Chapter 8

Multi-Higgs Doublet Models

Main motivation to go beyond one SU(2) scalar doublet is to provide a CP violation in the scalar sector, what seems to be needed also for baryogenesis... etc. Below I will follow mainly a book: G. C. Branco, L. Lavoura and J. P. Silva, CP Violation (Oxford Univ. Press, Oxford, England, 1999).


In Multi-Higgs Models typically one assumes $SU(2) \times U(1)$ gauge symmetry and $n_g$ families.

Lagrangian:

$$L = L_{SM} + L_H + L_Y, \quad L_H = T - V,$$

where $T$ corresponds to the kinetic term for scalar (Higgs) fields, $V$ is their potential, $L_Y$ describe a Yukawa interaction of scalar fields with fermions.

8.1 Multidoublets

In models with SU(2) doublets only, we consider

$$\Phi_a = \begin{pmatrix} \varphi_a^+ \\ \varphi_a^0 \end{pmatrix}, \quad a = 1 - n_d \ (n_d - \text{number doublets}), \quad Y = +1.$$  (8.1)
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8.2 Vacuum state and decomposition

We assume, that vacuum preserves $U(1)_{em}$, so that

$$< 0|\Phi_a|0> = \left( \frac{v_a}{\sqrt{2}} e^{i\theta_a} \right), \quad v_a = \text{real, non-negative.} \quad (8.2)$$

Using the $U(1)$ gauge transformation we can make vev of $\phi^0_1$ real ($\theta_1 = 0$) and positive as in the SM.

Decomposition of doublets:

$$\Phi_a = e^{i\theta_a} \left( \frac{\varphi^+_a + \rho_a + i\eta_a}{\sqrt{2}} \right), \quad (8.3)$$

and

$$\tilde{\Phi}_a = e^{-i\theta_a} \left( \frac{\varphi^+_a + \rho_a - i\eta_a}{\sqrt{2}} \right), \quad (8.4)$$

with $v = \sqrt{v_1^2 + ...} > 0$. Gauge boson masses are $M_W = \frac{g v}{2}$, with $M_Z$ as in SM.

8.3 Symmetry of the potential and number of parameters

The (hermitian) MHDM potential can be written in the following form:

$$V = Y_{ab} \Phi^\dagger_a \Phi_b + Z_{abcd} \Phi^\dagger_a \Phi^\dagger_b \Phi^\dagger_c \Phi_d, \quad \text{with } a, b, c, d = 1, 2 \quad (8.5)$$

with obvious symmetry relations

$$Y_{ab} = Y_{ba}^*, \quad Z_{abcd} = Z_{cdab}, \quad Z_{abcd} = Z_{badc}^*. \quad (8.6)$$

Taking into account these relations one can derive the maximal numbers of independent real parameters, namely:

$$n_Y = n^2_d \quad \text{for } Y_{ab} \text{ parameters}, \quad (8.7)$$

$$n_Z = n^2_d (n^2_d + 1) \quad \text{for } Z_{abcd} \text{ parameters}. \quad (8.8)$$
CHECK: How many in the case with \( n_d = 2 \)?

Additional constraints on number of these parameters are coming from the reparametrization freedom, arising from possible a \( U(n) = SU(n) \times U(1) \) transformations in the doublet \( (n_d \text{-dimensional}) \) space. \( SU(n) \) transformations are described \( n_r \) parameters. (In the simple case these transformations correspond to the rephasing of the doublets fields). This means that the number of real parameters, which are needed to define a physical situation, is equal to \( n_p = n_Y + n_Z - n_r \).

This freedom allows to chose a useful, for a particular consideration, basis for the doublets with adjusted forms of the vev’s for new doublets. Often it is useful to consider a basis called the Higgs (Georgi) basis, in which one doublet has zero vev.

### 8.4 Two Higgs Doublet Model (2HDM)

In case with two scalar doublets (2HDM), we can expect 5 Higgs bosons-three of them neutral and two charged.

