Exploring the Universe with gravitational waves: From Lensing to Testing General Relativity

Anna Liu

The Chinese University of Hong Kong

20 December 2024

香港中文大學
The Chinese University of Hong Kong

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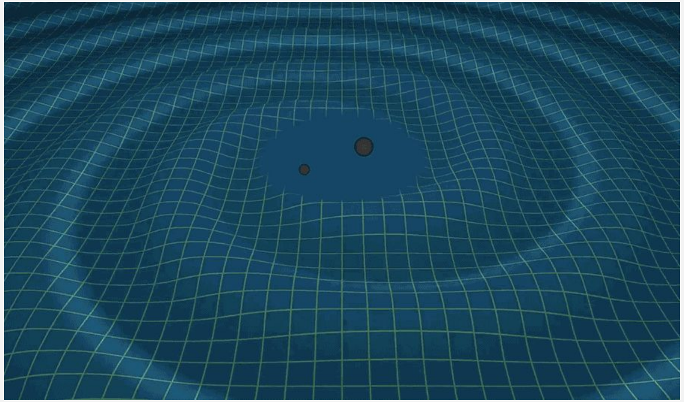
Gravitational waves



LIGO
VIRGO
KAGRA

Typical gravitational wave signal

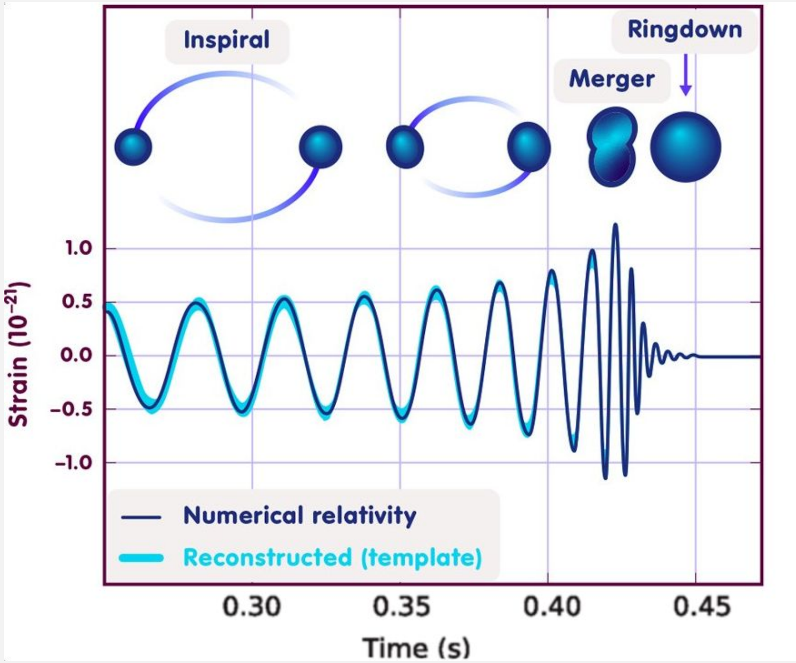


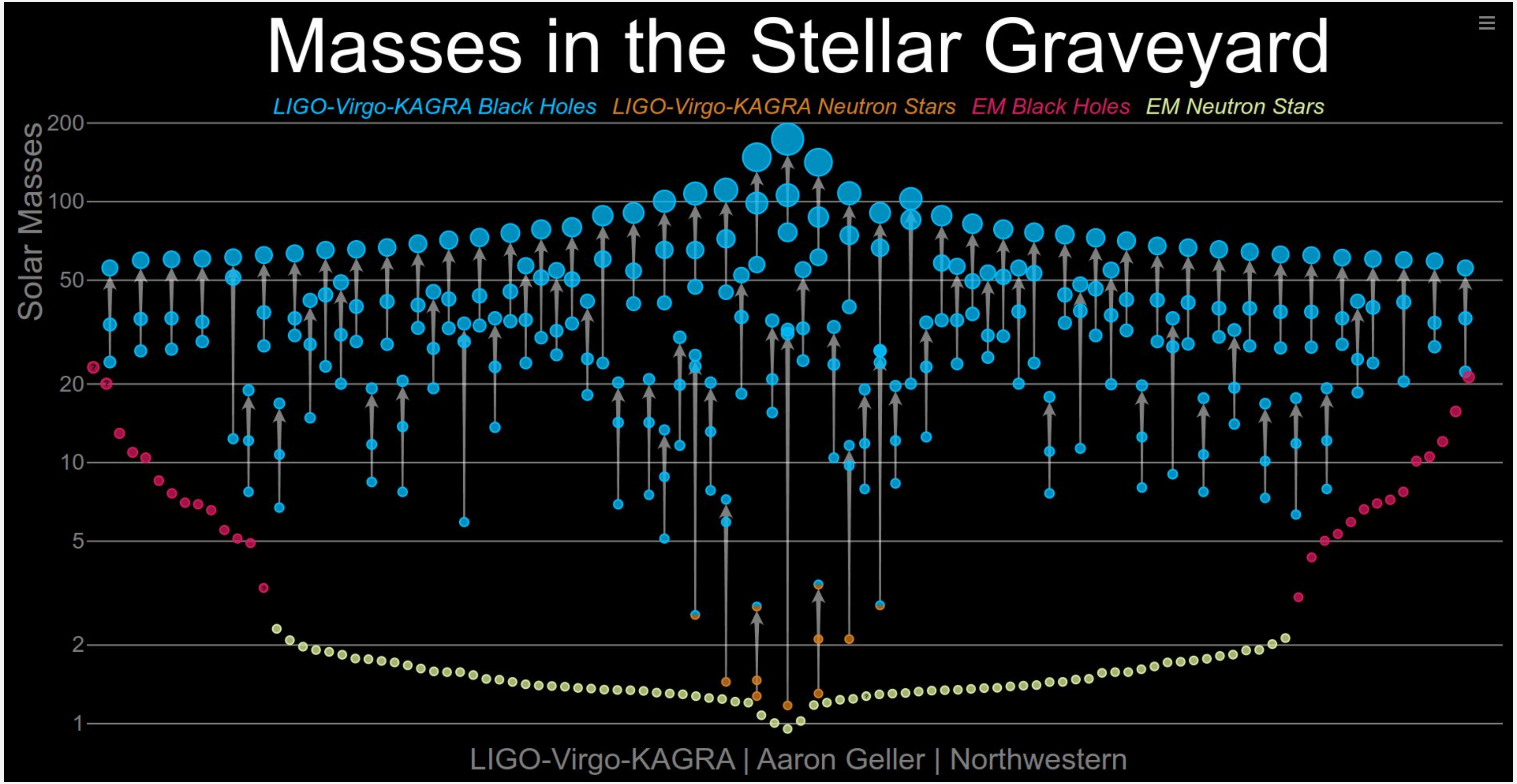
Image: LIGO

$$\tilde{h}(f) = A_{\mathrm{GR}}(f)e^{i\psi(f)}$$

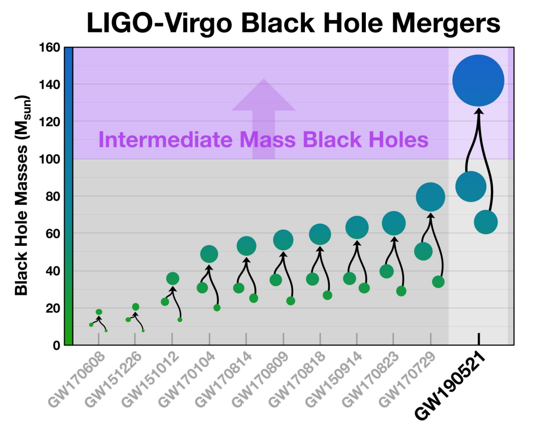
Two-body motion in the large velocity, highly dynamical, strong-field regime of gravity

GWs provide direct information from strong dynamical regions of spacetime

100 detections until today and more to come!



Many open questions



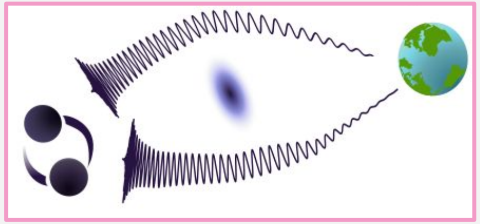
Astrophysics:
Black hole population properties
Binary black hole formation channels

Fundamental physics:
Nature of gravity (tests of General Relativity)

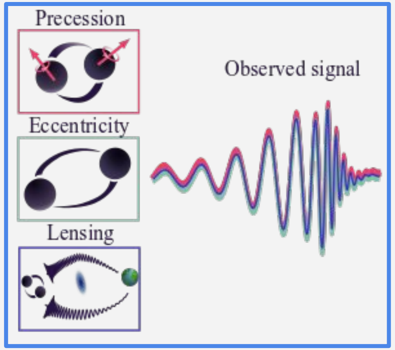
Cosmology:
Distribution of matter in the universe
Dark matter

