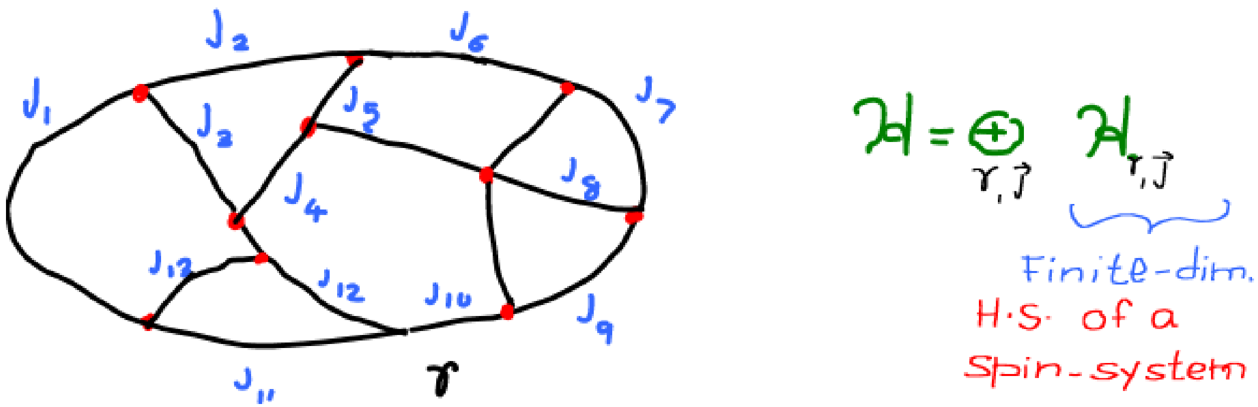
- Surprise: Quantum algebra $\mathfrak{a}$ generated by holonomies and triad-fluxes. It admits a unique *diff invariant* state $\Rightarrow$. Thanks to background independence, quantum kinematics is unique in LQG!
(Lewandowski, Okolow, Sahlmann, Thiemann; Fleischhack)

Surprisingly powerful role of diff invariance! (AA)

Polymer Geometry

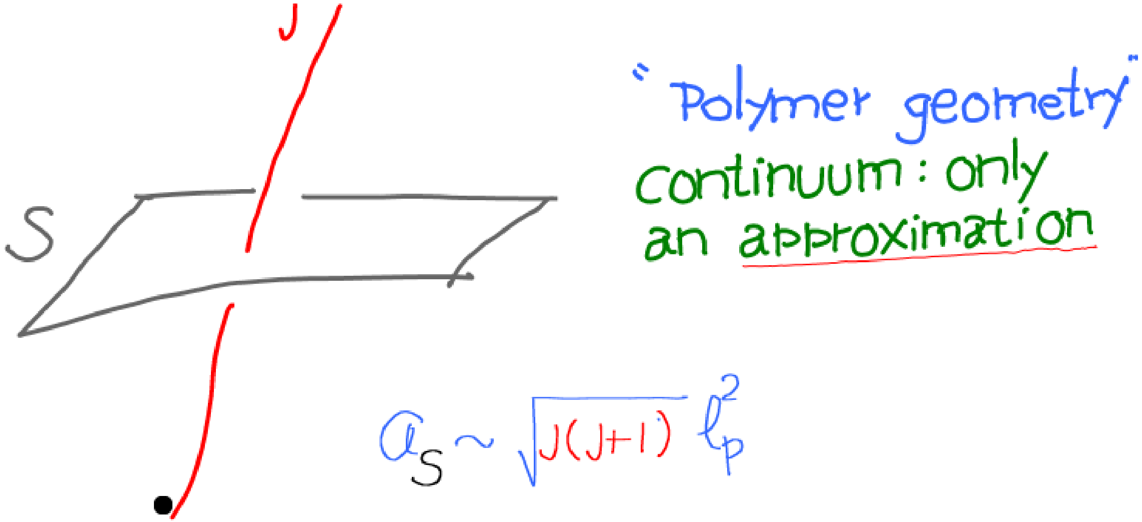
• This unique kinematics was first constructed explicitly in the early nineties. High mathematical precision. Provides a Quantum Geometry which replaces the Riemannian geometry used in classical gravity theories. (AA, Baez, Lewandowski, Marolf, Mourão, Rovelli, Smolin, Thiemann,...)
Details: monographs by Rovelli; Thiemann; Review by AA & Lewandowski

