Relativity of superluminal observers in $1 + 3$ spacetime

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Student's problem: find all linear transformations preserving the speed of light.

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$$c^2 \mathrm{d}t^2 - \mathrm{d}x^2 = -c^2 \mathrm{d}t'^2 + \mathrm{d}x'^2$$

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Quantum principle of relativity

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Special relativity in 1+3 dimensional spacetime

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$$\boldsymbol{r}' - \frac{\boldsymbol{r}' \cdot \boldsymbol{V}}{V^2}\boldsymbol{V} = \boldsymbol{r} - \frac{\boldsymbol{r} \cdot \boldsymbol{V}}{V^2}\boldsymbol{V}$$

Special relativity in 1+3 dimensional spacetime

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$$\frac{\boldsymbol{r}' \cdot \boldsymbol{V}}{V^2}\boldsymbol{V} = \frac{\frac{\boldsymbol{r} \cdot \boldsymbol{V}}{V^2} - t}{\sqrt{1 - \frac{V^2}{c^2}}}\boldsymbol{V}$$

Special relativity in 1+3 dimensional spacetime

$$\left\{\begin{aligned} \boldsymbol{r}' &= \boldsymbol{r} - \frac{\boldsymbol{r}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V} + \frac{\frac{\boldsymbol{r}\cdot\boldsymbol{V}}{V^2} - t}{\sqrt{1-\frac{V^2}{c^2}}}\boldsymbol{V}, \\ ct' &= \frac{ct - \frac{\boldsymbol{r}\cdot\boldsymbol{V}}{c}}{\sqrt{1-\frac{V^2}{c^2}}}. \end{aligned}\right.$$

Special relativity in 1+3 dimensional spacetime

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$$c^2\mathrm{d}t^2 - \mathrm{d}\boldsymbol{r}\cdot\mathrm{d}\boldsymbol{r} = c^2\mathrm{d}t'^2 - \mathrm{d}\boldsymbol{r}'\cdot\mathrm{d}\boldsymbol{r}'.$$

Superluminal branch of solutions

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Superluminal branch of solutions

$$\begin{cases} r' = \dfrac{Vt - \frac{\boldsymbol{r}\cdot\boldsymbol{V}}{V}}{\sqrt{\frac{V^2}{c^2} - 1}}, \\[2ex] c\boldsymbol{t}' = \boldsymbol{r} - \dfrac{\boldsymbol{r}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V} + \dfrac{\frac{\boldsymbol{r}\cdot\boldsymbol{V}}{Vc} - \frac{ct}{V}}{\sqrt{\frac{V^2}{c^2} - 1}}\boldsymbol{V} \end{cases}$$

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$$\boldsymbol{v}' = \frac{\mathrm{d}r'}{\mathrm{d}t'}\frac{\mathrm{d}\boldsymbol{t}'}{\mathrm{d}t'}$$

Superluminal branch of solutions

$$\boldsymbol{v}' = \frac{\sqrt{1-\frac{c^2}{V^2}}\left(\boldsymbol{v}-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V}\right)-\left(\frac{c^2}{V^2}\boldsymbol{V}-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V}\right)}{1-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}}\left(1-\frac{\left(1-\frac{c^2}{V^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\right)^2}\right)^{-1}$$

Superluminal branch of solutions

$$\boldsymbol{v}' = \frac{\sqrt{1-\frac{c^2}{V^2}}\left(\boldsymbol{v} - \frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V}\right) - \left(\frac{c^2}{V^2}\boldsymbol{V} - \frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\boldsymbol{V}\right)}{1 - \frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}} \left(1 - \frac{\left(1-\frac{c^2}{V^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\right)^2}\right)^{-1}$$

$$\left(1 - \frac{c^2}{v'^2}\right) = \frac{\left(1-\frac{c^2}{V^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{\boldsymbol{v}\cdot\boldsymbol{V}}{V^2}\right)^2}$$