#### 8.4.1 Potential

The most general renormalizable potential respecting the gauge symmetry \( SU(2) \times U(1) \) is given by

\[
V = m_1 \Phi_1^\dagger \Phi_1 + m_2 \Phi_2^\dagger \Phi_2 + m_3 [e^{i\delta_3} \Phi_1^\dagger \Phi_2 + h.c.]
\]

\[
+ a_1 (\Phi_1^\dagger \Phi_1)^2 + a_2 (\Phi_2^\dagger \Phi_2)^2 + a_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + a_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + [a_5 e^{i\delta_5}(\Phi_1^\dagger \Phi_2)^2 + h.c.]
\]

\[
+ [(a_6 e^{i\delta_6}(\Phi_1^\dagger \Phi_1) + \lambda_7 e^{i\delta_7}(\Phi_2^\dagger \Phi_2))(\Phi_1^\dagger \Phi_2) + h.c.]
\]

\( a_{1-7}, m_1 \) and \( m_2 \) are real (by hermiticity of the potential) and \( m_3, a_{5-7} \) can be taken non-negative. So altogether there are 14 parameters.

#### 8.4.2 Reparametrization freedom

Our Lagrangian contains two fields (doublets) with identical quantum numbers: a global unitary transformations \( \tilde{F} \) which mix these fields and change
the phases is given by:
\[
\left( \begin{array}{c} 
\phi_1' \\
\phi_2'
\end{array} \right) = \hat{\mathcal{F}} \left( \begin{array}{c} 
\phi_1 \\
\phi_2
\end{array} \right), \quad \hat{\mathcal{F}} = e^{-i\rho_0} \left( \begin{array}{cc} 
\cos \theta e^{i\rho/2} & \sin \theta e^{i(\tau-\rho)/2} \\
-\sin \theta e^{-i(\tau-\rho)/2} & \cos \theta e^{-i\rho/2}
\end{array} \right).
\]

(8.10)

So, \(\hat{\mathcal{F}}(\rho_0; \rho, \tau, \theta)\) with all parameters real and with \(\rho_0\) a overall phase.

These transformations contain phase rotations and mixings of fields and lead to the change of the field basis: the particular case with \(\theta = 0\) is a global transformation of fields with the independent phase rotations:

\[\phi_{1,2} \rightarrow e^{-i\rho_{1,2}} \phi_{1,2}; \quad \rho_1 = \rho_0 - \rho/2, \quad \rho_2 = \rho_0 + \rho/2, \quad \rho = \rho_2 - \rho_1\]

(rephasing of fields).

The transformations of fields lead to the corresponding change of parameters in the Lagrangian \(L_H\), without a change of the physical content of the model. This is a reparametrization invariance of \(L_H\).

This reparametrization transformation (or in short reparametrization) is given by three parameters \((\rho, \tau, c = \cos \theta, s = \sin \theta)\). A set of these physically equivalent Higgs Lagrangians forms the reparametrization equivalent space, which is subspace of the entire space of Lagrangians \(L_H\). The different physical situations are described by 11 parameters in the potential. Some specific set of parameters defines a specific reparametrization scheme.

The rephasing (and reparametrization) invariance can be extended to the description of a whole system of scalars and fermions if the corresponding transformations for the Yukawa terms (phases of fermion fields and Yukawa couplings) supplement the transformations.

Note, that in the potential \(V\) (8.9) the \(Z_2\) symmetry satisfied, i.e. the symmetry under the following transformation: \(\Phi_1 \rightarrow -\Phi_1, \Phi_2 \rightarrow \Phi_2\) (or vice versa), if \(a_6 = a_7 = m_3 = 0\).

Comment: The complex values of some of the parameters in \(L\) provide a necessary condition for the CP violation in the Higgs sector. Obviously, this violation does not appear if the Lagrangian with complex parameters can be transformed to a form with all real parameters by means of some reparametrization.

### 8.4.3 Constraints on the potential

**Stability of the potential** To have a stable vacuum the potential should be large at large values of the fields \(|\varphi_k|\). This leads to the following conditions...
called also positivity constraints (without proof):

$$a_1 > 0, a_2 > 0, 2\sqrt{a_1 a_2} + a_3 > 0 \text{ and } 2\sqrt{a_1 a_2} + a_3 + a_4 \pm 2a_5 > 0.$$  (8.11)

**Extremum, minimum and global minimum conditions** The extremum of the potential is obtained for vanishing first derivatives of the potential. This defines the vacuum expectation values (v.e.v) of the fields $\Phi$:  

$$\left. \frac{\partial V}{\partial \Phi_1} \right|_{\Phi_1 = \langle \Phi_1 \rangle, \Phi_2 = \langle \Phi_2 \rangle} = 0,$$

$$\left. \frac{\partial V}{\partial \Phi_2} \right|_{\Phi_1 = \langle \Phi_1 \rangle, \Phi_2 = \langle \Phi_2 \rangle} = 0.$$  (8.12)

To get a minimum, second derivatives should be positive, what is equivalent to having positive mass-squared for a physical Higgs particles.