• Quantum States: $\Psi \in \mathcal{H} = L^2(\bar{A}, d\mu_o)$
μ_o a diffeomorphism invariant, regular measure on the space $\bar{A}$ of (generalized) connections.



• Fundamental excitations of geometry 1-dimensional. Polymer geometry at the Planck scale. Continuum arises only in the coarse rained approximation.

- Flux lines of area. Background independence!



- Examples of Novel features:

⋆ All eigenvalues of geometric operators discrete. Area gap. Eigenvalues not just equally-spaced but crowd in a rather sophisticated way. Volume operator: Tamburino's talks. Geometry quantized in a very specific way.

⋆ Inherent non-commutativity: Areas of intersecting surfaces don't commute. Inequivalent to the Wheeler-DeWitt theory (quantum geometrodynamics).

Succinct Summary: AA & Lewandowski, Encyclopedia of Mathematical Physics

see the outreach pages at http://igc.psu.edu

Applications and Lectures at this School

- Quantum geometry used crucially in calculating the statistical mechanical entropy of isolated horizons. Encompass physically realistic black holes as well as cosmological horizons in one go. Because horizons are stationary, dynamical aspects of quantum gravity play a minimal role in *all* approaches.

- This unique kinematical arena provides a Quantum Riemannian Geometry to formulate dynamics, i.e. quantum Einstein's equations. Main challenge of LQG. Progress in the Hamiltonian theory (Giesel, Thiemann, Lewandowski...) and path integrals/spin foams (Baez, Barrett, Reisenberger, Rovelli, Freidel, ...).Two are complementary for addressing long standing problems. Hamiltonian methods well suited particularly for BH entropy, quantum cosmology, ... while spin foams for graviton propagators, low energy limit, ...

- As > 36 hours of lectures of Rovelli, Giesel, Sahlmann, Brunnemann, Krajewski, Speziale, Baratin, Singh, Perini, Fairbairn, Bianchi, and Kaminski will show in detail there have been impressive advances in each of these approaches. The work of the Warsaw group has started bridging the two sets of developments.

3. Outlook: Hamiltonian Theory

- Well established Dirac Program.

⋆ Old Problems: Frozen formalism; Issue of time; Physical Inner product; Dirac Observables; ...

⋆ Solutions: Relational time; Use matter as clocks and rods. (Giesel's lectures) Framework well-understood.

⋆ Realized in detail in cosmological models (Singh's lectures): Status very different from the old WDW theory. There is a physical Hilbert space, Dirac observables; A self adjoint operator for density which has a dynamically induced upper bound. Not only singularity resolved but Planck scale physics explored in detail.

- Quantum Constraints and their Solutions:

⋆ Diff constraint well controlled in LQG ✓

⋆ Hamiltonian constraint has been regulated: nontrivial

But, unfortunately, too many ambiguities !! Furthermore don't understand their physical implications.

⋆ Important lessons from LQC (AA, Pawlowski, Singh): 'Natural, obvious' strategies can lead to untenable physical theories. Won't know if we remain at a formal level. Should take LQC hints seriously.

Hamiltonian Theory: Opportunities

- Use scalar fields as clocks (and rods)

$\star$ Idea goes back to the 90s (Rovelli & Smolin). But not pursued: in the classical theory, works only locally both in space-time & phase space.

$\star$ New Viewpoint: (Warsaw Group) Construct an internally viable quantum theory and then investigate domain of applicability (i.e. right low energy limit). May be just a non-linear neighborhood of FLRW space-times. Still, would be very helpful in practice!!