Energy and momentum of superluminal particles

$$E \equiv \frac{\sigma m c^2}{\sqrt{\frac{v^2}{c^2} - 1}}$$

$$\boldsymbol{p} \equiv \frac{\sigma m \boldsymbol{v}}{\sqrt{\frac{v^2}{c^2} - 1}}$$

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$$\sigma' = \sigma \operatorname{sgn}\left(1 - \frac{\boldsymbol{v} \cdot \boldsymbol{V}}{c^2}\right)$$

„Anti-particles" are relative

Covariant dynamics

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$$S \equiv \int L[\psi] \mathrm{d}t = \int L[\psi'] \frac{\mathrm{d}r'}{c}$$

Covariant dynamics

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$$\psi \qquad\qquad \psi \;\rightarrow\; \psi' \qquad\qquad S \;=\; S'$$

$$S \equiv \int L[\psi]\mathrm{d}t = \int L[\psi']\frac{\mathrm{d}r'}{c}$$

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$$S \equiv \int L \, \mathrm{d}t \qquad\qquad S' \equiv \int L' \, \mathrm{d}^3 t'$$

$$\psi \qquad\qquad \psi \;\rightarrow\; \psi' \qquad\qquad \boxed{S = S'}$$

$$S \equiv \int L[\psi]\mathrm{d}t = \int L[\psi']\frac{\mathrm{d}r'}{c}$$

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$$\boxed{\psi(t, \boldsymbol{r})}$$

Example: Maxwell's theory

$$\boldsymbol{\nabla}_{t'} \cdot \boldsymbol{E}' = -\frac{1}{\varepsilon_0 c} j'$$

$$\boldsymbol{\nabla}_{t'} \cdot \boldsymbol{B}' = 0$$

$$\boldsymbol{\nabla}_{t'} \times \boldsymbol{E}' = -\partial_{r'} \boldsymbol{B}'$$

$$\boldsymbol{\nabla}_{t'} \times \boldsymbol{B}' = -\mu_0 c \boldsymbol{\varrho}' + \frac{1}{c} \partial_{r'} \boldsymbol{E}'$$

