Quantum Time

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Collaboration

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Introduction to introduction 1/2

For many years time was treated in physics as a universal parameter which allows the observer to divide the reality into past, present, and future. What is more, time was flowing always in one direction, called the arrow of time. This direction implied also the direction of changes that may spontaneously happen to any physical system, which ultimately leads to the notion of causality. We are used to the fact that past affects future, but future cannot affect the past, as this will act against the arrow of time.

Relativity

The development of relativity theory changed this picture in a substantial way. To obtain a realistic model one needs to treat time and space in a way consistent with the relativity theory, i.e., on the same footing

A.Góźdź,M.Góźdź,A.Pędrak, arXiv:1910.11198v2 [quant-ph]

Introduction to introduction 2/2

- One may ask if time and space positions behave in the same way in the macroscopic and microscopic scales?
- The so-called Pauli theorem states, that it is impossible to construct a self adjoint time operator which would be canonically conjugate to the Hamiltonian. It follows that time is not a physical observable but is introduced as a universal numerical parameter.

1. W.Pauli, Quantentheorie, Quanten, Handbuch der Physik, eds. H. Geiger and K Scheel, Springer, Berlin, Heidelberg, 1926, 1–278;
2. W. Pauli, Die allgemeinen Prinzipien der Wellenmechanik, Quantentheorie, Handbuch der Physik, eds. H. Geiger and K. Scheel, Springer, Berlin, Heidelberg, 1933, 83–272

This problem can be solved using more realistic and weaker assumption about quantum observables,e.g., POVM.

INTRODUCTION
(a few experiments)

Einstein's nonlocality 1/6

Principle of realism

Properties of objects are real and exist in our physical universe independent of our minds.

Local causality

- Any physical object can only be influenced by its immediate surroundings.
- Interactions mediated by physical fields can only occur at speeds no greater than the speed of light.

Einstein's locality

- Realism
- $+$
- Local causality.

Nonlocality 2/6

Is GR local? Unlike the QM, GR does not allow for an outside observer, because there is no "outside". Instead, all of reality is described in terms of relations between objects and between different regions of space.

Some type of nonlocalities in QM

- Breaking of Einstein's locality.
- Observables of objects (position, momenta, energy, mass etc.) are usually not sharp observable, they are smeared.
- Colaps of quantum states.
- Entangled states and quantum correlations (no causal influence).
- In standard approach to time in QM the simultaneity of events with respect to an observer is a source of paradoxes about locality.

Einstein's nonlocality is broken 3/6

Bell's inequalities

Bell's type inequality (CHSH) is broken in quantum world:

$$S = \langle A_a B_b \rangle - \langle A_a B_{b'} \rangle + \langle A_{a'} B_b \rangle + \langle A_{a'} B_{b'} \rangle$$

Should be $|S| \leqslant 2$, observed $|S| > 2$.

J.F. Clauser; M.A. Horne; A. Shimony; R.A. Holt (1969), "Proposed experiment to test local hidden-variable theories", Phys. Rev. Lett., 23 (15): 880–4

Early Test

Tested by the group of A. Aspect

Aspect, A. and Dalibard, J. and Roger, G., Experimental test of Bell's inequalities using time-varying analysers, Phys. Rev. Lett., 49, 1982, 1804–1807

Nonlocality in spacetime 4/6

"Delayed choice"

J.A. Wheeler proposed a Gedankenexperiment, the so called "delayed choice problem".

Wheeler, J.A., The "Past" and the "Delayed-Choice" Double-Slit Experiment, in Mathematical Foundations of Quantum Theory, ed. Marlow, A.R., Academic Press, New York, USA, 1978, 9–48; Wheeler, J.A., Law without law, in Quantum Theory and Measurement, Wheeler, J.A. and Zurek W.H., Princeton University Press, 1984, 182–213

"Quantum eraser"

Double slit experiment with quantum eraser.
The effect was visible even when the changes introduced to the experimental setup led to acausal events.

Ho Kim, Y. and Yu, R. and Kulik, S.P. and Shih, Y. and Scully, M.O., Delayed "Choice" Quantum Eraser, Phys. Rev. Lett., 84, 2000, 1–5

Nonlocality in spacetime 5/6

Teleportation

Teleportation – was conducted using entangled pairs of photons separated by 144 km. Even though the particles were causally disconnected, **the changes made in the first laboratory were affecting the second particle**.

