Computer Tools for Nuclear Physics

Initial state of collision within Glauber model

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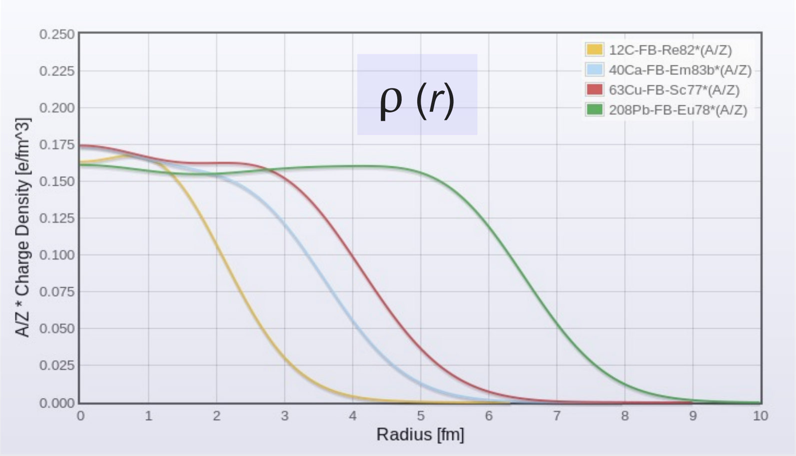
Basics of Glauber Model

- Glauber model: Relativistic Heavy-Ion collision
  - made of sum of independent NN collisions
  - density profile is diffused

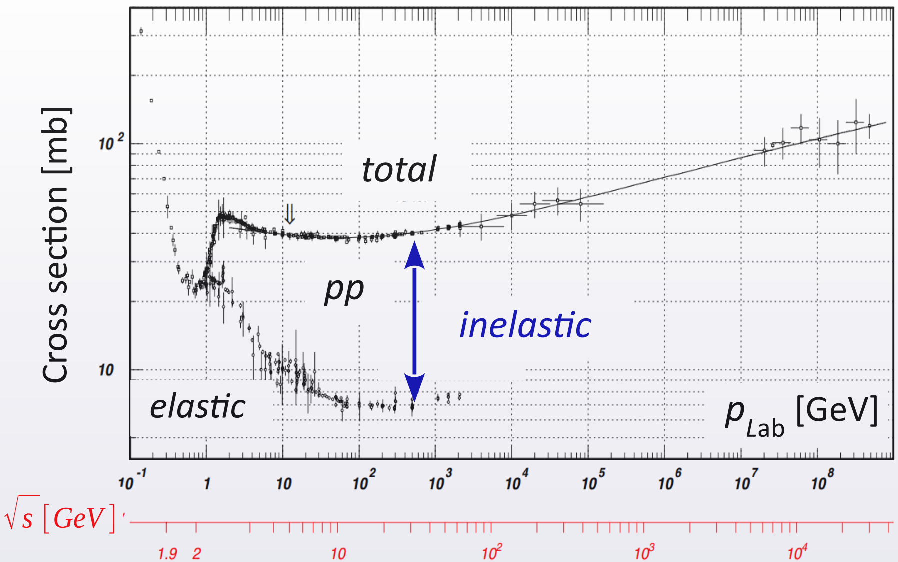
  Simplest density model: Fermi distribution
  $$\rho(r) = \frac{\rho_0}{\exp\left(\frac{r - r_0}{d}\right) + 1}$$

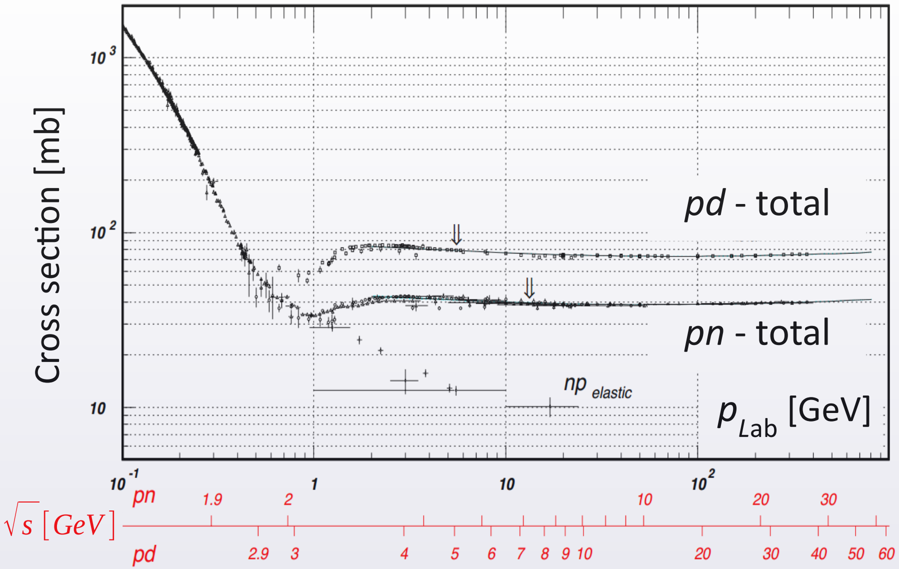
- Cross section for NN collision is not constant.



faculty.virginia.edu/ncd

journals.aps.org/prd/abstract/10.1103/PhysRevD.86.010001





$\sigma_{\mathrm{NN,\,inelastic}}$: swing + wide plateau $\sigma_{NN} \approx 30\,..\,50$ mb

Glauber Model: optical approach

- *Introductory paper:* M. Miller et al. Ann.Rev.Nucl.Part.Sci. 57, 205 (2007)
  Talk: J. Wilkinson Glauber modelling in hi-ener nucl. coll.

