Computer Tools for Nuclear Physics

Microscopic Transport Models
(outline)

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1. Type of kinematics and adequate degrees of freedom
2. Energy of nucleus and nuclear matter
3. BUU transport equation including collisions
4. QMD family. Interactions between nucleons
5. Summary of model properties and additional info
6. Characteristics of selected transport models:
   GiBUU, UrQMD, SMASH, PHSD, RQMD.RMF

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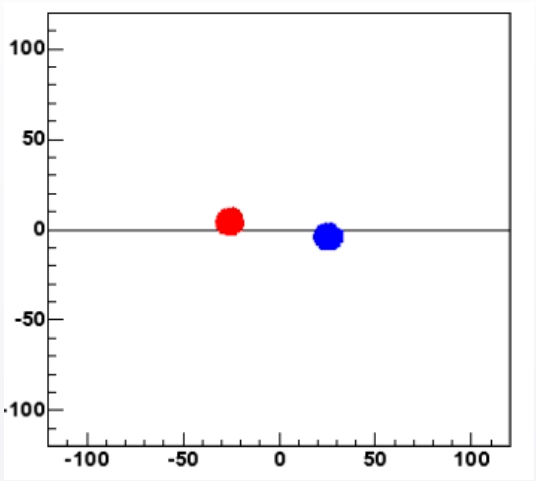
Microscopic Transport models

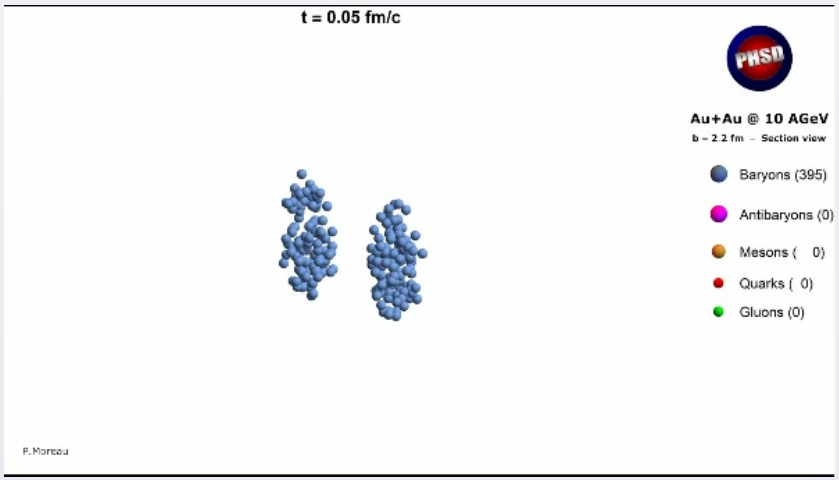
- **QMD** simulation of Au+Au collision at beam kinetic energy T_{Beam} = **15 MeV/nucleon** (15*A* MeV)

[www.fuw.edu.pl/~kpias/rhic/QMD_Tb.015_b08_010.mpg]



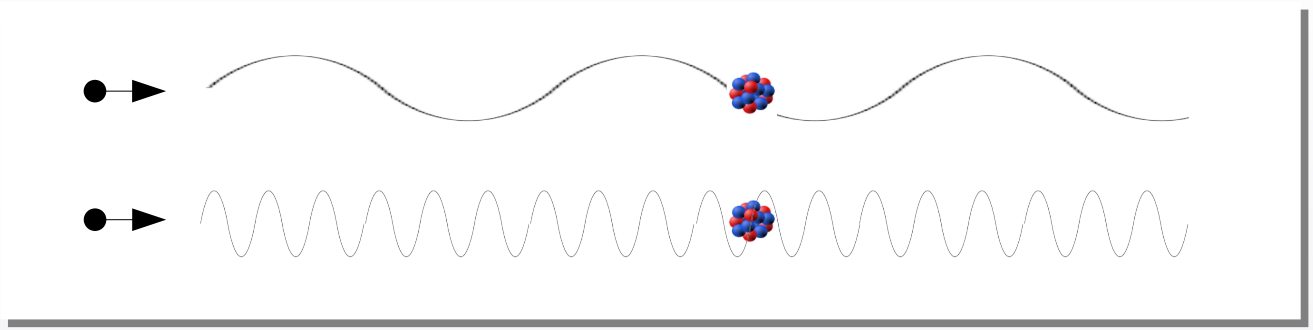
- **PHSD** simulation of Au+Au collision at beam kinetic energy T_{Beam} = **10 GeV/nucleon** (10*A* GeV)

[theory.gsi.de/~ebratkov/phsd-project/PHSD/documents/movie_AuAu_10AGeV.mp4]



Collision of heavy ions and beam energy

- Which basic parameters change with beam energy?



T_{Beam} / nucleon	γ	β	$\lambda_{de\ Broglie}$ [fm]
5 MeV	1.005	0.10	13
50 MeV	1.05	0.3	4
500 MeV	1.5	0.75	1
5 GeV	6	0.99	0.2
50 GeV	55	0.9998	0.02
500 GeV	530	0.999998	0.002

Mean field sector
Nucleon ”sees” nucleus as a whole

Hadron-hadron sector
Hadron production in hh channels

Quark-gluon sector

- How to calculate it?

$$T_{Beam} = m_N c^2 \left(\gamma - 1\right) \rightarrow \gamma = 1 + \frac{T}{m_N c^2}$$

$$\beta = \sqrt{1 - \gamma^{-2}}$$

$$\lambda_{de\ Broglie\ (N)} = \frac{h}{p_N} = \frac{\hbar c \cdot 2\pi}{m_N c^2 \cdot \gamma\beta}$$

Model framework adequate to collision description

- With increasing beam energy the **relativistic kinematics** becomes more and more relevant (e.g. for $T_{Beam} = 500 \cdot A$ MeV, coefficient $\gamma = 1{,}5$).

- Increase of beam energy causes contraction of de Broglie wavelength, and therefore also the objects adequate to the scale (so-called **degrees of freedom**) :

  At **low energies** the interaction of selected nucleon with nucleus is described by the **potential**, generated by the whole nucleus (so-called ***Mean field***) .

  At **higher energies** ($T_{Beam} \gtrsim 100 \cdot A$ MeV) the **nucleon-nucleon collisions** (NN) appear.
  These collisions may lead to production of new hadrons (h) , e.g.:

  mesons π , K , ϕ, J/ψ , D … baryons N, Δ , Λ , Σ , Ξ …

  These hadrons move further within the nuclear matter – and can collide with other hadrons.
  The occuring process can be **two-body** ($h_1h_2 \rightarrow h_3h_4$) , **three-body** ($h_1h_2 \rightarrow h_3h_4h_5$) , …
  Probabilities of these processes are quantified by **energy-dependent cross sections** .

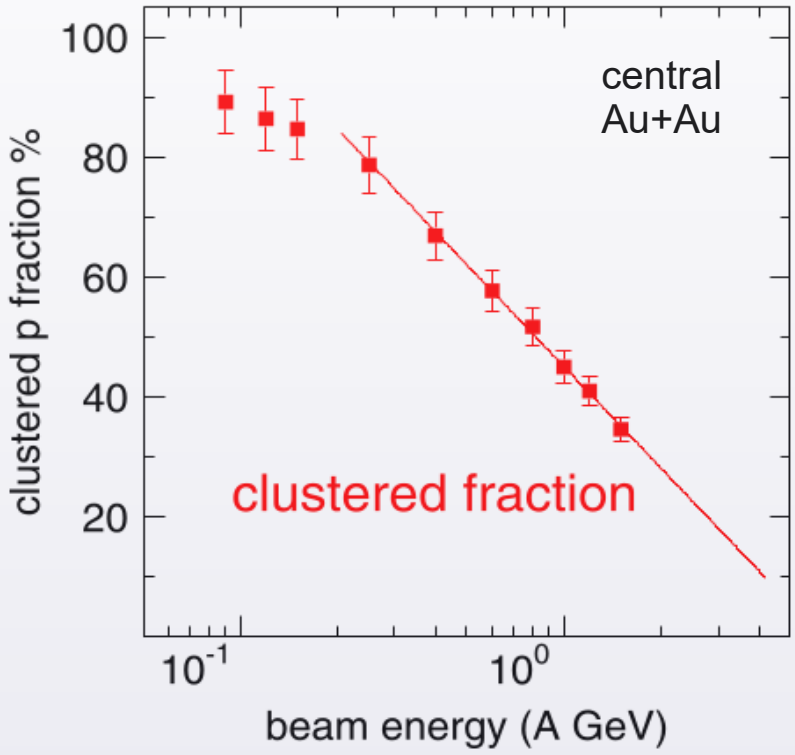
  At **ultrarelativistic energies** ($T_{Beam} \gtrsim 5 \cdot A$ GeV) the **quark-gluon** processes appear.

