



Calculators of two-body kinematics

Krzysztof Piasecki

[NRV]

I. Zagrebaev et al. , JINR Dubna

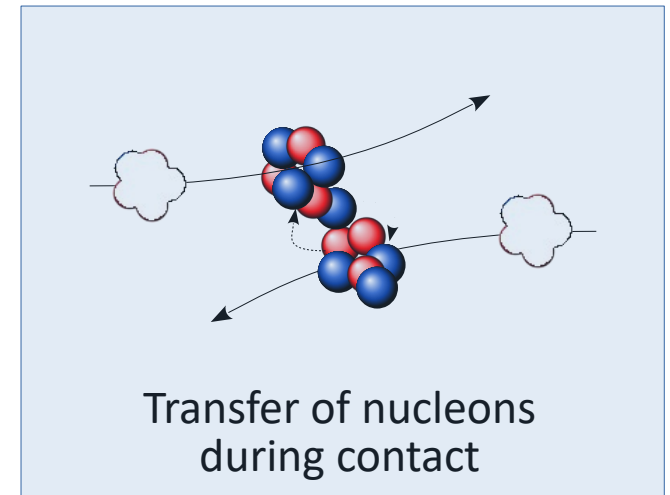
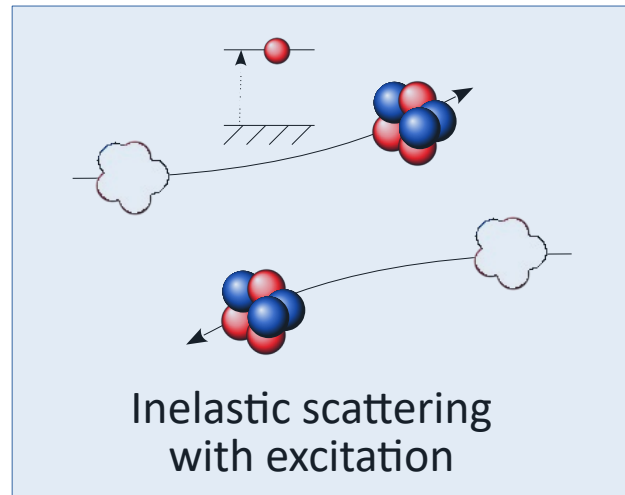
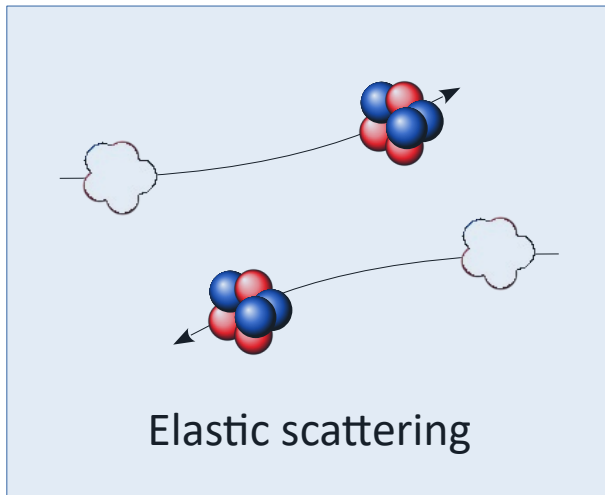
[LISE++]