- **Optical** ⇐ **Glauber Model** ⇒ **Monte Carlo**

Optical	**Monte Carlo**
continuous ϱ(r) distribution. Analytical formulae, Integrals are probed numerically.	Granulation of A+B nucleons. Their centers are positioned according to ϱ(r).

Step 1: average chance for 1 NN collision = ?

- 3D → 2D. **Thickness function T_A** [fm^{-2}] :

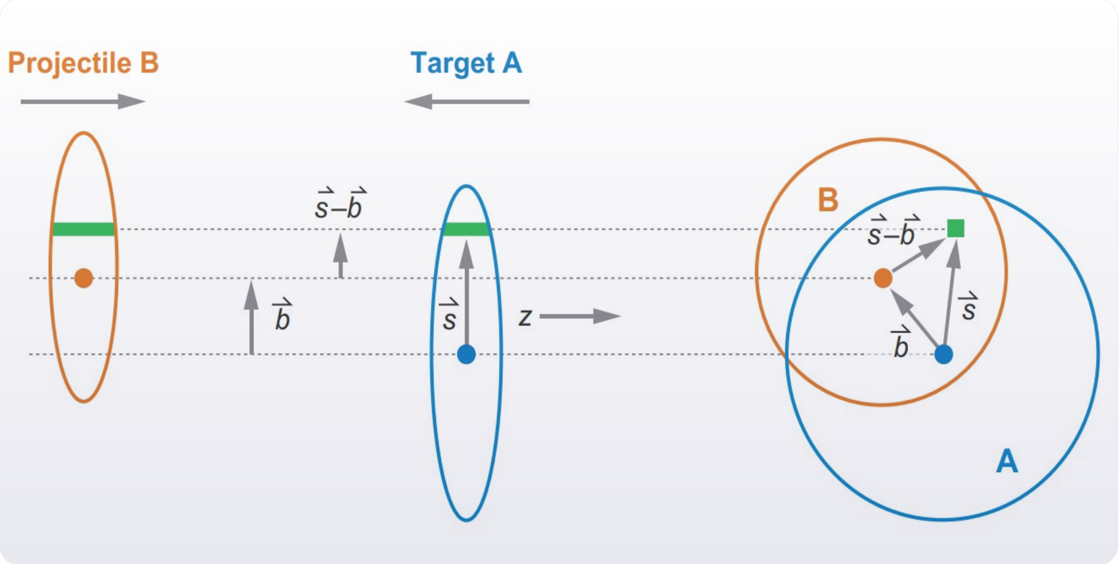
$$T_A(\vec{s}) = \int \rho_T\left(\sqrt{\vec{s}^{\,2}+z^2}\right)\, dz \qquad \int \rho_T\, d^3r = 1$$

(chance of finding 1 nucleon per 1 fm^2 of ⊥ area)

- **Overlap function T_{AB}** :

$$T_{AB}(\vec{b}) = \int d^2s\; T_A(\vec{s})\; T_B(\vec{s}-\vec{b})$$

(chance that at fixed *b*,
a nucleon from A meets a nucleon of B) [fm^{-2}]



⇒ $T_{AB}(b) \cdot \sigma_{NN,\,inel.}$ = chance for a *single* NN collision

Glauber Model: optical approach

- We set up two nuclei at the parameter *b*. Nucleons move forward and „try to collide", with success or failure.
  Average probability of *single* NN collision is : $T_{AB}(b) \cdot \sigma_{NN}$.
  Number of NN collisions = number of „successes" in A · B attempts ↔ follows the binomial distribution:

$$P(n,b) = \binom{AB}{n} \left[T_{AB}\sigma_{NN,inel}\right]^n \left[1 - T_{AB}\sigma_{NN,inel}\right]^{AB-n}$$

- Total cross section for nucleus-nucleus (AA) collision. The AA collision occurs if at least 1 NN collision is done:

$$P(\geq 1 \text{ NN collison}) = 1 - P(0 \text{ NN collisions}) \quad \Longrightarrow \quad \sigma_{AB,inel} = 2\pi \int_0^\infty \left\{1 - \left[1 - T_{AB}(b)\sigma_{NN,inel}\right]^{AB}\right\} bdb$$

- Number of „binary" NN collisions:

$$N_{coll}(b) = \sum_{n=1}^{AB} nP(n,b) = \ldots = AB \cdot T_{AB}(b)\sigma_{NN,inel}$$

- Number of participant nucleons:

$$N_{part}(b) = A\int T_A(\vec{s}) \left\{1 - \left[1 - T_B(\vec{s}-\vec{b})\sigma_{NN,inel}\right]^B\right\} d^2s \; +$$

$$B\int T_B(\vec{s}-\vec{b}) \left\{1 - \left[1 - T_A(\vec{s}) \sigma_{NN,inel}\right]^A\right\} d^2s$$

Inelastic NN cross sections

① σ(pp) is different from σ(pn) and σ(np)