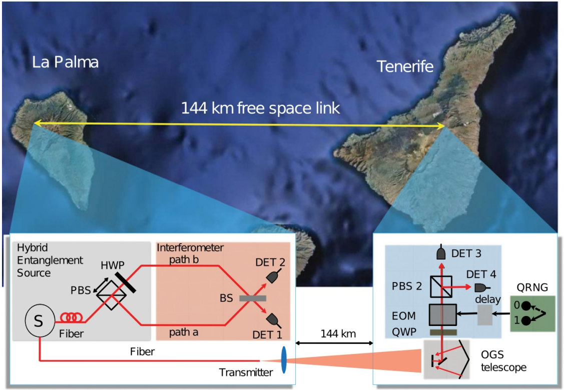
Ursin, R. and Tiefenbacher, F. and Schmitt-Manderbach, T. and Weier, H. and Scheidl, T. and Lindenthal, M. and Blauensteiner, B. and Jennewein, T. and Perdigues, J. and Trojek, P. and Ömer, B. and Fürst, M. and Meyenburg, M. and Rarity, J. and Sodnik, Z. and Barbieri, C. and Weinfurter, H. and Zeilinger, A., Entanglement-based quantum communication over 144 km, Nature Physics, 3, 2007, 481–486

More than 90 times faster signal than c is required to connect both events.

Nonlocality in spacetime 6/6

Xiao-Song Ma, et al., Quantum teleportation between the Canary Islands La Palma and Tenerife over both quantum and classical 143-km free-space channels.

Nature 489, 269–273 (13 September 2012) doi:10.1038/nature11472



Temporal interference 1/5

- If time in the quantum regime should be treated as a coordinate, and in fact a quantum observable, all physical objects' states have to have some "width" in the time direction, which is related to the energy-time (more precisely – the temporal component of the four momentum operator versus time) uncertainty relation.
- This means that it should be possible to observe the interference of quantum objects through their overlap in time

1. Houser, U. and Neuwirth, W. and Thesen, N., Time-dependent modulation of the probability amplitude of single photons, Phys. Lett. A, 49, 1974, 57–58
2. Lindner, F. and Schätzel, M.G. and Walther, H. and Baltuška, A. and Goulielmakis, E. and Krausz, F. and Milošević, D.B. and Bauer, D. and Becker, W. and Paulus, G.G., Attosecond Double-Slit Experiment, Phys. Rev. Lett., 95, 2005, 040401

Difraction on a time slit – exp. 1. 2/5

U. Houser, W.Neuwirth, N. Thesen: Phys. Lett. A49 (1974) 57.

- A beam of the Mösbauer photons is emitted with $E_\gamma = 14.4$ keV from the excited state (lifetime $\tau = 141$ ns) of ^{56}Fe.
- This beam is modulated by a chopper with 2500 wholes.
- One gets about 3000 γ-counts i.e. 1 γ passes the slit per $3000/\tau \approx 2000$ of lifetimes τ – these photons are well separated.
- One observes the interference fringes on the "energy screen"
- The only explanation: the single photon interfere with itself.
- The time cannot be interpreted as a parameter.

Difraction on a time slit – exp. 1. 3/5

Volume 49A, number 1 PHYSICS LETTERS 12 August 1974

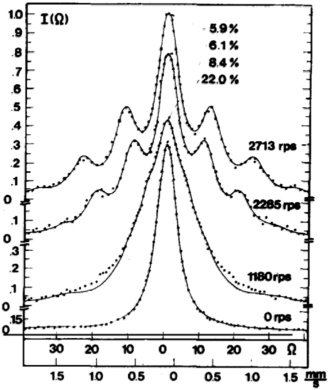


Fig. 2. Experimental results compared with theory. At the wings neighbouring channels are accumulated.

where $f(\Omega)$, $g(\Omega)$ and $h(\Omega)$ stand for

$$f(\Omega) = 1 - 2\exp(-T)\cos\{\Omega T\} + \exp(-2T) .$$

$$g(\Omega) = 1 - 2\exp(-T)[\sinh\{T\rho\}\cdot\cos\{\Omega T(1-\rho)\} + \sinh\{T(1-\rho)\}\cdot\cos\{\Omega T\rho\}] - \exp(-2T) ,$$

$$h(\Omega) = 2\exp(-T)[\cosh\{T\rho\}\cdot\sin\{\Omega T(1-\rho)\} + \cosh\{T(1-\rho)\}\cdot\sin\{\Omega T\rho\} - \sin\{\Omega T\}] .$$

The natural variables $T = \Delta/2\tau$, $\Omega = (E - E_0)/(\Gamma_0/2)$ refer to the lifetime $\tau = \hbar/\Gamma_0$ = 141 nsec and the linewidth Γ_0 of the excited state of the source. The formula represents a Lorentzian central line, reduced in weight by the duty cycle $\rho = \Delta_{open}/\Delta = 1 - \Delta_{closed}/\Delta$ plus oscillating, non-Lorentzian terms symmetric to $\Omega = 0$. The solid lines in fig. 2 represent the theoretical transmission spectra, normalized to $I(\Omega{=}0) = 1$. These transmission spectra are given by 95 % of the modulated emission spectra $I(\Omega,T,\rho)$ convoluted with the resolution of the Mössbauer absorber, plus an unmodulated contribution of 5 %. Fig. 2 includes also the absolute resonance absorption of the central peak as a function of rps. All experimental constants factors as ρ, the transmission and the energy calibration as