My research experience

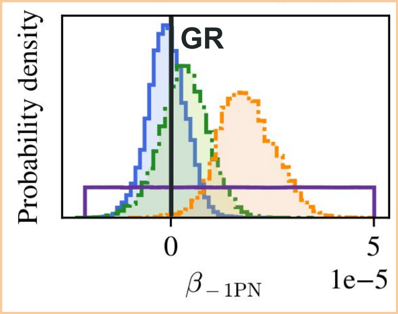
Gravitational-wave (GW) lensing



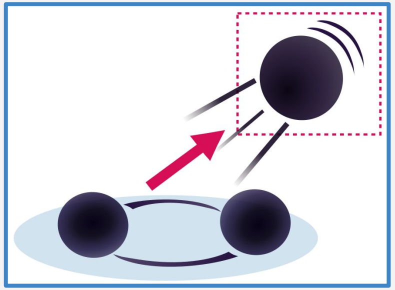
Systematic biases in GW inference



Tests of General Relativity with GWs



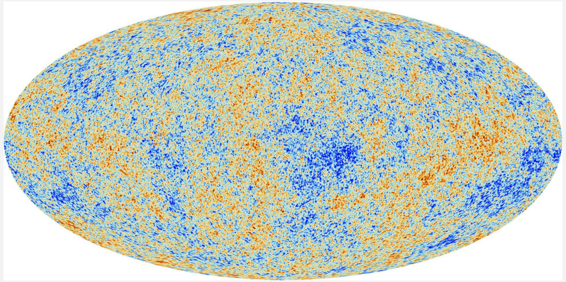
Binary black-hole formation



Bayesian inference with LIGO-Virgo-KAGRA (LVK) data

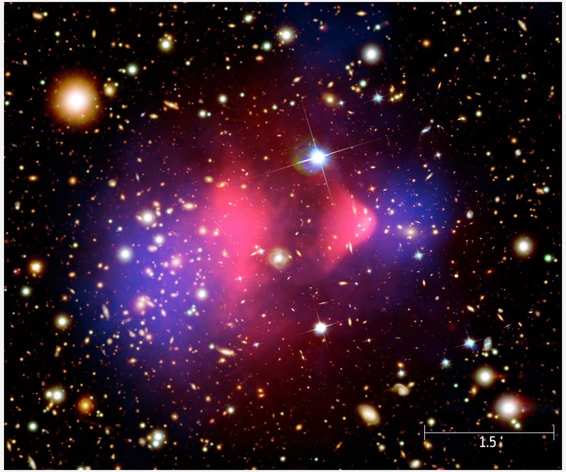
Gravitational waves as a probe of matter distribution in the Universe

Observational evidence for dark matter



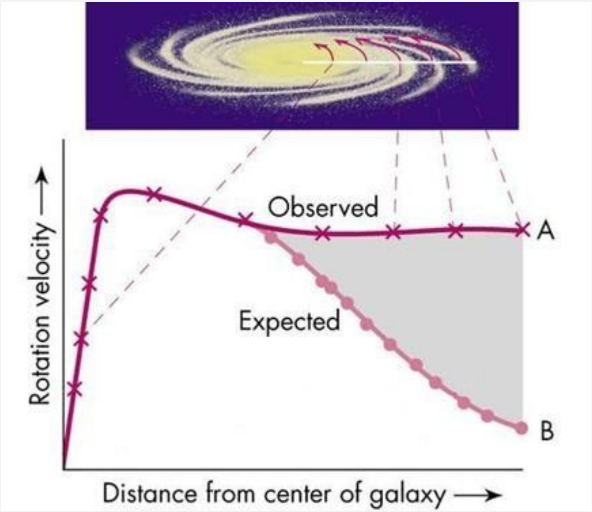
Planck

CMB temperature anisotropies



NASA

Bullet cluster



R. Evans

Rotation curves of galaxies

Observations at different scales provide compelling evidence for dark matter

What do we know about dark matter?

We know that dark matter is:

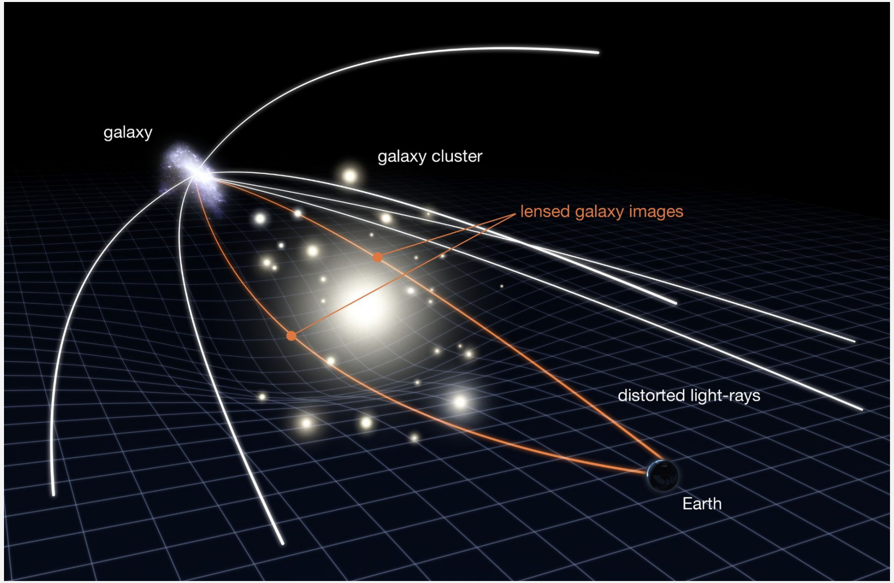
- Massive and collisionless
- Interacts via gravity → *lensing*
- ~84% of matter in the universe
- Not a Standard Model particle

We don't know yet:

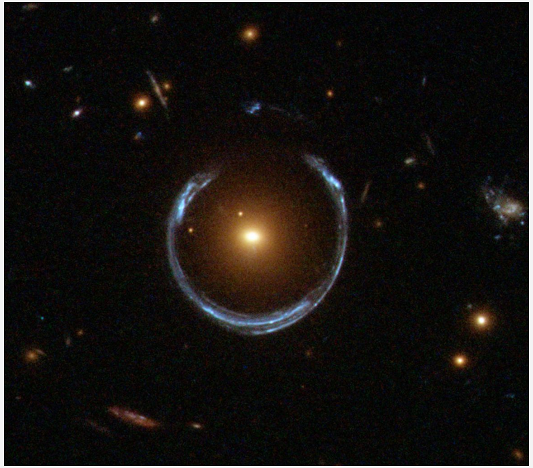
- Nature of dark matter
- Formation process
- Non-gravitational interactions

The unknown nature of dark matter is one of the key open questions in astrophysics & cosmology

Dark matter halos as gravitational lenses



ESA/Hubble

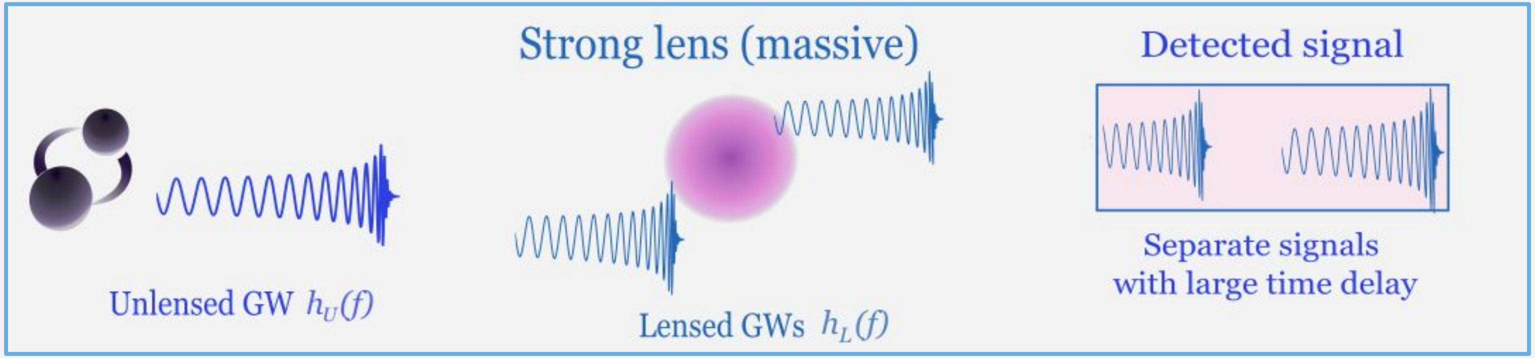


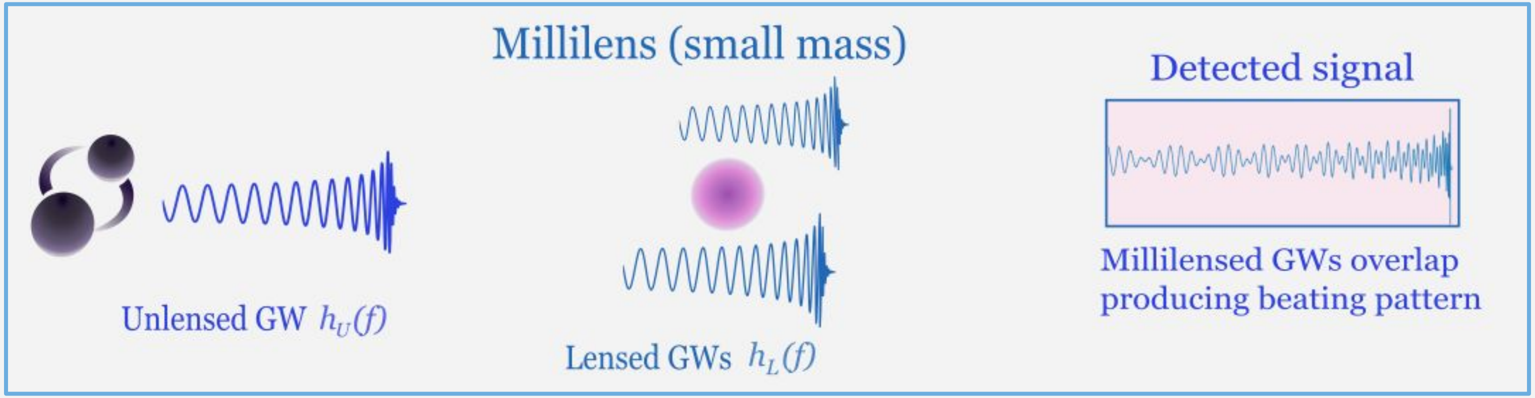
ESA/Hubble

Lensed images of background galaxy

Studying how dark matter bends light can tell us about its distribution

Gravitational-wave lensing regimes

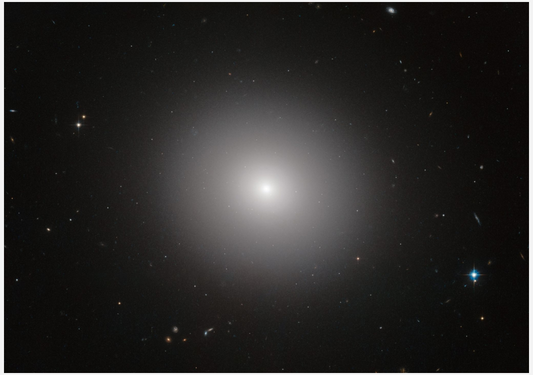




Typical lensing analyses

Point-mass lens

Singular isothermal sphere (SIS)