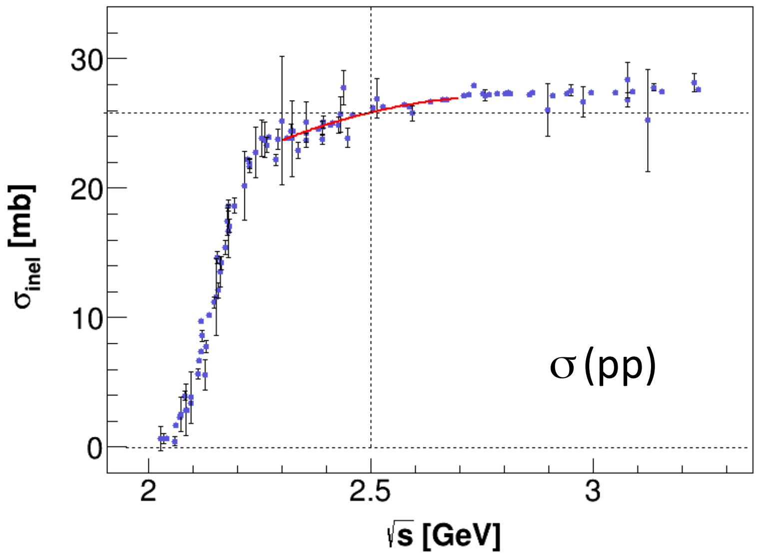
② Assumption: isospin symmetry [$\sigma_{nn} = \sigma_{pp}$]

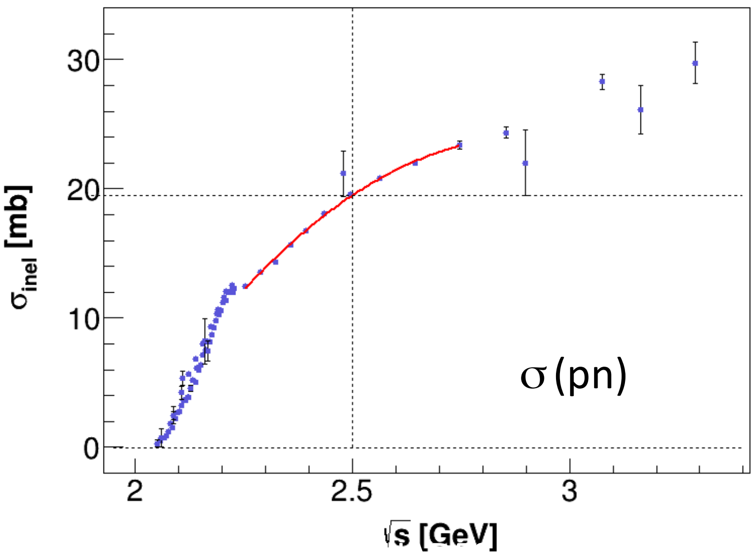
$$\sigma_{NN} = \frac{Z_p Z_t \sigma_{pp} + N_p N_t \sigma_{nn} + (Z_p N_t + N_p Z_t)\sigma_{np}}{A_p A_t}$$

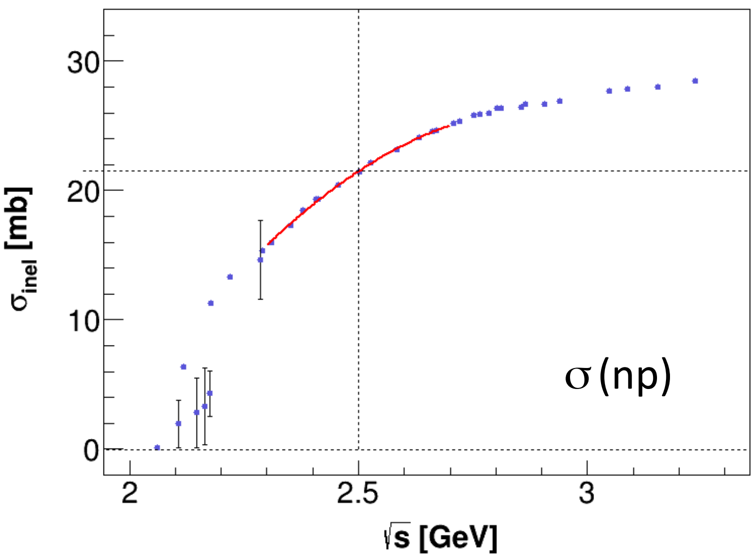
③ Experimentally, σ(pn) is not the same as σ(np)