O.B Tarasov, D. Bazin , MSU East Lansing

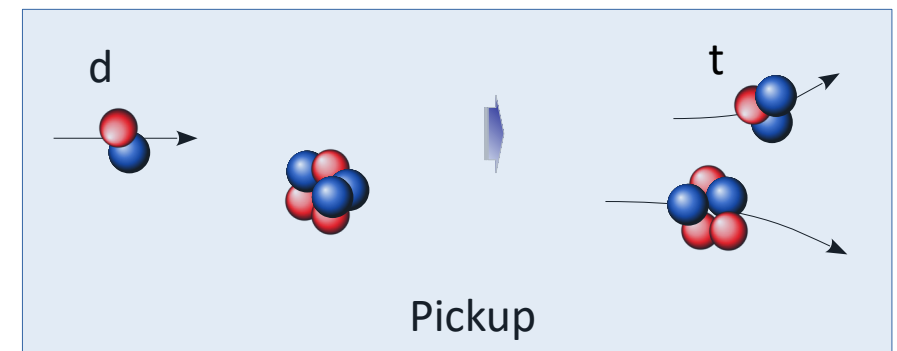
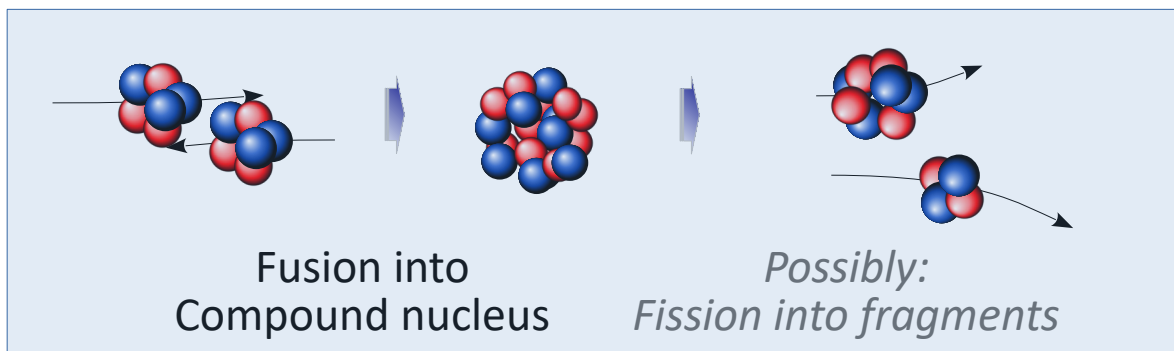
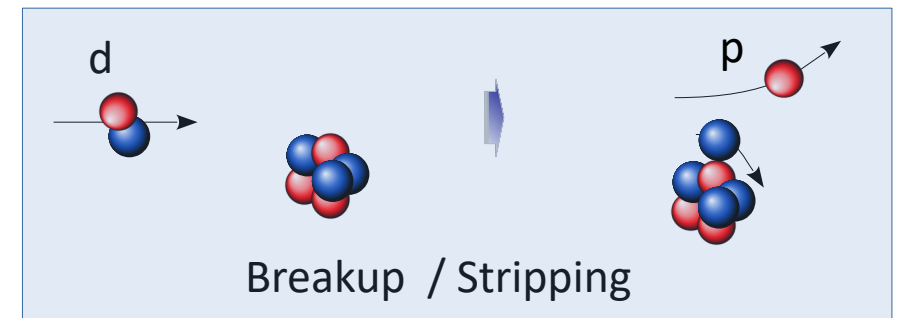
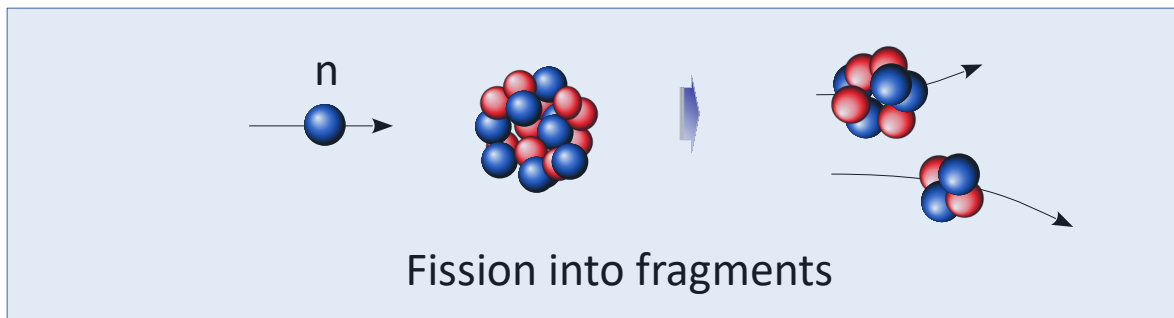


Scattering and reactions of two nuclei (lower energies)

