Beyond the Born rule in quantum gravity

Antony Valentini
Theoretical Physics Group
Imperial College London

What do we mean by ‘quantum gravity’?

-- We consider canonical quantum gravity.
Quantise the 3-metric on a spacelike hypersurface.
Wheeler-DeWitt equation for $\Psi[g_{ij}]$.

-- Not perturbatively renormalisable.
Perhaps valid in some approximation
(not too close to the Planck scale?)

-- Mostly consider models of quantum cosmology
with a small (finite) number of degrees of freedom.

-- Despite these limitations, we might learn something.
In particular: possible implications for the Born rule.

Main points

-- no such thing as the Born rule in quantum gravity

-- Born rule emerges only in semiclassical regime (quantum relaxation on a classical spacetime)

-- quantum-gravity corrections to the semiclassical regime can make the Born rule unstable

Main points

-- no such thing as the Born rule in quantum gravity

Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi = 0$

$|\Psi[g_{ij}]|^2$ is not a probability

-- Born rule emerges only in semiclassical regime (quantum relaxation on a classical spacetime)

time-dependent Schrödinger equation $i\frac{\partial \psi}{\partial t} = \hat{H}\psi$

$|\psi|^2$ can be a probability (after relaxation)

-- quantum-gravity corrections to the semiclassical regime can make the Born rule unstable

$\rho = |\psi|^2$ can evolve to $\rho \neq |\psi|^2$

No such thing as the Born rule in quantum gravity

Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi = 0$

$\left|\Psi[g_{ij}]\right|^2$ is not a probability

Solutions $\Psi\left[g_{ij}\right]$ are non-normalisable

Because: Wheeler-DeWitt is like Klein-Gordon

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0 \longleftrightarrow \left(-\frac{\partial^2}{\partial t^2} + \delta^{ij}\frac{\partial^2}{\partial x^i \partial x^j} - m^2\right)\psi = 0$$

(indefinite metric G_{ijkl} on superspace)

$$\int Dg \; |\Psi[g_{ij}]|^2 = \infty \longleftrightarrow \int d^3x \int_{-\infty}^{+\infty} dt \; |\psi(x,t)|^2 = \infty$$

Illustrate with model of quantum cosmology

Expanding universe: scale factor $a(t)$ ($d\tau^2 = dt^2 - a^2 d\mathbf{x}^2$)

Contains a homogeneous matter field ϕ with potential $\mathcal{V}(\phi)$

('minisuperspace' with two degrees of freedom (a, ϕ))

Wheeler-DeWitt eqn. for $\Psi(a, \phi)$ (Planck mass $m_{\mathrm{P}}^2 = 3/4\pi G$)

$$\frac{1}{m_{\mathrm{P}}^2}\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial \Psi}{\partial a}\right) - \frac{1}{a^2}\frac{\partial^2 \Psi}{\partial \phi^2} + 2a^4\mathcal{V}(\phi)\Psi = 0$$

or in terms of $\alpha = \ln a$

$$\frac{1}{m_{\mathrm{P}}^2}\frac{\partial^2 \Psi}{\partial \alpha^2} - \frac{\partial^2 \Psi}{\partial \phi^2} + 2e^{6\alpha}\mathcal{V}(\phi)\Psi = 0$$

Two-dimensional Klein-Gordon eqn. with a potential term

Illustrate with model of quantum cosmology

wave equation $$\frac{1}{m_{\rm P}^2}\frac{\partial^2\Psi}{\partial\alpha^2} - \frac{\partial^2\Psi}{\partial\phi^2} + 2e^{6\alpha}\mathcal{V}(\phi)\Psi = 0$$

free part (ignoring potential) has general solution

$$\Psi^{\rm free} = f(\phi - m_{\rm P}\alpha) + g(\phi + m_{\rm P}\alpha)$$

f and *g* are packets travelling with 'wave speed' $c = m_{\rm P}$

(in the minisuperspace (α,ϕ))

Clearly, in general:

$$\int\int d\alpha d\phi\ |\Psi|^2 = \infty \longleftrightarrow \int d^3x \int_{-\infty}^{+\infty} dt\ |\psi(x,t)|^2 = \infty$$

Probability $|\Psi[g_{ij}, \phi]|^2$ is naive

Well known in quantum-gravity circles

(reviews: Isham (1992), Kuchar (1992), Anderson (2017))

Called the "Naïve Schrödinger Interpretation"

(Unruh and Wald 1989)

(though extensively applied by Hawking *et al*. in the 1980s)

Problematic because "timeless" (fix by conditioning?)

In any case ultimately fails because not normalisable

A common explanation

Probability $|\Psi[g_{ij}, \phi]|^2$ is naïve because:

time is "hidden" in the 3-metric g_{ij}

For example, quantum cosmology: $\Psi(a, \phi)$

treat scale factor a as "time"

↑ *"time"*

Controversial:

-- recover standard quantum mechanics?

-- what happens to "time" if a expands and recontracts?

-- maybe "time" emerges only approximately and in certain conditions, not fundamental (DeWitt, Rovelli, …)

New explanation

In the deep quantum-gravity regime (Planck scale),
there is simply no such thing as the Born rule

We can talk about probabilities $P[g_{ij}, \phi, t]$

But they are not tied to the Born rule

In fact, *necessarily,* $P[g_{ij}, \phi, t] \neq |\Psi[g_{ij}, \phi]|^2$ *always*

Quantum gravity is in a perpetual state of 'nonequilibrium'

(true physical significance of non-normalisable $|\Psi[g_{ij}, \phi]|^2$)

To make sense of this

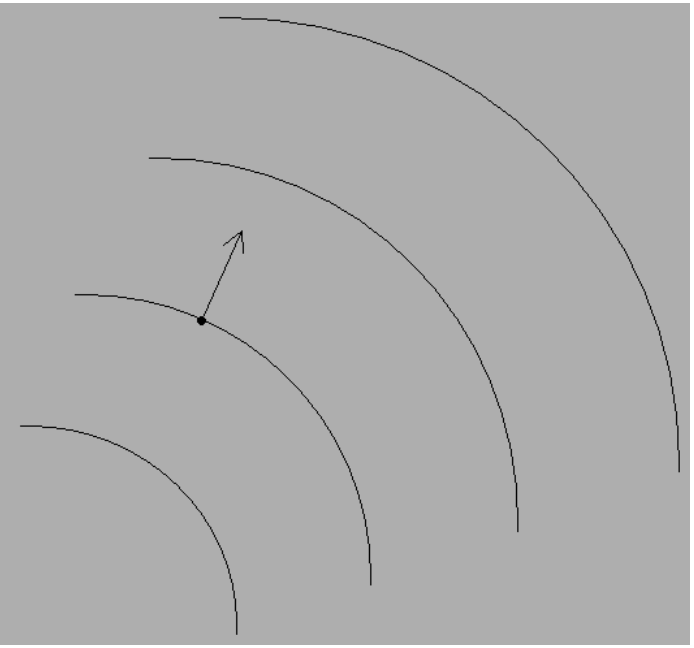
We need to look at the pilot-wave theory
of de Broglie (1927) and Bohm (1952)
(interpreted correctly)