$\star$ Still challenge remains: Which of the regulated Hamiltonian constraint? But now a greater chance to learn from the LQC ($\bar{\mu}$) dynamics.

- Relation to Non-commutative Geometry

$\star$ Geometry on the configuration space $\bar{A}$ of generalized connections. Geometry on $\bar{A}$ based on the Gel'fand Theory (AA & Lewandowski); Spectral triple based on $\bar{A}$ (Aastrup & Grimstrup)

$\star$ Spatial triads don't commute. (Brown AA, Corichi, Lewandowski, & Zapata). So, no triad representation in the standard QM sense. But a non-trivial Triad Representation (AEI Group): Action by $*$-product.

$\star$ Integrate out the gravitational DOF in 3-d; Left with a non-commutative field theory (again involving $\star$-products). (Freidel & Loupre)

Outlook: Spin-Foams

- Path Integral Approach to LQG

⋆ Avoids 3+1 decomposition. Local Lorentz invariance certainly possible in the Hamiltonian theory but not manifest. Here it is.

⋆ But a fundamental difference from other path integral approaches (e.g., Misner SOH, Regge Calculus, Cambridge School): paths are quantum geometries; based on decorated 2-complexes. uv-finiteness built in. It is the infra-red limit that is tricky.

- Transition amplitude $\longrightarrow$ Extraction amplitude.

⋆ Normally: SOH provides an amplitude for a state at time t to evolve to time t'. Here: Timeless framework to start with. SOH accomplishes two things: Extracts solutions to constraints from the initial and final states AND provides the physical inner product between them. Same goal as in the Hamiltonian theory. (Reisenberger, Rovelli, Baez, ...)
(Perturbative string theory also faces infra-red problems.)

⋆ Concrete proposals for what is summed over, i.e., the analog of e^{iS} for quantum histories (Barrett & Crane, EPRL, FK; KKLM, ...). Elegant formulation with succinct stream-lined logic (Bianchi, Rovelli). Principles to overcome ambiguities in the Hamiltonian constraint?

Spin-Foams: Opportunities

- Answer written as a series: One term for a given number of vertices in the quantum history: Vertex Expansion. Each term in the expansion is an infinite sum over decorations of the 2-complex. Ultra-violet finiteness assured. Infra-red is not.

- Open Issues: Physical Meaning & Convergence of Vertex Expansion

$\star$ Should one sum over 2-complexes with an arbitrary number of vertices or should we take a continuum limit instead? Intriguing direction: The two are equivalent (Rovelli & Smerlak)! But no direct control over the sum.

$\star$ Ideas from Group Field Theory (Oriti, Rivasseau, Gurau, Krajewski...). This is an auxilliary quantum field theory defined not on space-time but on a group manifold. The Lagrangian has a coupling constant λ and interaction term which is polynomial in the fields. Surprisingly, term by term, coefficient of the Feynman expansion in λ appears to agree with the vertex expansion in spin foams!! A new avenue for mathematical control.

$\star$ In LQC since full theory is under control, can address these issues. LQC vertex expansion is a convergent series (AA, Campiglia, Henderson; Bianchi, Rovelli, Vidotto, Wilson-Ewing). GFT $\lambda \leftrightarrow$ cosmological constant Λ in space-time. Can these considerations be generalized? Reverse: Use spin foams for cosmology (Bianchi, Rovelli, Vidotto)?

Spin-Foams: Applications

- Graviton propagator, n-point functions, Perturbation theory

$\star$ Conceptual issues: what does a propagator mean in a background independent context?

$\star$ Elegant solution (Ockel, Colossi, Rovelli, ...). n-point function requires: Boundary state and observables. Use for the boundary state a semi-classical one peaked at flat geometry and sum over all quantum geometries in between. Leading order: Standard flat space graviton propagator! (Perini's talks)

$\star$ Higher order corrections? State dependence? Lorentzian domain/ Why does standard perturbation theory in flat space fail?? Outstanding problems and opportunities. In relation to the importance of the problem, rather few people are working actively.