④ σ(np) at low √s and σ(pn) at higher √s are rare

[③, ④] contribute to systematic errors



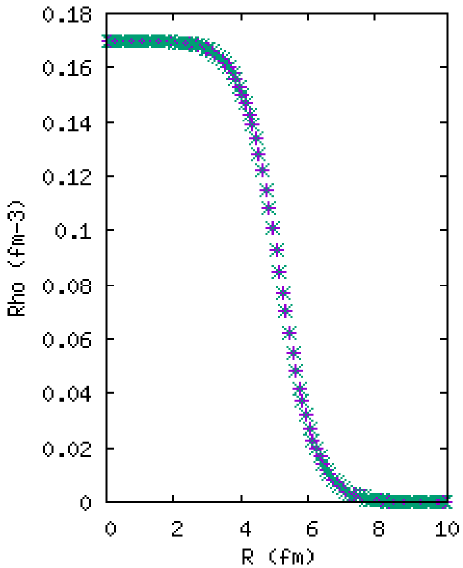


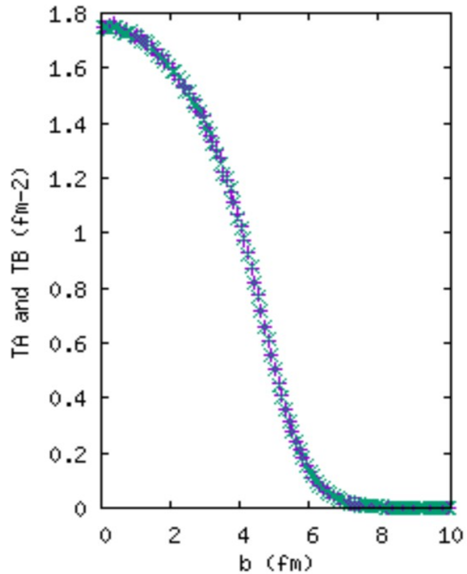


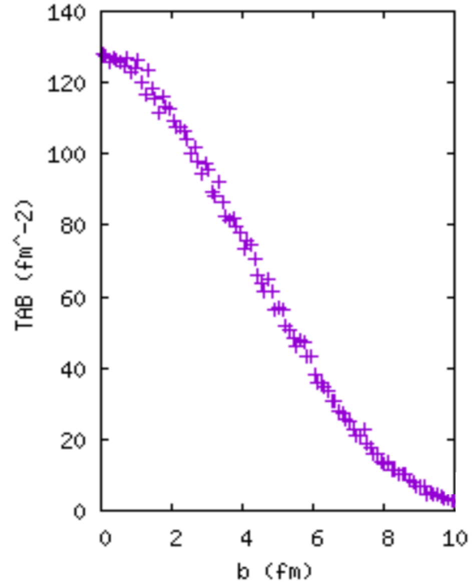
Refs. :

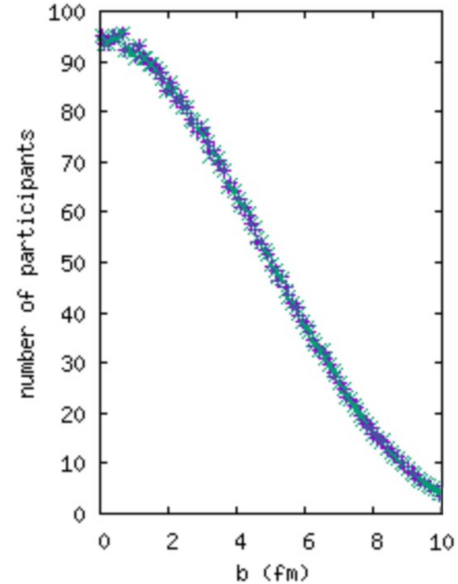
- PDG
- B. Kardan's Ms. th.

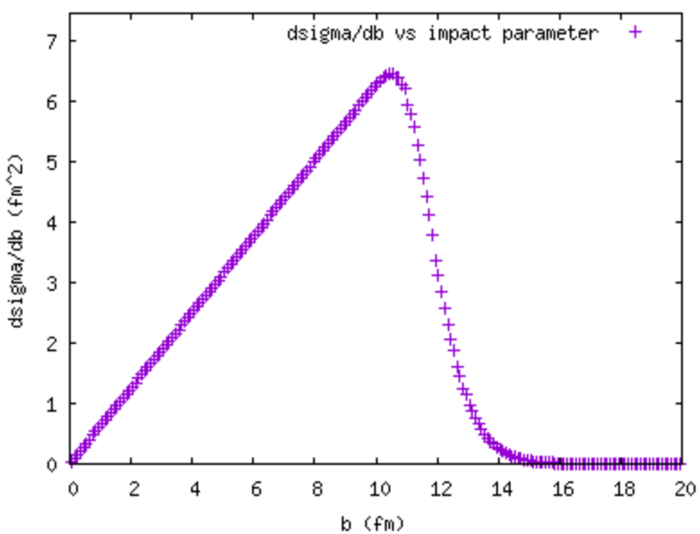
- **"Overlap":** optical Glauber model online **[click here]**.   E.g. for ^{108}Ag + ^{108}Ag collision (let's assume $\sigma_{NN,\ inel.}$ = 25 mb)











- Let's check how settings can influence $\langle A_{part}\rangle$ and $\langle N_{Coll}\rangle$:

	Uniform balls	Fermi distrib. σ_{NN} = 20 mb	Fermi distrib. σ_{NN} = 30 mb
$\langle A_{part}\rangle_b$	55	44.5	48.5
$\langle N_{coll}\rangle_b$	92	65	76

Glauber Monte Carlo: TGlauberMC

- TGlauberMC: ROOT-based simulator of initial conditions of AA collision within the Glauber MC approach.
  Authors: from PHOBOS @ RHIC

  Homepage: tglaubermc.hepforge.org
  Download: www.hepforge.org/downloads/tglaubermc
  Prerequisites: Root ≥ 4

  Main papers: [No. 1 @ 2008] [No. 2 @ 2019] [No. 3 @ 2019]
  User guide: Best description in paper 2.

  Applicability: (authors:) $\sqrt{s_{NN}} \in$ [200 GeV … 10 TeV] , but used also for $\sqrt{s_{NN}}$ = 2.5 GeV.

- Installing TGlauberMC on your account of NPD's training computer:

```
$  mkdir tglaubermc ; cd tglaubermc
$  cp -p ~kpiasecki/soft/TGlauberMC/tglaubermc_install.sh .
$  ./tglaubermc_install.sh
```

- Each time you open a new Terminal, do `cd tglaubermc` and then :

```
$  . ./tglaubermc_start.sh
```

  (*note*: dot at the beginning)

  Now if you just launch Root in this directory, you have all the TGlauberMC functions connected.

- Currently implemented nuclei: p d t ^{3,4}He C O Al Si S Ar Ca Ni Cu Xe W Au Pb U

  Basic form of parametrization of nuclear shape:

$$\rho(r) = \rho_0 \cdot \frac{1 + w\left(\frac{r}{R}\right)^2}{1 + \exp\left[\frac{r - r_0}{a}\right]}$$

  Shape deformation is an option for Al, Si, Cu, Xe, Au, U :

$$\rho(r) = \rho_0 \cdot \frac{1}{1 + \exp\left[\frac{r - R\left(1 + \beta_2 Y_{20} + \beta_4 Y_{40}\right)}{a}\right]}$$

  Possible nuclei and profiles can be checked or updated in the `TGlauNucleus::Lookup` method.