The Born rule is not an axiom or law,
but a state of statistical equilibrium
(analogous to classical thermal equilibrium)
(AV 1991, 1992)

De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial\psi}{\partial t} = -\sum_{n=1}^{N}\frac{1}{2m_n}\nabla_n^2\psi + V\psi \qquad \frac{d\mathbf{x}_n}{dt} = \frac{\nabla_n S}{m_n}$$

$$\psi(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, t) \qquad \psi = |\psi|\, e^{iS}$$



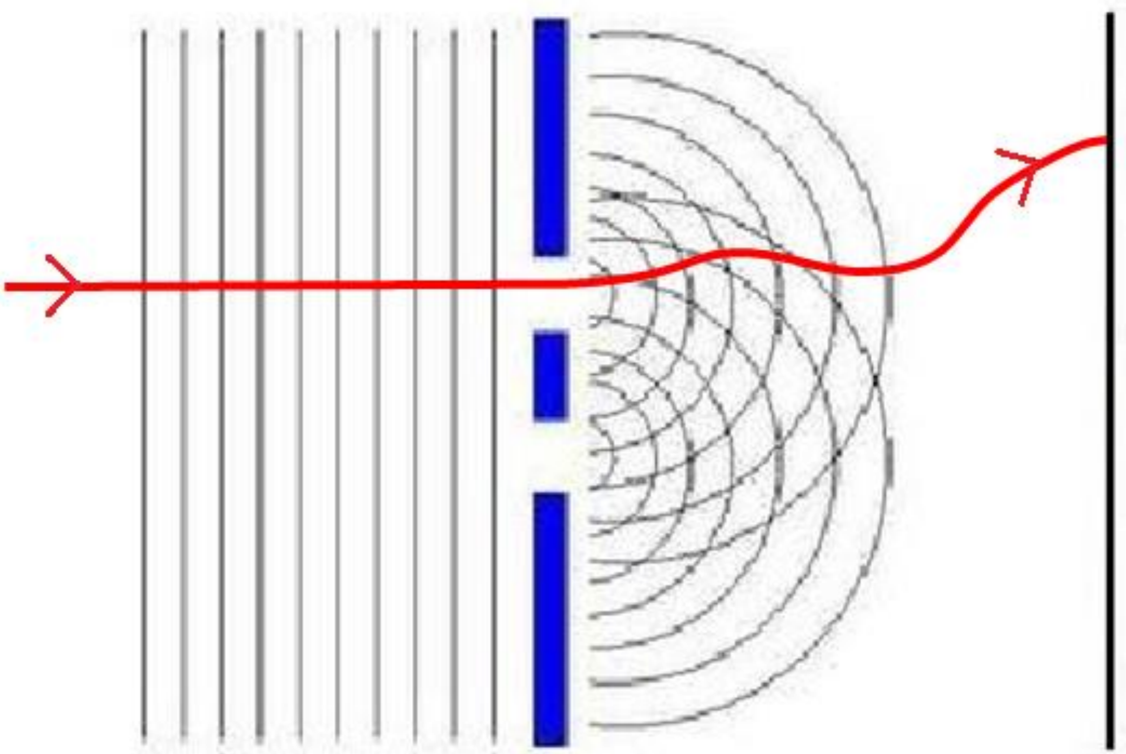
$$q = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N)$$

Motion of configuration $q(t)$
is determined by a 'pilot wave' ψ

(ψ is defined on configuration space)

An example: the two-slit experiment

Fire one particle at a two-slit screen



Given the wave function ψ and the initial position **x**(0), the particle trajectory **x**(*t*) is determined by de Broglie's *law of motion*

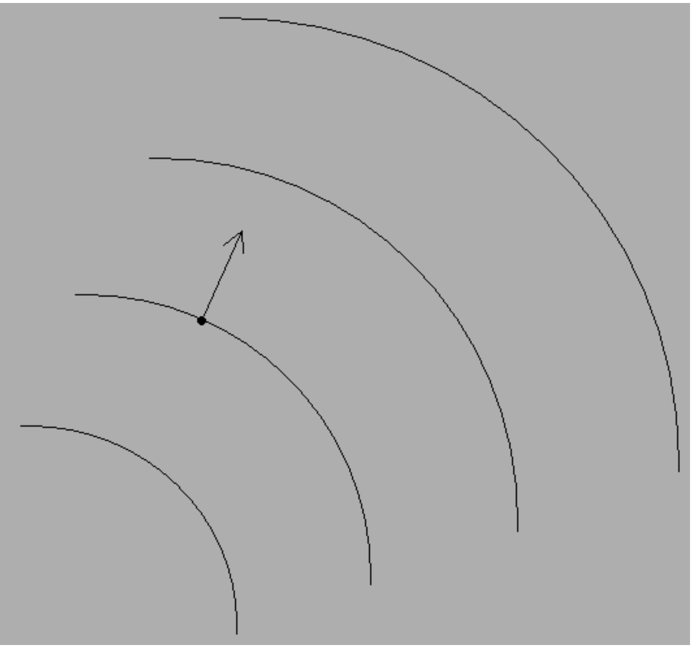
$$\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m} \qquad \psi = |\psi|\, e^{iS}$$

Postulated to be true, even if in practice we do not know **x**(0)

De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial \psi}{\partial t} = -\sum_{n=1}^{N} \frac{1}{2m_n}\nabla_n^2\psi + V\psi \qquad \frac{d\mathbf{x}_n}{dt} = \frac{\nabla_n S}{m_n}$$

$$\psi(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, t) \qquad \psi = |\psi|\, e^{iS}$$



Consider an ensemble of systems with the same ψ and different *q*'s

Ensemble distribution $\rho(q,t)$

Recover quantum mechanics *if* *assume* initial distribution $\rho = |\psi|^2$ (preserved in time by dynamics)

(shown fully by Bohm in 1952)

Illustration for one particle

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V\psi \quad \Rightarrow \quad \frac{\partial|\psi|^2}{\partial t} + \nabla\cdot\left(|\psi|^2\frac{\nabla S}{m}\right) = 0$$

$$\psi = |\psi|\,e^{iS}$$

Guidance equation $\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m}$ applied to an *ensemble* (same ψ, different $\mathbf{x}$'s)

Distribution $\rho(\mathbf{x},t)$ obeys $\frac{\partial\rho}{\partial t} + \nabla\cdot\left(\rho\frac{\nabla S}{m}\right) = 0$

THEOREM (quantum equilibrium):

If $\rho(\mathbf{x},0) = |\psi(\mathbf{x},0)|^2$, then $\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2$ for all t

(Generalisation: replace $\mathbf{x}$(t) by general configuration $q(t)$)