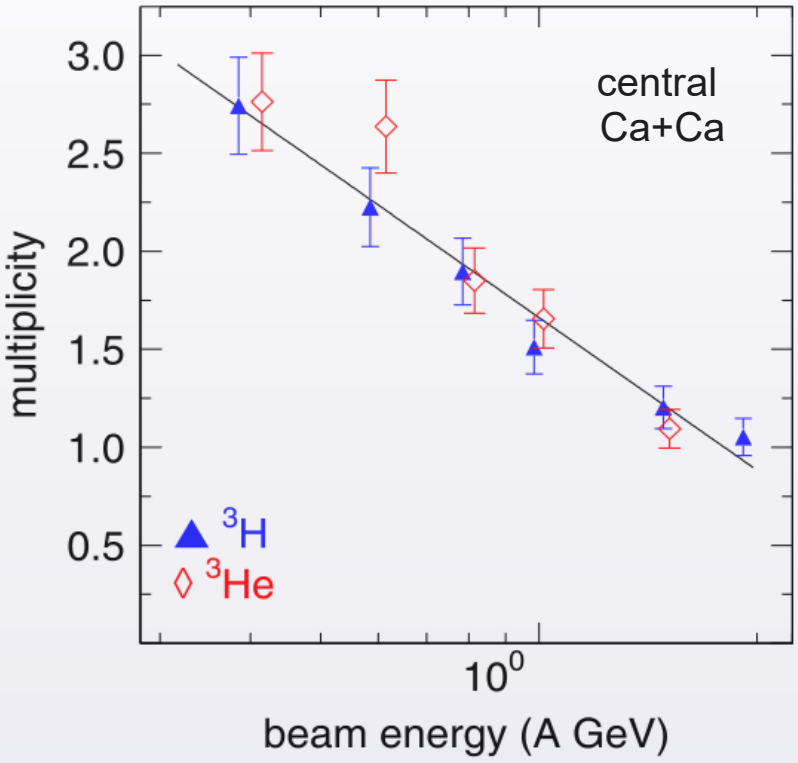
Selected model framework should be adequate to the beam energy.

Model framework adequate to collision description

- Proton-neutron (p-n) interaction may lead to the production of ***deuteron*** (d).
  Deuteron-neutron (d-n) interaction may create ***triton*** (t) and deuteron-proton (d-p) create ***^{3}He*** . Etc...
  These particles are called the ***Light Charged Particles*** (***LCP***, ***clusters***, ***small nuclear fragments***).

- At $T_{Beam} \sim 2 \cdot$ A GeV, as much as ¼ protons are bound in LCP. Trace amounts of deuterons are seen in LHC.



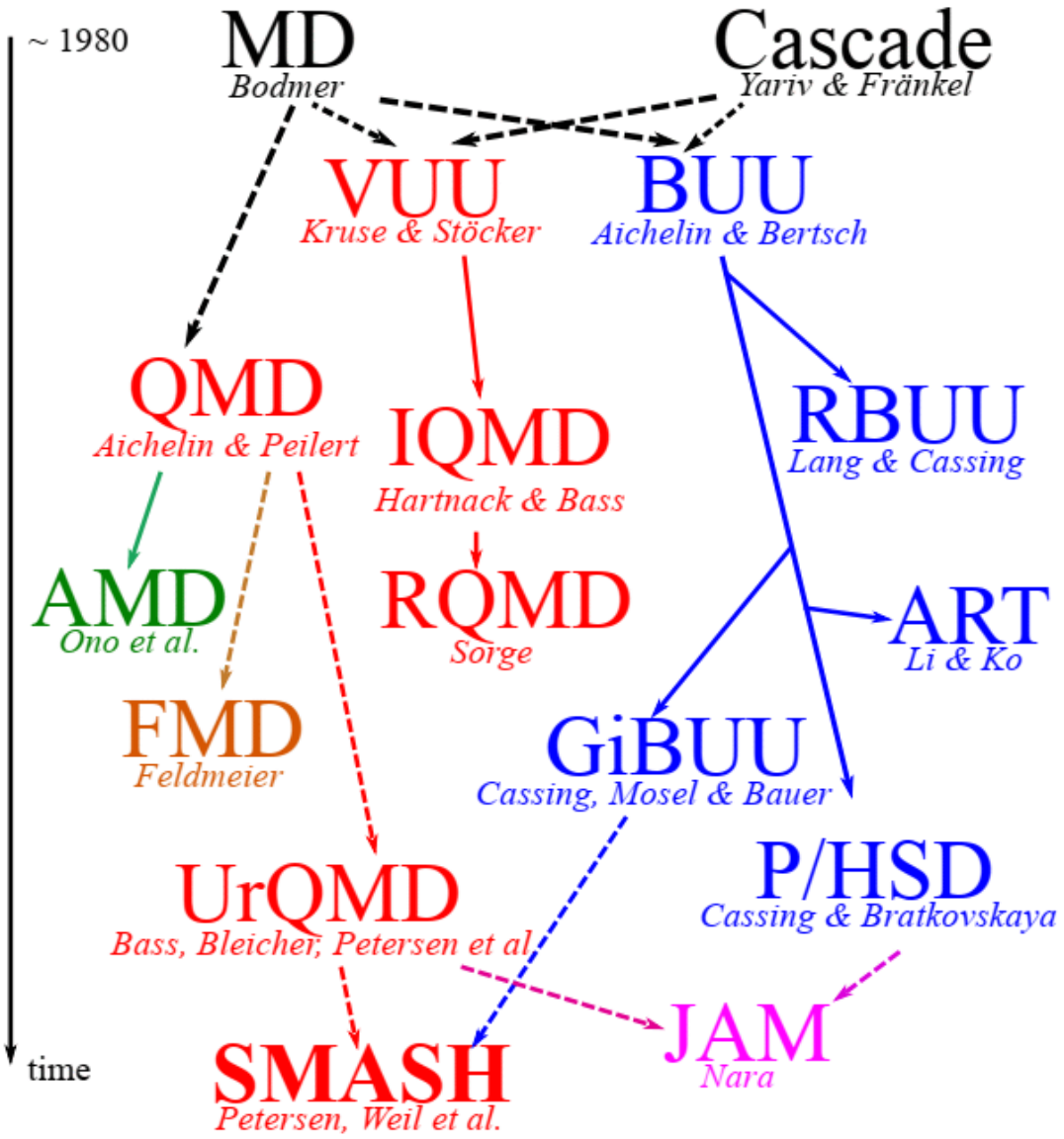


W.Reisdorf et al.,
Nucl.Phys. A 848, 366 (2010)

- Usually, if LCP appears in the models, they are assembled from p and n, already after the simulation.
  Full description of production and dynamics of LCP is currently absent.
  Attempts: ***PHQMD*** (*under development*).

➡ **If description of LCP is relevant, one should consider the predictive power of model-candidates.**

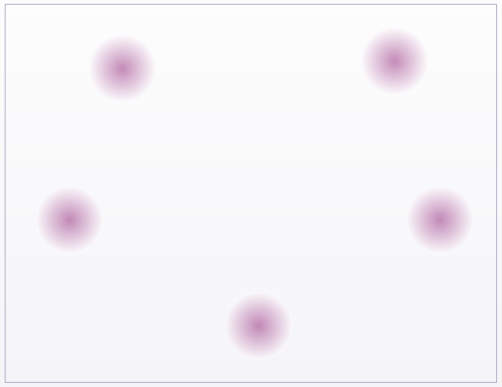
Transport models across decades



Many models were developed (more than shown above), but many of them inherit the main framework. We shall describe two branches: family of **BUU** and **QMD** models.

Energy in nucleus (schematic view)

In ∞

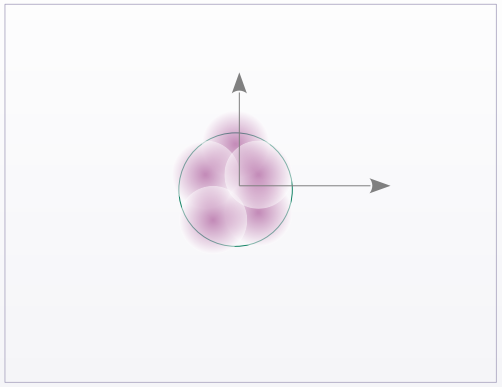


Nucleons are free,
$\bar{p}_i = \bar{0}$.

$$E_{System} = \sum_i m_{N,i} c^2$$

$$\frac{E_{System}}{A} = m_N c^2$$

Nucleus as a whole:



In nucleus' own frame, $\bar{p}_{nucleus} = \bar{0}$.

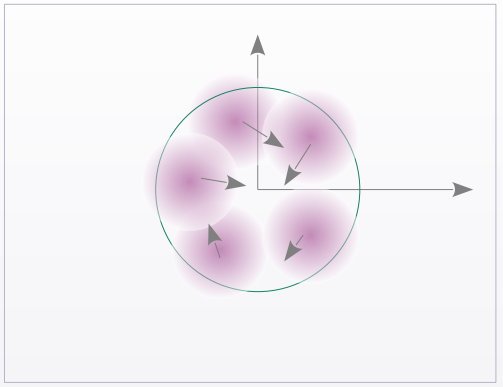
$$E_n = M_n c^2 \underbrace{=} \sum_i m_{N,i} c^2 - B_J$$

quantitatively

$$\frac{E_n}{A} = m_N c^2 - \frac{B_{nucleus}}{A}$$

$\approx$ 8 MeV

Nucleus as a sum of nucleons:



Pauli exclusion → nucleons must move,
$\bar{p}_i \neq \bar{0}$.

$$E_{nucl.} = \sum_i e_i$$

$$e_i = \sqrt{\left(m_{N,i} c^2\right)^2 + \left(\vec{p}_i c\right)^2} + \frac{U_i}{A}$$

In reality the formulae are more complex.

① Coulomb interaction must be added (φ, $\bar{A}$) .