- Two nuclei in the outgoing channel

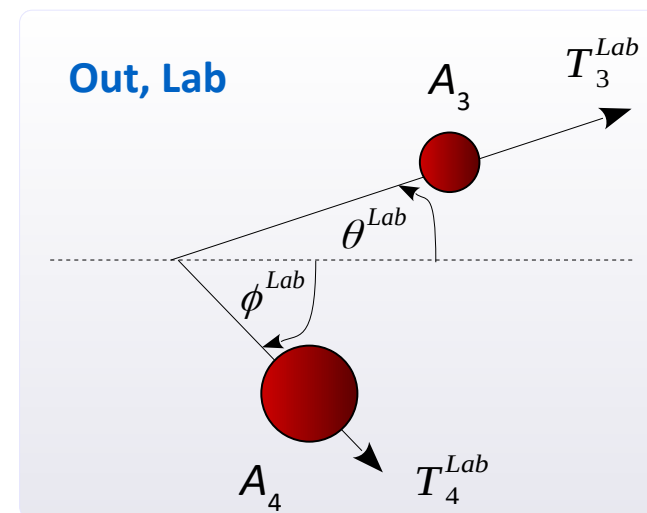
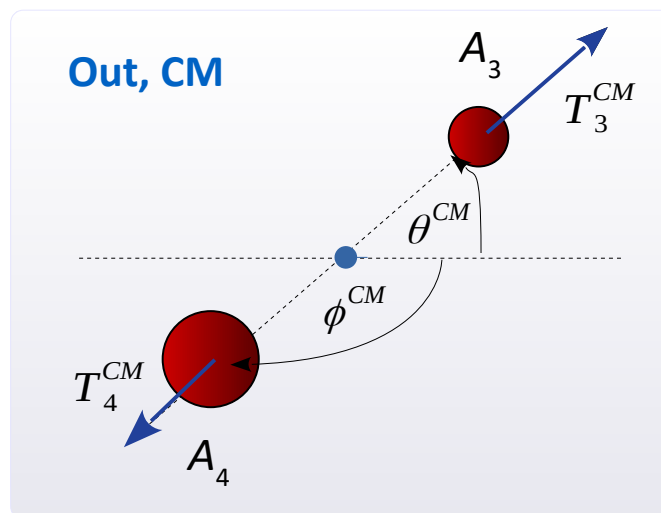
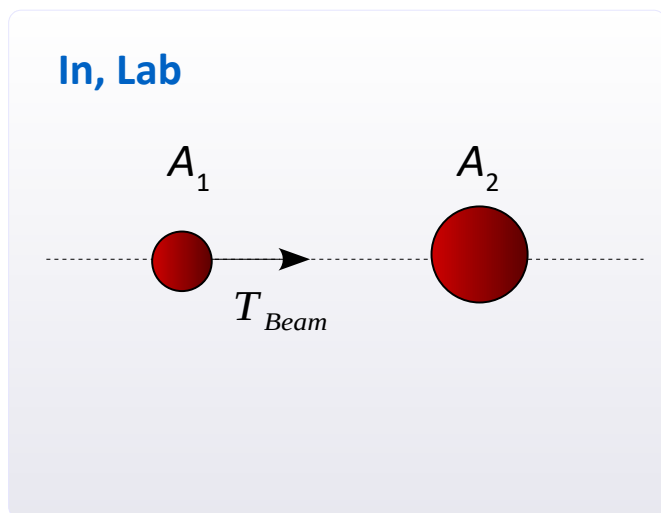


- Two / more nuclei in the outgoing channel + possible further emission of γ / n



Some facts on non-relativistic kinematics of elastic collision

- The beam nucleus A_1 is accelerated to kinetic energy T_{Beam} and hits the stationary target nucleus A_2 .



- If the scattering or collision is elastic, $A_3 = A_1$ and $A_2 = A_4$. Applying E and p conservation laws gives:

$$T_3^{\text{Lab}} = T_{\text{Beam}} \frac{(A_1 + A_2 \cos \theta^{\text{CM}})^2 + (A_2 \sin \theta^{\text{CM}})^2}{(A_1 + A_2)^2}$$

where

$$\tan \theta^{\text{Lab}} = \frac{\sin \theta^{\text{CM}}}{\cos \theta^{\text{CM}} + \frac{A_1}{A_2}}$$

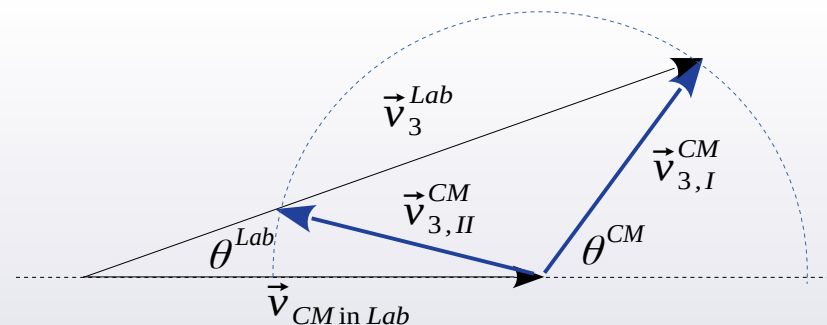
$$T_4^{\text{Lab}} = T_{\text{Beam}} \frac{A_1 A_2}{(A_1 + A_2)^2} (2 \sin \phi^{\text{Lab}})^2$$

$$\phi^{\text{Lab}} = \frac{\pi}{2} - \frac{\phi^{\text{CM}}}{2}$$

- Caution:** if $A_1 > A_2$, then:

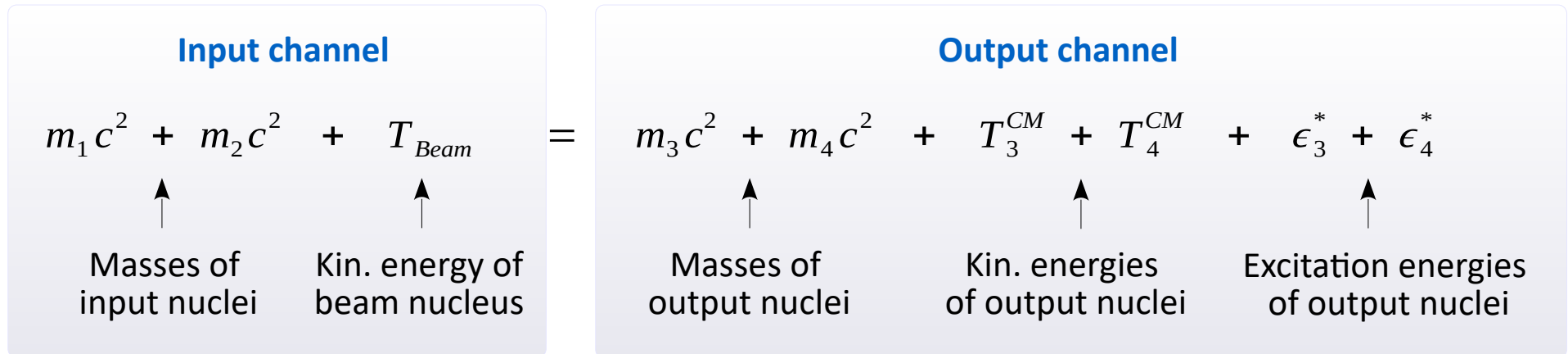
- one θ^{Lab} is realized by two θ^{CM}
- θ^{Lab} has an upper limit.

This situation is called **reverse kinematics**.



Heat of reaction Q

- **Energy balance** for a two-body process (not necessarily elastic one), in the CM frame:



- Imagine a specific output channel (nuclei of masses m_3, m_4 in excited states of energies $\epsilon_3^*, \epsilon_4^*$). Can this reaction undergo freely? Does it release (kinetic) energy spontaneously? Or should energy be pumped (via beam kinetic energy)?

- **Heat of reaction, Q:**

$$Q \stackrel{df}{=} (m_1 + m_2) - (m_3 + m_4) - (\epsilon_3^* + \epsilon_4^*)$$

$$\Rightarrow Q \stackrel{numerically}{=} T_3^{CM} + T_4^{CM} - T_{Beam}$$

If $Q \geq 0$, this process can occur **spontaneously**

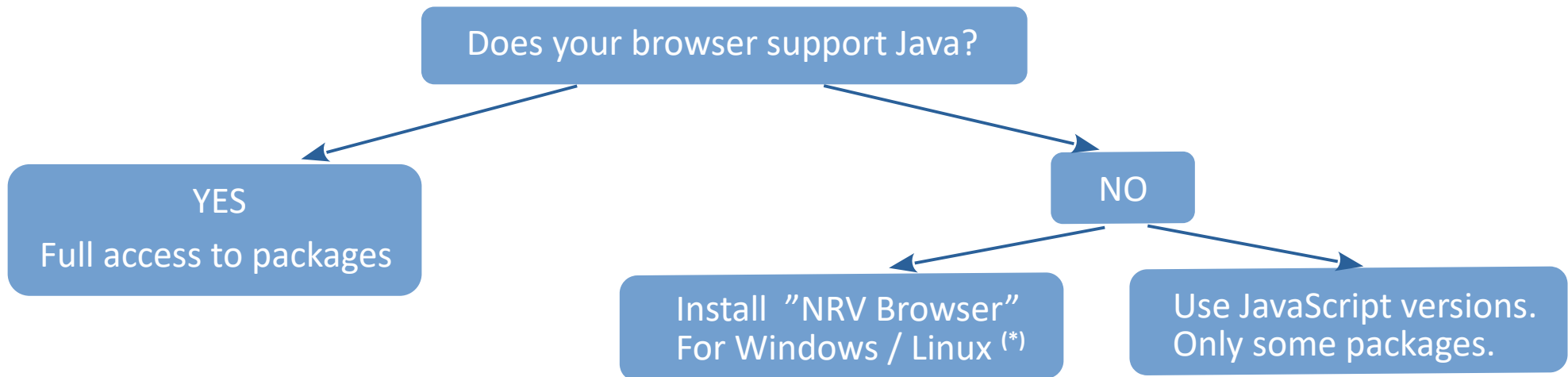
If $Q < 0$, it cannot. We must supply this amount of energy (via beam kinetic energy)

If the collision is **elastic**, then $Q = 0$.

- **Nuclear Reactions Video** by V. Zagrebaev's group @ JINR Dubna around 1999.

It is a "low-energy knowledge base" with nuclide chart, data on nuclei, nuclear processes, kinematics calculator.

