1. MOTIVATION

Several information theoretic approaches to foundations of quantum theory:

* Fuchs, Caves, Schack, ...
* D'андіо, Chiribella, Perinotti
* Bus, Clifton, Halvorson
* Abramsky, Cecke, Vicary, ...

aim to derive quantum theory from another theory.

BUT:

* What is the range of structures of "quantum theory" to be derived?
  (how about QFT? renormalisation? QG?
  how to construct Hamiltonians directly from experimental data?
  which non-unitary reduction method is correct?)

* Why choose this particular "another theory" as foundations?
  (why Kolmogorov's measure-theoretic probability?
  why symmetric monoidal 1-categories?)

NOTE:

* Different foundational approaches have different limits of applicability and leave many problems of quantum theory unresolved.
* Their own foundations should be more appealing than the quantum theory itself.
II. AIM & PLAN

- Having these issues in mind, we'll develop new approach to foundations of QM.
- Our aim: to cover most general forms and uses of quantum theory, including quantum field theory (and aiming at quantum gravity).
- Note: at such level, quantum theory still lacks (and asks for) mathematically strict form.
- We are less concerned with axiomatisation (at this stage). Without strict mathematics, axiomatisation is premature.

PLAN OF THE TALK:

1. Review of foundations of probability theory
2. New foundations for probability theory
3. Review of mathematical foundations of quantum theory
4. New mathematical framework: quantum information geometry
5. New conceptual framework: intersubjective coherence interpretation
6. Recovery of traditional quantum theory and probability theory as special cases
Borel-Kolmogorov:

- Measure spaces \((X, \mathcal{V}(X), \mu)\), probabilistic models
- \(M(X, \mathcal{V}(X), \mu) \leq L^1(X, \mathcal{V}(X), \mu)\)

- No generic notion of conditional updating of probs.

- Note: there are many measure spaces leading to \(L^1\) spaces isomorphic to \(L^1(\mathcal{V})\) space, when \(\mathcal{V}\) is a c.a.d.c Boolean algebra, constructed by \(\mathcal{V} = \mathcal{V}(X)/\{A \in \mathcal{V}(X) | \mu(A) = 0\}\). Thus, only \(\mathcal{V}\) is important for probabilistic models.

- Note: the description in terms of measures on \((X, \mathcal{V}(X))\) can be completely replaced by the description in terms of integrals on vector lattice \(A(\mathcal{V})\), canonically associated with \(\mathcal{V}\). Thus one can deal exclusively with integrals on \(\mathcal{V}\) instead of measures on \(\mathcal{V}(X)\).

Bayes-Laplace:

- Finitely additive Boolean algebras and cond. prob.'s \(p(A|B)\)
- Bayes' rule \(p(A|I) \to \text{prev}(A|I) = p(A|I) \frac{p(B|A, I)}{p(B|I)}\)

- No generic notion of probabilistic expectation over continuous domain

- Note: Bayes' rule is a special case of constrained relative entropy updating:

\[
p(x|\theta) \to \text{prev}(x|\theta) = \arg \min_{p} \left\{ SE(p, p') + F(q) \right\} \text{ for constraints } F(q) = \lambda \left( \int dx \theta q(x|\theta) - 1 \right) + \lambda_{2} \left( \int dx \theta q(x|\theta) - \mathcal{E}(x-b) \right), \text{ prior } E\theta(p, p') = D(p, p'|\theta) \text{ and } D \text{ given by the Kullback-Leibler divergence}.\]
IV. PROBABILITY THEORY: INTERPRETATIONS

Frequentist:  [von Mises, Wald, Church, Martin-Löf]
* still widely believed, but none mathematically strict and logically sound formulation exists
* note: the normalisation of probabilities is required only due to this interpretation

[Subjective] bayesian:  [de Finetti, Savage]
* conceptually consistent, but by definition it lacks any inter-subjectively valid rules relating the probability assignments (theoretical model construction) with numbers, words, and other knowledge about experiments (experimental setup construction, experimental data?)
* note: accusations of arbitrariness are unjustified, because any theoretical statement is arbitrary
* note: accusations of arbitrariness are justified, because scientific inquiry seems to be somewhat more than individual personal coherence of bets.