② Within QCD with strong interactions, so-called scalar and vector potentials appear, which e.g. change mass of nucleon inside nuclear matter.

$$e_i = \sqrt{\left(m_{N,i} c^2 - V_S\right)^2 + \left(\vec{p}_i c - \vec{V}_W\right)^2} + V_{W,0}$$

(not by heart :))

Nuclear matter equation of state (sketch)

- **Real atomic nucleus**: system of nucleons with finite *N* and *V* that interacts strongly, weekly and by Coulomb. Nucleons from ∞ fall into the potential well (binding energy E_B), wherein they acquire kinetic energy (Fermi motion). In the **liquid drop model**:

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm \delta(A,Z)$$

For average nucleus: $E_B / A \approx$ 8 MeV.

- **Nuclear matter**: abstract matter built of nucleons, of infinite *N* and *V*, but finite ρ . Depth of binding depends on: temperature, density, n/p ratio.

$$E_B(\rho = \rho_0,\ T = 0) = a_V A \quad \ldots \quad - a_A \frac{(N-Z)^2}{A}$$

(ρ_0 = 0.17 fm^{-3} , "normal density of nuclear matter")

For symmetric (*N* = *Z*) nuclear matter in normal state: $E_B / A \approx$ 16 MeV.

- During the collision the matter condenses and subsequently expands. How E_B / A depends on density ρ? Two points are fixed:

[1] for $\rho = 0$ (free nucleons) , $E_B / A =$ 0 MeV
[2] for $\rho = \rho_0$ (stable nucleus) , $E_B / A =$ –16 MeV

- One can derive models of interactions that aim to reproduce findings [1,2] .
One can also insert into the model some functional dependence that agrees to [1, 2].
The testing ground are the kinematic distributions of particles: if/when model agrees with experiments?

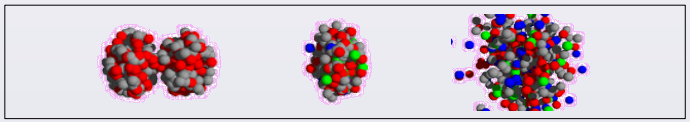
Nuclear matter equation of state (sketch)

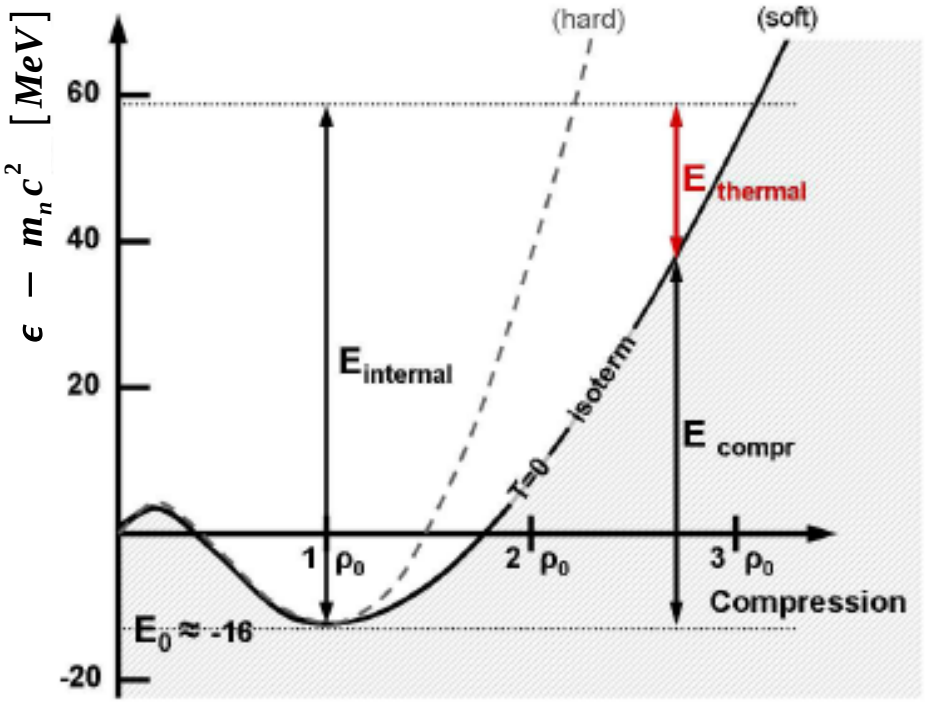
- **Nuclear matter equation of state ("EoS") :**

$$\epsilon \equiv \frac{E}{A}(\rho, T) = m_n c^2 + \epsilon_{Kin}(\rho, T=0) + u(\rho, T=0) + \epsilon(\rho, T>0)$$

- It's worth to compare the plot to the dynamics of the nuclear collision. Let's trace it:

$\rho_0 \rightarrow 3\rho_0 \rightarrow \rho_0$





- Thermodynamical quantities:

pressure $\quad p = \ldots = -\rho \left.\frac{\partial \epsilon}{\partial \rho}\right|_{T=0}$

Bulk modulus $\quad \kappa = \ldots = +\rho \left.\frac{\partial p}{\partial \rho}\right|_{T=0}$

- In nuclear studies:

Incompressibility modulus $\quad \kappa \equiv 9 \left.\frac{\partial p}{\partial \rho}\right|_{T=0}$

- In the state of normal nuclear density ($\rho = \rho_0$) :

$$\kappa(\rho_0) = \ldots = 9 \left.\frac{d^2 \epsilon}{d \rho^2}\right|_{\rho=\rho_0,\ T=0}$$

(or: ~curvature of ϵ at $\rho = \rho_0$)

- High (low) κ : "***hard***" ("***soft***") ***EoS***

Usually in papers: "soft" : $\kappa \approx 200$ MeV,
"hard": $\kappa \approx 380$ MeV

Framework of family of BUU models (outline)

- **B**olzmann–**Ü**hling–**U**hlenbeck transport equation. Exemplary introductions:

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B.Serot, J.Walecka, arXiv:nucl/th/9701058 section 7A
C.Hartnack et al., Eur. Phys. J. A 1, 151 (1998)
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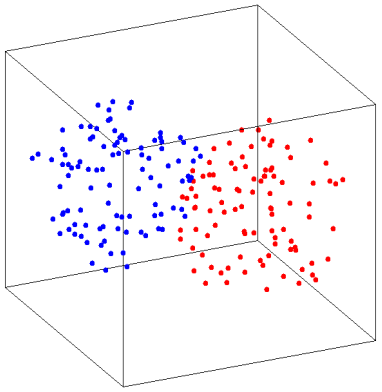
- Outline. We consider *N* particles moving in the phase space (x^3, p^3). Their distribution is described by the function $f(\mathbf{r}, \mathbf{p}, t)$. Particles are **in a field described by a potential *U*** (mean field; it's the BUU's feature).



$$N = \int d^3\boldsymbol{r} \int d^3\boldsymbol{p}\ f(\boldsymbol{r},\boldsymbol{p},t)$$

$$df \equiv \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial r_i}dr_i + \frac{\partial f}{\partial p_i}dp_i$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_r f \cdot \boldsymbol{v} - \nabla_p f \cdot \nabla_r U$$

$$\frac{\partial f}{\partial r_i}dr_i = (\nabla_r f)\cdot d\boldsymbol{r}_i \qquad d\boldsymbol{r} = \boldsymbol{v}\,dt \qquad d\boldsymbol{p} = -\nabla_r U\,dt$$

- Potential *U* of the mean field:

$$U = U(\rho) + U_{Coulomb} + U(\vec{p})$$

$$U(\rho) = \alpha\frac{\rho}{\rho_0} + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

(*nuclear matter "equation of state", typical parameterization*)

$$U(\vec{p}) = \delta\frac{8}{\rho_0(2\pi)^3}\int d^3p'\frac{f(\boldsymbol{r},\boldsymbol{p}')}{1+\left(\frac{\boldsymbol{p}-\boldsymbol{p}'}{\Lambda}\right)^2}$$

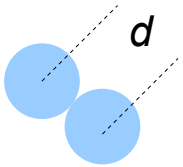
(*momentum-dependent term of potential typical parameterization*)

- If *N = const*, we would have: $\frac{df}{dt} = 0$

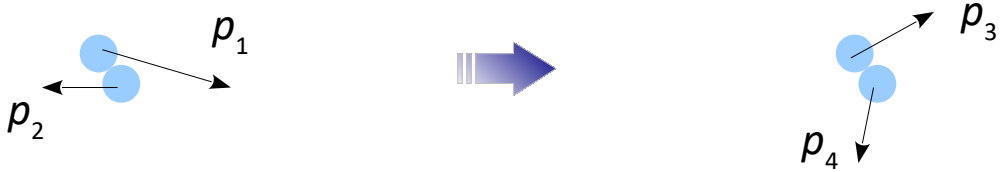
Framework of family of BUU models (outline)

- … but there are also **collisions**. Let's consider a two-body process (1,2) → (3,4).
  Probability of collision is described by the cross section σ (more precisely, dσ/dΩ) .
  A collision occurs if:

$$d < \sqrt{\frac{\sigma_{NN}}{\pi}}$$



- A collision generates a single loss in cells of momenta p_1 and p_2 for the sake of gain in p_3 and p_4.



- But one should account for the Pauli exclusion (momenta cells p_3 i p_4 are not always available).
  It is included probabilistically:

f (**r**,**p**,t)	is a probability of occupation of momentum cell p.
$1 - f$(**r**,**p**,t)	is a probability that momentum cell p is free.

