Yang-Mills theory of the conformal Cartan connection: applications to gravity

Adam Bac, Wojciech Kamiński, Jerzy Lewandowski, Michalina Broda

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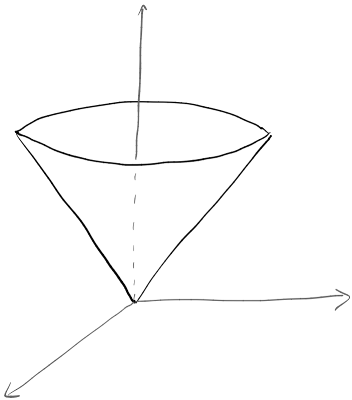

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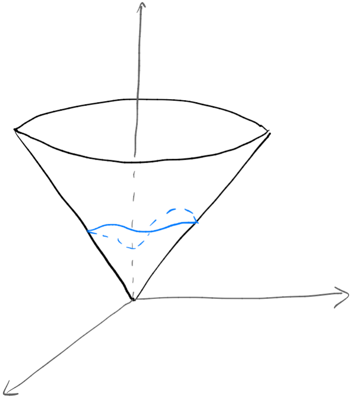

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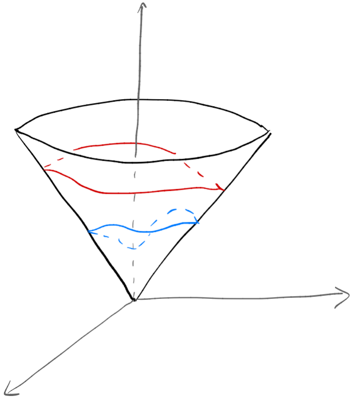

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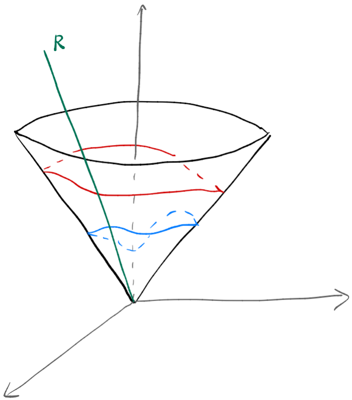

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to

$$P \xrightarrow{H} M, \quad \dim G = \dim P, \quad \hat{A} \in \mathsf{T}^* P \otimes \mathfrak{g}, \quad \mathrm{d}\hat{A} + \hat{A} \wedge \hat{A} =: \hat{F}$$

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where $\Lambda \in \mathrm{O}(\eta)$, $c \in \mathbb{R}^+$, $b \in \mathbb{R}^n$.
Corresponding actions on $(\tilde{\theta}^a, \tilde{\phi})(g(m)) \in P$:

- rotate $\tilde{\theta}^a$ by $\Lambda^a{}_b$
- rescale $\tilde{\theta}^a$ by c and $g(m)$ by c^2
- $\tilde{\phi} \mapsto \tilde{\phi} + b_a \tilde{\theta}^a$

Then $P \xrightarrow{\pi} M$ is a principal H-bundle and $\dim P = \dim G$

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For each local section $M \supset U \ni m \mapsto \sigma(m) = (\tilde{\theta}^a, \tilde{\phi})(g(m)) \in P$, we have

$$A := \sigma^* \hat{A}, \quad F := \sigma^* \hat{F}$$

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$$A := \sigma^* \hat{A}, \quad F := \sigma^* \hat{F}$$

Under a change of sections given by a point-dependent transformation $h \in C^\infty(U, H)$,

$$A \mapsto h^{-1} A h + h^{-1}\,\mathrm{d}h\,, \quad F \mapsto h^{-1} F h$$

NCC connection in the natural gauge

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A choice of a co-frame $\theta^a \in \mathrm{L}^* U$ such that $\eta_{ab} \theta^a \theta^b \in [g]$ determines fields $\tilde{\theta}^a := \pi^* \theta^a$ and $\tilde{g} := \eta_{ab} \tilde{\theta}^a \tilde{\theta}^b$.

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However, we can choose any $\tilde{\phi}$, subject to the condition $\tilde{\phi}(\tilde{\zeta})$. There is a unique choice, such that the resulting section $\sigma_N : U \to P$ satisfies

$$A := \sigma_N^* \hat{A} = \begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{bmatrix}$$

NCC connection in the natural gauge

$$A = \begin{bmatrix} 0 & \theta_b & 0 \\ P^a & \Gamma^a{}_b & \theta^a \\ 0 & P_b & 0 \end{bmatrix}$$

$$\mathrm{d}\theta^a + \Gamma^a{}_b \wedge \theta^b = 0, \quad \Gamma_{ab} = -\Gamma_{ba}$$

$$P_a = P_{ab}\theta^b, \quad P_{ab} = \frac{1}{12} R\eta_{ab} - \frac{1}{2} R_{ab}$$