[Objective] bayesian:  [Cox, Jaynes]
* claims to provide general rules of assignment of probabilities (= model construction), but fails to provide sound justification for these rules which would be neither epistemic-personalist (subjective bayesian) nor ontological-idealist (frequentist)
* note: the intersubjective coherence of relationship between theory and experiment is a crucial idea!
V. PROBABILITY THEORY: NEW APPROACH

Mathematical framework:
* Take the best insights from B-K and B-L approaches, and unify kinematic (probabilistic, evaluational) and dynamic (statistic differential, relational) components.
* Replace measure spaces \((X, \mathcal{V}(X), \mu)\) by (causal) boolean algebras \(\mathcal{V}\).
* Information kinematics: models \(\mathcal{M}(\mathcal{V}) \leq \mathcal{L}(\mathcal{V})\) defined as spaces of finite positive integrals on \(\mathcal{V}\).
* Information dynamics: updating by constrained relative entropy maximization on \(\mathcal{M}(\mathcal{V})\) with divergence \(D\), prior \(E\), and constraints \(F\).

Interpretation:
* Take the best insights from 'subjective' and 'objective' bayesianism & take lessons from Fleck's analysis of the structure of scientific theory.
* Require that the particular rules of probability assignment should correspond to the particular methods of relating experimental setup construction with the theoretic model construction.

Only some specific constraints are used to check whether some particular (very complicated) apparatus is an intersubjectively acceptable instance of the "ideal experimental setup of a given type."

Beyond community of intersubjective methods of experimentation, the particular model construction rules are irrelevant (arbitrary, personalistic, 'subjective'), but within this range they are indispensable (necessary, scientific, 'objective').
VI. QUANTUM THEORY: HILBERT SPACE FRAMEWORK

Formalism:
* Kinematics: Hilbert spaces, algebra of operators, density operators
* Dynamics: two different forms of temporal evolution
  (unitary Schrödinger, non-unitary von Neumann-Lüders)
* Quantification: Spectral representation in terms of Hilbert space
  based on Kolmogorov's probability measures

Problems:
* It lacks any definite methods of model construction that
  would correspond to the particular experimental
  situation (⇒ quantization, perturbation, renormalization,
  and other ad hoc techniques)
* It lacks any definite relationship between two temporal
  evolutions
* It fails to describe relativistic quantum field models
  and continuous statistical mechanics models

Semi-spectral (novel) perspective:
* Requires more direct relationship between theoretical
  model construction and experimental construction
  [Busch Grabowski: Lahti '95, de Huguet '02]
* Shows the breakdown of bijection between the notion
  of (measurable quantity) and self-adjoint operators
  but shares all problems of von Neumann's approach.
VII. QUANTUM THEORY: ALGEBRAIC FRAMEWORK

**Formalism:**
- **Kinematics:** $C^*$-algebras of operators, states on these algebras
- **Dynamics:** *-automorphisms of $C^*$-algebras and corresponding unitary operator
- **Quantification:** Spectral representations again

**Virtues:**
- Improves over Hilbert space approach allowing mathematically strict relativistic quantum field models and continuous quantum statistical models

**Problems:**
- It lacks any definite methods of model construction that would correspond to the particular experimental situation ($\Rightarrow$ applied QFTs use ad hoc methods)
- It ignores the non-unitary (reduction), leaving the problem unsolved.
Insights:

Positive:
* the theoretic model construction should be strongly tied with the experimental setup construction,
* if one regards from interpreting the self-adjoint elements of algebra as (measurable quantities),
* the relativistic quantum field theory and continuum quantum statistical mechanics can be dealt with, if one works with abstract C*-algebras (or W*-algebras) and states on these algebras.

Negative:
* two conflicting temporal evolutions in a single time
* no general method of theoretical model construction associated with a given experimental situation
* appealing to Kolmogorov's framework of probabilistic measures for quantitative analysis of quantum theoretic models:
  - Kolmogorov's framework also does not possess any general method of theoretical model construction (= ad hoc methods of statistical inference: estimation, testing ...)
  - Kolmogorov's framework lacks any sound conceptual justification
  - awkward contextualism related with the necessity of selection of commutative subalgebra for the purpose of probabilistic interpretation
IX. NEW MATHEMATICAL FOUNDATIONS FOR Q.T.