→ Additional term in BUU equation, describing collisions: $\frac{df}{dt} = I_{coll}$

$$I_{coll} = \sum_{\substack{\text{reaction}\\ \text{channels}}} \int \frac{d^3p_2\, d^3p_3\, d^3p_4}{(2\pi)^6} \frac{d\sigma_{12\to 34}}{d\Omega'} \cdot \Big[\underbrace{(1-f_1)(1-f_2)f_3f_4}_{\text{Gains}} - \underbrace{f_1f_2(1-f_3)(1-f_4)}_{\text{Losses}}\Big] \cdot \underbrace{\delta(p_1+p_2-p_3-p_4)}_{\substack{\text{Momentum}\\ \text{conservation}}}$$

- The **BUU** (Boltzmann-Ühling-Uhlenbeck) equation in short:

$$\frac{df}{dt} = I_{coll}$$

BUU-type transport models: working principle

- For each collision a distribution (of nucleons) $f(\boldsymbol{r}, \boldsymbol{p}, t)$ *in the initial state* is prepared:

$$f(\boldsymbol{r},\boldsymbol{p},t) = \sum_{i=1}^{A_{beam}+A_{target}} \delta(\boldsymbol{r}-\boldsymbol{r}_i)\ \delta(\boldsymbol{p}-\boldsymbol{p}_i)$$

- **Initialization of positions**

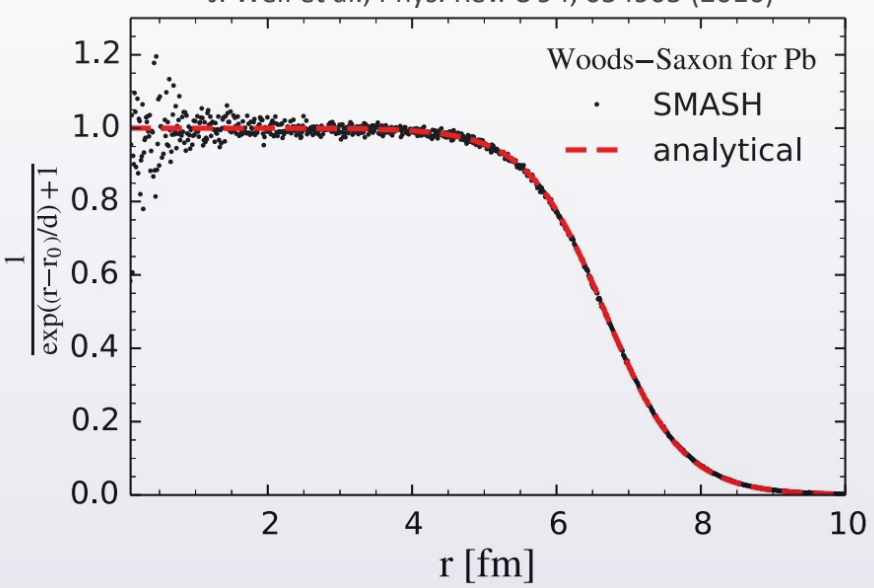
Usually: **Woods-Saxon** distribution with parameters based on experimental charge distribution

$$\frac{d^3N}{dr^3} = \frac{\rho_0}{\exp\left(\frac{r-r_0}{d}\right) + 1}$$

TABLE I. This table summarizes the specific parameters used in the Woods–Saxon initialization for some nuclei.

Nucleus	A	r_0 [fm]	d [fm]
U	238	6.86	0.556
Pb	208	6.67	0.54
Au	197	6.38	0.535
Cu	63	4.20641	0.597

J. Weil et al., Phys. Rev. C 94, 054905 (2016)



- **Initialization of momenta**

Usually: for nucleons away by r from the center, p is pulled randomly from unit sphere, i.e.: $p \in [0, p_{Fermi}(r)]$

$$p_{\text{Fermi}}(\vec{r}) = \hbar \left[\frac{3}{2}\pi^2\, \rho(\vec{r}) \right]^{1/3}$$

Usually, Fermi momentum in the middle of a nucleus $\approx$ 270 MeV/c .

BUU-type transport models: working principle

- Motion of a given particle is governed by the *1-body hamiltonian* :

Non-relativistic variant: $H = \frac{p^2}{2m} + U$

Relativistic variant: $H = \sqrt{p^2 + m^2} + U$

(it's a simplified picture: without EM and strong interactions)

- *In consecutive time steps* (n) position and momentum of i-th particle evolves due to: (non-relat. variant)

$$\begin{cases} \dot{\boldsymbol{r}}_i = \nabla_{\boldsymbol{p}_i} H \\ \dot{\boldsymbol{p}}_i = -\nabla_{\boldsymbol{r}_i} H \end{cases} \Rightarrow \begin{cases} \boldsymbol{r}_i(n+1) = \boldsymbol{r}_i(n) + \frac{\boldsymbol{p}_i(n+\frac{1}{2})}{m} \cdot \Delta t + \nabla_{p_i} U_i \cdot \Delta t \\ \boldsymbol{p}_i(n+\frac{1}{2}) = \boldsymbol{p}_i(n-\frac{1}{2}) - \nabla_{\boldsymbol{r}_i} U_i(n) \cdot \Delta t \end{cases}$$

⊕ collisions: if $d < d_{min}$ and no Pauli blocking → then $\delta(p_1)$ and $\delta(p_2)$ changes into $\delta(p_3)$ and $\delta(p_4)$

- <u>Caution</u>: Within BUU the only field-type (continuous) interaction – is through the mean field.
No nucleon–nucleon interactions (except for collisions).

→ BUU usually knows nothing about adjacent nucleons coalescing into LCPs (d, t, ^{3}He, …).

- In **QMD** models the objects are the particle wave packets:

$$\Psi = \prod_i \psi_i \sim \prod_i \exp\left[-\frac{(\boldsymbol{x_i}-\boldsymbol{r_i}(t))^2}{L}\right] \cdot \exp\left[i\,\boldsymbol{x_i}\,\boldsymbol{p_i}(t)\right]$$

e.g. C.Hartnack et al. Eur. Phys. J. A 1, 151 (1998) arxiv.org/abs/nucl-th/9811015

Parameter *L* describes the packet size. It is found such that Ψ describes the density drop at the nucleus' skin.

- An *i*-th particle moves in the potential ⊕ collides with neighbours.

However, the potential is built from a sum of potentials from '*j-th*' particles surrounding the '*i*-th' one.

Quantum hamiltonian:

$$\langle H\rangle = \langle T\rangle + \langle V\rangle = \sum_i T_i + \sum_i \sum_{j>i} \int \psi_i^* \psi_j^*\; V^{ij}(x_1,x_2)\;\; \psi_i \psi_j \;\; dx_1 dx_2$$

Equation of motion:

$$\begin{cases} \dot{\boldsymbol{r}}_i = \dfrac{\boldsymbol{p_i}}{m} + \nabla_{p_i} \sum_j \langle V_{ij}\rangle \\ \dot{\boldsymbol{p}}_i = -\nabla_{r_i} \sum_{j\neq i} \langle V_{ij}\rangle \end{cases}$$

(in fact, the simulation traces the centroids of wave packets)

- **N_i–N_j interactions :**

$$V_{ij} = V_{ij}^{Skyrme} + V_{ij}^{Yukawa} + V_{ij}^{p-dependent} + V_{ij}^{Coulomb} + V_{ij}^{p-n\ assymetry}$$

$$= \left[t_1 + t_2\,\rho^{\gamma-1}(\boldsymbol{x_i})\right] \cdot \delta(\boldsymbol{x}_i - \boldsymbol{x}_j) + t_3 \frac{\exp\left\{-|\boldsymbol{x}_i - \boldsymbol{x}_j|/\mu\right\}}{|\boldsymbol{x}_i - \boldsymbol{x}_j|/\mu} +$$

$$+ t_4 \ln^2\left(1 + t_5(\boldsymbol{p}_i - \boldsymbol{p}_j)^2\right) \cdot \delta(\boldsymbol{x}_i - \boldsymbol{x}_j) + \frac{Z_i Z_j e^2}{\{\boldsymbol{x}_i - \boldsymbol{x}_j\}} + t_6 \frac{1}{\rho_0}\, T_i^3 T_j^3 \cdot \delta(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

- From the *talk* of Ch. Hartnack and J. Aichelin (2015), the authors of **IQMD**:

Definition of the potentials

$$
\begin{aligned}
V^{ij} &= G^{ij} + V^{ij}_{\rm Coul} \\
&= V^{ij}_{\rm Skyrme} + V^{ij}_{\rm Yuk} + V^{ij}_{\rm mdi} + V^{ij}_{\rm Coul} + V^{ij}_{sym} \\
&= t_1\delta(\vec{x}_i-\vec{x}_j) + t_2\delta(\vec{x}_i-\vec{x}_j)\rho^{\gamma-1}(\vec{x}_i) + t_3\frac{\exp\{-|\vec{x}_i-\vec{x}_j|/\mu\}}{|\vec{x}_i-\vec{x}_j|/\mu} + \\
&\quad t_4\ln^2(1+t_5(\vec{p}_i-\vec{p}_j)^2)\delta(\vec{x}_i-\vec{x}_j) + \frac{Z_iZ_je^2}{|\vec{x}_i-\vec{x}_j|} + \\
&\quad t_6\frac{1}{\varrho_0}T_3^iT_3^j\delta(\vec{r}_i-\vec{r}_j)
\end{aligned}
$$

Bethe Weizsaecker –mass formula:

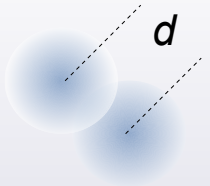
Volume term +Surface term +Coulomb term +symmetry term

(nucl. eos) (+pairing term not included) (asy- eos)

Outline of transport models. QMD Family

- **Hadron-hadron collision**. Occurs if the centroids between 2 nucleons approach closer than:

$$d < \sqrt{\frac{\sigma_{NN}}{\pi}}$$



Pauli exclusion is accounted for.