Flagship paper: A.V. Karpov *et al.*, "NRV web of knowledge base on low-energy nuclear physics", [Nuclear Instruments and Methods A 859, 112 \(2017\)](#)



- **Nuclear properties** – chart of nuclides. Data for each nuclide: binding energies, deformations, excited states (E, spins) and decay properties (BR, Q-values).
- **Systematics** – provide graphs of: separation / binding energy, size, half-life, deformation, fission barriers as function of A / Z / N for a selected one of these values.
- **Kinematics calculator** – provides translations between energies and emission angles of nuclei emitted in 2-body nuclear collisions:
 - elastic scattering (from Coulomb or nuclear interaction)
 - inelastic scattering (with excitation of any of nuclei; via Coulomb or nuclear forces)
 - transfer of some nucleons between nuclei during the collision

(*) On NPD training computer, type: ① `cp -r ~kpiasecki/soft/nrv/ctnp/* .` ② `./run.sh`

Kinetic energy of beam nucleus
Caution: press enter!

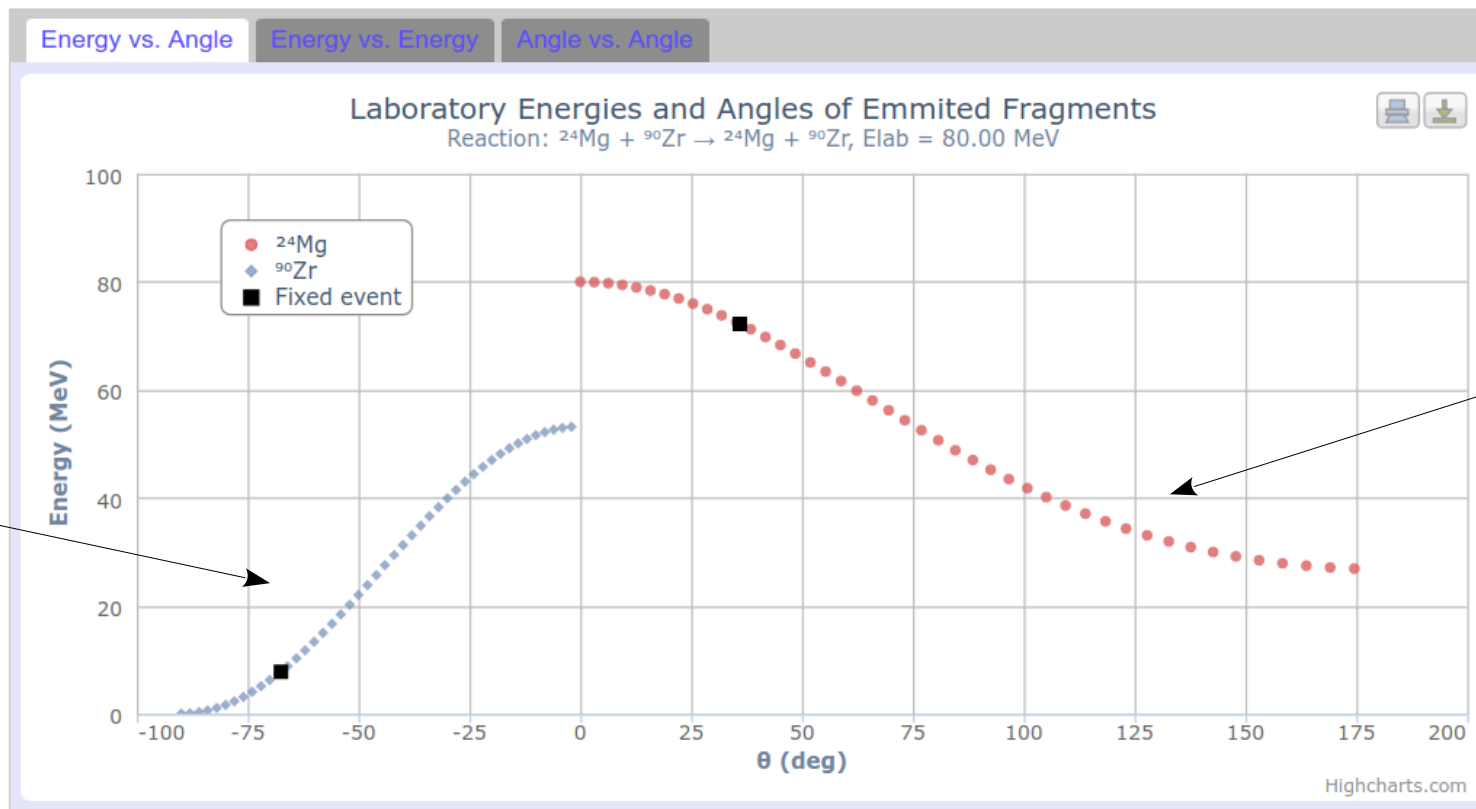
Projectile: Mg 24 + Target: Zr 90 → Projectile-like fragment: Mg 24 + Target-like fragment: Zr 90 + $Q_{tot} (= 0.0 \text{ MeV}) [1]$
 $E_{lab} = 80.00 \text{ MeV}$
 $\epsilon_{PLF} = 0.0 \text{ MeV}$ $\epsilon_{TLF} = 0.0 \text{ MeV}$ $E_{cm(out)} = 63.16 \text{ MeV}$
 Calculate $\Delta\theta_{cm} = 4.00 \text{ deg}$