Singular isothermal ellipsoid (SIE)

All the models assume isolated lens & well-defined symmetries

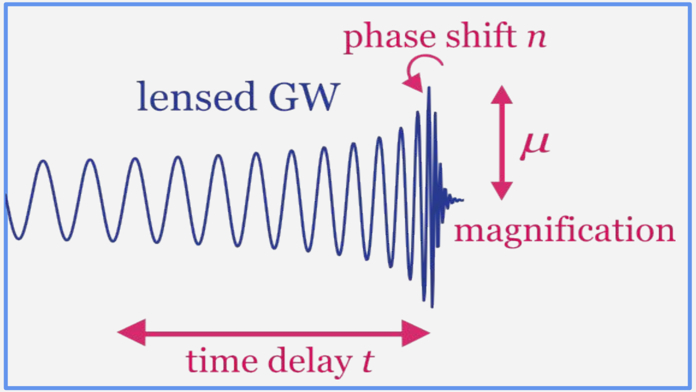
which is **physically not realistic**

Alternative: phenomenological approach without prior assumption on lens mass

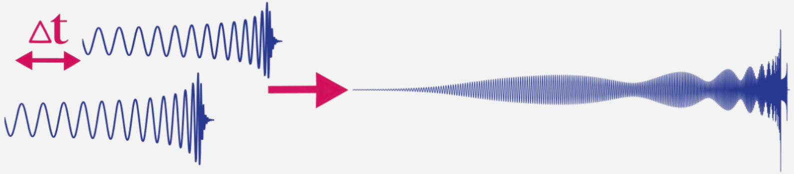
Phenomenological framework to study milillensed GWs

$$\tilde{h}_L(f;\theta_L) = F(f;\theta_L)\tilde{h}(f;\theta)$$

Lensing effect



- Each signal characterised by 3 parameters
- Arbitrary number of overlapping signals
- Trans-dimensional sampling



[AL, Wong, Leong et al. (2023) MNRAS]

Millilensed gravitational waveform

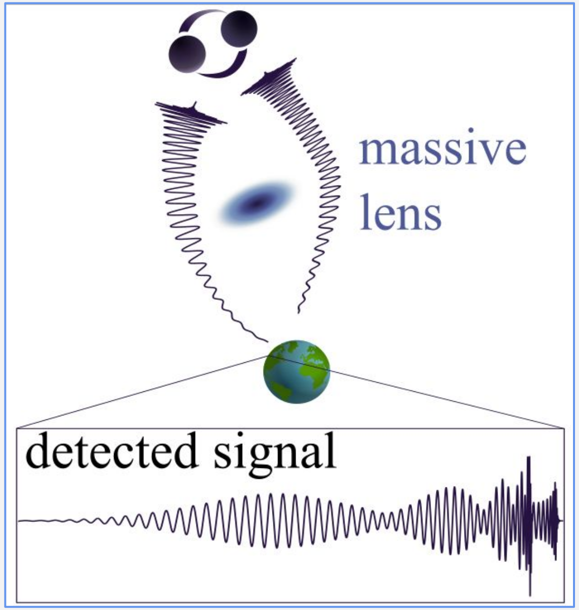
$$\tilde{h}_L(f;\theta_L) = F(f;\theta_L)\tilde{h}(f;\theta)$$

Lensing effect

Sum of component lensed signals arriving at the detector

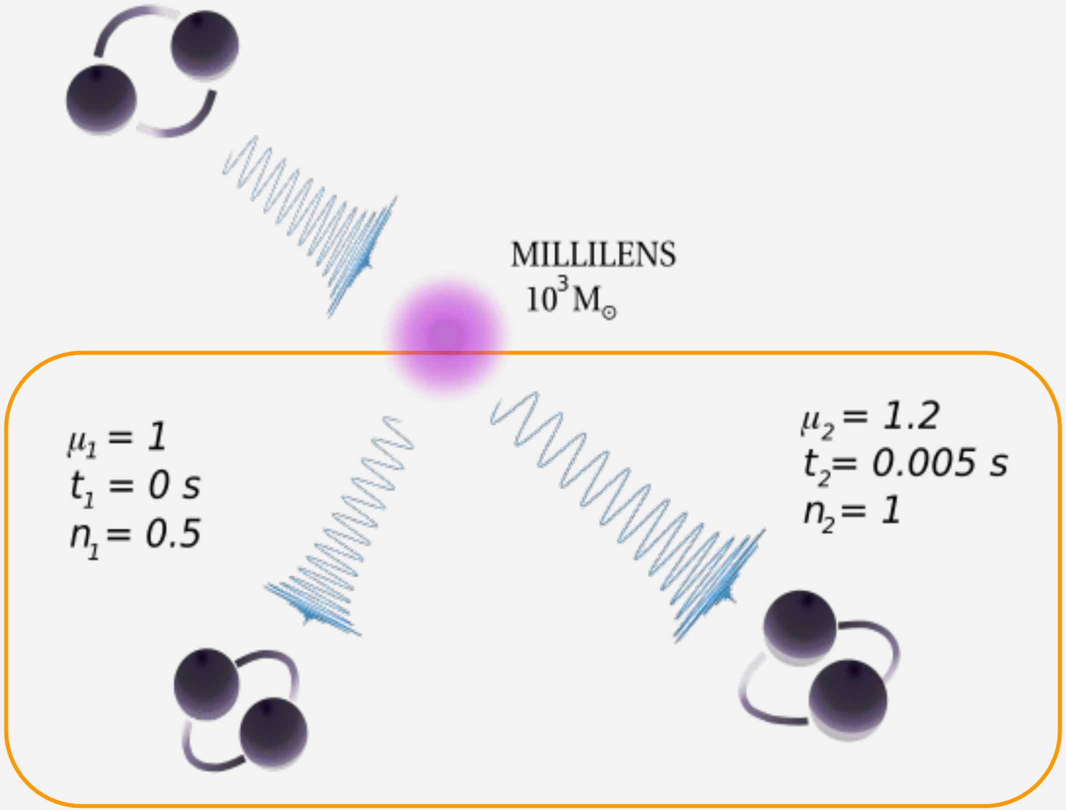
$$F(f;\theta_L) = \Sigma_j |\mu_j|^{1/2} \exp[2\pi i f t_j - i\pi n_j]$$

magnification   time delay   phase shift



Parameter estimation

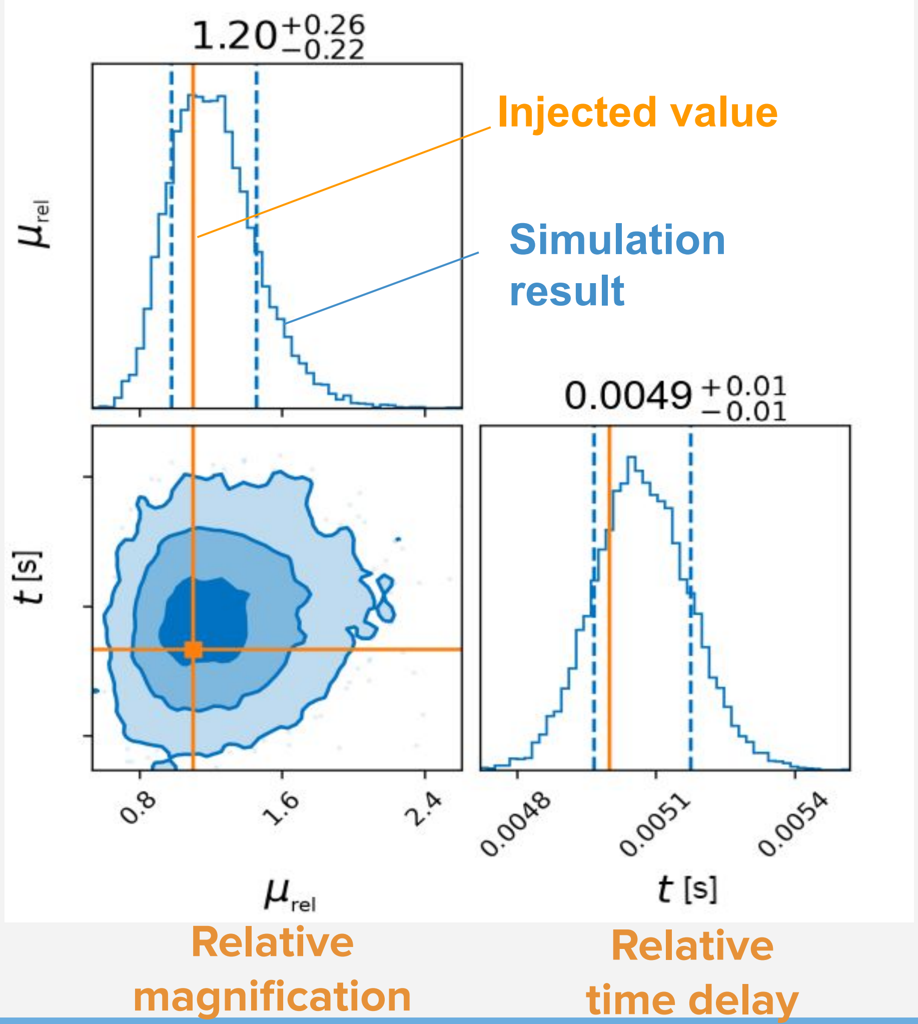
1. Inject overlapping millilensed GW signal into O3 detectors
2. Perform full parameter estimation
3. Obtain BBH source & lensing parameters