- By selecting components V_{ij} : $V^{\text{Skyrme}} + V^{\text{Yukawa}} + V^{\text{p-dependent}}$, through the respective convolution one can "reproduce" the equation of state of nuclear matter.

$$U(\rho) = \alpha \frac{\rho_{int}}{\rho_0} + \beta \left(\frac{\rho_{int}}{\rho_0}\right)^{\gamma} + U^{\text{p-dependent}}$$

$$U^{p-dependent} = \delta \cdot \ln^2\left(\epsilon \cdot (\Delta p)^2 + 1\right)$$

Δp : particle momentum w.r.t $\langle p \rangle$ of medium

One can choose the V_{ij} parameters so that the simulation reproduces given Equation of State, (including "recreation" of the hard/soft scenarios)

	α (MeV)	β (MeV)	γ	δ (MeV)
S	-356	303	1.17	—
SM	-390	320	1.14	1.57
H	-124	71	2.00	—
HM	-130	59	2.09	1.57

- The **Isospin-QMD** (**IQMD**) variant :
  - Separate treatment of n, p, Δ, π
  - Possible proton-neutron interactions
  - one can describe the asymmetric (n ≠ p) part of EoS

IQMD provides the framework for description of binding of nucleons into LCPs (clusters).

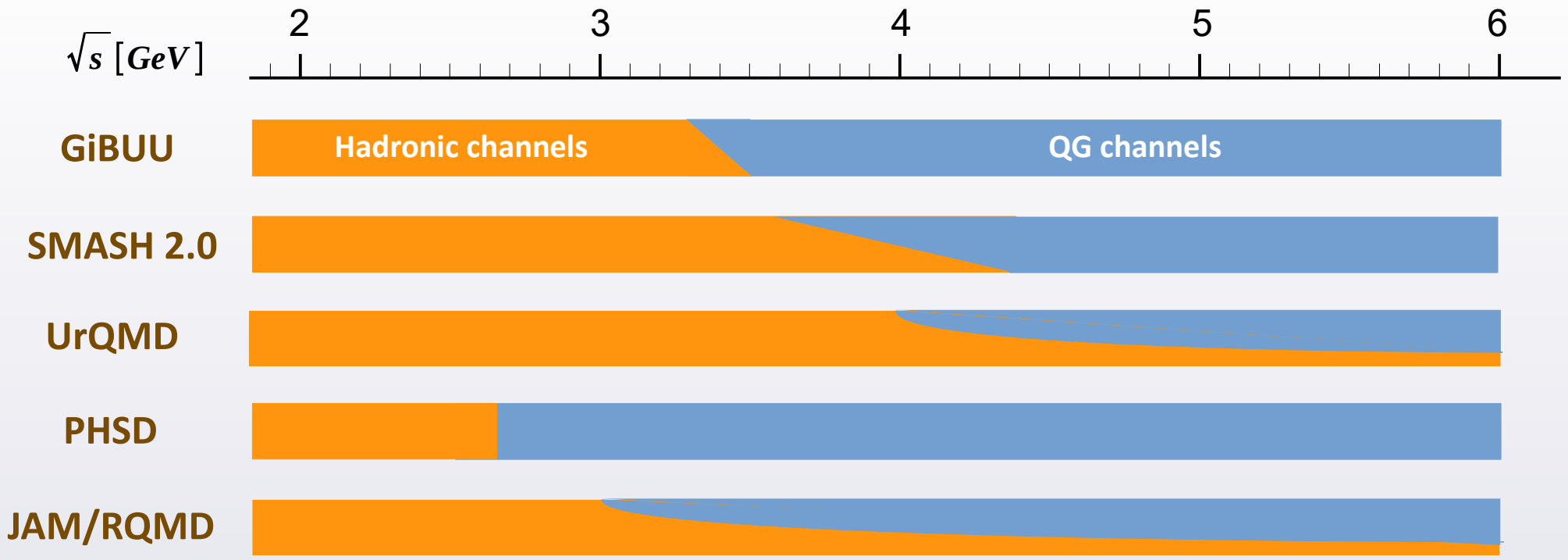
Hadron vs quark sectors

- Due to shortening of $\lambda_{\text{de Broglie}}$ with beam E, with increasing energy we move from the hadron sector to that dominated by quarks and gluons (QG). Transport models aim to describe both these dynamics.

  Some models propose the insertion of the QGP phase as the initial stage of collision, by transitioning to the **hydronamical description**, after which the **hadronization** occurs (QG fluid → hadron gas) .

  In addition, most of models also at the hadronic stage switch on the QG degrees of freedom.
  It is possible if in the individual hadron-hadron collision the $\sqrt{s}$ crosses the threshold value.

- For the baryon-baryon (BB) channels:



- Description of QG channels: so-called ”strings”. Between 2 quarks a segment is created (”string”) with homogeneous energy distribution. This string is divided into portions, which generate hadrons.

- Characteristics of main properties of selected (”currently on market”) transport models

Property	GiBUU	IQMD	UrQMD	RQMD.RMF	SMASH	PHSD
Relativistic Kinematics	optionally	n,p : no K : yes	yes	yes	yes	yes
Potential = Mean field	yes	n,p : no K : yes	no	optionally	yes	yes
Potential = sum of nucleon pots.	no	yes	yes	optionally	no	no
Electromagnetic potential	yes	yes	yes for baryons no for π	optionally	no	no
Momentum-dependent potential	yes	yes	no	yes	no	yes
Creation of LCP (clusters)	at end of simulation	at end of simulation	no	at end of simulation	at end of simulation	no
modifications of hadron mass in the medium	yes	yes	no	baryons: yes K, π: no	no	yes
quark-gluon phase described by ”strings”	yes	no	yes	yes	yes	yes

- **Fact:** each scalar product of two four-vectors is invariant w. r. t. Lorentz Transform.

$$p_\mu p^\mu = E^2 - \vec{p}^2 = const \quad \rightarrow \text{ quite obvious, as: } \quad E^2 - \vec{p}^2 = m^2$$

$$\left(\sum_i E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2 = \text{invariant} \equiv s$$

at the same time:

$$\begin{cases} \sum_i E_i = const \\ \sum_i \vec{p}_i = const \end{cases}$$

→ "*s*" is not only invariant.
It also is the integral of motion
[= const(*t*)].

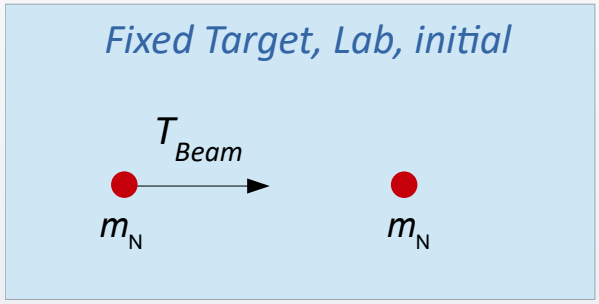
- Centre of mass frame (CM) : such frame that $\sum_i \vec{p}_{i,CM} = \vec{0}$

$$s \equiv \left(\sum_i E_{i,CM}\right)^2 - \left(\sum_i \vec{p}_{i,CM}\right)^2 = \left(\sum_i E_{i,CM}\right)^2$$

$$\rightarrow \quad \sqrt{s} = \sum_i E_{i,CM}$$

$\sqrt{s}$ is called the "**available energy**". It amounts to the sum of energy of the system in the CM frame.

- For fixed-target collisions, in order to heat up the system (in CM), we provide T_{Beam} energy (in Lab). Let's consider the nucleon-nucleon (NN) collision at the fixed target. How to translate $T_{\text{Beam}} \rightarrow \sqrt{s}$?



In **Lab**:

$$s \equiv \left(2\,m_N + T_{Beam}\right)^2 - \vec{p}^{\,2}_{Beam}$$

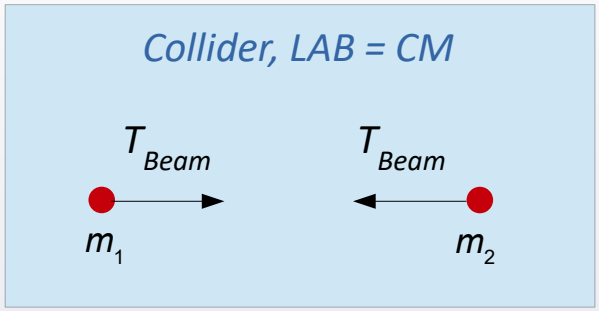
. . .

$$\sqrt{s} = \sqrt{2 \cdot m_N \left(2\,m_N + T_{Beam}\right)}$$

$\sqrt{s}$ rises as $\sqrt{T_{Beam}}$

E.g. for T_{Beam} = 1 GeV, $\sqrt{s}$ = 2.3 GeV

- $\sqrt{s}$ is commonly used in the collisions of two opposing beams. Why so?



Momenta sum up to $\vec{0}$. We obtain:

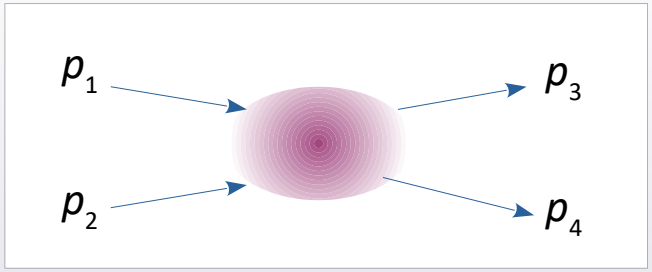
$$\sqrt{s} = \sum_i E_{i,CM} = 2 \cdot \left(m_N + T_{Beam}\right)$$

Nb. in the collider mode it's easier to inject much energy to the system of colliding nuclei:

$$\sqrt{s} \sim T_{Beam}$$

What else should we know? Mandelstam variables

- Consider process with initially 2 particles with 4-momenta p_1, p_2 and finally also 2, with 4-momenta p_3, p_4.