After typing nuclei and E , Q is shown automatically.

Excitation energies of PLF and TLF

PLF = Projectile-like fragment

TLF = Target-like fragment



PLF

TLF

In geom. model maximal (output) scattering angle before nuclei touch each other →

	θ_{lab} (deg)	E_{lab} (MeV)
^{24}Mg	$0.0^\circ < \theta_{lab} < 180.00^\circ$	$26.81 < E_{lab} < 80.00$
^{90}Zr	$0.0^\circ < \theta_{lab} < 90.00^\circ$	$0.0 < E_{lab} < 53.19$

Grazing collision ^{[4], p.7}: $\theta_{gr(cm)} \approx 143.3 \text{ deg}$, $\theta_{gr(lab)} \approx 131.9 \text{ deg}$, $b_{gr} \approx 1.7 \text{ fm}$, $L_{gr} \approx 13 \hbar$
Coulomb barrier ^{[4], p.326, 332}: $V_C \approx 62.2 \text{ MeV}$, $R_C \approx 10.3 \text{ fm}$

Fixed event parameters
 $E_{PLF} = 72.21 \text{ MeV}$ $\theta_{PLF} = 35.99 \text{ deg}$
 $E_{TLF} = 7.79 \text{ MeV}$ $\theta_{TLF} = -67.50 \text{ deg}$
 Show/Hide $\theta_{cm} = 45.00 \text{ deg}$

Give some input values and you'll get output & black point on the plot.

Save data as ASCII file

- **LISE++** by O.B Tarasov and D. Bazin @ Michigan State University (MSU)

Its main goal concerns **fragment separators** (transmission and yields of fragments produced / collected there). However, it offers several calculators, including the **Relativistic Kinematics Calculator**.

WWW: <https://lise.nscl.msu.edu>

Papers: [2008], [2016], [2023] .

Installation:

- ▶ Download: [[HERE](#)]
- ▶ NPD training computer: first install on your account:
 - ① `mkdir lise; cd lise`
 - ② `cp ~kpiasecki/soft/lise++/ctnp/lise_setup.sh`
 - ③ `./lise_setup.sh`Then run using wine:
 - ④ `nice wine ./LISE++.exe`

- **Instructions** on **Relativistic Kinematics calculator**: [[HERE](#)] and [[HERE](#)]

- **How to open Calculator**:

▶ From the upper toolbar, choose:



- Different **types of processes**:

Scattering: $A + B \rightarrow A + B$, nuclear convention: $B(A, A)B$

Two body: $A + B \rightarrow C + D$, nuclear convention: $B(A, C)D$

Breakup (fission), but also: decay.

Understood as „spontaneous” process, i.e. symbol x means none.

Either fission of nucleus ($A \rightarrow B C$), or decay from excited state ($A^* \rightarrow B C$)

Reactions

TWO BODY $B(A, C)D$

SCATTERING $B(A, C=A)D=B$

BREAKUP (FISSION) $x(A, C D)x$
(or γ -emission)

- Different **types of beam**:

Beam

Heavy ion **Neutron** **Gamma**

- Target has **thickness**
⇒ substrates and products **lose energy**.

Lise needs to be told,
where in the target the reaction occurs.

Also, if energy of products should be given
just after reaction, or after crossing the target.

Reaction takes place at the target

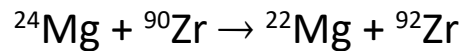
Entrance **Middle** **Exit**

For Kinematic Plots use energy values

after reaction

at entrance of detectors

● **Example**



$$T_{\text{Beam}} = 30A \text{ MeV}$$

Target thickness: 1.5 μm

Reaction: mid-target

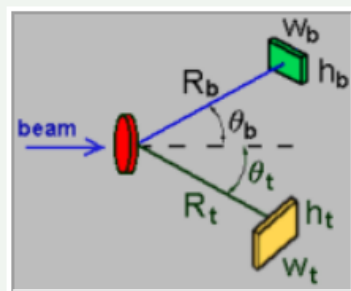
Reactions

TWO BODY B (A, C) D

SCATTERING B (A, C=A) D=B

BREAKUP (FISSION) x (A, C D) x (or γ -emission)