A vacuum corresponds to the minimum with the lowest energy (a global minimum).

**Unitarity and perturbativity constraints** The quartic terms in the potential are leading to the quartic self-interactions of Higgs bosons. In physical processes like $HH \to HH$ (or in scattering of gauge bosons) one should respect unitarity limit for some amplitudes. This so called the tree-level unitary constraint limits the upper value of $a_i$ parameters. Similar role plays perturbative conditions (to allow to perform a perturbative expansions). Both lead typically to a condition $|a_i| < 4\pi$.

### 8.4.4 The quark Yukawa Lagrangian

### 8.4.5 Natural suppression of the Flavour Changing Neutral Yukawa Interaction

In the original basis we define Model II, as such in which $\Phi_1$ couples to $d$-type quarks and charged leptons $\ell$, while $\Phi_2$ couples to $u$-type quarks (we take neutrinos to be massless). In Model I only the $\Phi_1$ couples to fermions.

In both Model I and II FCNC are killed at the tree level.
8.4.6 2HDM in the Higgs basis

In the Higgs basis we have after SSB the following decomposition

\[ H_1 = \left( \begin{array}{c} \phi^+ \\ v + h + i \chi \end{array} \right), \quad H_2 = \left( \begin{array}{c} H^+ \\ H + iA \end{array} \right), \quad (8.13) \]

\[ \left( \begin{array}{c} H_1 \\ H_2 \end{array} \right) = \hat{F}_H \left( \begin{array}{c} \Phi_1 \\ e^{-i\theta} \Phi_2 \end{array} \right), \quad \hat{F}_H = \frac{1}{v} \left( \begin{array}{cc} v_1 & v_2 \\ v_2 & -v_1 \end{array} \right). \quad (8.14) \]

and

\[ V = \mu_1 H_1^\dagger H_1 + \mu_2 H_2^\dagger H_2 + (\mu_3 H_1^\dagger H_2 + h.c.) \quad (8.15) \]

\[ + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_1^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_1^\dagger H_1) \]

\[ + [\lambda_5 (H_1^\dagger H_2) + \lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)](H_1^\dagger H_2) + h.c. \]

8.4.7 Condition for CP violation

The gauge-kinetic term in the Lagrangian is CP symmetric, and we use this fact to introduce the most general CP transformation. Below we focus on scalar sector only. Note, that the vev’s properties are also important in the analysis.

We write (at the beginning for the original basis with \( n_d \) doublets):

\[ \phi_a^0 = v_a e^{i\theta_a} + H_a^0, \quad a = 1..n_d. \quad (8.16) \]

From the requirement of CP invariance of the gauge interaction of the scalars, we find that

\[ (CP)\phi_a^+(t, \vec{x})(CP)^\dagger = U_{ab}^{CP} \phi_b^-(t, -\vec{x}), \quad (8.17) \]

\[ (CP)H_a^0(t, \vec{x})(CP)^\dagger = U_{ab}^{CP} H_b^1(t, -\vec{x}), \quad (8.18) \]

and \( U^{CP} \) should be chosen such that

\[ v_a e^{i\theta_a} = U_{ab}^{CP} v_b e^{-i\theta_b}. \quad (8.19) \]

Then we have

\[ (CP)\Phi_a(t, \vec{x})(CP)^\dagger = U_{ab}^{CP} (\Phi_b^1)^T(t, -\vec{x}). \quad (8.20) \]
8.4. TWO HIGGS DOUBLET MODEL (2HDM)

In the Higgs basis, from the definition, one can derive that the form of this matrix should be:

\[
U^{CP} = \begin{pmatrix}
1 & 0_{1 \times (n_d - 1)} \\
0_{1 \times (n_d - 1)} & K^{CP}
\end{pmatrix},
\]  

(8.21)

where \(K^{CP}\) - an arbitrary \((n_d - 1) \times (n_d - 1)\) matrix.