In face of above insights and new foundations for probability, we deny the following principles:

* quantisation
* spectral representation in terms of measure spaces and Bochner-Kolmogorov probability theory
* two evolutions (Schr., W*-algebras) in a single time
* Hilbert spaces

Instead of this, we propose:

* replace Hilbert space (too rigid) and C*-algebras (too general) by W*-algebras $\mathcal{N}$, because only these non-commutative algebras allow integration theory
* information kinematics: models $M(\mathcal{N}) \subseteq L_1(\mathcal{N}) \cong \mathbb{N}_0^\infty$, defined as spaces of positive finite integrals on $\mathcal{N}$
* information dynamics: updating by constrained quantum relative entropy maximisation on $M(\mathcal{N})$ with divergences $D_\mu$, prior $E_\mu$, and constraints $F$
* quantitative model construction: based on quantum information geometry on $M(\mathcal{N})$ and its representation $M(\mathcal{N}) \ni \phi \rightarrow \rho_{\phi} \in L_1(\mathcal{N})$ in $\mathbb{N} \in L_1(\mathcal{N})$ spaces.
* two different times: external dynamics $t$ and internal kinematics
* external time evolution: non-unitary, uniquely associated with given constraints $F(\omega) = f(\omega)$, and generated by relative entropy updating on $M(\mathcal{N})$
* internal time evolution: unitary, uniquely associated with every faithful state information state on $M(\mathcal{N})$, and generated by Tomita-Takesaki modular hamiltonian.
X. RECONSTRUCTION OF STANDARD FRAMEWORKS

Probability theory:
* if $\mathcal{W}$-algebra is commutative, then it naturally generates
  a $\mathcal{D}C_0$ boolean algebra $\mathcal{B}$
* all corresponding structures reduce to their commutative
  counterparts in our new framework for probability theory,
  except the internal time evolution, which disappeared
* our generalization of probability theory, in appropriate
  limits, reduces to Borel-Kolmogorov and Bayes-place
  frameworks

Hilbert space based quantum theory:
* by Gel'fand-Naimark-Segal theorem, each $\rho \in \mathcal{M}(\mathcal{W})$
  generates a unique associated Hilbert-space $\mathcal{H}_\rho$ equipped
  with a representation $\mathcal{T}_\rho(\mathcal{W}) \cong \mathcal{B}(\mathcal{H}_\rho)$ of $\mathcal{W}$.
* by Tomita-Takesaki theorem, each $\rho \in \mathcal{M}(\mathcal{W})$ is
  faithful, this representation is uniquely equipped
  with a unitary evolution $\mathcal{U}_\rho(t) = e^{-i t \mathcal{H}_\rho}$.
* all representations $\{ \mathcal{T}_\rho(\mathcal{W}) \mid \rho \in \mathcal{M}(\mathcal{W}) \}$ are unitary.
  Isomorphic (what is the case, by Stone-von Neumann
  theorem, in orthodox QM), then all fibers of $\mathcal{W}$ $\mathcal{M}(\mathcal{W})$
  can be identified with a single Hilbert-space $\mathcal{H}$.
* in such case, the unitary operators generating these
  isomorphisms become joined with the Tomita-Takesaki
  evolution, and recover the unitary evolution
* the perturbations of this evolution by (already determined)
  (interaction) and the $\mathcal{W}$-Lie rule come directly from
  constrained maximum entropy updating.
XI. PICTURE

\[ U \xrightarrow{\mathcal{M}_N} \]

\[ U_{w_1}(s) \xrightarrow{\mathcal{M}_2} U_{w_2}(s) \xrightarrow{\mathcal{M}_3} U_{w_3}(s) \]

\[ \text{(TR) internal-time evolution in the bundle of (GNS) Hilbert spaces} \]

\[ \text{information model} = \text{quantified knowledge} \]

\[ \text{abstract language of qualities (algebra) subjected to quantitative evaluation (finite positive integration)} \]

\[ w(D, E, F(t)) \]

\[ w_3 \]

\[ w(D, E, F) \]

\[ \text{continuous information dynamics} \]
XII. INFORMATION SEMANTICS & INTERSUBJECTIVE INTERPRETATION

Semantics:
* \( W^\#, \) algebras \( W \) and \( \theta = \text{boolean algebra, } V \) represent
  the abstract qualitative language used as a reference for intersubjective communication
* \( M_W \) models \( M(W) \) and \( M(V) \) represent
  the quantified intersubjectively shared knowledge
* \( \text{information geometry, } M \) server for description
  (in terms of: metrics, connections, divergences, convex sets...)
  and quantification (in terms of \( L_p \) space representations)
  of kinematics and dynamics of information models
* \( \text{external time is a } \) time of becoming \( = \)
  = an epistemically definable and controllable parameter
* \( \text{internal time is a } \) time of being \( = \)
  = a mathematical parameter
  arising from representation of qualitative language in context of quantitative knowledge

Interpretation:
* 'Quantum theory' does not govern anything!
* 'Quantum theory' is just a particular mathematical formalism for statistical inference plus particular methods of intersubjective experimentation plus particular methods of relating these two, that are relative to the community of users, which set the range of 'intersubjectivity'
* No paradoxes due to strict denial of ontological idealism