Fission (breakup) batch mode



For Kinematic Plots use energy values

after reaction

at entrance of detectors

Click



- Kinematics plots
- Rutherford plot
- 2D fragment plot (Monte Carlo)

Beam

Heavy ion Neutron Gamma

Participants

		ME [MeV]	Excitation Energy	E(CM) = 565.74 MeV	
A	Beam	²⁴ Mg	-13.93	0	Beam energy 30 MeV/u
B	Target	⁹⁰ Zr	-88.77	0	Intensity 1 kW
C	Fragment	²² Mg	-0.4	0	Target thickness 1.5 micron
D	Residual	⁹² Zr	-88.46	0.001	Q_value = -13.85 MeV

Q_opt= +0.0 & Ex= -13.8 MeV

Reaction takes place at the target

Entrance **Middle** Exit

Set-up

Search an angle in CM

from 0 degrees and up

from 180 degrees and down

	fragment (C)	residual (D)
R =	10000 cm	10000
w =	1 cm	1
h =	2 cm	2

Selected participant

LAB "C" LAB "D" **CM "C"** CM "D"

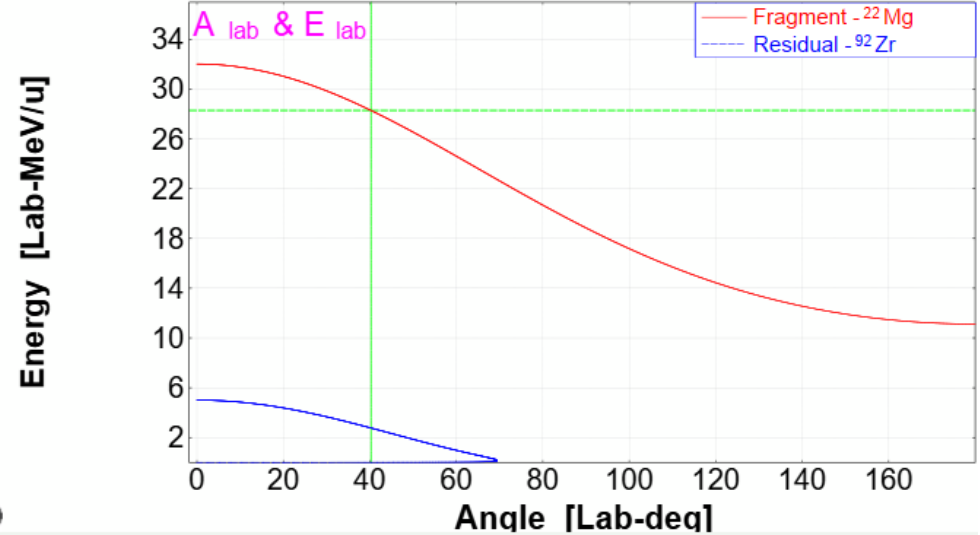
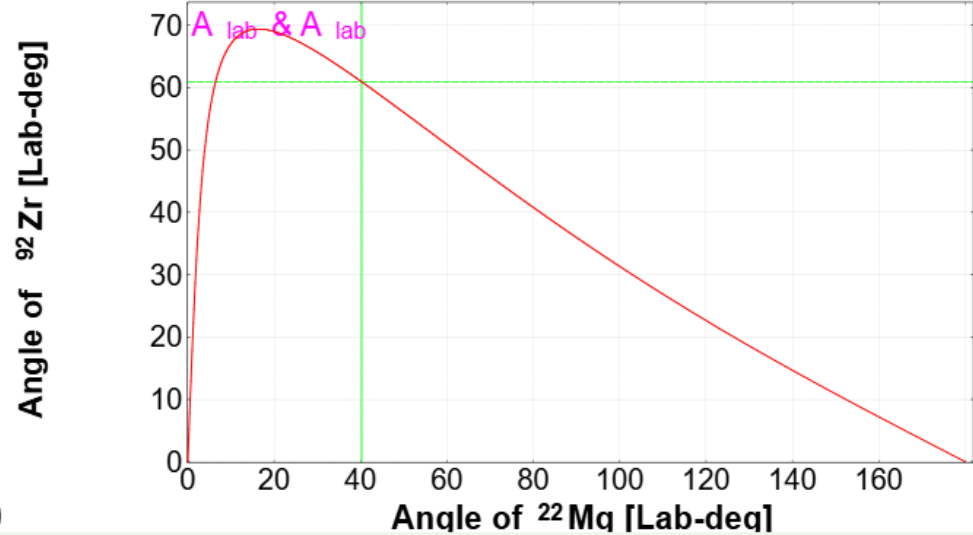
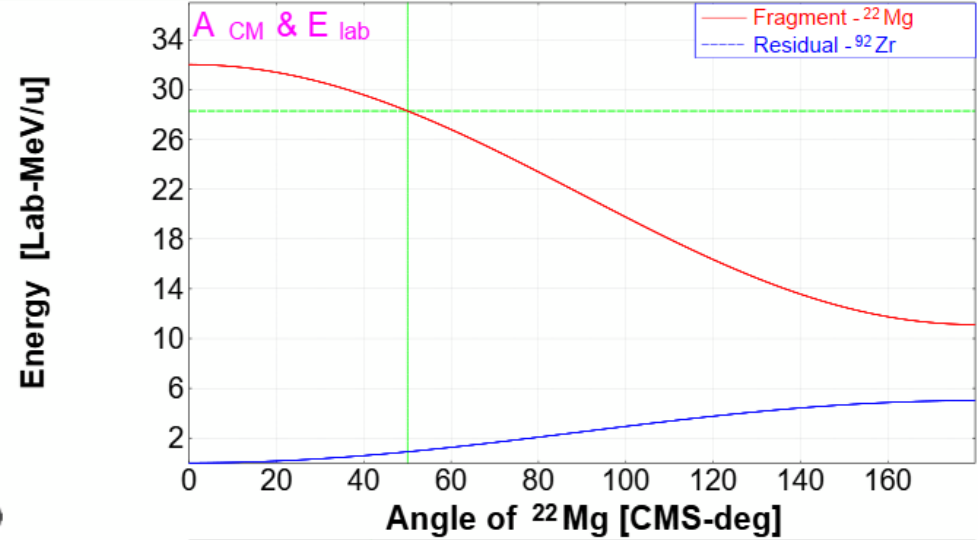