Figure: Hauser exp., Emission through the time slits

Interference on the time slits – exp. 2. 4/5

quant-ph/05033165 v2, 2005, F. Lindner et al., PRL 95 (2005) 040401-1

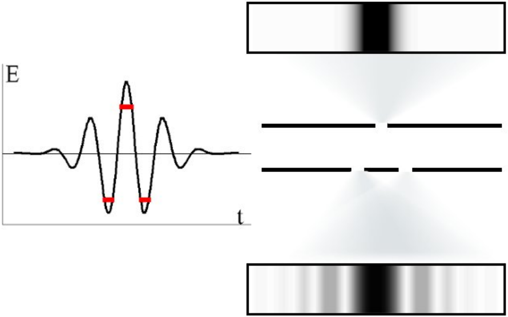


Figure: Emission through the time slits (Fig. O.P.)

Interference on the time slits – exp. 2. 5/5

quant-ph/05033165 v2, 2005, F. Lindner et al.

Attosecond double-slit experiment

F. Lindner,[1] M. G. Schätzel,[1] H. Walther,[1,2] A. Baltuška,[1] E. Goulielmakis,[1]
F. Krausz,[1,2,3] D. B. Milošević,[4] D. Bauer,[5] W. Becker,[6] and G. G. Paulus[1,2,7]
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[2] *Ludwig-Maximilians-Universität München, 85748 Garching, Germany*
[3] *Institut für Photonik, Technische Universität Wien, Gusshausstr. 27, A-1040 Wien, Austria*
[4] *Faculty of Science, University of Sarajevo, Zmaja od Bosne 35, 71000 Sarajevo, Bosnia and Hercegovina*
[5] *Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[6] *Max-Born-Institut, Max-Born-Str. 2a, 12489 Berlin, Germany and*
[7] *Department of Physics, Texas A&M University, College Station, TX 77843-4242*
(Dated: June 30, 2005)

A new scheme for a double-slit experiment in the time domain is presented. Phase-stabilized few-cycle laser pulses open one to two windows ("slits") of attosecond duration for photoionization. Fringes in the angle-resolved energy spectrum of varying visibility depending on the degree of which-way information are observed. A situation in which one and the same electron encounters a single and a double slit at the same time is discussed. The investigation of the fringes makes possible interferometry on the attosecond time scale. The number of visible fringes, for example, indicates that the slits are extended over about 500 as.

The conceptually most important interference experi-
ment is the double-slit scheme, which has played a pivotal

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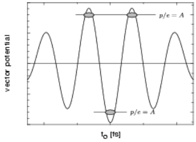


FIG. 4: Vector potential of a −sine-like few-cycle pulse. The temporal slits are given by the condition $p - eA(t_0) = 0$. For a −sine-like pulse, this leads to a double slit in the negative direction (since $e = -|e|$) and a single slit in the opposite direction. Each slit can be resolved into a pair of slits.

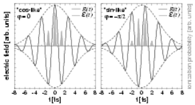


FIG. 1: Temporal variation of the electric field $\mathcal{E}(t) = \mathcal{E}_0(t)\cos(\omega t + \varphi)$ of few-cycle laser pulses with phase $\varphi = 0$ ("cosine-like") and $\varphi = -\pi/2$ ("sine-like"). In addition, the field ionization probability $R(t)$, calculated at the experimental parameters, is indicated. Note that an electron ionized at $t = t_0$ will not necessarily be detected in the opposite direction of the field $\mathcal{E}$ at time t_0 due to deflection in the oscillating field.

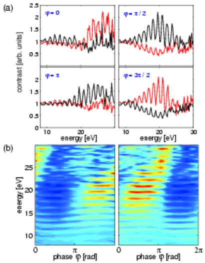
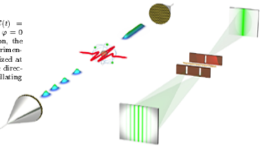


FIG. 2: Photoelectron spectra of argon measured with 6-fs laser pulses for intensity 1×10^{14} W/cm^2 as a function of the

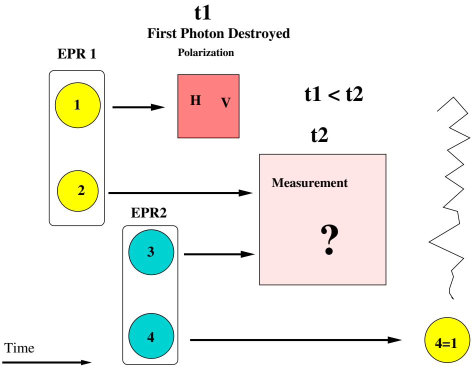


EPR between two moments 1/1

Temporal Entanglement

There exists entanglement over the time dimension.