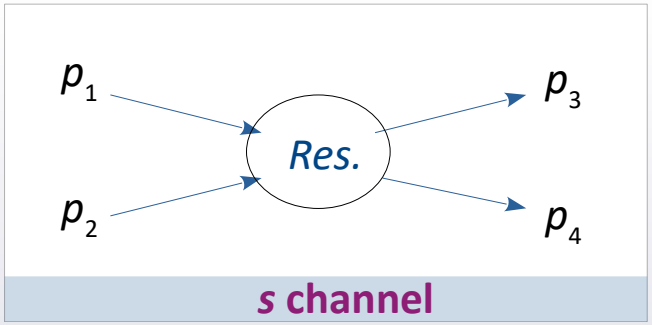
Energy + momentum conservation ≡ 4-momentum conservation.

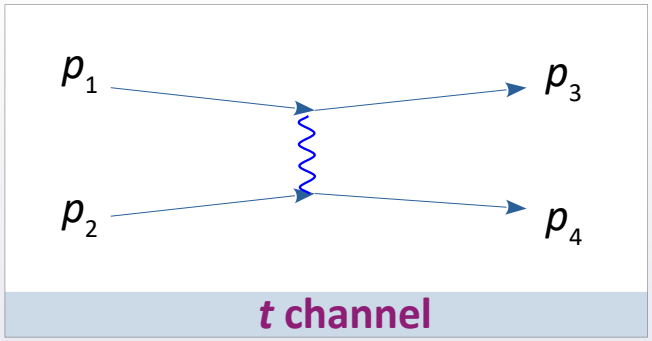
$$p_1 + p_2 = p_3 + p_4$$

- So it's worth introducing the "**available energy**" :

$$s \equiv (p_1 + p_2)^2 = (p_3 + p_4)^2$$

If a process is the fusion of particles into a resonance, then its mass $m = \sqrt{s}$. We say a reaction is in the "**s channel**".





- But the 4-momentum conservation law can be rearranged:

$$p_1 - p_3 = p_4 - p_2$$

and it's worth introducing here "**4-momentum transfer**":

$$t \equiv (p_1 - p_3)^2 = (p_4 - p_2)^2$$

If a process involves an exchange of a virtual particle, then its squared 4-momentum equals to *t*. A reaction is "**in *t* channel**"

- Example: Process $\pi^- \Lambda \to K^- N$ is a "t-channel reaction".
  Process $\pi^- \Lambda \to \Sigma^{*-} \to K^- N$ is a "s-channel reaction".

Before we analyse data in Root...

- **Simple tools in Bash for previewing text data:**

```
cat [file]                                  : To display file contents
head  -[No. of lines]  [file]               : To display first lines of file
tail  -[No. of lines]  [file]               : To display last  lines of file
less [file]                                 : File reader  (handy hotkeys:  g G / q )
wc -l [file]                                : To count the number of lines

cat mydata.txt | wc -l                      : The | (pipe) symbol redirects an output A → input B
grep [phrase] [file]                        : To search for a (line with) phrase in a file

cat myfile | awk 'NF==8 && $5=="Z" '        : Accepts only lines with 8 words, where 5. word is Z
cat myfile | awk 'NF==8 {print $3}'         : Accepts only lines with 8 words ⊕ prints 3. word only
```

- **Example:**

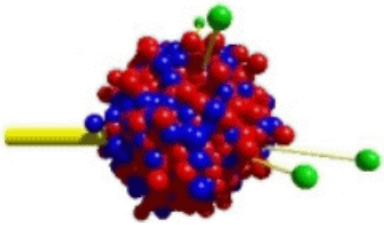
```
grep "# event" EventOutput.Real.oscar | wc -l
cat particle_lists.oscar | awk 'NF == 12 && $10 == 321' > mykaons.dat
```

- **Comment:**

Shells like Bash offer many text-based tools that e.g. sort, position, substitute phrases etc.
They allow for manipulation using variables, conditional instructions,
loops – including these over files and directories. One can code in scripts.

Many tutorials on Bash and extensive forums are available on the web.
You can find an interesting cheat-sheet here.
The introduction to Bash on the pages of UW Faculty of Physics is e.g. here.

GiBUU Model



GiBUU
The Giessen Boltzmann-Uehling-Uhlenbeck Project

- **Home page:** gibuu.hepforge.org

Download.
Download instructions: gibuu.hepforge.org/trac/wiki/download
Prerequisites: gibuu.hepforge.org/trac/wiki/tools
Download: gibuu.hepforge.org/downloads

Installation. Compilation instructions: gibuu.hepforge.org/trac/wiki/compiling

How to run: gibuu.hepforge.org/trac/wiki/running

Help.
Tutorial: gibuu.hepforge.org/trac/wiki/tutorial
Presentations: gibuu.hepforge.org/trac/wiki/Talks
Animations: gibuu.hepforge.org/trac/wiki/MovieMain

Table of PID codes: gibuu.hepforge.org/trac/wiki/ParticleIDs

Main papers:

O. Buss *et al.* ”Transport-theoretical description of Nuclear Reactions”, Physics Reports 512, 1 (2012), arXiv: 1106.1344

- **On the neutronx computer:**

```
mkdir gibuu ; cd gibuu
cp -r /home/kpiasecki/soft/gibuu/nkfj/*   .
```

Let's inspect the input file (c.f. next page), e.g:

```
less 001_CaCa1.91_test.job
```

Rich collection of input options (*please consider them thoughtfully*) **:**

http://gibuu.hepforge.org/Documentation/

Rich collection of demonstrative input files in this folder:

```
/home/kpiasecki/soft/gibuu/testRun/jobCards/
```

Let's perform a simulation (here: in the background . Log goes into file: `caca_1.91_test.log`)

```
nohup ./GiBUU.x <001_CaCa1.91_test.job 1>caca_1.91_test.log 2>&1 &
```

Many output files. Among them – file with events containing particles (see next page):

```
less EventOutput.Real.oscar
```

- **Input file `001_CaCa1.91_test.job` :**

```
! file: ./inputOutput/input.f90
&input
        eventtype             =      1
        numEnsembles          =     80
        num_runs_sameEnergy   =     10
        delta_T               =     0.25
        numTimeSteps          =   160
        printParticleVectorTime = T
        timeForOutput         = 0.
        timeSequence          = 20.
        path_To_Input    = '{...}/buuinput'
/
{....}
```

```
! file: ./init/initHeavyIon.f90
&heavyIon
      impact_parameter    = -7.
      impact_profile      = 0
      distance            = 16.
      coulomb             = F
      ekin_lab_Target     = 0.00
      ekin_lab_Projectile = 1.91
      adjustGridFlag      = T
      cmsFlag             = T
/
! file: ./density/nucleus.f90
&projectile
       projectile_Z= 20, projectile_A= 40
/
! file: ./density/nucleus.f90
&target
       target_Z= 20, target_A= 40
/
{....}
```

- **Output file `EventOutput.Real.oscar` :**

Table of PDG codes

```
#!OSCAR2013 particle_lists t x y z mass p0 px py pz pdg ID
# Units: fm fm fm fm GeV GeV GeV GeV GeV none none
# File generated by GiBUU (https://gibuu.hepforge.org)
# event     1 out   80
  0.000000E+0   7.183791E+0   3.961299E+0   2.780585E+1   9.227111E-1  1.225751E+0   2.703973E-2 -9.247542E-2   8.011161E-1  2212   100000
  0.000000E+0   5.118310E+0  -7.210583E-1   2.525251E+1   7.657091E-1  1.101021E+0  -5.729713E-3 -1.723748E-2   7.909611E-1  2112   100001
  0.000000E+0   2.584306E+0  -1.719659E-1   2.539883E+1   7.726719E-1  1.088114E+0  -1.631816E-1 -5.991384E-2   7.461942E-1  2112   100002
  0.000000E+0   2.133399E+0   3.397612E-1   2.692126E+1   7.944067E-1  1.045015E+0  -2.850862E-2 -9.578418E-2   6.717517E-1  2112   100003
  0.000000E+0   6.169842E+0  -1.890743E+0   2.453204E+1   8.264268E-1  1.030645E+0  -6.987948E-2   1.303097E-1   5.971482E-1  2112   100004
....
  0.000000E+0  -7.235081E+0   1.162180E+0  -2.613539E+1   7.677036E-1  1.057389E+0   3.919098E-2   1.860452E-1  -7.017274E-1  2212   103239
# event     1 end     0
# event     2 out   80
  0.000000E+0   3.456383E+0   6.567166E-1   2.607133E+1   7.648380E-1  8.782891E-1  -6.281280E-3   4.809546E-2   4.290245E-1  2212   100040
  0.000000E+0   5.786570E+0   1.426497E-1   2.657666E+1   7.663178E-1  1.130114E+0  -1.239606E-1   9.851192E-2   8.153864E-1  2112   100041
  0.000000E+0   6.166525E+0  -3.298759E+0   2.666215E+1   8.635919E-1  1.227330E+0   9.365502E-2  -4.166068E-2   8.660087E-1  2112   100042
....
```