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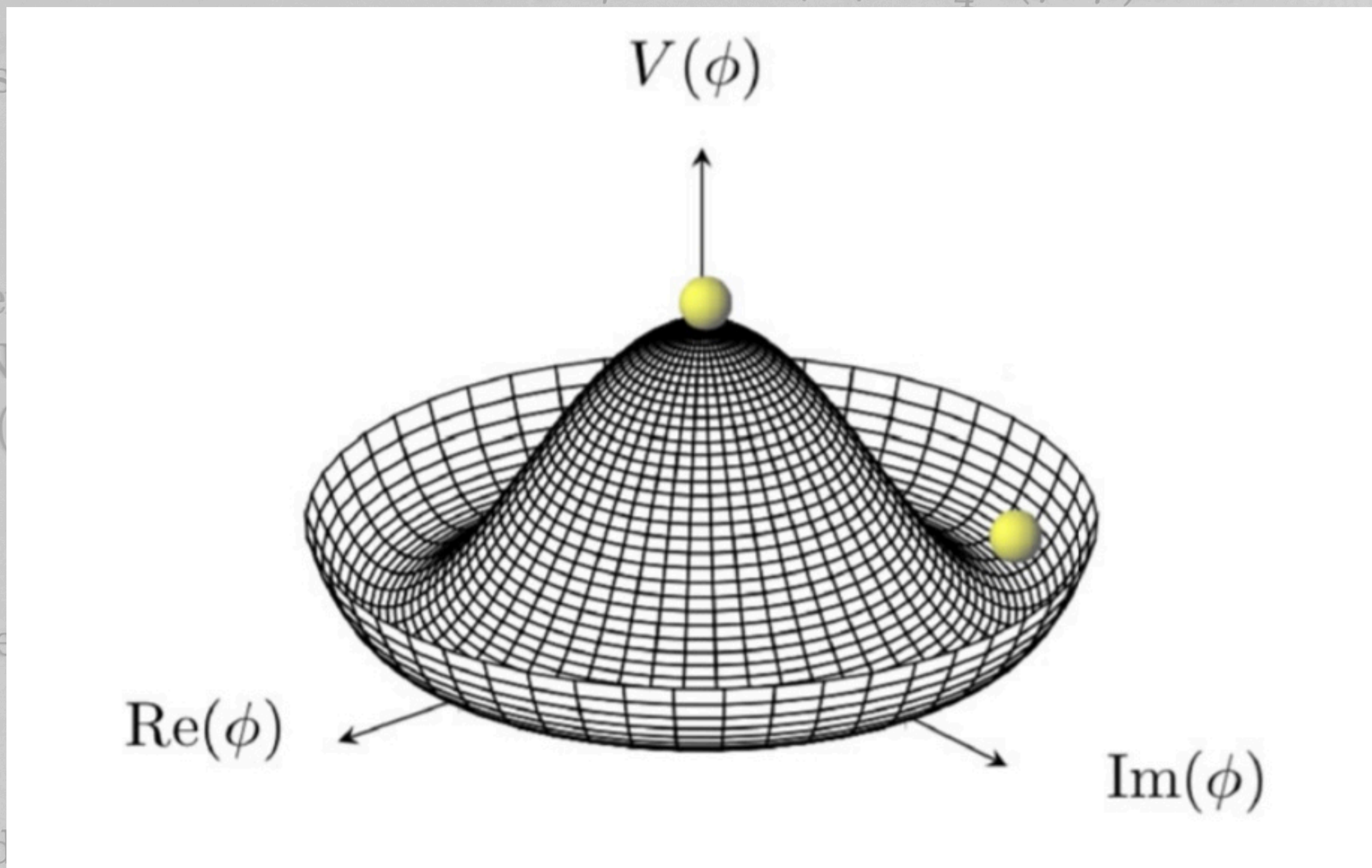
$$\partial_{r'} \boldsymbol{E}' = \frac{\boldsymbol{\varrho}'}{\varepsilon_0}$$

"Gauss law"

Do tachyons exist?

Consider the theory (introduced in section 22) of a complex scalar field φ with

$$\mathcal{L} = -\partial^\mu\varphi^\dagger\partial_\mu\varphi - m^2\varphi^\dagger\varphi - \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2\,. \quad (32.1)$$



tional. Thus we have a continuous family of minima of the potential, parameterized by θ. Under the U(1) transformation of eq. (32.2), θ changes to $\theta + \alpha$; thus the different minimum-energy field configurations are all related

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$$\mathcal{L} = \ldots \quad (32.1)$$