- First, *b* parameter is pulled randomly from ⊿ distribution. Centers of nuclei are defined as [-*b*/2, 0, 0] and [*b*/2, 0, 0] within frame as shown on this Figure.

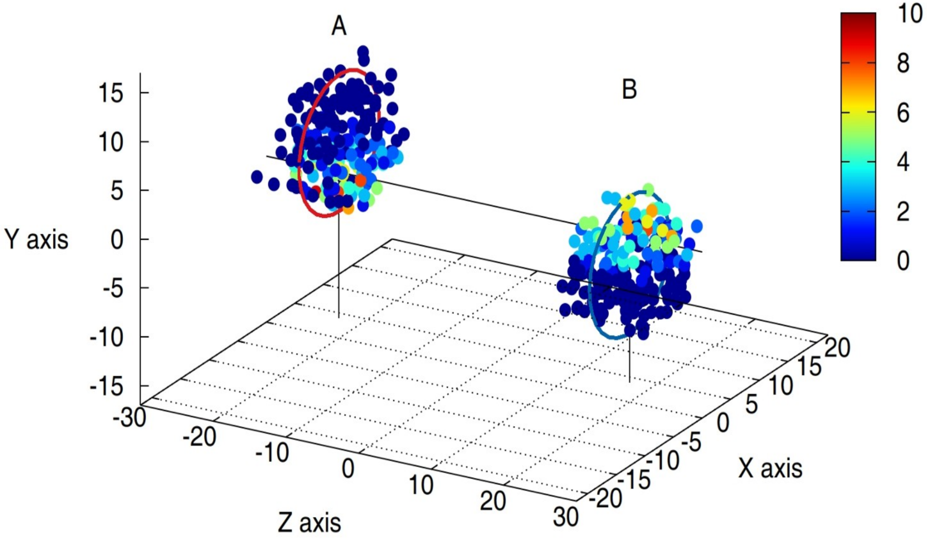
  Positions of nucleons are pulled from shape distributions. A minimum distance between N-N centroids is required (d_{min} = 0.4 fm by default).

- Deformed nuclei are rotated randomly by φ and ϑ .

  Nucleons are 'traversing' head-on ('eikonal' approximation) and undergo collisions independently.

- A single NN collisions is counted if the distance *d* between centroids is less than:

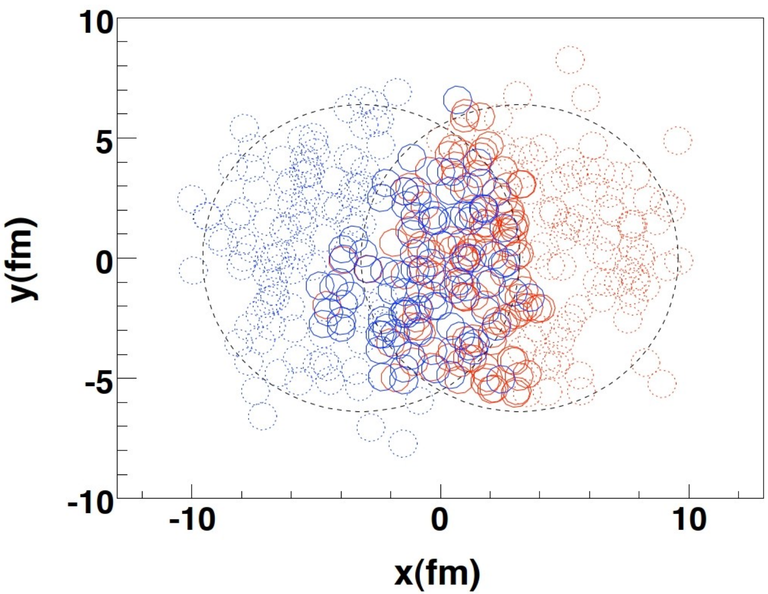
$$d < \sqrt{\frac{\sigma_{NN}}{\pi}}$$



J.Adamczewski-Musch et al., Eur. Phys. A 54, 85 (2018)

- Code offers a few classes and macros.
  Class `TGlauNucleon` represents a single nucleon: position, type (p/n), origin (nucleus A/B), No. of collisions,...
  Class `TGlauberMC` is the main simulation manager.

- `root[1] TGlauberMC gmc ("Pb","Pb",30.)` — Initialize Pb+Pb collision with σ_{NN} = 30 mb
  `root[2] gmc.NextEvent ( 5. )` — Simulate 1 collision at *b* = 5 fm. For -1 , random *b*.
  `root[3] gmc.Draw ()` — Visualize of the collision (wounded/participants)



Get (inelastic) collision cross section based on this event:

```
root[4] gmc.GetTotXSect ()
12.57
```

Getting No. of participants and collisions:

```
root[5] cout << gmc.GetNpart() <<'\t'<<
               gmc.GetNcoll() << endl;
370 786
```

Get array of nucleons:

```
root[6] gmc.GetNucleons ()
```

Simulate e.g. 1000 collisions at once.

```
root[7] gmc.Run (1000)
```

Retrieve event-wise ntuple (can be saved in Root file)

```
root[8] TNtuple* ntu = gmc.GetNtuple ()
```

- Btw. you can apply range of *b* pars:

```
gmc.SetBmin ( 4. );
gmc.SetBmax ( 7. );
```

Glauber Monte Carlo: TGlauberMC

- Macro `runAndSaveNtuple` simulates and stores the resulting event-wise ntuple in file:

`root [0] runAndSaveNtuple (100, "Ni", "Ni", 25.)` simulate 100 Ni+Ni collisions with σ_{NN} = 25 mb