Overlap at the detector

Parameter estimation

We can recover individual lensing parameters

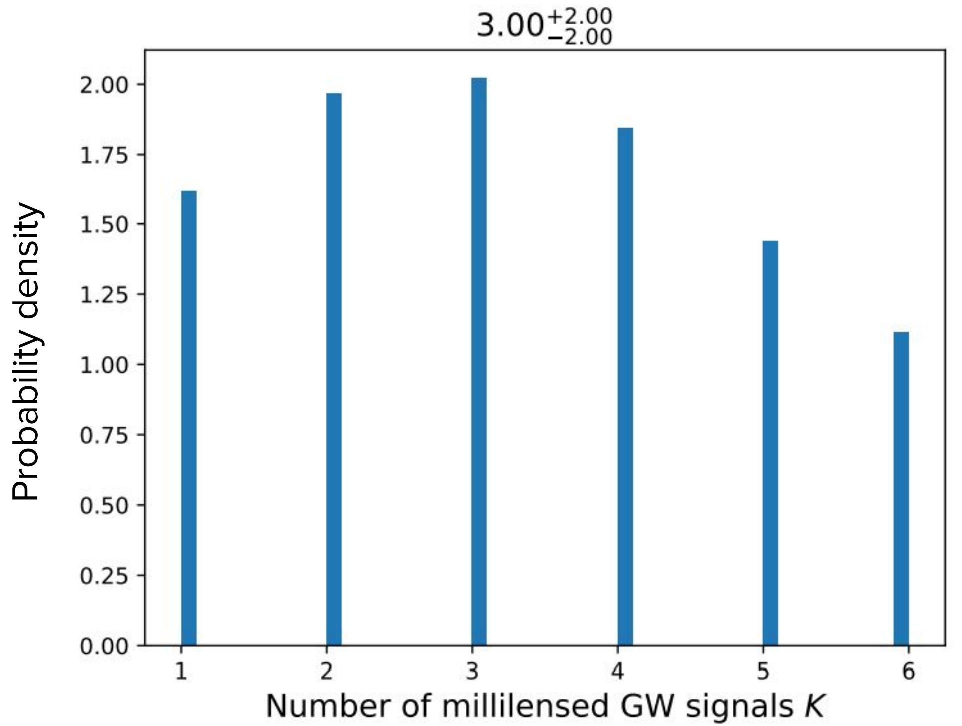


Analysing GW data

No compelling evidence for millilensing in O3 data

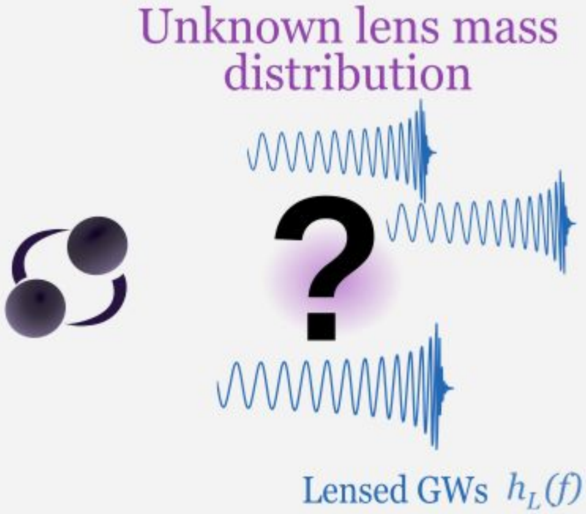
Model	$\ln(\mathcal{B})$
Two signals	35.18
Three signals	35.31
Four signals	35.40
Multi-signal	35.73
No lensing	33.27

GW200208



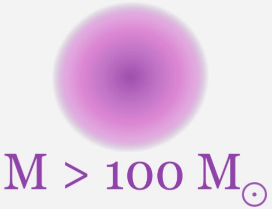
Advantages of the phenomenological approach

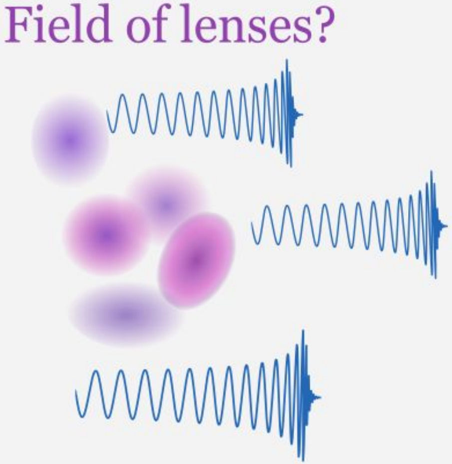
- No lens model assumed - arbitrary lens configuration
- More realistic than widely used isolated lens models
- Any number of GW millisignals



Limitations of the phenomenological approach

- Valid for lenses $\gtrsim 100\ M_\odot$ (geometric optics approximation)
- Limited to single lens scenarios (future work)
- No lens model - physical picture not immediately clear
  → lens mapping

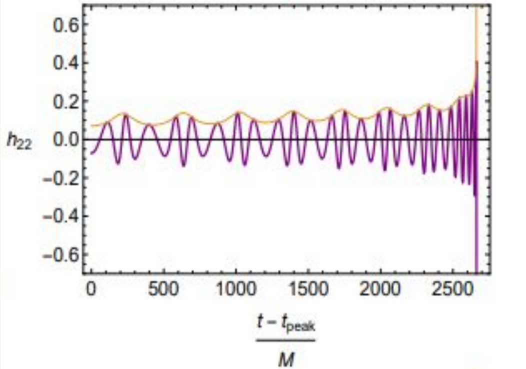




Systematic biases and model confusion challenges in gravitational-wave data analysis

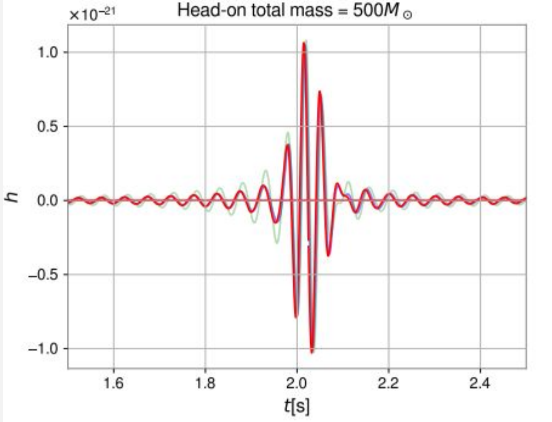
How to tell if it's lensing?

Eccentric binary orbits



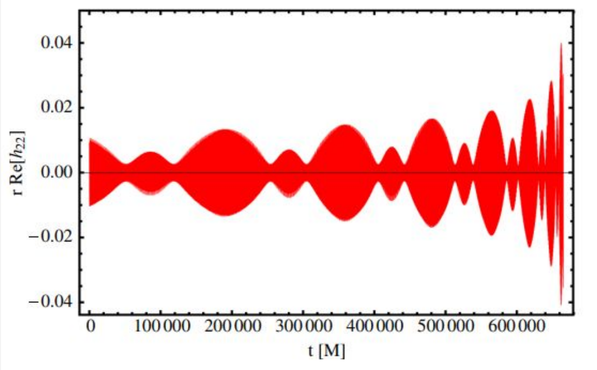
[Chowdhury & Khlopov, 2022]

Head-on mergers



[J. Calderón Bustillo et al., 2021]

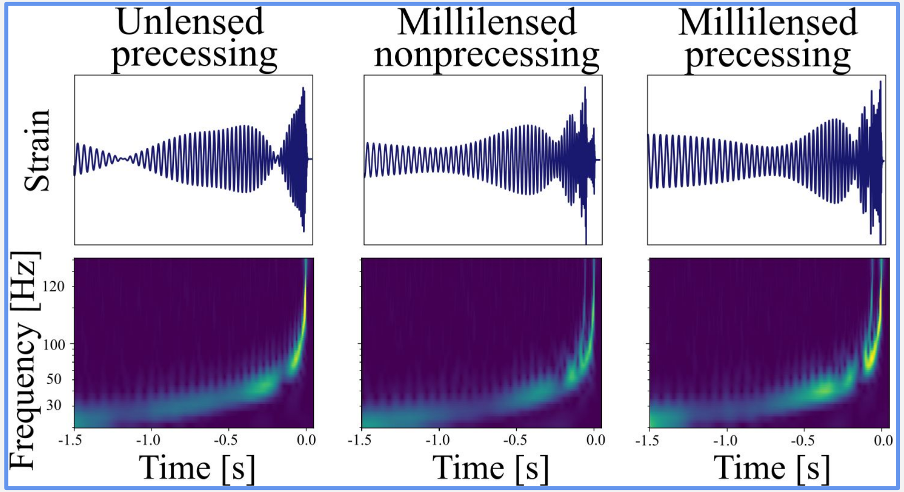
Precessing binaries



[P. Schmidt et al., 2012]

Different physical effects can mimic lensing signatures in GW signals

Precessing or millilensed?