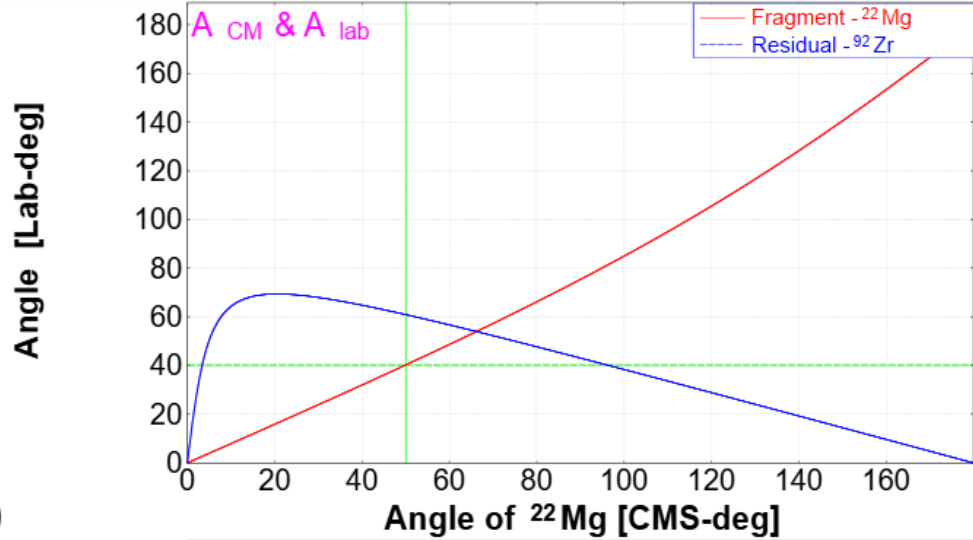
	fragment (C)	residual (D)	fragment (C)	residual (D)
Angle (deg) =	40.246	60.914	50	130

Calculations

	LAB		CM		
Counting in monitor =	1.61e-01	2.42e-01			pps
Differential Cross Section =	142.547	214.689	100	100	mb/sr
Energy after reaction =	28.288	0.8988	20.2076	1.170	MeV/u**
Energy at det. entrance =	28.239	0.6427			MeV/u (** MeV for γ)

Reaction's Kinematics

$^{24}\text{Mg} + ^{90}\text{Zr} \rightarrow ^{22}\text{Mg} + ^{92}\text{Zr}$ or $^{90}\text{Zr}(^{24}\text{Mg}, ^{22}\text{Mg})^{92}\text{Zr}$; Reaction at the "middle" of the target
 Projectile Energy at the reaction place: 29.97 MeV/u; Grazing angle: CMS = 6.65 deg; Lab = 5.27 deg
 $Q_{\text{reaction}} : -13.85 \text{ MeV}$ (Excitations 0.0+0.0 \rightarrow 0.0+0.0); Plotted Energy option is "after reaction"



Click  to save data into text file.