- Variables available in the ntuple include:

`Npart`	No. of participants	`AreaA`	Area defined by "and" of participants
`Ncoll`	No. of collisions	`AreaO`	Area defined by "or" of participants
`Nhard`	No. of hard-core collisions	`MeanX`	$\langle x \rangle$ of wounded nucleons
`B`	Impact parameter	`MeanY`	$\langle y \rangle$ of wounded nucleons
`BNN`	Average NN impact parameter	`MeanX2`	$\langle x^2 \rangle$ of wounded nucleons
`Ncollpp`	No. of pp collisions	`MeanY2`	$\langle y^2 \rangle$ of wounded nucleons
`Ncollpn`	No. of pn collisions	`MeanXY`	$\langle xy \rangle$ of wounded nucleons
`Ncollnn`	No. of nn collisions	`MeanXSystem`	$\langle x \rangle$ of all nucleons
`VarX`	Variance of X of wounded nucleons	`MeanYSystem`	$\langle y \rangle$ of all nucleons
`VarY`	Variance of Y of wounded nucleons	`MeanXA`	$\langle x \rangle$ of nucleons in nucleus A
`VarXY`	Covar. between X and Y of wounded nucleons	`MeanYA`	$\langle y \rangle$ of nucleons in nucleus A
`NpartA`	No. of wounded nucleons in nucleus A	`MeanXB`	$\langle x \rangle$ of nucleons in nucleus B
`NpartB`	No. of wounded nucleons in nucleus B	`MeanYB`	$\langle y \rangle$ of nucleons in nucleus B
`Npart0`	No. of singly-wounded nucleons	`PhiA`	ϕ angle nucleus A (applied if deformed)
`AreaW`	Area defined by width of participants	`ThetaA`	ϑ angle nucleus A (applied if deformed)
`PsiN`	Event plane angle of n-th harmonic	`PhiB`	ϕ angle nucleus B (applied if deformed)
`EccN`	Participant eccentricity for n-th harmonic	`ThetaB`	ϑ angle nucleus B (applied if deformed)

- *Caution*: `PsiN` and `EccN` are constructed from weighted positions of wounded nucleons.

Geometry of wounded nucleons

- Centroids of nuclei are initially fixed to the XZ plane (see Fig. on p. 8) .

  Ideally:
  - area occupied by wounded nucleons looks like a vertical almond (∼ellipse)
  - angle ψ between the longer axis and X axis: $\psi_{EP} = 90°$
  - shape has given eccentricity ε

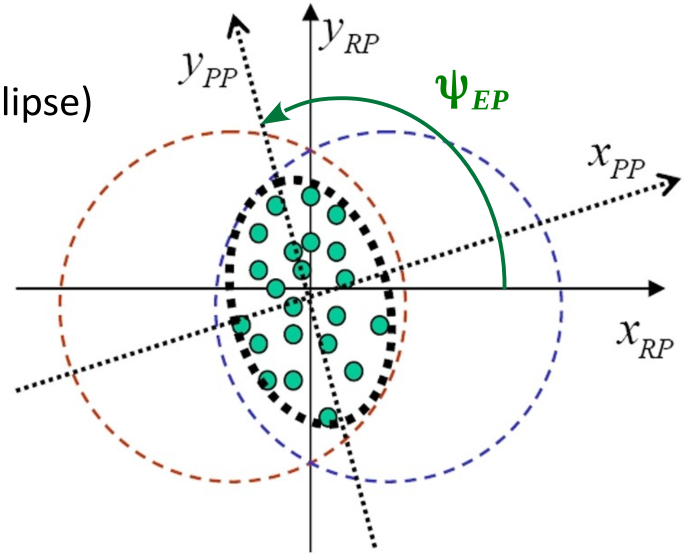
  However, in an event, positions of nucleons are pulled randomly.
  - shape deviates from an “ideal” almond
  - angle ψ_{EP} differs from 90°.
    A de-facto plane is called “**event plane**” or “**participant plane**”
  - shape has given **eccentricity ε** (deviated from ideal ε)

  ψ_{EP} is found from positions (x_i, y_i) by:

  $$\psi_{EP} \equiv atan\left(\frac{\sum y_i}{\sum x_i}\right)$$

  ε is also found from wounded nucleons:

  $$\varepsilon \equiv \frac{\sqrt{\langle r^2\cos(2\varphi)\rangle^2 + \langle r^2\sin(2\varphi)\rangle^2}}{\langle r^2\rangle}$$

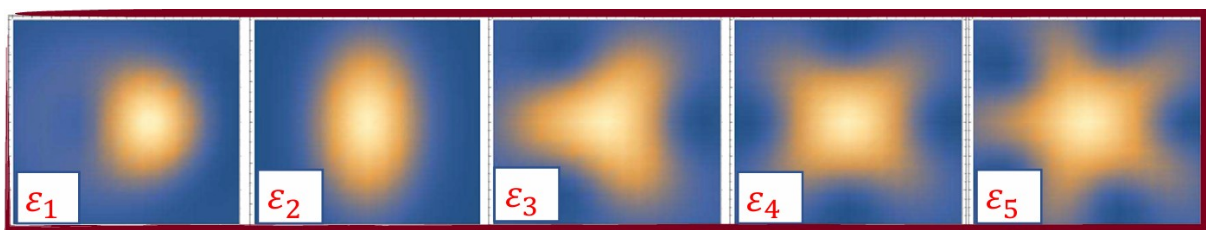


[S. Voloshin et al.]

- For more refined analyses of shape and fluctuations, ψ and ε of higher harmonics is available.