Similarly for a general system

System with configuration $q(t)$ and wave function(al) $\psi(q,t)$

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi \qquad\qquad \frac{dq}{dt} = v = \frac{j}{|\psi|^2}$$

where $j = j\left[\psi\right] = j(q,t)$ is the Schrödinger current

By construction $\rho(q,t)$ will obey

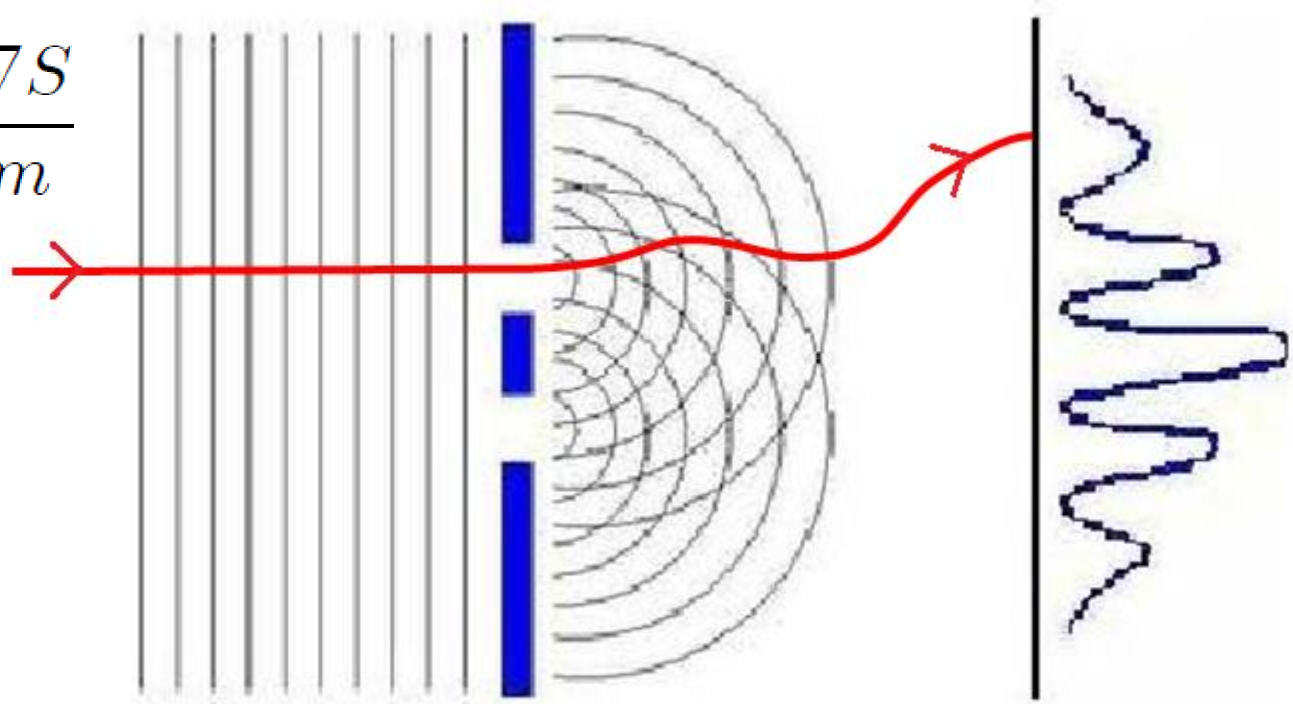
$$\frac{\partial\rho}{\partial t} + \partial_q\cdot(\rho v) = 0$$ (*same* as $\frac{\partial\left|\psi\right|^2}{\partial t} + \partial_q\cdot\left(\left|\psi\right|^2 v\right) = 0$)

and $\rho(q,t) = \left|\psi(q,t)\right|^2$ preserved in time (Born rule).

If $\rho = \left|\psi\right|^2$ at *t* = 0 then $\rho = \left|\psi\right|^2$ at *t* > 0

Consider the example of the two-slit experiment

$$\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m}$$



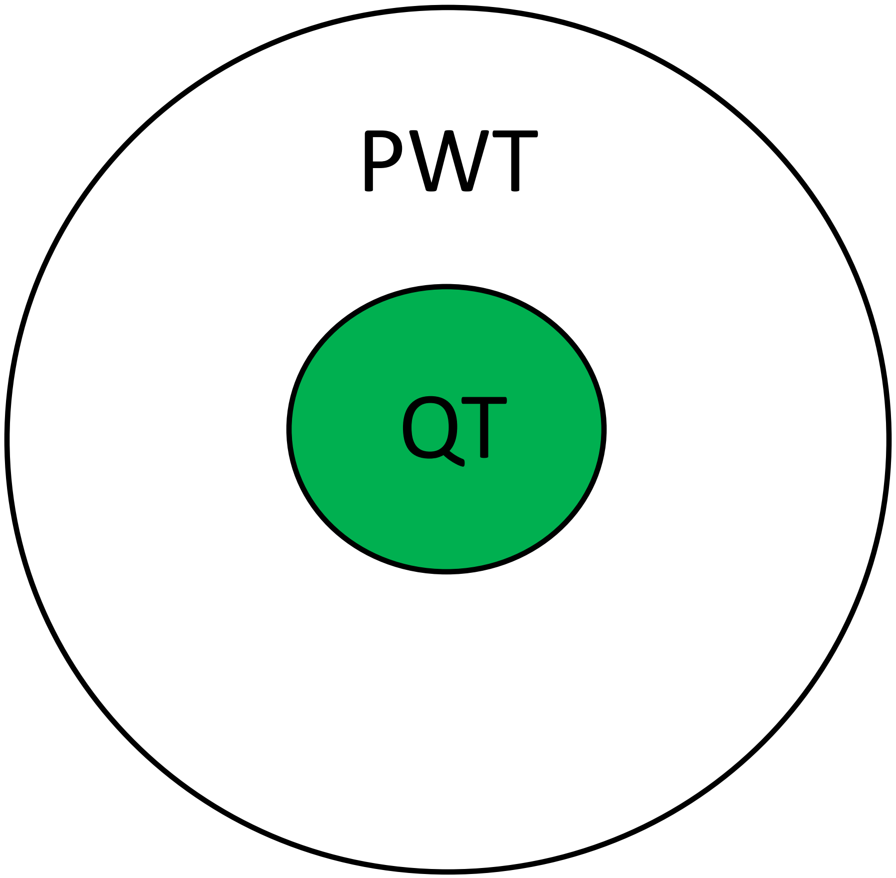
Quantum Equilibrium $\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2$