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NCC connection in the natural gauge – curvature

$$F = \mathrm{d}A + A \wedge A = \begin{bmatrix} 0 & 0 & 0 \\ DP^a & C^a{}_b & 0 \\ 0 & DP_b & 0 \end{bmatrix}$$

$$DP^a = \mathrm{d}P^a + \Gamma^a{}_b \wedge P^b \ (\sim \text{Cotton tensor}), \quad C^a{}_b = \frac{1}{2} C^a{}_{bcd}\, \theta^c \wedge \theta^d$$

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NCC connection in the natural gauge – summary

$$A = \begin{bmatrix} 0 & \theta_b & 0 \\ P^a & \Gamma^a{}_b & \theta^a \\ 0 & P_b & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & 0 \\ DP^a & C^a{}_b & 0 \\ 0 & DP_b & 0 \end{bmatrix}$$

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The Bianchi identity

$$D_A F = \mathrm{d}F + A \wedge F - F \wedge A = 0$$

encodes the differential identities encoded by the Weyl and Schouten tensors.

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$$g := \eta_{ab}\theta^a\theta^b, \quad \epsilon_{abcd} := \sqrt{|\det\eta|}\varepsilon_{abcd}, \quad \varepsilon_{0123} := 1$$

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$$B_{ab} := 2\nabla^c\nabla_{[b}P_{c]a} - 2P^{cd}C_{cadb}$$

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In particular, for Einstein spacetimes:

$$R_{ab} = \Lambda\eta_{ab} \implies B_{ab} = 0 \implies D_A\star F = 0$$

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\end{aligned}$$

$$A = \begin{bmatrix} 0 & \theta_b & 0 \\ P^a & \Gamma^a{}_b & \theta^a \\ 0 & P_b & 0 \end{bmatrix}, \quad D_A \star F = \begin{bmatrix} 0 & 0 & 0 \\ B^{ac} \star \theta_c & 0 & 0 \\ 0 & B_{bc} \star \theta^c & 0 \end{bmatrix}$$

$$\implies \delta L_{\mathrm{CYM}}(\theta) = 2\delta\theta^a \wedge B_{ab} \star \theta^b + \mathrm{d}\,\mathrm{Tr}\,(\delta A \wedge \star F)$$

Presymplectic potential current

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Decomposition of Θ_{CYM}

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$$L_{\mathrm{CYM}} = \frac{1}{4}\mathcal{E} + L_1$$

$\mathcal{E}$ – Euler form

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$$\begin{aligned}\Theta_1 &:= \Theta_{\rm CYM} - \frac{1}{4}\Theta_{\mathcal{E}} \\ &= 2\delta\theta^a \wedge \star DP_a + \frac{1}{2}\epsilon^{abcd}\delta\Gamma_{ab} \wedge (C_{cd} - \mathcal{R}_{cd})\end{aligned}$$

Decomposition of Θ_{CYM}

$$L_{\mathrm{CYM}} = \frac{1}{4}\mathcal{E} + L_1$$

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Decomposition of Θ_{CYM} on Einstein spacetimes

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When $R_{ab} = \Lambda\eta_{ab}$, $P_a = -\frac{\Lambda}{6}\theta_a$ and $DP_a = 0$. Then

$$\Theta_1 \cong -\frac{\Lambda}{3}\delta\Gamma_{ab} \wedge \star\big(\theta^a \wedge \theta^b\big)$$

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$$\Theta_{\mathrm{CYM}} \cong \frac{1}{4}\Theta_{\mathcal{E}} - \frac{16\pi G\Lambda}{3}\Theta_{\mathrm{EH}}$$

Θ_{CYM} on the conformal boundary

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Conformal compactification $(\hat{M}, \hat{g})$

$$g = \eta_{ab}\theta^a\theta^b, \quad \hat{\theta}^a := \Omega\theta^a, \quad \hat{g} = \Omega^2 g = \eta_{ab}\hat{\theta}^a\hat{\theta}^b$$

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$\hat{\theta}^a$ finite at $\mathscr{I} \implies \Theta_{\mathrm{CYM}}(\hat{\theta}, \delta\hat{\theta}) = \Theta_{\mathrm{CYM}}(\theta, \delta\theta)$ also finite

Θ_{CYM} on the boundary of asymptotically de Sitter

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$$g = \frac{\ell^2}{\rho^2}\left(-\,\mathrm{d}\rho^2 + \sum_{n=0}^{\infty} g_{ij}^{(n)}\,\mathrm{d}x^i\,\mathrm{d}x^j\right)$$

$i, j, k, \ldots = 1, 2, 3$, lowered / raised with $g_{ij}^{(0)}$ / $g_{(0)}^{ij} = \left(g_{ij}^{(0)}\right)^{-1}$