E. Megidish et al., Entanglement Between Photons that have Never Coexisted. arXiv:1209.4191v1 [quant-ph] 19 Sep 2012



Quantum Motion Algebra

AG., A. Pędrak, in preparation.

Quantum Motion Algebra 1/2

- The group of motions G provide the configuration space of our quantum system and create the group algebra with involution QMA(G).
  - A linear functional $\langle \rho, S \rangle$ on QMA(G) defines the probability amplitudes in QMA(G).
  - The quotient structure $\mathcal{K} = \text{Hilbert}(\text{QMA(G)}/I_\rho)$ determine the state space of the system uniquely (GNS construction). The scalar product is determined as:

$$\langle S_2 | S_1 \rangle := \left\langle \rho; S_2^\sharp \circ S_1 \right\rangle$$

  - Elementary states $|g\rangle = g|e_G\rangle$ represent quantum counterparts of points in the configuration space (vacuum states).

Wave packets realization of QMA(G) 2/2

$$\hat{S} = \int_G d\mu(g) f(g) T(g) + \sum_{g \in G} \tau(g) T(g) \ ,$$

$T(g)$ represents here a unitary representation (action) of the group G in a given Hilbert space $\mathcal{K}$.
The discrete sum over the group ensures (it is important in physical applications) that the state from which the quantum state space is generated also belongs to this state space, though it is not always needed.

Time as an observable 1/1

To summarize:

- Quantum Gravity requires time to be considered on the same footing as other observables.
- ____
- Quantum time should be a part of the spacetime position quantum observable.
- ____

Possible solution:

Projection evolution, PEv

Projection evolution of quantum states, A.G, M. Góźdź, A. Pędrak, arXiv:1910.11198v2 [quant-ph] 25 Mar 2020

Projection evolution 1/3

!!! The changes principle:

The evolution of a system is a random process caused by the spontaneous changes in the Universe.

- The projection evolution operators at the evolution step τ_n are defined as a family of transformations:

$$\text{₣}(\tau_n;\nu,\cdot) : \mathcal{T}_1^+(\mathcal{K}(\tau_{n-1})) \to \mathcal{T}^+(\mathcal{K}(\tau_n)),$$

  where $\mathcal{T}^+(\mathcal{K}(\tau_n))$ is the quantum state space at the evolution step τ_n.
- τ_n enumerates subsequent changes of quantum states – it is a global ordering parameter – it is not TIME !
- PEv approach allows to treat time as a quantum observable – as it is required.

Projection evolution, chooser 2/3

The generalized Lüders projection postulate is proposed as the principle for the evolution (chooser):

$$\rho(\tau_n;\nu_n) = \frac{₣(\tau_n;\nu_n,\rho(\tau_{n-1};\nu_{n-1}))}{\mathrm{Tr}\left(₣(\tau_n;\nu_n,\rho(\tau_{n-1};\nu_{n-1}))\right)}.$$

Probability distribution

The probability distribution for the chooser is given by the quantum mechanical transition probability from the previous to the next state.

This probability for pure quantum states is determined by the appropriate probability amplitudes in the form of scalar products. The transition probability among mixed states remains an open problem.

₣ are some quantum operations, e.g., K. Krauss: States, Effects and Operations: Fundamental Notions of Quantum Theory, Springer Verlag 1983

Projection evolution, chooser 3/3

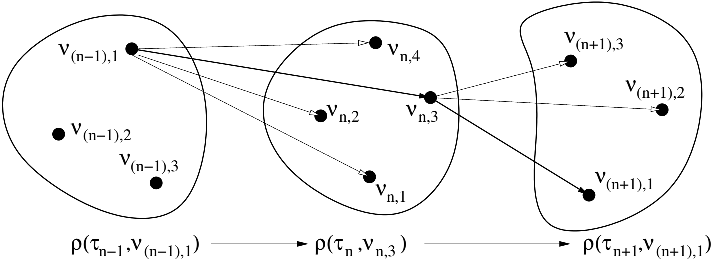


Figure: The density matrix ρ is randomly chosen at each evolution step τ from the possible states labeled by $\mathcal{Q}_m = \{\nu_{m,1}, \nu_{m,2}, \dots\}$, where $m = n-1, n, n+1$.