same statistical predictions as quantum mechanics

Quantum Nonequilibrium $\rho(\mathbf{x},t) \neq |\psi(\mathbf{x},t)|^2$

statistical *deviations* from quantum mechanics

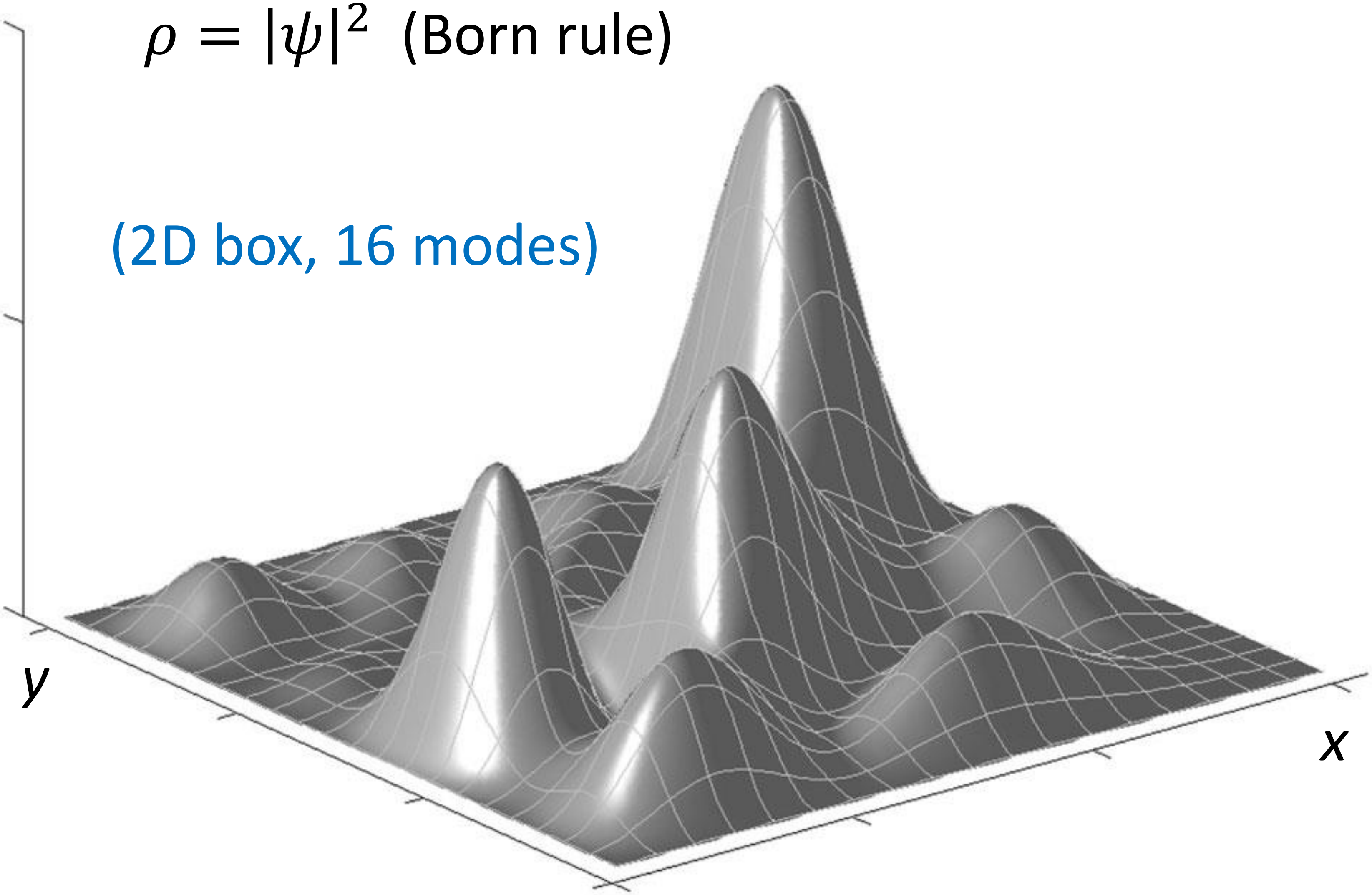
Quantum theory = special case of a wider physics



BUT: *experimentally* we always find the "quantum equilibrium" distribution:

$\rho = |\psi|^2$ (Born rule)

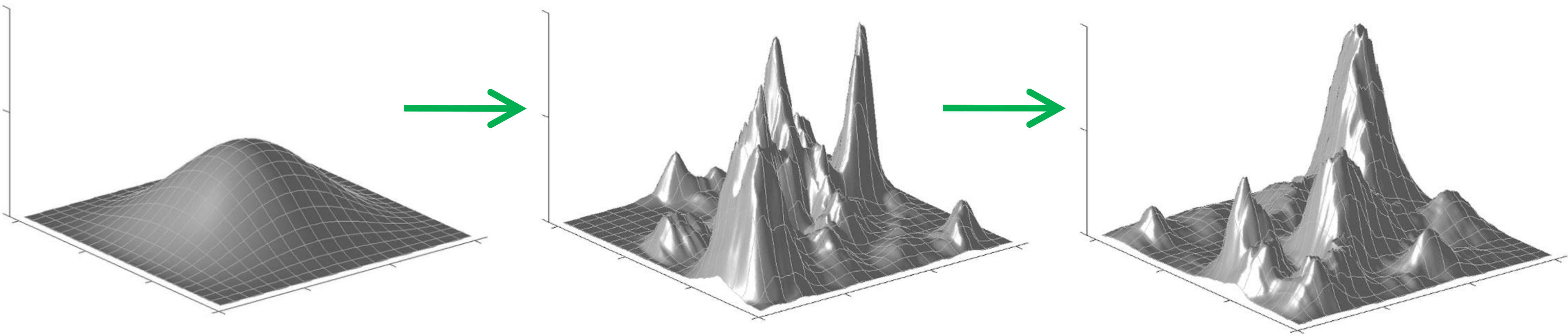
(2D box, 16 modes)



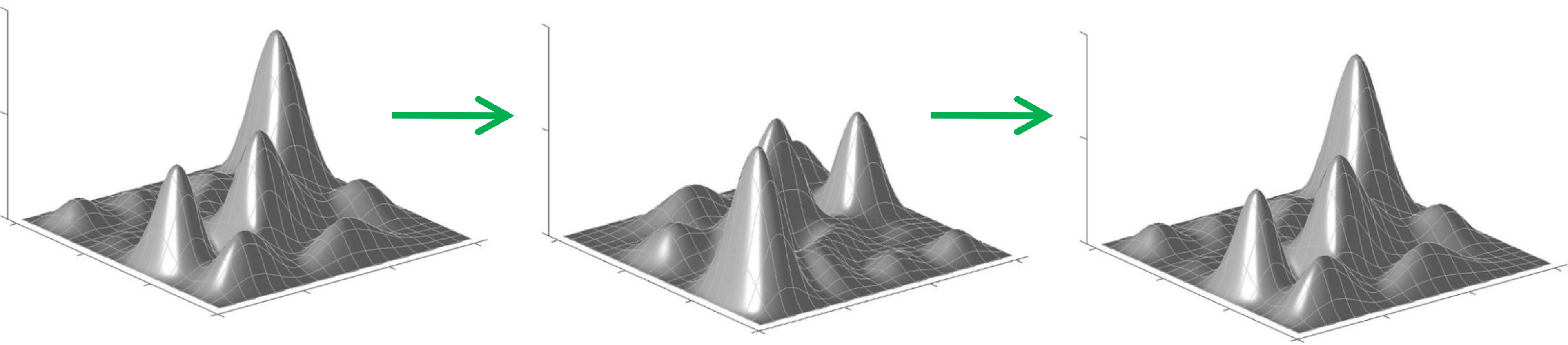
Why?