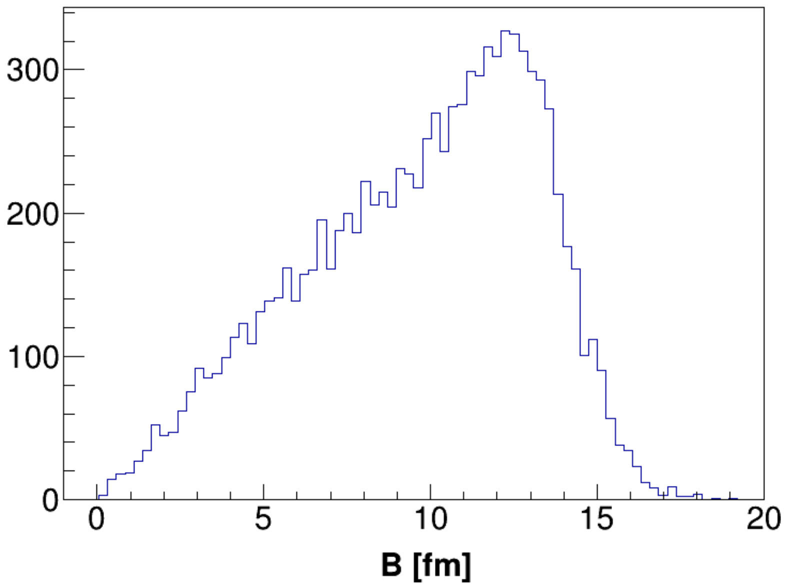
$$\varepsilon_n \equiv \frac{\sqrt{\langle r^n\cos(n\varphi)\rangle^2 + \langle r^n\sin(n\varphi)\rangle^2}}{\langle r^n\rangle}$$

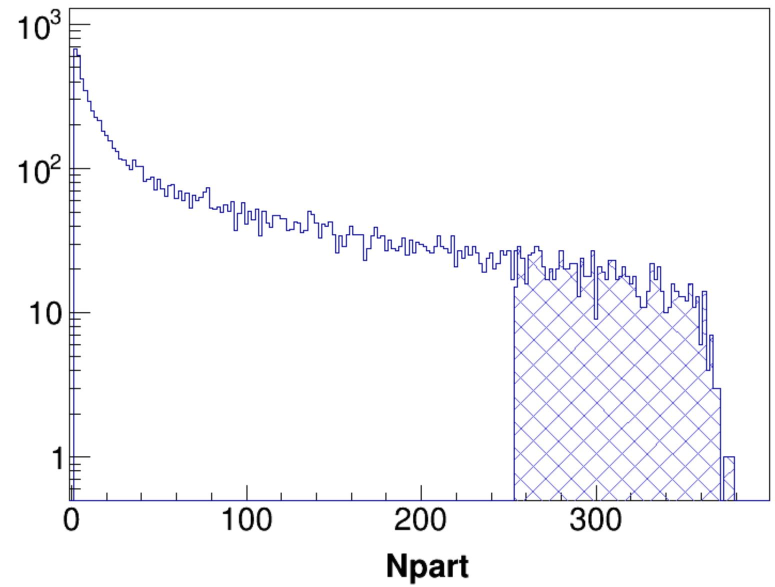
$$\psi_n \equiv \frac{1}{n}\, atan\left(\frac{\sum r^n\cos(n\phi)}{\sum r^n\sin(n\phi)}\right)$$

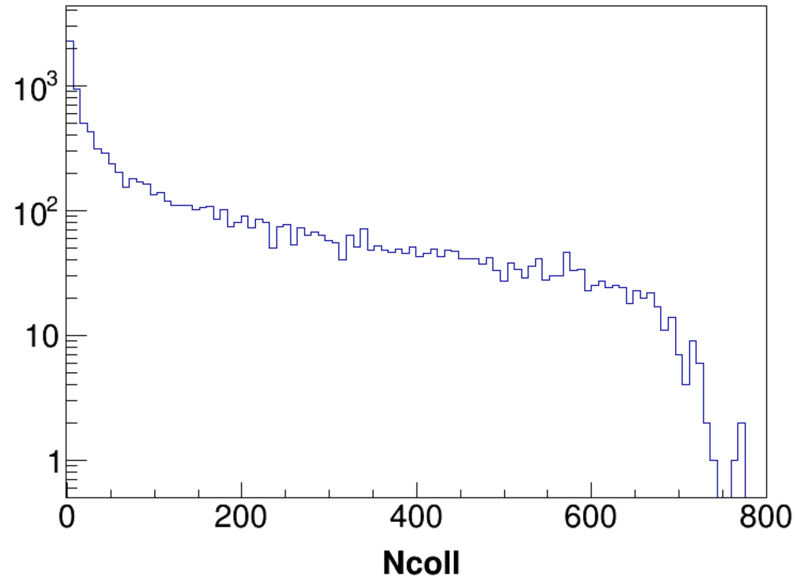


[Source: M. Stefaniak]

- Simulation: Au+Au, σ_{NN} = 23.7 mb .







Glauber Monte Carlo: TGlauberMC

- Let's focus on `gmc.GetNucleons ().` It returns the address to the array of `TGlauNucleon` objects.