**Example: 2HDM with \(Z_2\) symmetry**  In this case \(m_3 = a_6 = a_7 = 0\). The potential at the vacuum \(V_0\) contains one term with \(\theta\) (the phase of the second doublet) equal to \(1/2a_5v_1^2v_2^2\cos(\delta_5 + 2\theta)\). The lowest value of \(V_0\) is obtained for

\[
\cos(\delta_5 + 2\theta) = -1.
\]  

(8.22)

Using the \(U^{CP}\) in the form

\[
U^{CP} = \begin{pmatrix}
1 & 0 \\
0 & e^{2i\theta}
\end{pmatrix}
\]  

(8.23)

and the following form of the CP-transformation (from the (8.20))

\[(\mathcal{CP})(\Phi_1^\dagger \Phi_2)^2(\mathcal{CP})^\dagger = e^{4i\theta}(\Phi_2^\dagger \Phi_1)^2,\]

we can check that CP-invariance of the \(a_5\) term (other terms are already CP invariant). It requires

\[
e^{i(\delta_5 + 4\theta)} = e^{-i\delta_5},\]  

(8.24)

what it is already guaranteed by (8.22).

**2HDM with a softly broken \(Z_2\) symmetry**  If in the potential there is a \(m_3\) term, then \(Z_2\) symmetry is broken softly.

The CP-invariance would require

\[
e^{2i(\delta_5 + 2\theta)} = 1\]  

(8.25)

\[
e^{2i(\delta_3 + \theta)} = 1\]  

(8.26)

what is not automatic, as it was before.
8.5 Physical Higgs bosons

In case of CP conservation neutral Higgses are \( h, H, A \) (\( h \) being lighter than \( H \)). \( h, H \) CP-even (with parameter \( \alpha \) describing mixing among them), and \( A \) CP-odd. For CP violation we have \( h_1, h_2, h_3 \) - with no definite CP properties. Charged Higgses are denote as \( H^\pm \).

8.6 Couplings

The \( v_i \) obey SM constraint: \( v_1^2 + v_2^2 = v^2 \), with \( v = (\sqrt{2}G_F)^{-1/2} = 246 \) GeV.

The another parameterization of these v.e.v.’s (via parameters \( v \) and \( \beta \)) is also used:

\[
v_1 = v \cos \beta, \quad v_2 = v \sin \beta, \quad \beta \in \left( 0, \frac{\pi}{2} \right).
\] (8.27)

There are a number of useful relations between the relative couplings of neutral Higgs particles to gauge bosons and fermions (basic relative couplings).

1. The pattern relation holds among the basic relative couplings of each neutral Higgs particle \( h_i \):

   a) \((\chi_u^{(i)} + \chi_d^{(i)})\chi_V^{(i)} = 1 + \chi_u^{(i)} \chi_d^{(i)}, \quad \text{or} \quad \text{b) } (\chi_u^{(i)} - \chi_d^{(i)}) (\chi_V^{(i)} - \chi_d^{(i)}) = 1 - (\chi_V^{(i)})^2, \)

   which has the same form for each Higgs boson \( h_i \) (in particular also for \( h, H, A \) in the case of CP conservation).

2. For each neutral Higgs boson \( h_i \) a horizontal sum rule holds:

\[
|\chi_u^{(i)}|^2 \sin^2 \beta + |\chi_d^{(i)}|^2 \cos^2 \beta = 1.
\] (8.29)

3. A vertical sum rule for each basic relative coupling \( \chi_j \) to all three neutral Higgs bosons \( h_i \) holds:

\[
\sum_{i=1}^{3} (\chi_j^{(i)})^2 = 1 \quad (j = V, d, u).
\] (8.30)

For couplings to the gauge bosons this sum rule takes place independently on a particular form of the Yukawa interaction.
8.7 Minimal Supersymmetric Standard Model (MSSM)

The Higgs sector in MSSM contains two doublets (with $Y=+1$ and -1) and has the same form of potential as 2HDM, however with the quartic couplings given by the gauge couplings. The realization of the Yukawa interaction is according to the Model II. Sum rules hold as in the non-supersymmetric 2HDM.