Quantum relaxation (cf. thermal relaxation)

Non-equilibrium ($\rho \neq |\psi|^2$) relaxes to equilibrium

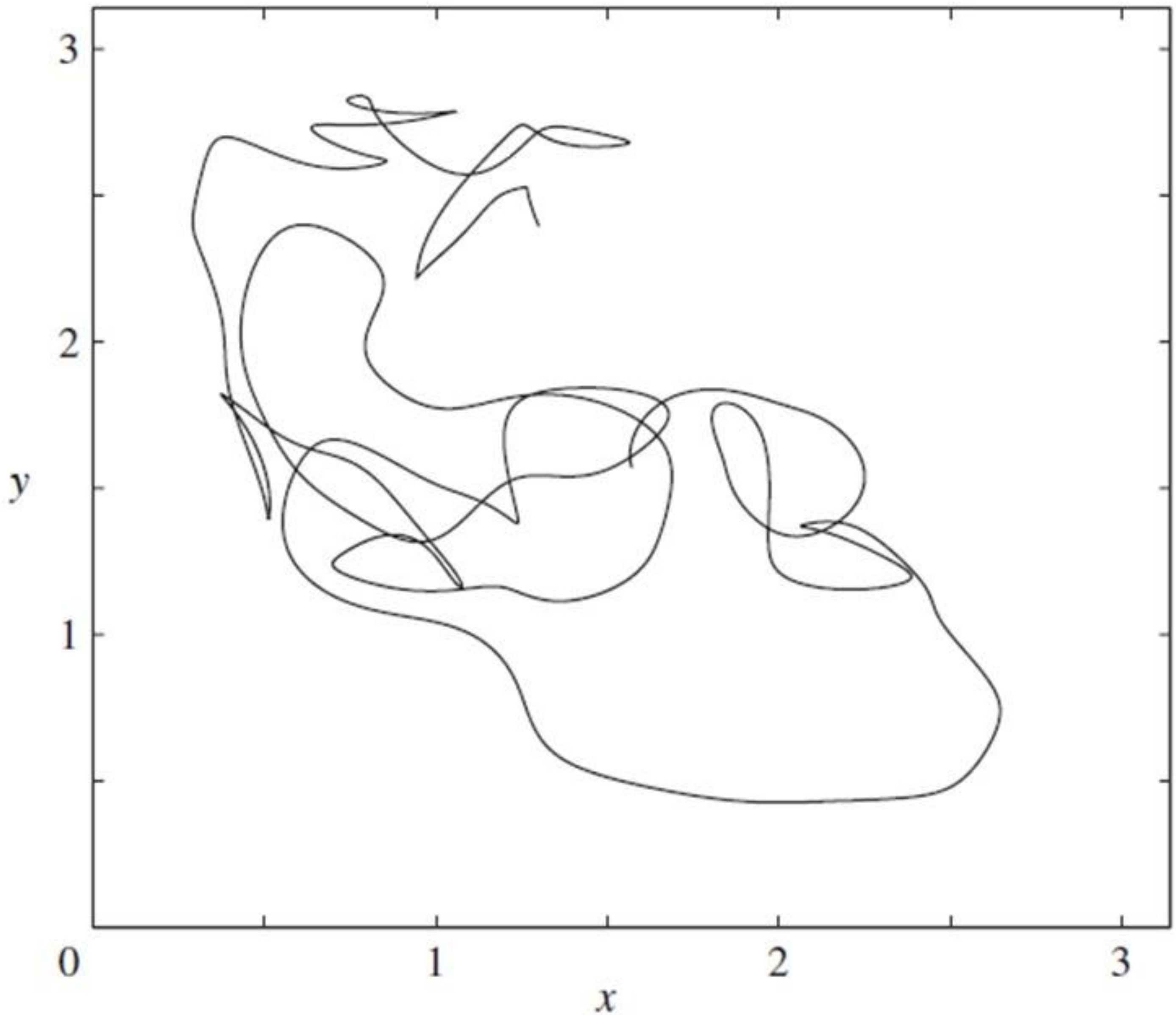


Compare with time evolution of equilibrium $\rho = |\psi|^2$



(Valentini and Westman, Proc. Roy. Soc. A 2005)

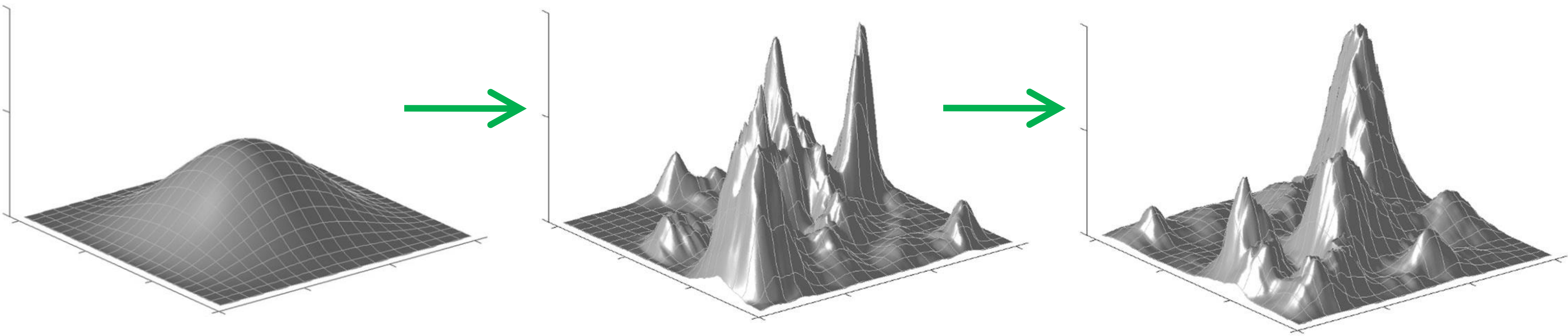
Superposed energies give rapidly-varying velocity fields



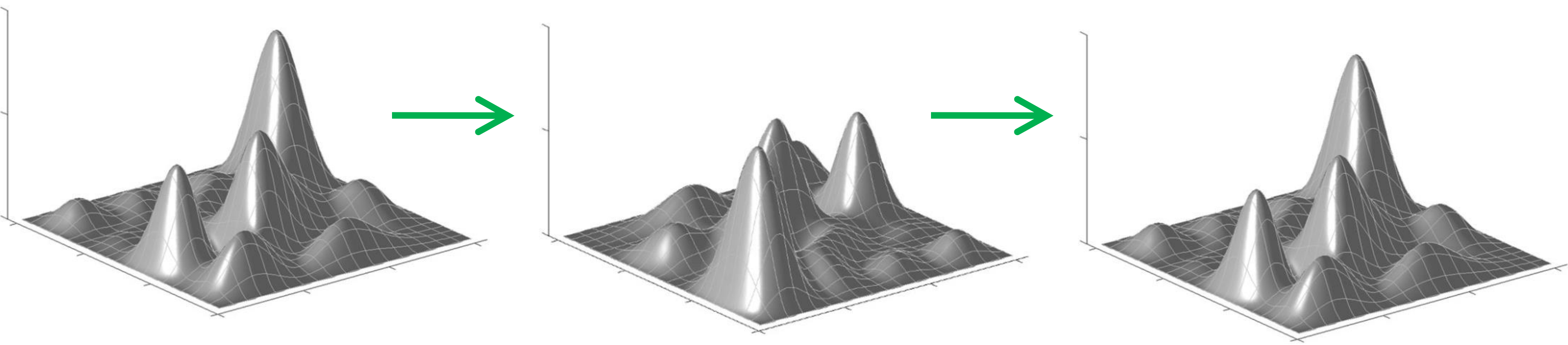
Trajectories are erratic and tend to explore the region