Evolution of the vacuum: $\{|\tau_{n-1}; g\rangle \in \mathcal{K}(\tau_{n-1}) : g \in \mathrm{G}\} \rightarrow \{|\tau_n; g\rangle \in \mathcal{K}(\tau_n) : g \in \mathrm{G}\} \rightarrow \{|\tau_{n+1}; g\rangle \in \mathcal{K}(\tau_{n+1}) : g \in \mathrm{G}\}$

Spacetime 1/3

Classical spacetime

Let us assume, we are able to identify the subgroub $\overline{\mathrm{G}} \subset \mathrm{G}$ of "intrinsic symmetries" of the configuration space.
The remaining quotient structure $X = \mathrm{G}/\overline{\mathrm{G}}$ represents support of classical spacetime $\Rightarrow$ $g = g(x, \xi)$, where x parametrizes $X = \mathrm{G}/\overline{\mathrm{G}}$ and ξ parametrizes $\overline{\mathrm{G}}$.

Quantum spacetime at τ_n – vacuum

Every point x of the classical spacetime is represented by the set of quantum degenerated states (quantum spacetime point):
$x \to X_Q := \{|\tau_n; g(x, \xi)\rangle : \xi \in \text{parameters}(\overline{\mathrm{G}})\}$

Paths and amplitudes, at τ_n 2/3

- Path in the state space $\mathcal{K}(\tau_n)$:
  $\kappa_{AB} :< 0, 1 > \to \mathcal{K}(\tau_n)$.
  $|S_A\rangle \equiv |S_A[0]\rangle$ and $|S_B\rangle \equiv |S_B[1]\rangle$.
- L_{AB} denotes a set of all paths from $|S_A\rangle$ to $|S_B\rangle$.
- The amplitude of the path connecting $|S_A\rangle$ and $|S_B\rangle$:

  $\mathrm{Amp}(\kappa_{AB}) :=$
  $\lim \ \langle S_B|S[\theta_N]\rangle\langle S[\theta_N]|S[\theta_{N-1}]\rangle \ldots \langle S[\theta_2]|S[\theta_1]\rangle\langle S[\theta_1]|S_A\rangle$

  where $\kappa_{AB}(\theta) = |S[\theta]\rangle \in \mathcal{K}(\tau_n), \ \theta \in < 0, 1 >$.
- Denote by $\bar{\kappa}_{AB}$ the path (or set of paths) representing maximum of transition probability from $|S_A\rangle$ to $|S_B\rangle$, i.e., $\bar{\kappa}_{AB}$ is the most probable path:
  $|\mathrm{Amp}(\bar{\kappa}_{AB})|^2| = \max\{\mathrm{Amp}(\kappa_{AB})|^2 : \text{ all possible } \kappa_{AB} \ \}$.

Quantum geometry, at τ_n 3/3

Dominating geometry

Geometry is generated by all maximum probability paths in $\mathcal{K}(\tau_n)$.

- Usually: "longer" path κ_{AB} smaller transition probability.
- For trial particles the transition probability from $|S_A\rangle$ to $|S_B\rangle$ is $|\sum_{\kappa_{AB}} \mathrm{Amp}(\kappa_{AB})|^2$ – interference of paths.

TOY MODEL

$\mathrm{QMA}(\mathrm{T}^4)'$

$\mathrm{QMA}(\mathrm{T}^4)'$ 1/9

- The group of motions $\mathrm{G} = \mathrm{T}^4 \sim \mathrm{R}^4$ and no discrete part in the algebra is $\mathrm{QMA}(\mathrm{T}^4)'$.
- The metastate kernel $\langle \rho; x \rangle = \delta(x)$ implies the scalar product.

$$\langle f_2 | f_1 \rangle = \int_{\mathrm{T}^4} d^4x f_2(x)^\star f_1(x) \ .$$

- The state space $L^1(\mathrm{T}^4) \cap L^2(\mathrm{T}^4)$.
- Because $\langle x' | x \rangle = 0$, for $x' \neq x$, there is no natural geometry generated by the metastate kernel – let us assume Minkowski structure.
- Every spacetime state $|x\rangle$ has no intrinsic structure (like a geometric point).
- The model is invariant under change of T^4 parametrization.

QMA$(\mathrm{T}^4)'$ 2/9

The Minkowski space is generated by a set of the spacetime Lorentz four–vector position operator (time + 3-space operators)

$$\hat{x}^\mu = \int_{\mathrm{T}^4} d^4x |x\rangle x^\mu \langle x|$$

$$\hat{x}^\mu f(x^0, x^1, x^2, x^3) = x^\mu f(x^0, x^1, x^2, x^3)$$

with respect to a fixed but arbitrary observer $\mathcal{O}$.

Time operator

$$\hat{t} \equiv \hat{x}^0 = \int_{\mathrm{T}^4} dx^0 x^0 \hat{M}_T(x^0)$$

$$\hat{M}_T(x^0) := \int_{\mathrm{T}^3} d^3\mathbf{x} |x\rangle\langle x|$$

$\hat{M}_T(x^0)$ projects onto space of simultaneous events.