The mass limit of lightest neutral $h$ boson is predicted to be lighter than $M_Z$, with radiative corrections lighter than 135 GeV.

8.8 Constraints on the Higgs sector in SM, MSSM and 2HDM

The newest data can be found here http://plhc2010.desy.de/ (eg. talk by Watts).
**TASKS**

### 8.9 Tasks 2009-10

**Tasks 1** Show that the $O(2)$ group of rotation in $(x,y)$ plane is equivalent to $U(1)$ transformation on $w = x + iy$.

**Tasks 2** Consider the Lagrangian with complex scalar (spin zero) field

$$\mathcal{L} = (\partial^\mu \phi)^*(\partial_\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2,$$

symmetric under the global $U(1)$ transformations $\phi(x) \to e^{i\theta} \phi(x)$. With negative $\mu^2$ and positive $\lambda$ spontaneous symmetry breaking is realized with vev $v = \sqrt{-\mu^2/\lambda}$.

**a/Introduce new fields**

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x)),$$

with vanishing vev for $\eta$ and $\zeta$.

Write $\mathcal{L}$ using these fields, including a constant term.

Calculate a scattering amplitude for the elastic $\eta\zeta$ (it was done during lecture) and $\zeta\zeta$ process at the Born level.

**b/Introduce new fields**

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x))e^{i\zeta(x)}/v,$$

with vanishing vev for $\eta$ and $\zeta$.

Write $\mathcal{L}$ using these fields, including a constant term.

Calculate a scattering amplitude for the elastic $\eta\zeta$ process at the Born level.
Tasks 3 Consider the same case as above in terms of $\phi_1$ and $\phi_2$ real fields, $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$.

Using a current $j_0 = q[(\partial_0 \phi_1)\phi_2 - (\partial_0 \phi_2)\phi_1]$ calculate a corresponding charge $Q$.

b/commutators of $Q$ with $\phi_1$ and with $\phi_2$.

Assuming that vev for $\phi_1 \neq 0$, while $\phi_2 = 0$, calculate

$\langle 0 | [Q, \phi_1] | 0 \rangle$, $\langle 0 | [Q, \phi_2] | 0 \rangle$.

Tasks 4 Check that for Yang-Mills theory for the n-plet of Dirac fields

$$(D_\mu \Psi)' = U(x)(D_\mu \Psi),$$

where $U(x)$ - n x n unitary matrix with Det $U=1$.

Tasks 5 For the same case as above verify, that

$$(D_\mu T_a A^a_{\mu})' = U(x)(D_\mu T_a A^a_{\mu}) U(x)^{-1},$$

where $T_a, a = 1, \ldots, n^2 - 1$ - generators of SU($n$), $A^a$-gauge bosons.

Tasks 6 Consider in the SM a doublet of quarks

$$Q_L = \begin{pmatrix} p \\ n \end{pmatrix}_L.$$

Mass generation for down-type quarks is due to the term in the Lagrangian

$$\mathcal{L} = -(Q_L \Gamma \Phi) n_R + \text{h.c.},$$

where $L,R$ describe the the left-handed and right-handed components of spinors, eg. $n_L = (1 - \gamma_5)/2 \ n$, $n_R = (1 + \gamma_5)/2 \ n$, $\Phi$ is a SU(2) doublet of spin-0 complex fields and $\Gamma$ is an arbitrary $n_g \times n_g$ matrix in the space of generations, ($n, p$ have $n_g$ components). Write the corresponding term needed to generate mass for up-type of quarks.

Tasks 7 Check that for SM with $n_g$- generations the Neutral Current (NC) interaction in terms of the quark $p,n$ states reads:

$$\mathcal{L}^{\text{NC}} = -\frac{g}{2\cos\theta_W} Z_\mu (\bar{p}_L \gamma^\mu p_L - \bar{n}_L \gamma^\mu n_L - 2\sin^2\theta_W J^\mu_{\text{em}}),$$

where

$$J^\mu_{\text{em}} = \frac{2}{3} \bar{p} \gamma^\mu p - \frac{1}{3} \bar{n} \gamma^\mu n.$$