Simulating Many Accelerated Strongly-interacting Hadrons

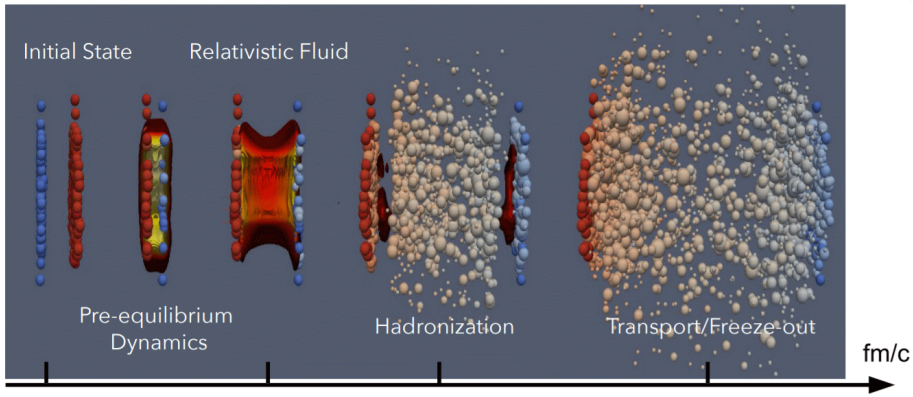
A relativistic hadronic transport approach

- **Home page:** theory.gsi.de/~smash/userguide/current
  smash-transport.github.io
- **Download** github.com/smash-transport/smash
- **Installation** theory.gsi.de/~smash/userguide/2.0/md_README.html
- **How to run**: theory.gsi.de/~smash/userguide/2.0/page_smash_invocation.html
- **Help.** Tutorial: theory.gsi.de/~smash/userguide/current/
  Presentations: D. Oliinychenko (2020) , A. Schäfer (2019)
  Animations: smash-transport.github.io/img/movie.mp4
- **Table of PDG codes**: github.com/smash-transport/smash/blob/master/input/particles.txt

- **Main papers**:

J. Weil *et al.* "Particle production and equilibrium properties within a new hadron transport approach for heavy-ion collisions" Physical Review C 94, 054905 (2016), arXiv: 1106.1344

H. Petersen *et al.* "SMASH – A new hadronic transport approach" Nuclear Physics A 982, 399 (2019), arXiv: 1808.06832



- **On the neutronx computer:**

```
mkdir smash ; cd smash
cp -r /home/kpiasecki/soft/smash/nkfj/*  .
```

Let's inspect the input file, e.g. :

```
less config.yaml
```

Rich collection of input options (*please consider them thoughtfully*) **:**

theory.gsi.de/~smash/userguide/2.0/input.html

Let's perform a simulation (here: in the background . Log goes into file: myrun .log)

```
nohup nice ./smash -i config.yaml -o outdir/ 1>myrun.log 2>&1 &
```

File with events containing particles can be found in folder `outdir` **:**

```
less outdir/particle_lists.oscar
```

- **Input file `config.yaml` :**

```
{....}
General:
    Modus:           Collider
    Time_Step_Mode:  Fixed
    Delta_Time:      0.1
    End_Time:        50.0
    Randomseed:      -1
    Nevents:         10

Output:
    Output_Interval: 20.0
    Particles:
        Format:          ["Oscar2013"]
        Only_Final:      false
```

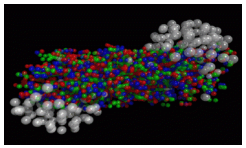
```
Modi:
    Collider:
        Projectile:
            Particles: {2212: 79, 2112: 118} #Gold197
        Target:
            Particles: {2212: 79, 2112: 118} #Gold197
        Impact:
            Sample: "quadratic"
            Range: [0.0, 12.0]

        E_Kin: 1.20
        Calculation_Frame: "center of mass"
        Fermi_Motion: "on"
        Collisions_Within_Nucleus: True
```

- **Output file `particle_lists.oscar` :**

Table of PID codes

```
#!OSCAR2013 particle_lists t x y z mass p0 px py pz pdg ID charge
# Units: fm fm fm fm GeV GeV GeV GeV GeV none none e
# SMASH-1.7
# event 1 in 394
-2.22563 -3.64001 -1.32594 -3.67708 0.938 2.236845 -0.03016814  0.00476330 2.030443 2112 0 0
-2.22563 -5.40173 -3.32581 -2.98890 0.938 2.101389  0.05289568 -0.05137847 1.878977 2112 1 0
-2.22563 -0.84595 -2.10703 -5.13988 0.938 2.523251  0.22803293  0.04328960 2.330897 2112 2 0
…
# event 1 out 394
     0    -3.67003 -1.32120 -1.65681 0.938 2.236845 -0.03016814  0.00476330 2.030443 2112 0 0
     0    -5.34571 -3.38022 -0.99825 0.938 2.101389  0.05289568 -0.05137847 1.878977 2112 1 0
```



Ultrarelativistic Quantum Molecular Dynamics

- **Home page:** urqmd.org

  Download. Home page → contact with the authors of the code.

  Installation + how to run: urqmd.org/en/compiling-and-running-urqmd/

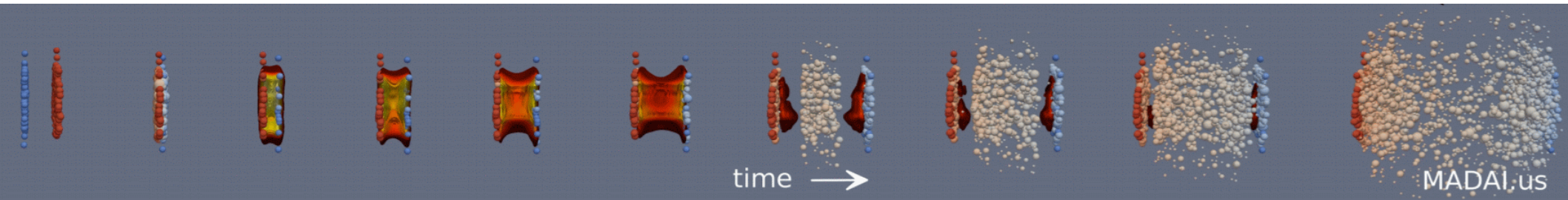
  Help. Manual: Copy on the web
  Presentation: M. Bleicher (2018)

  Table of PID codes: urqmd.org/en/particle-ids

- **Main papers**:

S.A. Bass *et al.* *"Microscopic models for ultrarelativistic heavy ion collisions"*
Progress in Particle and Nuclear Physics 41, 225 (1998), arXiv: nucl-th/9803035

M. Bleicher *et al.* "Relativistic hadron-hadron collisions in the ultra-relativistic quantum molecular dynamics model"
Journal of Physics G: Nuclear and Particle Physics 25, 1859 (1999), arXiv: hep-ph/9909407



- **On the neutronx computer:**

```
mkdir urqmd ; cd urqmd
cp -r /home/kpiasecki/soft/urqmd-3.4/nkfj/*   .
```

Let's inspect 2 input files, e.g. :

```
less inputfile_NiNi1.5
less runqmd.bash
```

Rich collection of input options is in Chapter 5 of Manual (*please consider them thoughtfully*) **:**

Copy on the web

Let's perform the simulation (here: in the background. Log goes into file: `mysim.log`)

```
nohup nice ./runqmd.bash 1>mysim.log 2>&1 &
```

We've got a collection of files with events.
In between the file with extension *f14* with events (and subsequent timesteps), containing particles :

```
less test.f14
```

- **Input file `inputfile.NiNi1.5` :**

```
pro 58 28
tar 58 28
nev 10
imp -10.
elb 1.5
tim 50 20
cto 18 1

f13
#f14
f15
#f16
f19
f20
{....}
```

- **Input file `runqmd.bash` :**

```
{....}
export ftn09=inputfile_NiNi1.5
export ftn13=test.f13
export ftn14=test.f14
export ftn15=test.f15
export ftn16=test.f16
export ftn19=test.f19
export ftn20=test.f20
{....}
```

- **Output file `test.f14` :**

Table of PID codes

charge

```
UQMD   version:       30400   1000  30400  output_file  14
projectile:  (mass, char)    58  28   target:  (mass, char)    58  28
transformation betas (NN,lab,pro)      0.0000000  0.6665679 -0.6665679
impact_parameter_real/min/max(fm):    6.28  0.00 10.00  total_cross_section(mbarn):     3141.59
equation_of_state:    0  E_lab(GeV/u): 0.1500E+0  sqrt(s)(GeV): 0.2517E+0  p_lab(GeV/u): 0.2250E+0
event#         1 random seed:  1600419089 (auto)   total_time(fm/c):       40 Delta(t)_O(fm/c):       20.000
(....)
pvec: r0         rx         ry          rz          p0          px          py          pz          m      ityp 2i3 ch lcl ncl or
    119      20
     73      47      22       4       7      19       0       0
 0.20000E+2  0.74819E+2  0.36667E+0  0.10799E+2  0.12222E+2 -0.81322E-2  0.91387E-1  0.80353E+0  0.92473E+0   1   1   1   0   0   0
 0.20000E+2  0.34167E+2  0.27792E+0  0.59270E+2  0.11642E+2  0.18126E-1 -0.53284E-1  0.68819E+0  0.93800E+0   1   1   1  79   1   0
 0.20000E+2  0.78149E+0 -0.30699E+2  0.10397E+2  0.13424E+2  0.88550E-1  0.91699E-2  0.97368E+0  0.92038E+0   1   1   1   0   0   0
{....}
        119          40
      83       54      22       7       7      19       0       0
 0.40000E+2  0.74819E+1  0.36667E+0  0.24239E+2  0.12282E+1 -0.81322E-2  0.91318E-1  0.80353E+0  0.92437E+0   1   1   1   0   0   0
```

Parton-Hadron-String Dynamics
Transport approach

- **Home page:** theory.gsi.de/~ebratkov/phsd-project/PHSD/

  Download. *[contact with authors]* theory.gsi.de/~ebratkov/phsd-project/PHSD/index4.html

  Installation + how to run: § 1.2 of Manual. Authors insist on using Intel Fortran compiler.