If detected by LVK detectors,
can we distinguish the three types of signals?

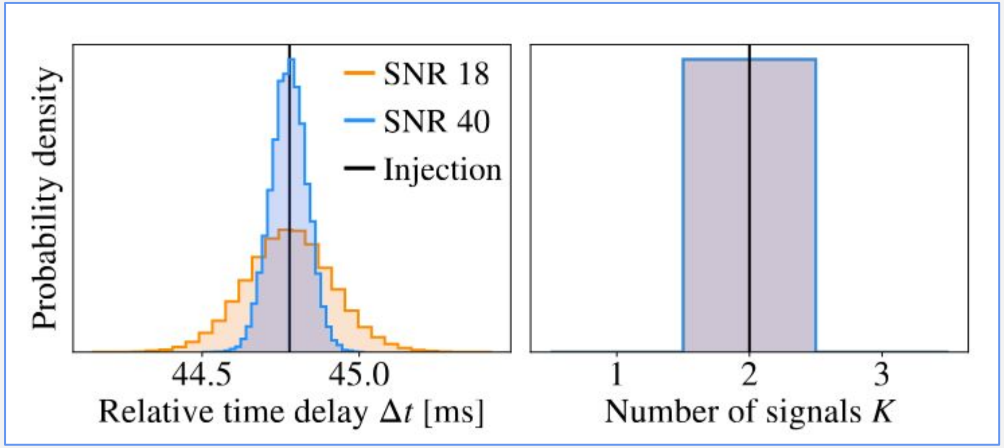
Parameter Estimation and SNR computation

Can we correctly recover lensing and precession parameters in the presence of one or both effects?

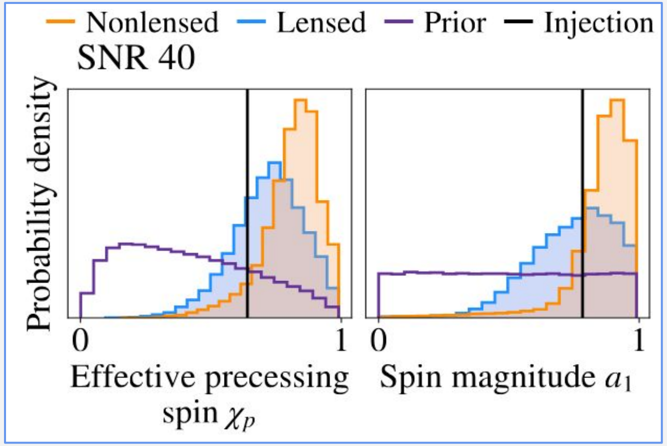
[AL and K. Kim, PRD 110.123008 (2024)]

Analysing a lensed precessing signal

Correct recovery of lensing parameters in the presence of precession



Preference for high spins when lensing neglected in the recovery model



Takeaway: Neglected lensing effects can affect spin recovery

[AL and K. Kim, PRD 110.123008 (2024)]

Science Objective:

Tests of General Relativity with Gravitational Waves

Modified theories of gravity - why modify at all?

Einstein's theory of General Relativity (GR)

- ✔ Solar system tests
- ✔ Binary pulsars
- ✔ Cosmological tests

} Quasi-stationary quasi-linear weak field regime of General Relativity

In other words, GR works when

Gravitational field weak cf. mass-energy of the system,
characteristic velocities small cf. speed of light,
gravitational field stationary w.r.t. characteristic size of the system

Modified theories of gravity - why modify at all?

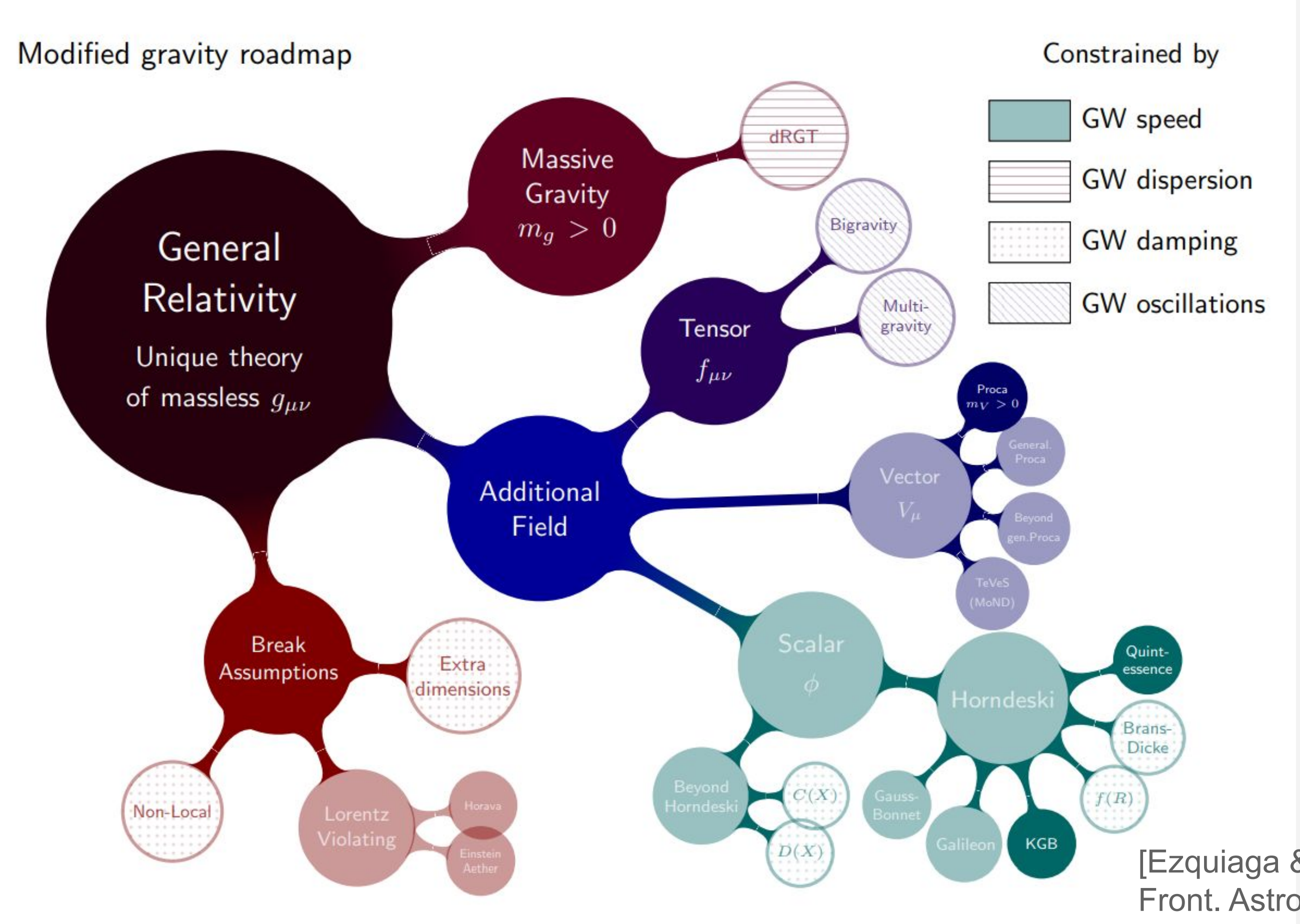
Einstein's theory of General Relativity (GR)

- ✔ weak gravity regions
- × unexplained problems

Modified theories of gravity

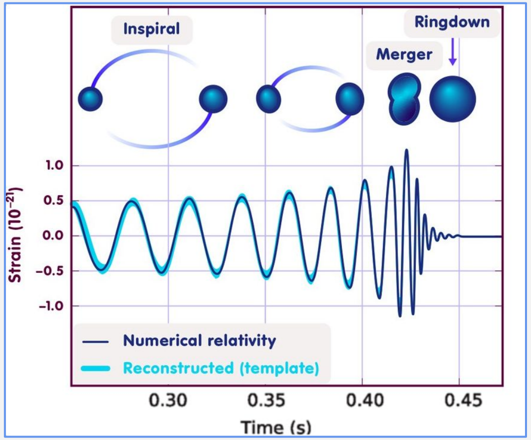
- ✔ recover GR in the weak field regime
- ✔ preserve symmetries (in general)
- ✔ can differ in the **strong curvature regime** → black holes

How to modify GR?



[Ezquiaga & Zumalacárregui, Front. Astron. Space Sci. (2018)]

Parameterized test of GW propagation: inspiral phasing



Inspiral waveform

$$h_{\rm GW}(f) = \mathcal{A}_{\rm GW}(f) e^{i\varphi_{\rm GW}(f)}$$

Introduce deviations to the GW phase at each PN order:

$$\varphi_{\rm GW}(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + v^{-5}\left[\sum_{n=-2}^{7} \varphi_n^{\rm (GR)}\left(1+\delta\hat{\varphi}_n\right) v^n + \sum_{n=5}^{6} \varphi_{n\ell}^{\rm (GR)}\left(1+\delta\hat{\varphi}_{n\ell}\right) v^n \log v\right]$$

$$v = (\pi M f)^{1/3}$$

Depend on the (conservative and dissipative) dynamics of the system

Science Question:

Can GW millilensing be mistaken as false positive deviations of General Relativity?

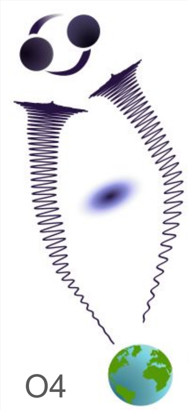
Can ignoring millilensing lead to false positive deviations of GR?