Quantum relaxation (cf. thermal relaxation)

Non-equilibrium ($\rho \neq |\psi|^2$) relaxes to equilibrium



Compare with time evolution of equilibrium $\rho = |\psi|^2$



(Valentini and Westman, Proc. Roy. Soc. A 2005)

Quantify relaxation with a coarse-grained *H*-function

$$\bar{H} = \int \mathrm{d}q\, \bar{\rho} \ln(\bar{\rho}/\overline{|\psi|^2}),$$

(minus the relative entropy)

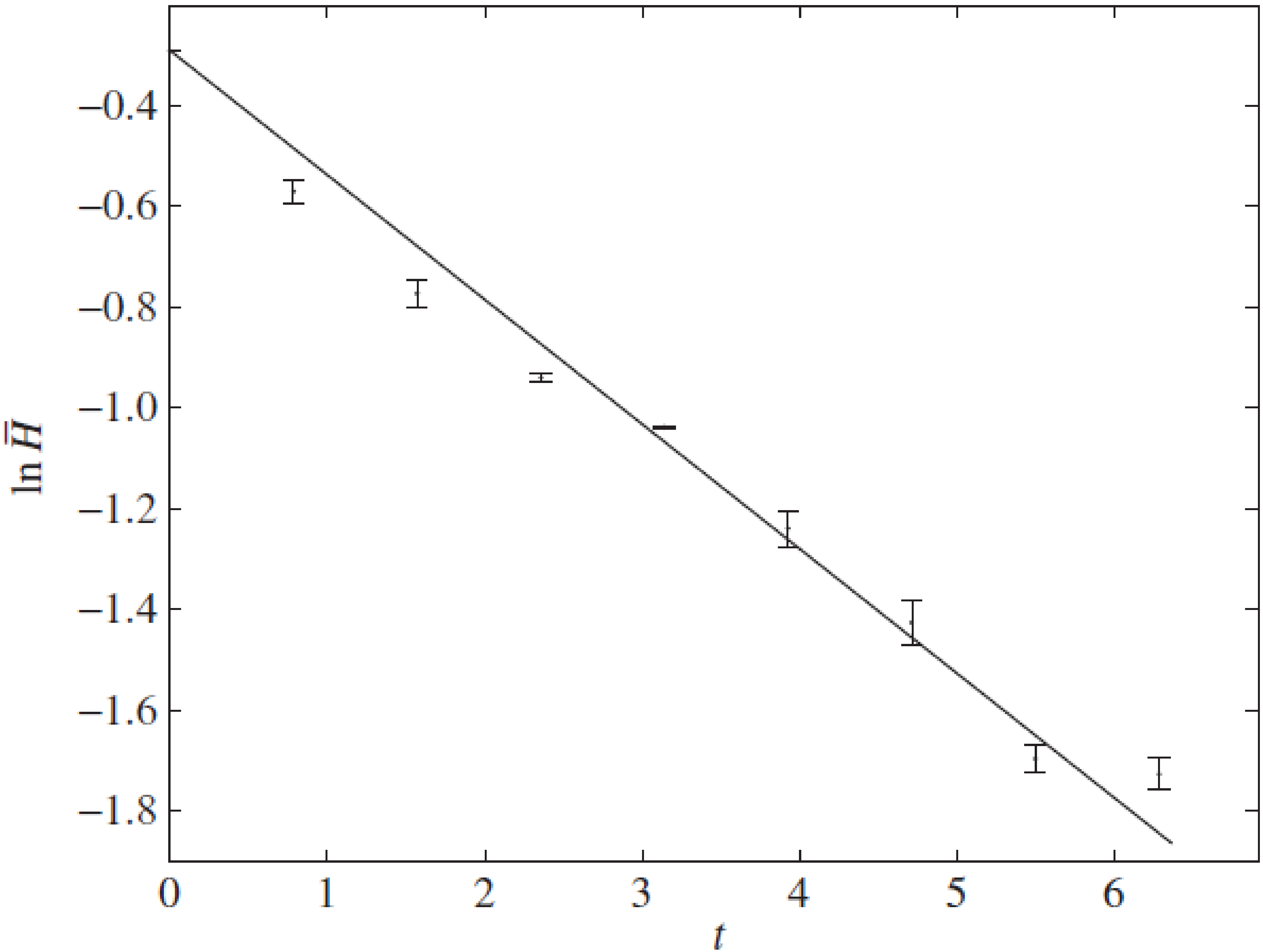
Obeys the *H*-theorem (Valentini 1991, 1992)

$$\bar{H}(t) \leqslant \bar{H}(0)$$

(cf. classical analogue)