Dual (momentum) operators 3/9

The four-translation generators represent the momentum operators

$$\hat{p}_\mu := i\hbar \frac{\partial}{\partial x^\mu}$$

Note the "canonical" commutation relations

$$[\hat{p_\mu}, \hat{x^\nu}] = i\hbar \delta_\mu^\nu$$

To keep a consistent interpretation, the zero component $\hat{p}_0$ describes the temporal momentum of the system under consideration. It gives:

- Arrow of time: either $p_0 > 0$ or $p_0 < 0$.
- By analogy to 3D, the value of p_0 determines "temporal inertia" × "speed in time" of motion in time.

Operator $\hat{p}_0$ 4/9

Sign of $\hat{p}_0$ determines arrow of time.
Spectral decomposition

$$\hat{p}_0 = \hbar \int_{\mathrm{T}^4} d^4k \, k_0 |k\rangle\langle k|$$

Projections onto positive and negative time direction:

$$\hat{M}_{T+} = \int_{\mathrm{T}^4} d^4k \, \delta(k_0 \geqslant 0) |k\rangle\langle k|$$

$$\hat{M}_{T-} = \int_{\mathrm{T}^4} d^4k \, \delta(k_0 \leqslant 0) |k\rangle\langle k|$$

Time reversal 5/9

Two time reversal operations:

- Racah's time reversal operator $\mathcal{T}_R$ is unitary:

$$\mathcal{T}_R \Psi(t) := \Psi(-t)$$

- Wigner's time reversal operator $\mathcal{T}_R$ is antiunitary:

$$\mathcal{T}_W \Psi(t) = \Psi(-t)^\star$$

Time reversal of time arrow projection operators $\hat{M}_{T\pm}$ 6/9

Racah's time reversal changes time arrow direction:

$$\mathcal{T}_R \hat{M}_{T+} \mathcal{T}_R = \hat{M}_{T-}$$
$$\mathcal{T}_R \hat{M}_{T-} \mathcal{T}_R = \hat{M}_{T+}$$

If $\hat{M}_{T+}\Psi = \Psi$ then $\hat{M}_{T+}(\mathcal{T}_R\Psi) = 0$, i.e., Racah's $\mathcal{T}_R$ change the time direction.
If $\hat{M}_{T+}\Psi = \Psi$ then $\hat{M}_{T-}(\mathcal{T}_W\Psi) = 0$, i.e. Wigner's $\mathcal{T}_W$ does not change the time direction.

Example: function moving in positive time direction 7/9

Nearly general form of the function moving in positive time direction:

$$\Psi_+(x) = \int_0^\infty dk_0 \gamma(k_0, \vec{x}) e^{-ik_0 t}$$

Temporal rectangular pulse:

$$\gamma(k_0, \vec{x}) = N \chi_{[0,k_{0M}]}(k_0) \Phi_0(\vec{x})$$

Then

$$\Psi_+(x) = \Phi_0(\vec{x}) \sqrt{\frac{k_{0M}}{2\pi}}\, e^{-i\frac{k_{0M}}{2}\,t} j_0(\frac{1}{2} k_{0M}\, t)$$

Example: Temporal rectangular pulse 8/9

Racah's time reversal:

$$\mathcal{T}_R \Psi_+(x) = \Phi_0(\vec{x}) \sqrt{\frac{k_{0M}}{2\pi}}\; e^{+i \frac{k_{0M}}{2}\, t} j_0(\frac{1}{2} k_{0M}\, t)$$

Wigner's time reversal do not change arrow of time, but gives complex conjugated spatial part (Kramer's degeneration):

$$\mathcal{T}_W \Psi_+(x) = \Phi_0(\vec{x})^{\star} \sqrt{\frac{k_{0M}}{2\pi}}\; e^{-i \frac{k_{0M}}{2}\, t} j_0(\frac{1}{2} k_{0M}\, t)$$

In both cases probability distribution is unchanged.

Energy versus temporal momentum 9/9

$E \leftrightarrow p_0$

The traditional interpretation of p_0 as the energy holds only in the case when the equations of motion relate p_0 directly to the energy of the system, e.g.,
on solutions of the Schrödinger equation $\hat{p}_0 = \hat{\mathcal{H}}$
or
on solutions of the Klein-Gordon equation $p_0^2 = m_0^2 + \vec{p}^2$, etc.

Measurement of p_0

Equation of motions allow for indirect measurement of the temporal momentum p_0.

Heisenberg uncertainty principle 1/1

The operators $\hat{x}_\nu, \hat{p}_\mu$ obey the Heisenberg uncertainty principle in the Robertson form

$$\mathrm{var}(\hat{p}_\mu)\ \mathrm{var}(\hat{x}^\nu) \geqslant \frac{1}{4} \langle i[\hat{p}_\mu, \hat{x}^\nu] \rangle^2 = \frac{\hbar^2}{4} \delta^\nu_\mu, \tag{1}$$

where $\mathrm{var}(\hat{A}) := \left\langle \hat{A}^2 \right\rangle - \left\langle \hat{A} \right\rangle^2$ denotes variance of $\hat{A}$.