This lagrangian is obv… transformation

$$(32.2)$$

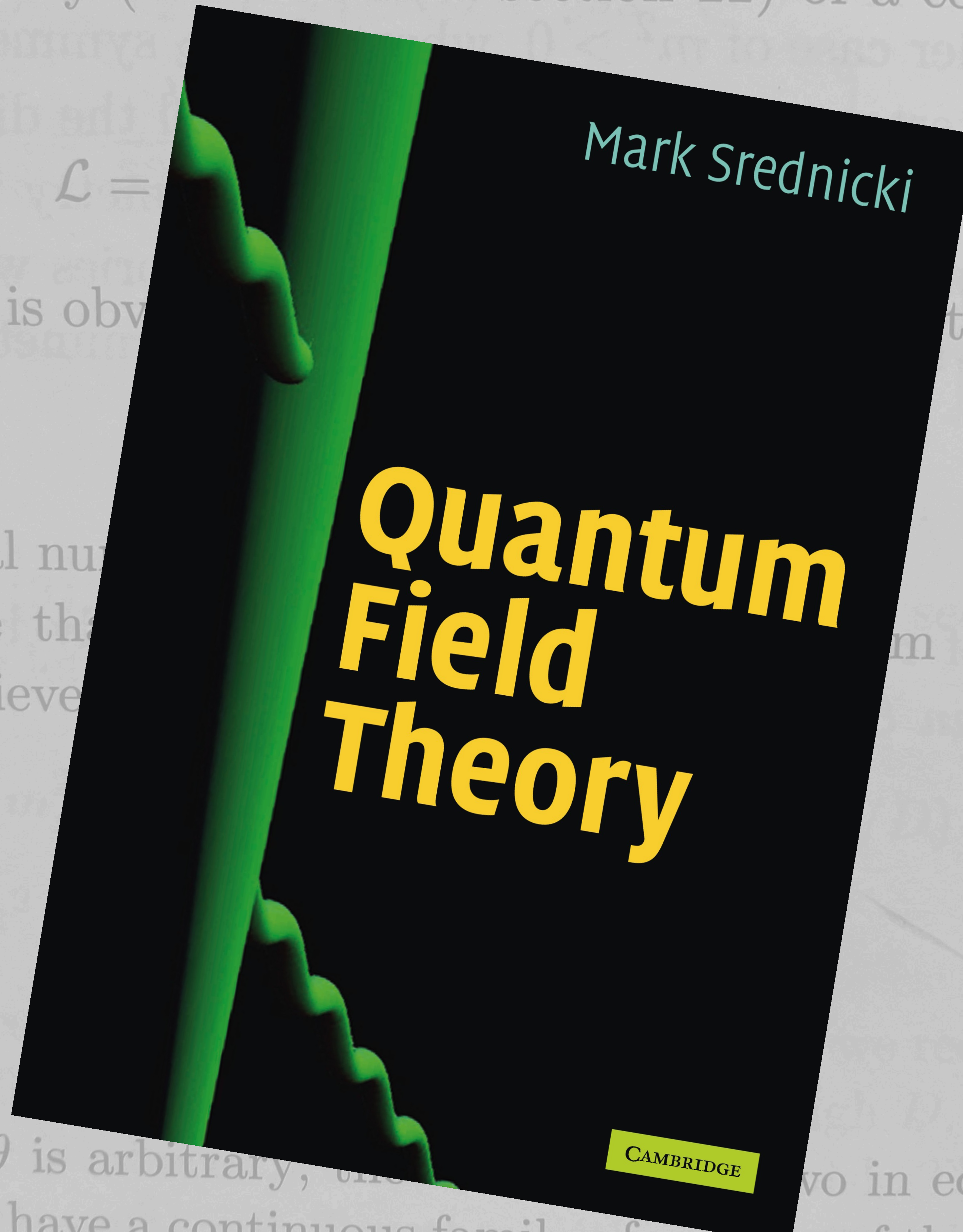
where α is a real nu…

Now suppose th… m of the potential of eq. (32.1) is achieve…

$$(32.3)$$

where

$$(32.4)$$



and the phase θ is arbitrary, … vo in eq. (32.3) is conventional. Thus we have a continuous family of minima of the potential, parameterized by θ. Under the U(1) transformation of eq. (32.2), θ changes to $\theta + \alpha$; thus the different minimum-energy field configurations are all related

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$$\mathcal{L} = -\partial^\mu \varphi^\dagger \partial_\mu \varphi - m^2 \varphi^\dagger \varphi - \tfrac{1}{4}\lambda(\varphi^\dagger \varphi)^2 . \tag{32.1}$$

This lagrangian is obviously invariant under the U(1) transformation

$$\varphi(x) \to e^{-i\alpha}\varphi(x) , \tag{32.2}$$

where α is a real number.

Now suppose that m^2 is negative. The minimum of the potential of eq. (32.1) is achieved for

$$\varphi(x) = \tfrac{1}{\sqrt{2}} v e^{-i\theta} , \tag{32.3}$$

where

$$v = (4|m^2|/\lambda)^{1/2} , \tag{32.4}$$

and the phase θ is arbitrary; the factor of root-two in eq. (32.3) is conventional. Thus we have a continuous family of minima of the potential, parameterized by θ. Under the U(1) transformation of eq. (32.2), θ changes to $\theta + \alpha$; thus the different minimum-energy field configurations are all related

Thanks for listening