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$$\begin{aligned}\Theta_{\mathrm{CYM}}(\theta, \delta\theta) &= 2\delta\theta^a \wedge \star DP_a + \delta\Gamma^a{}_b \wedge \star C^b{}_a \\ &= \delta\Gamma^a{}_b \wedge \star C^b{}_a, \text{ since } R_{ab} = \Lambda\eta_{ab}\end{aligned}$$

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Result:

$$\iota^*\Theta_{\mathrm{CYM}}(\theta, \delta\theta) = \frac{8\pi G}{\ell}\delta\mathring{g}_{ij}\, T^{ij}\mathring{\mathrm{Vol}}$$

where

$$\delta\mathring{g} := \iota^*\delta\hat{g}, \qquad T_{ij} := \frac{3\ell}{16\pi G} g_{ij}^{(3)}$$

Θ_{CYM} on the boundary of asymptotically de Sitter

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On the other hand, from the holographically renormalized action

$$S_{\mathrm{REN}} = \frac{1}{16\pi G}\left(\int_M (R - 2\Lambda)\mathrm{Vol} + \int_{\mathscr{I}} \left(2K + \frac{4}{\ell} - \mathring{R} \right) \mathring{\mathrm{Vol}} \right)$$

one obtains

$$\iota^* \Theta_{\mathrm{REN}} = -\frac{\ell}{2} \delta \mathring{g}_{ij} T^{ij}$$

Θ_{CYM} on the boundary of asymptotically de Sitter

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$\Theta_{\rm CYM}$ on the boundary of asymptotically de Sitter

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$$\iota^* \Theta_{\rm CYM} = -\frac{16\pi G \Lambda}{3} \iota^* \Theta_{\rm REN}$$

$$\Theta_{\rm CYM} \cong \frac{1}{4} \Theta_{\mathcal{E}} - \frac{16\pi G\Lambda}{3} \Theta_{\rm EH}$$

Noether currents

$$\delta L_{\mathrm{CYM}} = 2\delta\theta^a \wedge B_{ab} \star\theta^b + \mathrm{d}\Theta_{\mathrm{CYM}}$$

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For symmetries:

$$\delta_S L_{\mathrm{CYM}} = \mathrm{d}Z_S$$

The Noether current

$$J_S(\theta) \coloneqq \Theta_{\mathrm{CYM}}(\theta, \delta_S\theta) - Z_S(\theta)$$

is conserved whenever $B_{ab} = 0$ (in particular when $R_{ab} = \Lambda\eta_{ab}$).

Noether currents - diffeomorphisms

$$J_S(\theta) \coloneqq \Theta_{\mathrm{CYM}}(\theta, \delta_S \theta) - Z_S(\theta), \quad \delta_S L_{\mathrm{CYM}} = \mathrm{d} Z_S$$

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$$\mathcal{L}_\xi L_{\mathrm{CYM}} = \mathrm{d}(\xi \lrcorner L_{\mathrm{CYM}}) \implies Z_\xi = \xi \lrcorner L_{\mathrm{CYM}} = \mathsf{Tr}\left((\xi \lrcorner F) \wedge \star F\right)$$

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$$\Theta_{\mathrm{CYM}}(\theta, \mathcal{L}_\xi\theta) = \mathrm{Tr}\left(\mathcal{L}_\xi A \wedge * F\right)$$

$$J_\xi = \mathrm{Tr}\left(D_A(\xi \lrcorner A) \wedge \star F\right) = \mathrm{d}\,\mathrm{Tr}\left((\xi \lrcorner A)\star F\right) - \mathrm{Tr}\left((\xi \lrcorner A) \wedge D_A \star F\right)$$

$$Q_\xi = \mathrm{Tr}\left((\xi \lrcorner A)\star F\right)$$

Noether currents - gauge transofrmations

$$J_S(\theta) \coloneqq \Theta_{\mathrm{CYM}}(\theta, \delta_S\theta) - Z_S(\theta), \quad \delta_S L_{\mathrm{CYM}} = \mathrm{d}Z_S$$

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Conformal rescalings ($\delta\theta^a = c\theta^a$, $c \in \mathbb{R}$):

$$\delta L_{\mathrm{CYM}}(\theta) = 0, \quad \Theta_{\mathrm{CYM}}(\theta, \delta\theta) = 0$$

$$J_c = 0, \quad Q_c = 0$$

Noether currents - gauge transofrmations

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Lorentz transformations ($\delta\theta^a = \omega^a{}_b\theta^b$, $\omega \in \mathfrak{o}(p,q)$):

$$\delta L_{\mathrm{CYM}}(\theta) = 0, \quad \Theta_{\mathrm{CYM}}(\theta, \delta\theta) = \mathrm{d}\left(\omega^a{}_b \star C^b{}_a\right)$$

$$J_\omega = \mathrm{d}\left(\omega^a{}_b \star C^b{}_a\right), \quad Q_\omega = \omega^a{}_b \star C^b{}_a$$

Summary

Thank you!