Remark

In the standard approach to QM, where time is a parameter, for $\mu = \nu = 0$, i.e. for $\hat{x}^0, \hat{p}_0$, this inequality does not exist.

Causality 1/1

Broken causality

The functions $\Psi(x) := \langle x^0, x^1, x^2, x^3 | \Psi \rangle \in \mathcal{K}_X$ in their general form connect also events with space-like intervals $(x^0)^2 - \vec{x}^2 < 0$ $\Rightarrow$ causality is broken.

Causality can be easily recovered by constraints, HOWEVER,

Experiment

Yin et. al., Lower Bound on the Speed of Nonlocal Correlations without Locality and Measurement Choice Loopholes, Phys. Rev. Lett. 110, 2013, 260407

suggests, that it is a natural phenomenon that the classical causality is broken in the quantum world.

Within the PEv approach the quantum causality is realized by keeping the correct sequence of the subsequent steps of the evolution, ordered by the parameter τ.

Evolution generators 1/2

$\hat{\mathsf{W}}(\tau)$

For a given evolution step τ the projection evolution generator $\hat{\mathsf{W}}(\tau)$ is defined as a self-adjoint operator which spectral decomposition gives the orthogonal resolution of unity representing the set of evolution operators.

Typical form of the evolution generator for quantum relativistic equations of free object:

$$\hat{\mathsf{W}} \stackrel{C}{=} a^{\mu}\hat{p}_{\mu} + a^{\mu\nu}\hat{p}_{\mu}\hat{p}_{\nu}, \tag{2}$$

Evolution generators 2/2

Examples:

- Schrödinger motion

$$\hat{\mathsf{W}}_S = \hat{p}_0 - H,$$

- Klein–Gordon motion

$$\hat{\mathsf{W}}_{KG} \stackrel{C}{=} \hat{p}_\mu \hat{p}^\mu,$$

- Dirac motion

$$\hat{\mathsf{W}}_D \stackrel{C}{=} \gamma^\mu \hat{p}_\mu.$$

- Generalized Schrödinger motion

$$\hat{\mathsf{W}}_{GS}(\tau) = \hat{p}_0 - \hat{\mathcal{H}}(\tau) + \left[\frac{1}{2} B_T^{-1}(\tau)\hat{p}_0^2 + V_T(\tau, x^0)\right],$$

Wave functions normalization 1/1

Standard normalization of $\Psi(t,\vec{x})$ in 3D for a single particle (conditional probability):

$$\int_R d^3\vec{x}|\Psi(t,\vec{x})|^2 = 1.$$

$\Psi(t=t_0,\vec{x}) = 0$ to $\Psi(t,\vec{x}) \equiv 0$ for every t.

If $\Psi(t_0,\vec{x}) = 0$ then $\Psi(t,\vec{x}) = 0$ for all t.

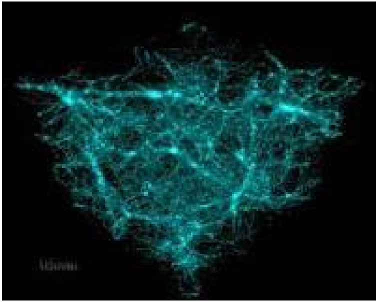
Normalization of $\Psi(t,\vec{x})$ in 4D for a single particle (density probability):

$$\int_R d^4x|\Psi(t,\vec{x})|^2 = 1$$

$\Psi(t,\vec{x})$ can be zero even in large regions of the spacetime and $\Psi(t,\vec{x}) \not\equiv 0$.

Dark Matter
Dark Energy
???????????????

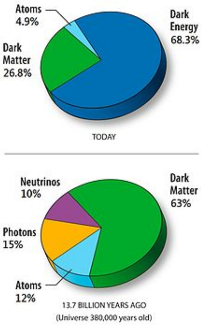
Dark Matter 1/1



Dark mater ?

Exotic states different from zero only for very short pulses can be candidate for dark matter.

Dark energy 1/1



Dark energy

Interaction energy of two parts of the Universe: one with $\langle p_0 \rangle > 0$ second with $\langle p_0 \rangle < 0 \Rightarrow$ Total $\langle P_0 \rangle = 0$.