  Help. Manual: Copy on the web
  Presentations: E. Bratkovskaya, Home page
  Animations: Au+Au @ 10A GeV , Au+Au @ 35A GeV , Pb+Pb @ 158A GeV

  Table of PID codes: Tables 1.1 i 1.2 in Manual

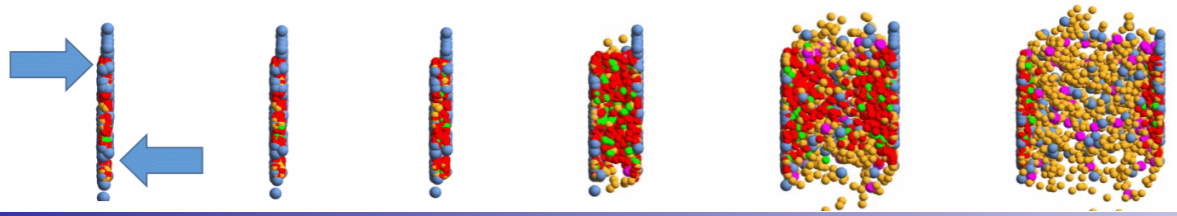
- **Main papers**:

  ⊙ PHSD:

  W. Cassing, E.L. Bratkovskaya "Parton transport and hadronization from the dynamical quasiparticle point of view", Physical Review C 78, 034919 (2018), arXiv: 0808.0022

  ⊙ HSD:

  W. Cassing, E.L. Bratkovskaya "Hadronic and electromagnetic probes of hot and dense nuclear matter" Physics Reports 308, 65 (1999), arXiv: 1808.06832



- **On the neutronx computer:**

```
mkdir phsd ; cd phsd
cp -r /home/kpiasecki/soft/phsd/nkfj/*  .
```

Let's inspect the input file, e.g. :

```
less inputPHSD.NiNi1.9
```

Rich collection of input options is in Section 1.4 of Manual (*please consider them thoughtfully*) **:**

Copy on the web

Let's perform the simulation (here: in the background. Log goes into file: `mysim.log`)

```
nohup nice ./run_phsd.sh 1>mysim.log 2>&1 &
```

We've got a collection of files with events.
In between the file `phsd.dat` with subsequent events containing particles :

```
less phsd.dat
```

- **Input file `inputPHSD.NiNi1.9` :**

```
58,     MASSTA: target mass
28,     MSTAPR: protons in target
58,     MASSPR: projectile mass
28,     MSPRPR: protons in projectile
1.91,   ELAB:  (=4060000. Lab energy per nucleon LHC),=21300 RHIC,=13433049 (5 TeV) = 26120000 (7 TeV)
0.0,    BMIN:   minimal impact parameter in fm ! no effect for p+A
7.0,    BMAX:   maximal impact parameter in fm ! no effect for p+A
0.5,    DeltaB: impact parameter step in fm (used only if IBweight_MC=0)
150,    NUM:    optimized number of parallel ensambles ("events")
10,     ISUBS:  number of subsequent runs
{....}
```

- **Output file `phsd.dat` :**

Table of PDG codes

```
125        1     1  0.6427918E+01     1
  27 -0.204536E+00  0.418460E+00  0.317638E+00  0.675407E+00  0.318581E+00  0.446779E+00  0.137598E+00  0.671028E+00
  2212     1 -0.25568587E+00 -0.16134205E+00 -0.12625135E+01  0.16016214E+01        1         51
  2112     0 -0.29324052E+00  0.86835183E-01 -0.75108749E+00  0.12399625E+01        1          0
  2212     1  0.61904673E-01  0.12047570E+00 -0.11011121E+01  0.14528037E+01       -1          0
  2212     1 -0.25516313E+00  0.11847670E+00 -0.90409690E+00  0.13328092E+01       -1          0
  2212     1  0.43485938E-02  0.11743490E-01 -0.73762423E+00  0.11933526E+01       -1          0
  2212     1 -0.69573373E-01 -0.43003834E-02 -0.66405290E+00  0.11513771E+01       -1          0
(....)
   111     0  0.37781116E-01  0.11493234E+00 -0.78976639E-01  0.19979531E+00        2         20
129        1     2  0.64279118E+01     1
  39  0.840226E+00  0.295869E+00 -0.752309E+00  0.163571E+00 -0.783561E+00  0.221714E+00 -0.245801E+00  0.586710E+00
  2212     1 -0.14489351E+00  0.92953674E-01 -0.11530347E+01  0.14963180E+01        1          0
  2212     1 -0.10142475E+00 -0.41412741E-01 -0.11190853E+01  0.14643080E+01        1          0
  2212     1  0.14764933E+00 -0.58835067E-01 -0.90158129E+00  0.13107077E+01        1          0
```

p_x p_y p_z E

charge

RQMD.RMF Model (QMD family ; based on JAM code)

- **Home page:** [Link] (retrieved page)

  Download. Versions 1.x (Fortran) . Versions 2.x (C++)

  Installation + how to run: [Link] (retrieved page) + files INSTALL and README

  Help. Manual: copy on the web
  Cheat-sheet about input file: README file

  Table of PDG codes: [File 1] , [File 2] (retrieved pages) or Appendix A in Manual.

- **Main papers**:

  ⊙ RQMD.RMF:

  Y. Nara, "Sensitivity of the excitation functions of collective flow to relativistic scalar and vector meson interactions in the relativistic quantum molecular dynamics model RQMD.RMF"
  Physical Review C 100, 054902 (2019), arXiv: 1906.03537

  ⊙ JAM:

  Y. Nara et al. "Relativistic nuclear collisions at 10A GeV energies from p+Be to Au+Au with the hadronic cascade model"
  Physical Review C 64, 024901 (1999), arXiv: nucl-th/9904059

RQMD.RMF Model (QMD family ; based on JAM code)

- **On the neutronx computer:**

```
mkdir rqmd.rmf ; cd rqmd.rmf
cp -r /home/kpiasecki/soft/rqmd.rmf/nkfj/*   .
```

Let's inspect the input file, e.g. :

```
less jam.cfg
```

Rich collection of input options is in Section 4.6 of Manual (*please consider them thoughtfully*) **:**

Copy on the web

Let's perform the simulation (here: in the background . Log goes into file `mysim.log`)

```
nohup nice ./jamexe 1>mysim.log 2>&1 &
```

We've got a collection of files with events.
In between the file `phase.dat` with subsequent events containing particles :

```
gzip -d phase.dat.gz
less phase.dat
```

RQMD.RMF Model (QMD family ; based on JAM code)

- **Input file `jam.cfg` :**

```
event=10
proj =108Ag
targ =108Ag
win=1.58gev
bmin=0.0
bmax=-10.
dt = 0.2
timestep = 150     # total number of time steps.
frame= nn          # computational frame  cm, nn, lab, collider
mstc(41)=0         ! 1: force resonance decays after sim, 0: let them remain
mstc(42)=1         ! 0: force    weak   decays after sim, 1: let them remain
{....}
```

- **Output file `phase.dat` :**

Table of PDG codes

```
#      2000      0.82165      0.67597      1.35698    2
#         1       223          25        21  9.10      36      58      43
1  2112  1   9.3960E-2  -7.4691E-2   4.6286E-3 -9.9425E-2   1.3700E+0  -7.8427E+0   3.5348E+0 -1.6458E+1   3.0200E+1   3.0200E+1
1  2112  1   9.3960E-2  -4.2849E-2   2.0180E-2 -8.9590E-2   1.3145E+0  -6.7592E+0   9.4156E+0 -1.7980E+1   3.0200E+1   3.0200E+1
1  2212  1   9.3960E-2  -1.1528E-2   2.2750E-2 -8.4282E-2   1.2676E+0  -7.5301E+0   5.3435E+0 -1.6708E+1   3.0200E+1   3.0200E+1
1  2112  5   9.3957E-2   3.0711E-2   2.4778E-2 -2.6403E-2   1.0527E+0   4.7449E+0   5.2862E+0 -4.7811E+0   3.0200E+1   3.0200E+1
{....}
1   111  8   1.3498E-1   1.4090E-1  -1.5388E-1  -1.4472E-1   2.8757E-1   2.5947E+0  -5.0680E+0  -8.0321E+0   3.0200E+1   3.0200E+1
#         2       227          78          62  7.12      85      152     124
1  2212  1   9.3960E-1  -2.1659E-2   1.1828E-1  -5.13744E-1  1.0776E+0  -6.4530E+0   4.1473E+0  -9.0628E+0   3.0200E+1   3.0200E+1
1  2212  1   9.3960E-1   4.0508E-2  -1.0861E-1  -7.09055E-1  1.1827E+0   6.2874E-2  -6.4578E-1  -1.1831E+1   3.0200E+1   3.0200E+1
```

mass

p_X p_Y p_Z E