- What does a single `TGlauNucleon` object contain ?
  - **position**: You can `SetXYZ` and `GetX/Y/Z`, but also `RotateXYZ(_3D)` it.
    Also it knows if it `IsInNucleusA/B`.
  - **type**: `IsProton, IsNeutron, Get/SetType.` Also energy, but it has no effect.
  - **collision status**: if it `IsWounded` and how badly: `GetNcoll.`
    Alternatively, if it `IsSpectator.`
    You can also "make" it `Collide` or heal fully: `Reset.`

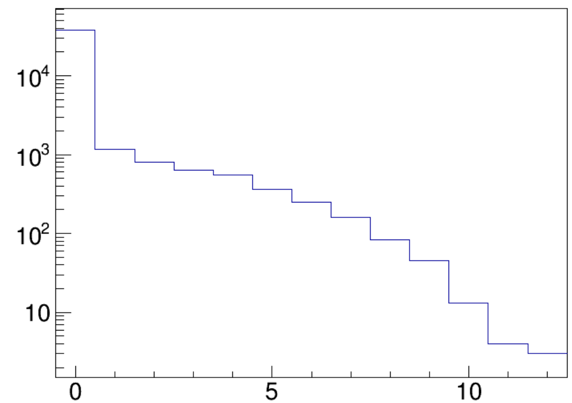
- Looping over nucleons in an event.

```
R__LOAD_LIBRARY (libMathMore)
R__LOAD_LIBRARY (runglauber_v3.2_C)

int nucloop () {
 TGlauberMC gmc      ( "Pb" , "Pb" , 25. );
 TH1F       hNucColl ("hnuccoll", "", 13, -0.5, 12.5);
 TObjArray*    nucArray ;
 TGlauNucleon* nuc ;

 for (int iEvent = 0; iEvent < 100 ; iEvent++)
 {
  gmc.NextEvent ( -1 );
  nucArray = gmc.GetNucleons ();
  for (int iNuc = 0; iNuc < nucArray->GetEntries(); iNuc++)
  {
   nuc = (TGlauNucleon*) nucArray->At (iNuc) ;
   hNucColl->Fill ( nuc->GetNColl() );
  }
 }
 hNucColl->Draw ();
 return 0;
}
```

- Create the simulator of Pb+Pb collision @ σ_{NN} = 25 mb
- Loop over events
- Simulate 1 coll. at random *b*
- Get array of nucleons
- Loop over nucleons
- Get given nucleon



Glauber Model vs experimental data

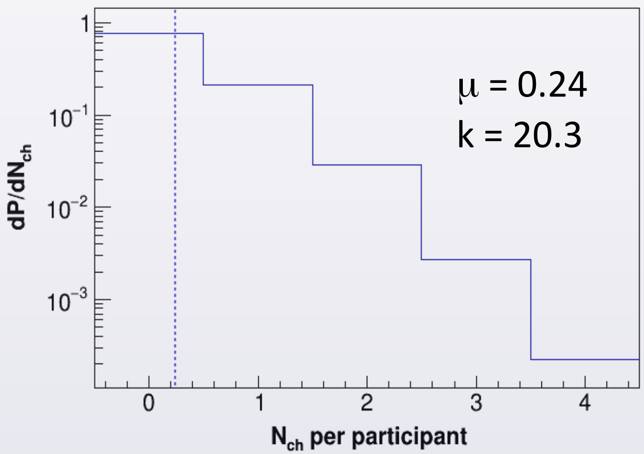
- Obtained A_{part} distribution is not yet a place of comparison with experiment:
  1. usually only charged particles are measured (protons)
  2. detector acceptance is limited $\rightarrow$ will measure only part of particles
  3. even if particle gets inside: setup has some inefficiency of track measurement (efficiency $\equiv \varepsilon$) .

- N_{part} distribution from TGlauberMC simulation has to be projected to N_{ch} measured particles, Accounting for fluctuations. Each participant is associated with „*P* of creating registered particle".

- One usually applies the **negative binomial distribution, NBD** with mean μ and width parameter k:

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\,\Gamma(k)} \cdot \frac{(\mu/k)^n}{(\mu/k+1)^{n+k}}$$

Example for Au+Au @ 1.2A GeV (HADES):



- Plot shows the distrib. of multiplicity of charged particles.

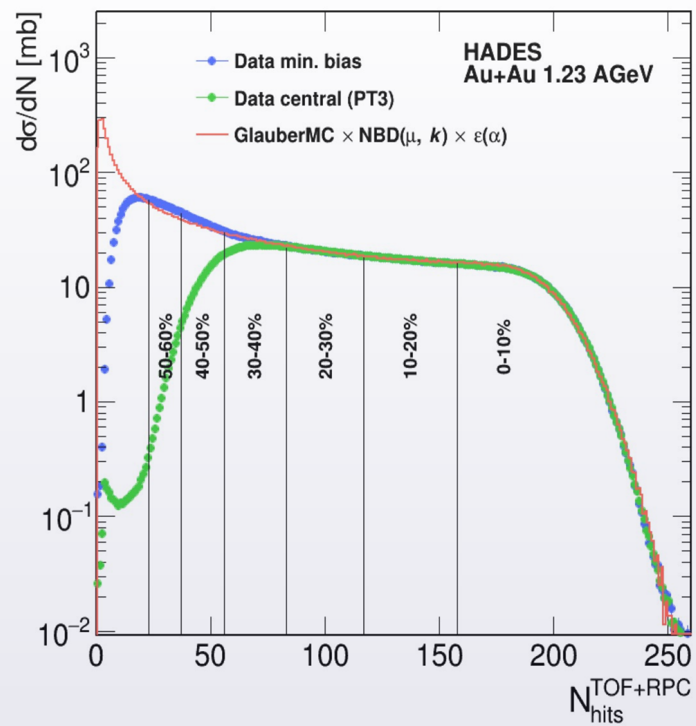
  — Glauber MC

  — Experiment, "minimum bias"

  — Experiment "central trigger"

  Good agreement between model and experiment.

  Experimentally one defines the „**centrality classes**" on this plot.



J. Adamczewski-Musch et al. (HADES), EPJA 54, 85 (2018)