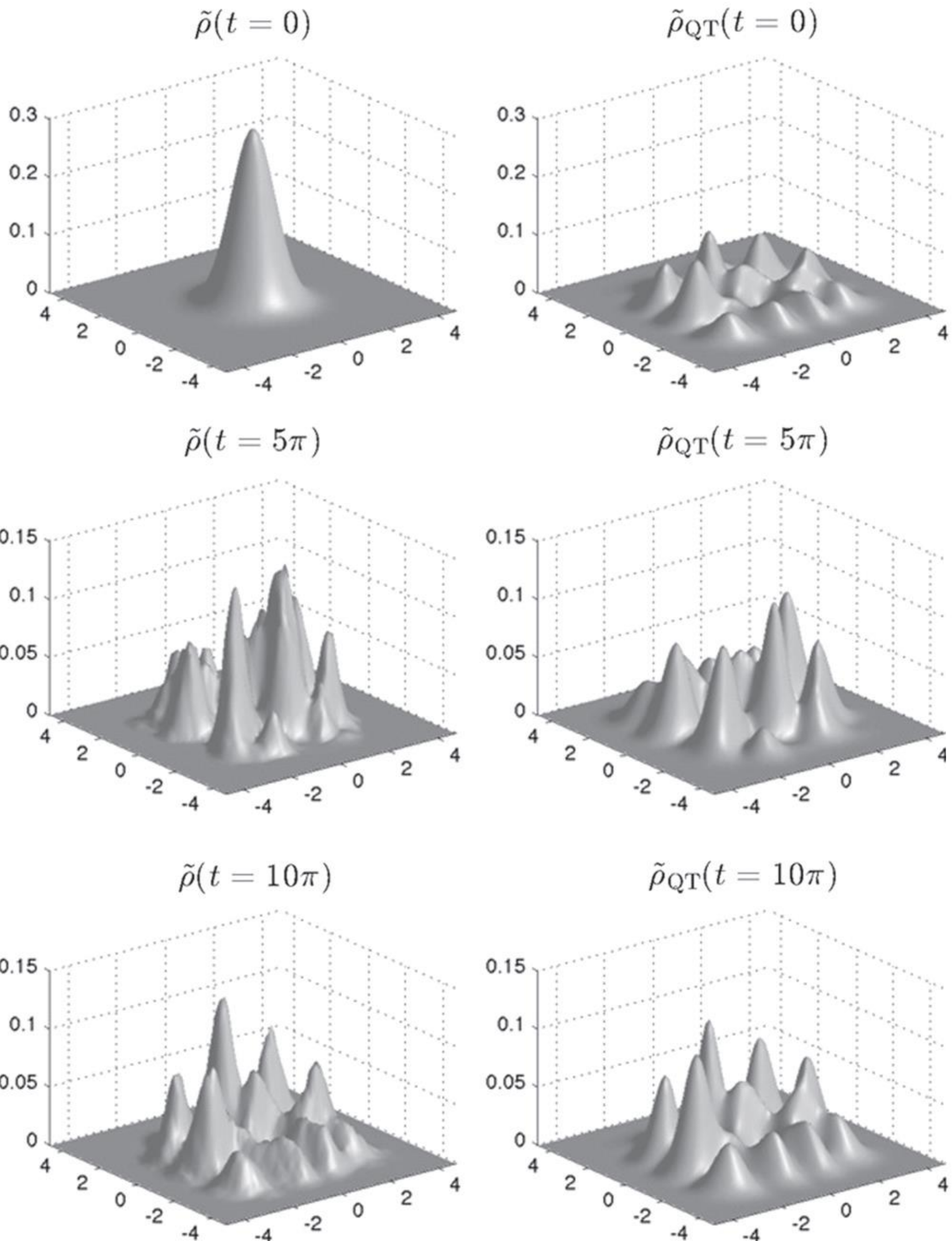
assuming no initial fine-grained structure in ρ and $|\psi|^2$

Simulations show *exponential decay* of *H*-function

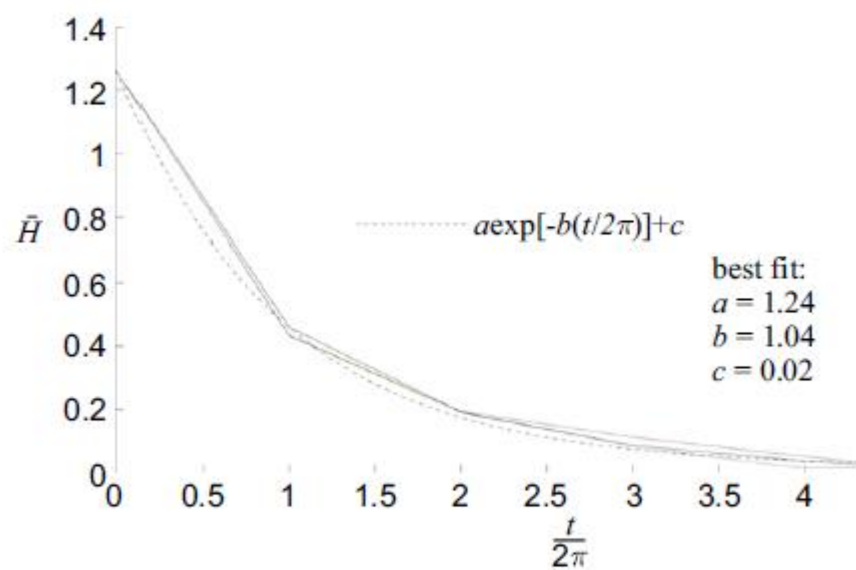


(Valentini and Westman, Proc. Roy. Soc. A 2005)

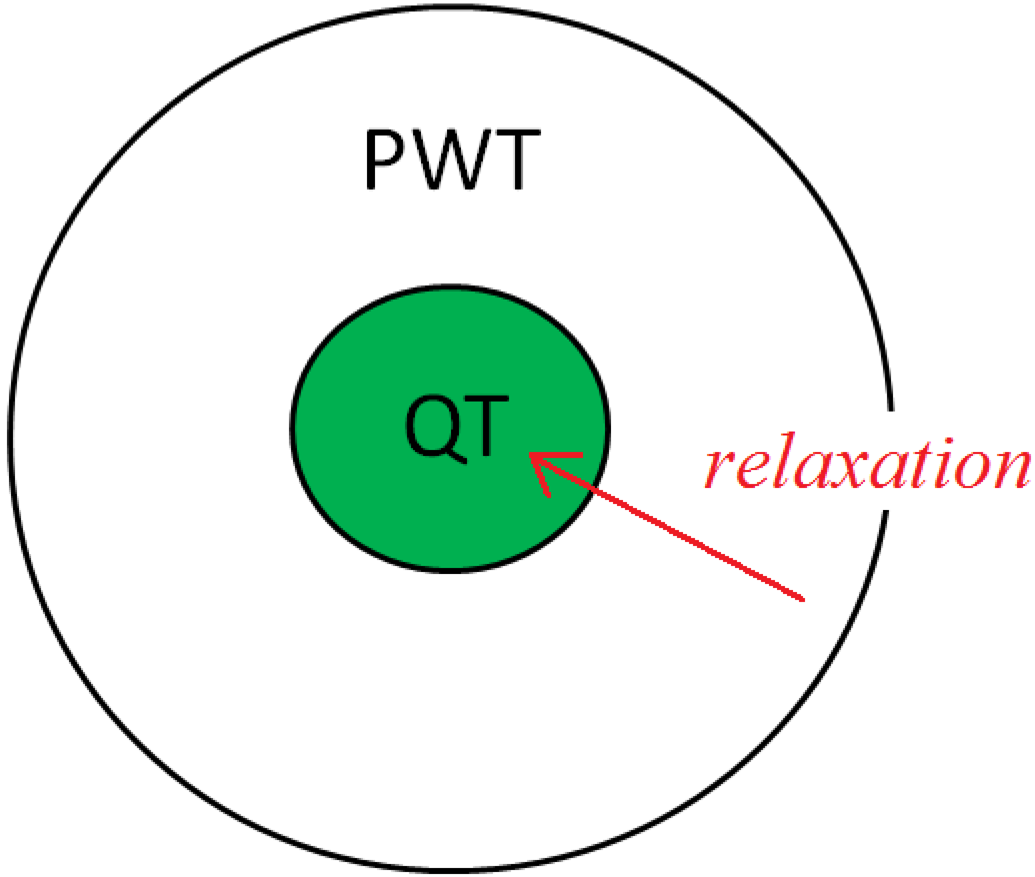
Confirmed and extended by many independent simulations



2D oscillator, 25 modes in superposition (Abraham, Colin and Valentini, J. Phys. A 2014)



Quantum theory = special case of a wider physics



Quantum Theory is the effective description of a special state of statistical equilibrium