Write $\mathcal{L}^{\text{NC}}$ in terms of $u,d$ (mass-eigenstates) and check if there is a mixing between various quark mass-eigenstates.
Tasks 8 Check that the following $CP$ transformations
\[
(CP)\phi^+(t, \bar{x})(CP)^\dagger = e^{i\xi_W} \phi^-(t, -\bar{x}) \quad (8.31)
\]
\[
(CP)\phi^-(t, \bar{x})(CP)^\dagger = e^{i\xi_W} \phi^+(t, -\bar{x}) \quad (8.32)
\]
\[
(CP)W^{+\mu}(t, \bar{x})(CP)^\dagger = -e^{i\xi_W} W^{-\mu}(t, -\bar{x}) \quad (8.33)
\]
\[
(CP)W^{-\mu}(t, \bar{x})(CP)^\dagger = -e^{i\xi_W} W^{+\mu}(t, -\bar{x}), \quad (8.34)
\]
with arbitrary $\xi_W$, are in agreement with $CP$ invariance of the gauge-kinetic terms of the SM lagrangian.

Hint: Gauge kinetic term for $\Phi$, after inserting relation $M_W = gv/2$, leads to terms $\ldots M_W^2 W^+W^- + iM_W (W^-\partial\phi^+ - W^+\partial\phi^-)$.

Tasks 9 How the term in the Yukawa interaction in the SM
\[
\phi^+ \frac{\sqrt{2}}{v}[\bar{u} M_u V \gamma_L d],
\]
with a diagonal $M_u$ matrix and the CKM matrix $V (\gamma_L = (1 - \gamma_5)/2)$, transforms under the standard $CP$ transformation (defined without a mixing among generations)?

Tasks 10 Check that for the SM with $n_g = 3$ generations of quarks all quartets $Q_{\alpha,i,\beta,j}$ (where $\alpha, \beta$ correspond to the up-type quarks, $i, j$ - down-type quarks), have the same imaginary part up the sign.

Tasks 11 Calculate amount of fine-tuning in the SM for $M_h = 200$ GeV for $\Lambda = 1$ TeV and $\Lambda = 2$ TeV.

Hint: Use the following expressions for the quantum corrections to the mass squared of the Higgs boson
\[
\text{for top contribution: } -\frac{3}{8\pi^2} h_t^2 \Lambda,
\]
Tasks 12 Check how many independent real parameters is in $3HDM$?

Tasks 13 In the $2HDM$ consider the generic neutral vacuum with nonzero $v_1, v_2 e^{i\theta} (v^2 = v_1^2 + v_2^2)$, corresponding to the vacuum expectation values for the neutral components of $\Phi_1, \Phi_2$, respectively. The Higgs basis for scalar doublets is defined as a basis in which only one scalar doublet, called it $H_1$, has nonzero vacuum expectation values for a neutral vacuum. What is the matrix $U$ realizing such change of basis (unitary transformation) for $2HDM$? Is Higgs basis defied in a unique way?
Tasks 14 For the 2HDM in the Higgs basis calculate the extremum condition (vanishing of the first derivative of $V$ with respect to scalar fields) for parameters of the quadratic terms: $\mu_1$ and $\mu_3$.

Hint: In the Higgs basis quadratic terms in $V$ have parameters $\mu_1, \mu_2, \mu_3$, while all quartic terms - $\lambda_i$, for the corresponding products of SU(2) scalar doublets as in a standard form of potential with $m_i, a_i$ parameters.

Tasks 15 Write for 2HDM in the Higgs basis the Lagrangian for Yukawa interactions for quark mass-eigenstates and show that there is a possibility of FCNC.

Hint: Use decomposition

$$H_1 = \left( \frac{1}{\sqrt{2}} (\varphi^+ + i\chi) \right), \quad H_2 = \left( \frac{1}{\sqrt{2}} (H + iA) \right).$$

Tasks 16 Consider 2HDM with an exact $Z_2$ symmetry in a generic basis. Show that

$$(\mathcal{CP})(\Phi_1^\dagger \Phi_2)^2(\mathcal{CP})^\dagger = e^{i\theta} (\Phi_2^\dagger \Phi_1)^2.$$ 

Hint: Use the most general CP transformation as introduced during the last lecture, with $U = \text{diag}(1, e^{2i\theta})$. 