Parameterized post-Einsteinian (ppE) framework

$$\tilde{h}_{\mathrm{ppE}}(f) = \tilde{h}_{\mathrm{GR}}(f)\,(1 + \alpha_{\mathrm{ppE}} u^a)\, e^{i\beta_{\mathrm{ppE}} u^b}$$

- Non-GR corrections added order-by-order at each PN term
- Our study: inspiral-only corrections to GW phase

[Yunes & Pretorius, PRD (2009)]
[Yunes, Yagi & Pretorius, PRD (2016)]

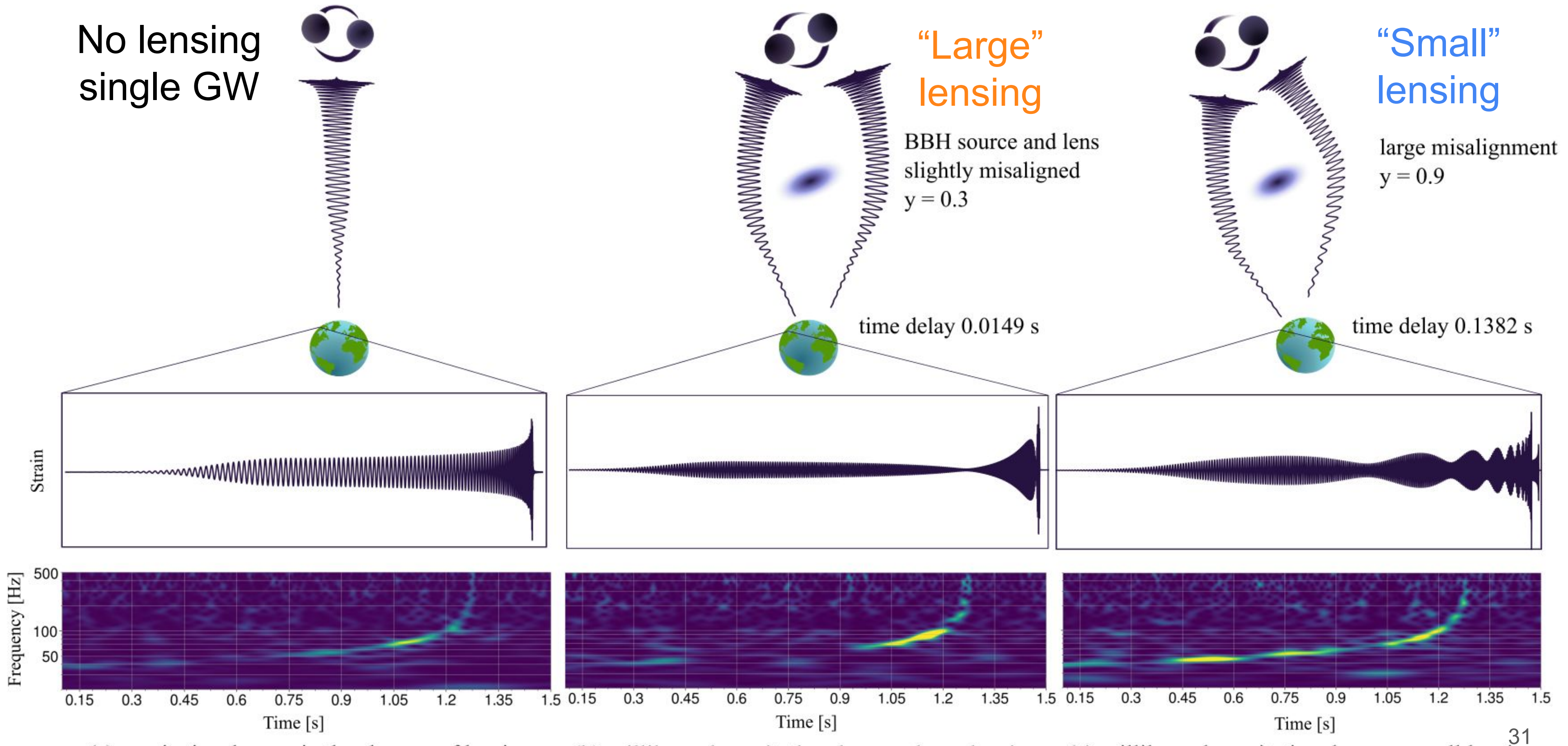


TGR Parameter Estimation

Violation of GR?

[AL, R.S. Chandramouli et al., 2410.21738]

Relative source-lens position as a measure of lensing effect



(a) gravitational-wave in the absence of lensing (b) millilensed gravitational wave: large lensing (c) millilensed gravitational wave: small lensing

Simulated signals

BH masses

$(12, 8)M_\odot$ $(24, 16)M_\odot$

How does the bias depend on BBH masses (signal duration)?

Lensing effect strength

y = (0.3, 0.6, 0.9)

(small y = large lensing effect)

How does the bias change with lensing effect?

PN orders

{-1PN, 1PN, 2PN}

At which PN order is the bias most prominent?

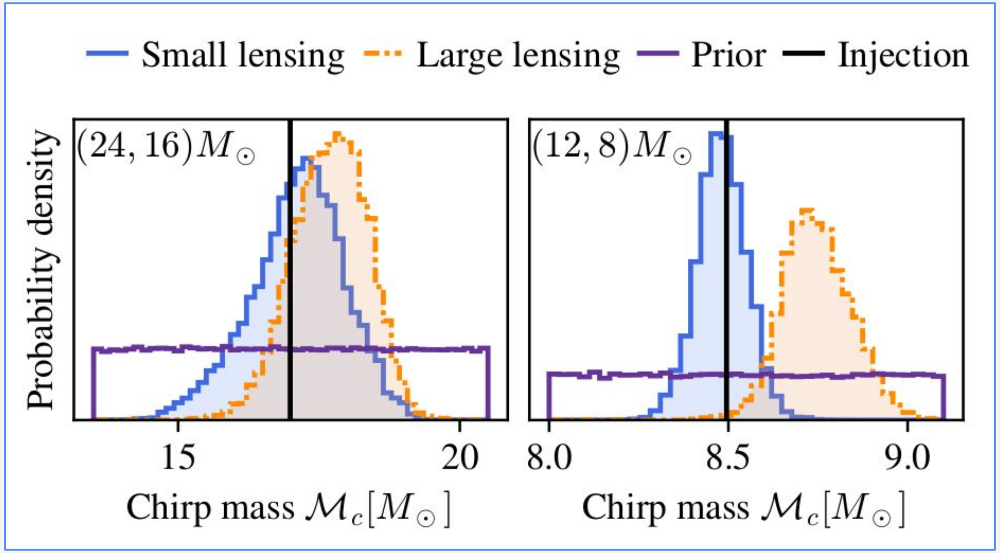
Detectors and SNR

O4 (L1, H1, V1): 10, 30, 60

At what SNR is the systematic bias significant?

[AL, R.S. Chandramouli et al., 2410.21738]

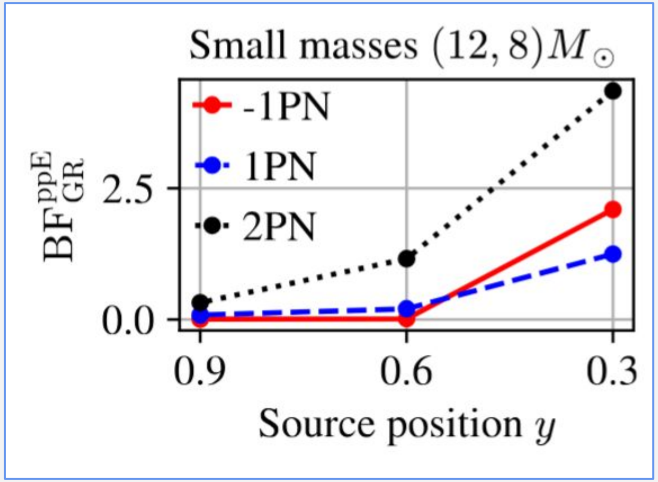
How are the parameters biased?



- ➔ Lensing effect builds up for longer signals
- ➔ Systematic bias more prominent for smaller mass system

[AL, R.S. Chandramouli et al., 2410.21738]

Should we worry? Quantifying the biases



- Weak preference for ppE model over GR in large lensing scenario (small masses, SNR 30)
- Significant SNR loss in the recovered signal: suggests model inaccuracies, rather than actual GR deviation (recovery model not a good fit for the signal)
- Conclusion: no strong inference of GR deviation

[AL, R.S. Chandramouli et al., 2410.21738]

Testing General Relativity with Black Hole Ringdown

Black hole ringdown

Excitation radiated as GWs

Perturbed black hole oscillates with characteristic frequencies

$$(\omega_{\ell m}, \tau_{\ell m})$$

Ringdown

Ringdown remnant: stabilizes to equilibrium after merger

Damped normal modes emission (perturbation theory)

Quasinormal modes

Ringdown waveform can be described as **superposition of damped sinusoids**

$$h = \sum_{\ell mn} A_{\ell mn} \cos(\omega_{\ell mn} t + \phi_{\ell mn}) \cdot e^{-t/\tau_{\ell mn}}$$

Characterized by **quasinormal modes (QNMs)**:

$$\tilde{\omega}_{\ell mn} = \omega_{\ell mn} + i/\tau_{\ell mn}$$

Oscillation frequency — Damping time

In GR (2, 2, 0) is the fundamental (loudest) mode.