Interference in Time

Temporal interference 1/4

- Initial state: $\mathbb{E}(\tau_0;\nu) = |\Psi_\nu\rangle\langle\Psi_\nu|$
- Szczeliny czasoprzestrzenneSpacetime slits:

$$\mathbb{E}(\tau_1;\nu) = \begin{cases} \int_\Delta dt\, d\vec{x}\ |t,\vec{x}\rangle\langle t,\vec{x}|, & \text{for } \nu = \Delta \text{ (e.g. } \Delta_1 + \Delta_2) \ , \\ \text{something}(\nu), & \text{in other cases} \end{cases}$$

- Final states: $\mathbb{E}(\tau_2;\mu) = |\Phi_\mu\rangle\langle\Phi_\mu|$

Probability for fixed path::

$$\begin{aligned} \text{Prob}\,(\tau_2;\nu,\Delta,\mu) &= |\langle\Phi_\mu|\mathbb{E}(\tau_1;\Delta_1)|\Psi_\nu\rangle|^2 + |\langle\Phi_\mu|\mathbb{E}(\tau_1;\Delta_2)|\Psi_\nu\rangle|^2 \\ &+ 2\text{Re}\,(\langle\Phi_\mu|\mathbb{E}(\tau_1;\Delta_1)|\Psi_\nu\rangle^\star\langle\Phi_\mu|\mathbb{E}(\tau_1;\Delta_2)|\Psi_\nu\rangle)\,. \end{aligned}$$

Interferencja temporalna 2/4

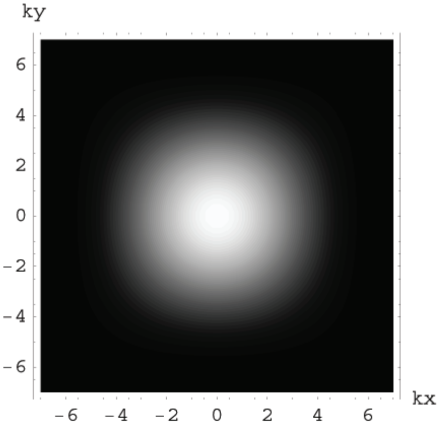
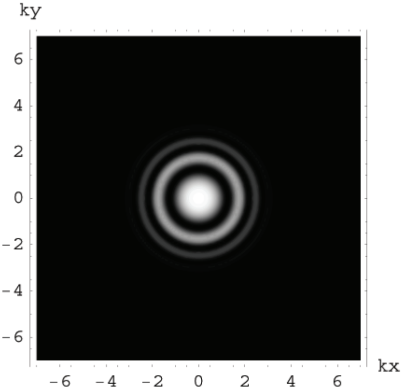


Figure: Momentum distribution (k_x, k_y) without temporal slits, diffraction on spatial slit.

Interferencja temporalna 3/4



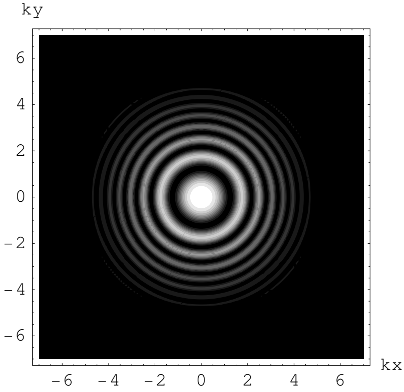


Figure: Momentum distribution (k_x, k_y) with temporal slits: $\delta_T = 1$ and $0.1 \times \delta_T$, respectively.

Interferencja temporalna 4/4

Probability distribution of the final (k_0, k) in the F. Lindner et al. experiment, on the energy shell $\hbar k_0 = \frac{\hbar^2}{2m}\vec{k}^2$

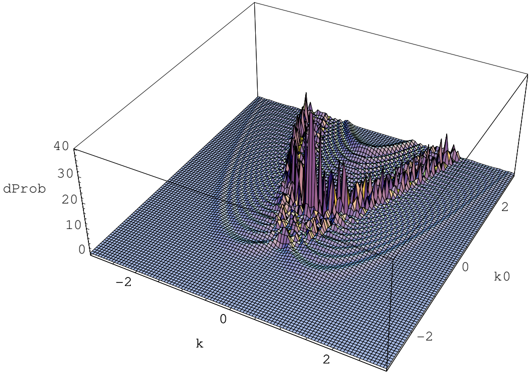


Figure 4: Probability distribution $(k0, k)$, "art. view"

SUMMARY

- (The group of motions G) + (Elementary probability amplitude function $\langle \rho(\tau); g \rangle$) creates for every step τ of the evolution a state space $\mathcal{K}(\tau)$
  $\Rightarrow$ Background independence.
- Quantum evolution is a stochastic process (PEv).
- Treating the quantum time on the same footing as the other observables (PEv)
  $\Rightarrow$ Covariance of spacetime position operator.
- Classical configuration space is represented by $\{|\tau; g\rangle\}_{\mathrm{G}}$
  $\Rightarrow$ Generation of the spacetime as a part of the configuration space.
- A natural geometry generated by transition amplitudes (not considered).
- New observables and phenomena in the time domain.