Outlook: Cosmology

• LQC started with a seminal paper by Bojowald in 2001. Over 500 papers by now. There are three outstanding directions/opportunities.

• Conceptual: Resolution of general space-like singularity?

⋆ Recall the history of space-time singularities in GR.

⋆ BKL Conjecture: As one approaches space-like singularities, "space derivatives become negligible compared to time derivatives" and ""With the exception of scalar fields, Matter does not matter". So dynamics well approximated by that of Bianchi models possibly with a scalar field. Lot of analytical and numerical evidence.

⋆ Singularities resolved in Bianchi models (AA, Wilson-Ewing) and a precise formulation of the BKL conjecture now exists in the LQG variables.

Theorems that general space-like singularities are resolved?

• Structural Issue: Contact with QFT in curved space-times?

⋆ Early work: Sahlmann & Thiemann. More recently, QFT in Quantum Cosmological Space-times constructed. Shown to reduce to QFT in curved space-times in a series of successive approximations (AA, Kaminski, Lewandowski). But motivation came from cosmology. Functional analytical issues of QFT not faced head-on.

⋆ Systematic treatment? Anomalies? Mean field theory with fluctuations starting from the Warsaw Group work in Hamiltonian LQG?

Inflation & Loop Quantum Gravity

• Initial motivations for inflation not really strong. Criticized by prominent relativists. But success with structure formation is spectacular. We can'nt afford to keep ignoring it!

• Essentially, just 4 assumptions:
i) Universe underwent an accelerated expansion (inflation) in its early history;
ii) During this phase, FLRW space-time with a scalar field (at zero temperature) and 1st order quantum perturbations thereon provide a good approximation;
iii) initial state of QFs describing matter+gravitational perturbations is given by the Bunch-Davis (adiabatic) Vacuum;
iv) Quantum fluctuations can be treated by classical perturbation equations once they exit the Hubble horizon.

• In LQC with a wide class of potential, can start with *all possible* data at the bounce. Almost all (probability > 0.9999) lead to a slow roll inflation compatible with the WMAP observations! (AA, Sloan)

Cosmology: Opportunities

- Can naturally explain the structure of the CMB inhomogeneities, including the scalar spectral index, etc! Furthermore, these when evolved by Einstein equations, lead to the observed large scalae structure. Everything in the universe arose from quantum fluctuations in the vacuum!!

- Like the Bohr atom: Assumptions ad-hoc but one explains so much more than what is put in that there must be an essential germ of truth.

- Like Bohr atom complete description likely to be very different. What is it? Can LQG provide this more complete description, starting from well motivated initial conditions at the bounce? Predictions for future Missions? Our best chance for making direct contact with observations in coming years.

Directions for Future Advances

• Future success will require us to be Outward Bound. Solid formalism essential. But not an end. Have to go beyond. Have to solve other peoples' problems!! Cosmology offers excellent opportunities.

• Can't afford to do a first stab, get a desired result and declare victory. Doesn't help to have a new model every year. Need sustained progress. Have to pursue & develop ideas in detail.

Examples: Derive LQC from the Warsaw LQG framework of scalar field; Graviton propagator in the Lorentzian domain; State dependence: Any invariant information? Why exactly does the standard perturbation theory fail? Can group field theory really control the sum in the vertex expansion? Matter fields in Spin foam? Control on ambiguities in the Hamiltonian constraint? Their physical meaning? Fate of generic singularities in LQG?

• Practical but all important points:

⋆ Sustained success in research requires a fine balance between optimism without bars and critical evaluation!

⋆ You cannot be more successful that the whole research program. Have to support each other.

A *Really* practical matter

Register for Loops11 - Madrid as soon as possible. Deadline in 10 days. Significant support has been procured. But only those who have registered will qualify. You will not be charged registration fee etc until AFTER support decisions are made.