Extracting $(\omega_{220}, \tau_{220})$ from GW data → estimate the mass and spin of the final BH (M_f, χ_f)

N modes → Tests of GR (if can be measured)

Testing GR with BH ringdown

No-hair theorem: In GR, black holes are Kerr and can be fully characterized by their mass and spin

$$\omega_{\ell mn} = \omega_{\ell mn}^{(\mathrm{GR})}(M, \chi)$$
$$\tau_{\ell mn} = \tau_{\ell mn}^{(\mathrm{GR})}(M, \chi)$$

Deviations from Kerr:

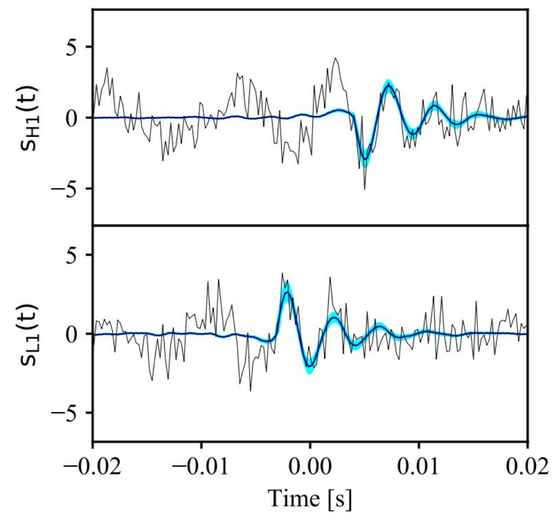
$$\omega_{\ell mn} = \omega_{\ell mn}^{(\mathrm{GR})}(M, \chi)\,(1 + \delta\omega_{\ell mn})$$
$$\tau_{\ell mn} = \tau_{\ell mn}^{(\mathrm{GR})}(M, \chi)\,(1 + \delta\tau_{\ell mn})$$

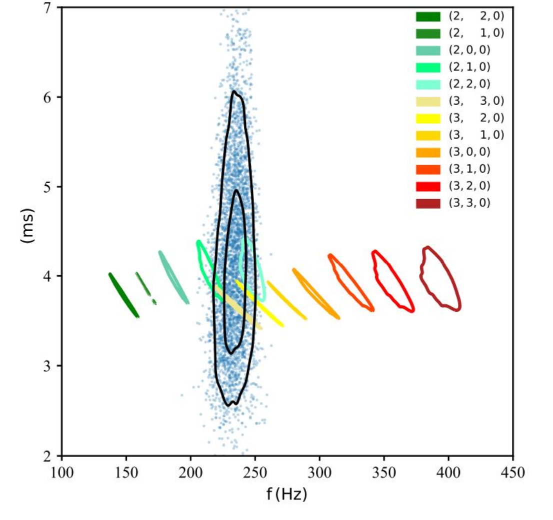
BBH merger as a testing ground for Kerr hypothesis

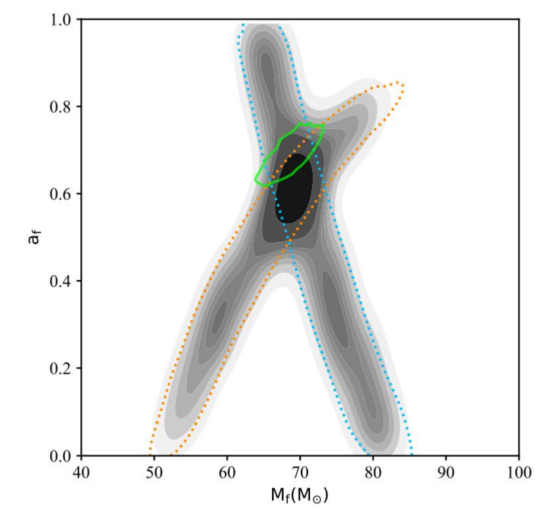
Black hole spectroscopy

$$h = \sum_{\ell m} A_{\ell m} e^{-i\omega_{\ell m} - t/\tau_{\ell m}} {}_2Y_{\ell m}$$

GW Data → QNM frequencies → BH mass and spin



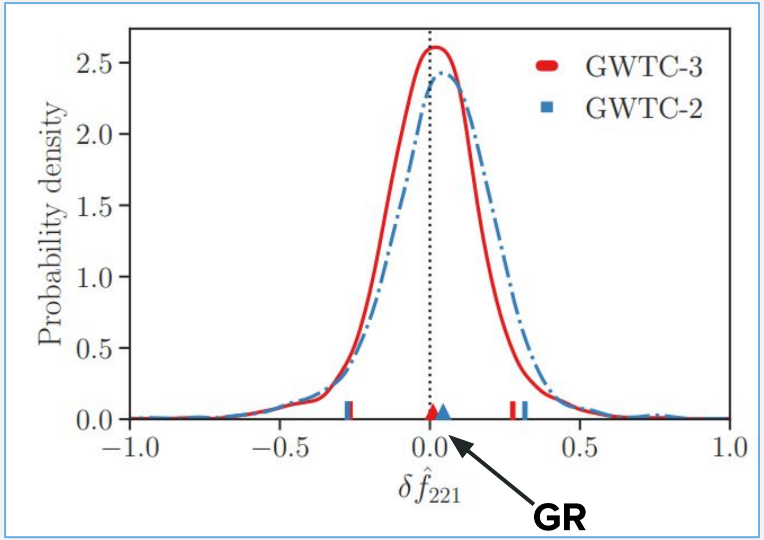


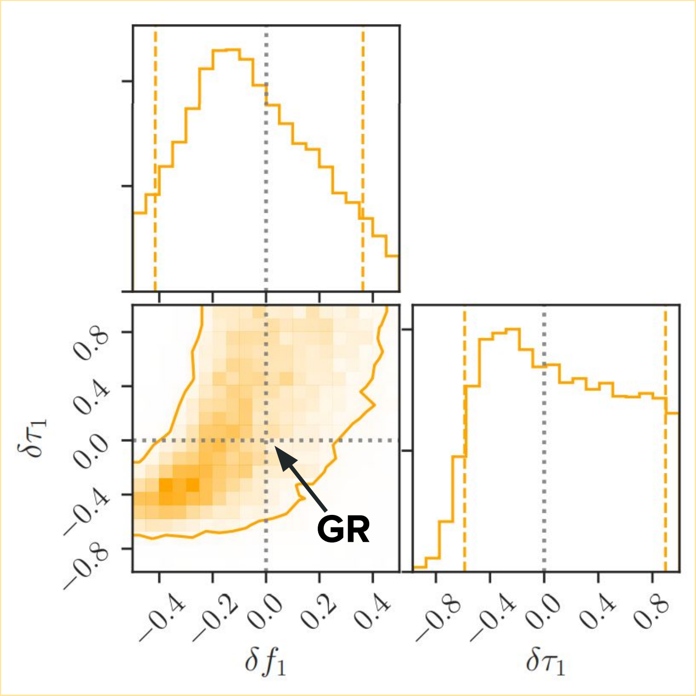


[Carullo et al., PRD 99.12 (2019)]

What has been constrained so far?

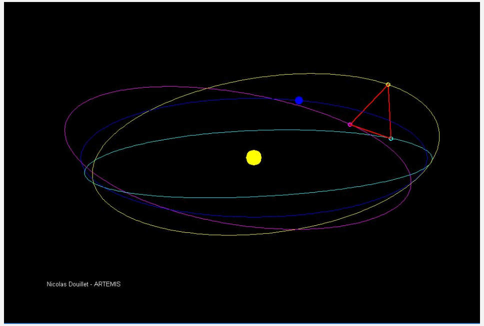
$$\omega_{\ell mn} = \omega^{(\mathrm{GR})}_{\ell mn}(M,\chi)\,(1+\delta\omega_{\ell mn})$$
$$\tau_{\ell mn} = \tau^{(\mathrm{GR})}_{\ell mn}(M,\chi)\,(1+\delta\tau_{\ell mn})$$

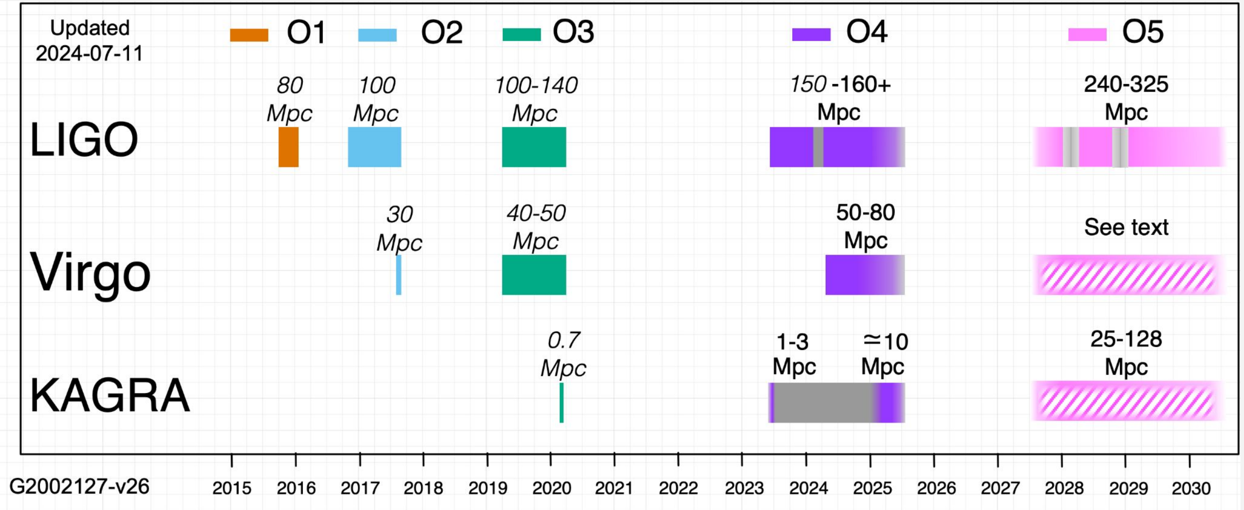




[Abbott et al. (2021), 2112.06861]

More exciting physics soon





https://observing.docs.ligo.org/plan/

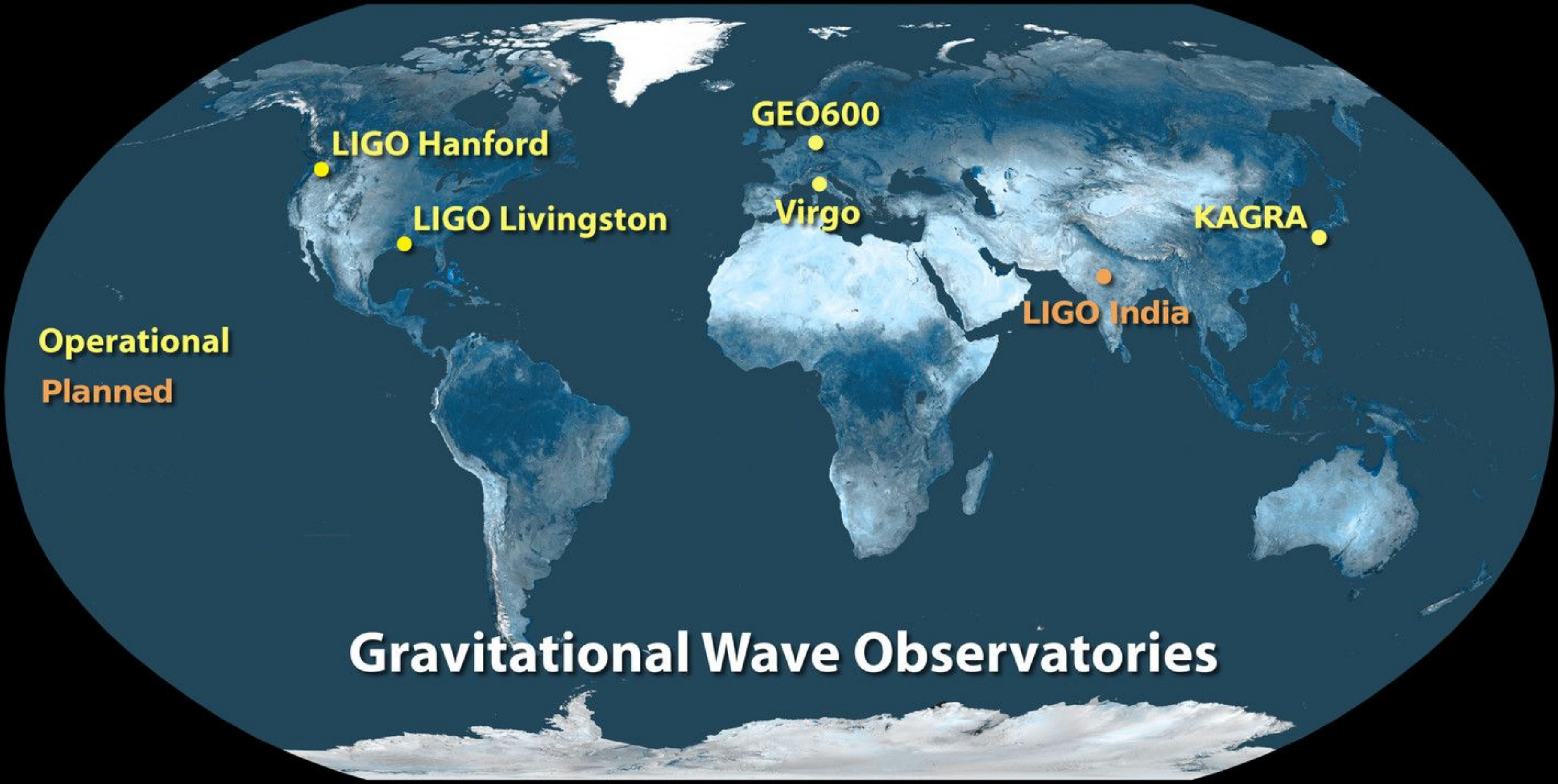
More detections from higher redshifts and wider frequencies predicted in the coming years

Summary

- GWs can be used to probe dark matter via GW lensing
- No confident lensed GWs found to date, ongoing search in O4 run
- GWs provide a unique laboratory to test gravity, e.g. with parameterised inspiral tests and ringdown tests
- No compelling evidence for violations of gravity found to date
- New observations from current and future detectors will reveal more signals with higher SNRs from a wide range of currently inaccessible sources

Thank you!

Backup slides


GEO600
LIGO Hanford
Virgo
LIGO Livingston
KAGRA
LIGO India
Operational
Planned
Gravitational Wave Observatories

Selecting lens models

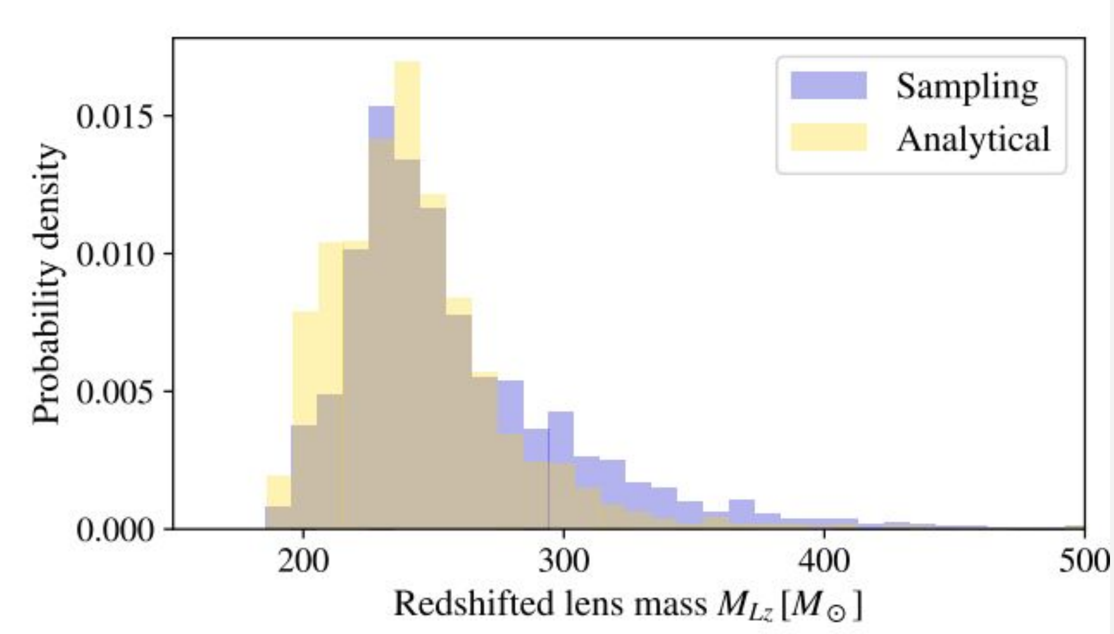
Singular Isothermal Sphere (SIS) lens

$$(\mu, t_d) \longleftrightarrow (y, M_{Lz})$$

Phenom. parameters

SIS model parameters

$$(\mu(y), t_d(y, M_{Lz}))$$



We can map results to different lens models to select the most favourable one

Probing small-scale structures

Warm dark matter (WDM):

- Light, fast particles
- Smoothens small structures "clump-smoother"

Cold dark matter (CDM):

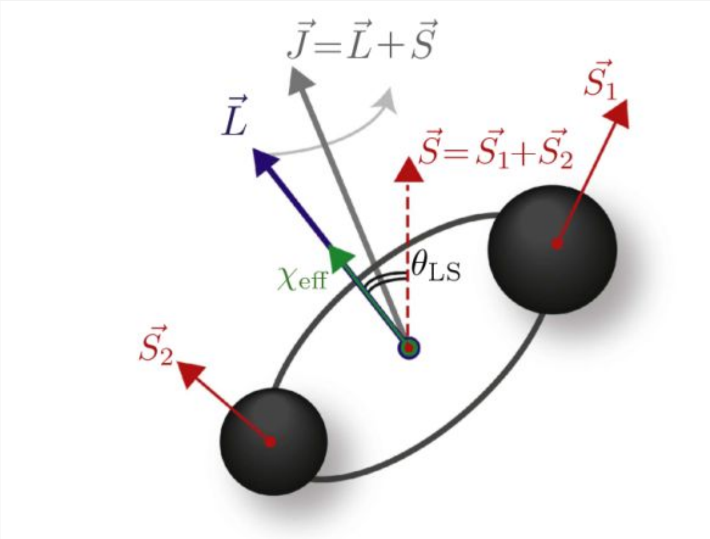
- Massive, slow particles
- Forms small structures, e.g. small DM halos $\rightarrow$ "clump-builder"

Problem: they do not agree at small scales!

↓

Small-scale DM structures as a ground for testing DM models

Precessing black hole binaries



credit: Vijay Varma

Spin-induced orbital precession leads to GW amplitude and phase modulations

Black hole spectroscopy

$$h = \sum_{\ell m} A_{\ell m} e^{-i\omega_{\ell m} - t/\tau_{\ell m}} {}_2Y_{\ell m}$$

multiple modes of vibration

excitation amplitudes

exponentially damped harmonic oscillations

spin-weighted spherical harmonics

$(\omega_{\ell m}, \tau_{\ell m})$ One mode → BH mass and spin (M_f, a_f)

N modes → Tests of GR (if can be measured)