We are in that state now because of past relaxation

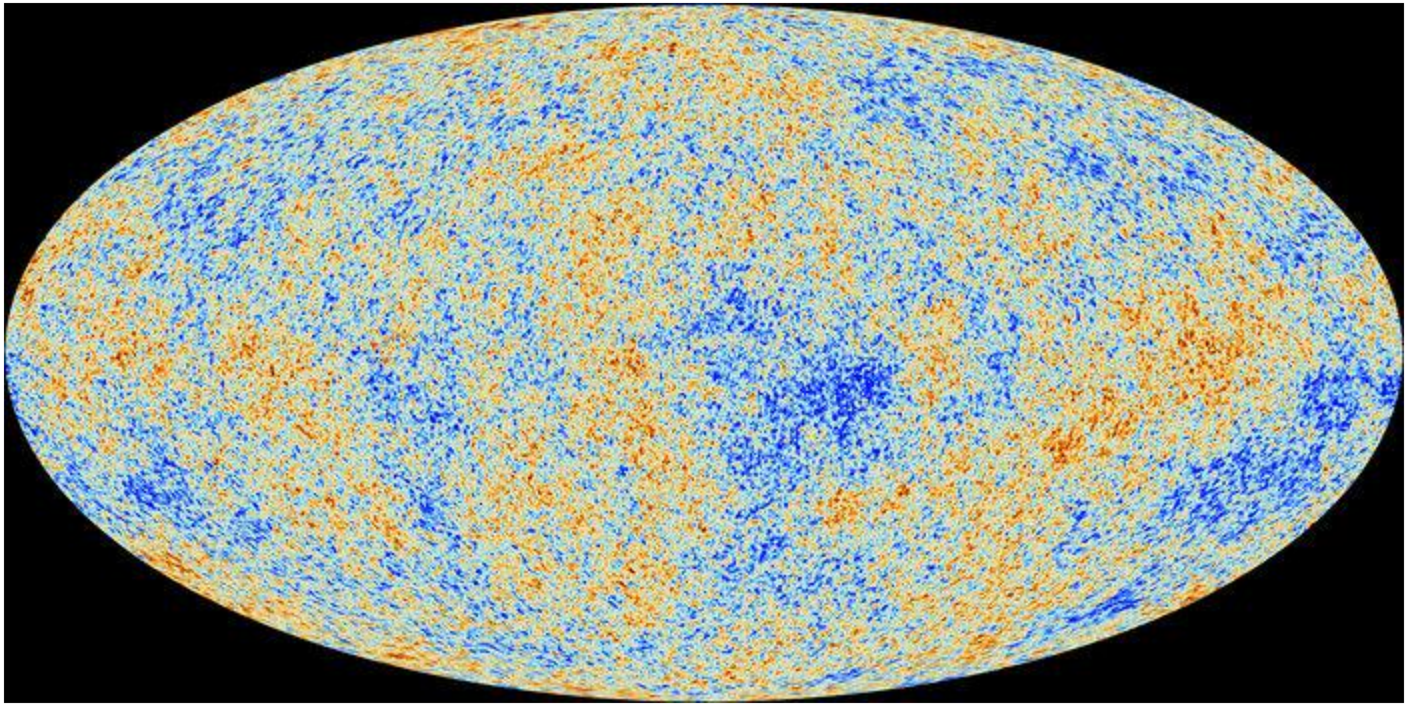
When did quantum relaxation happen?

Presumably, a long time ago, in the very early universe, soon after the big bang

If so, we can expect that quantum theory will break down close to the big bang.

This can be tested using inflationary cosmology

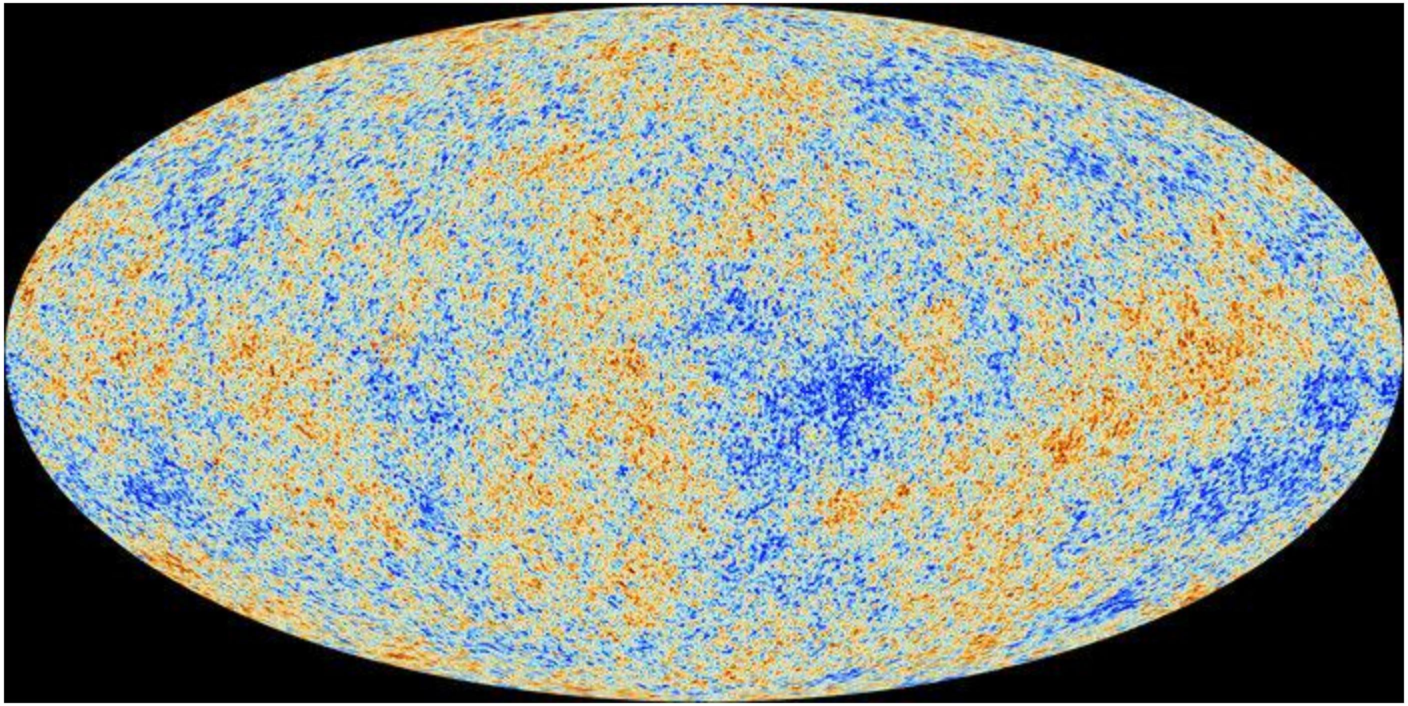
CMB anisotropies are ultimately generated by early quantum noise (inflationary vacuum)



We are testing the Born rule in the early universe

This can be tested using inflationary cosmology

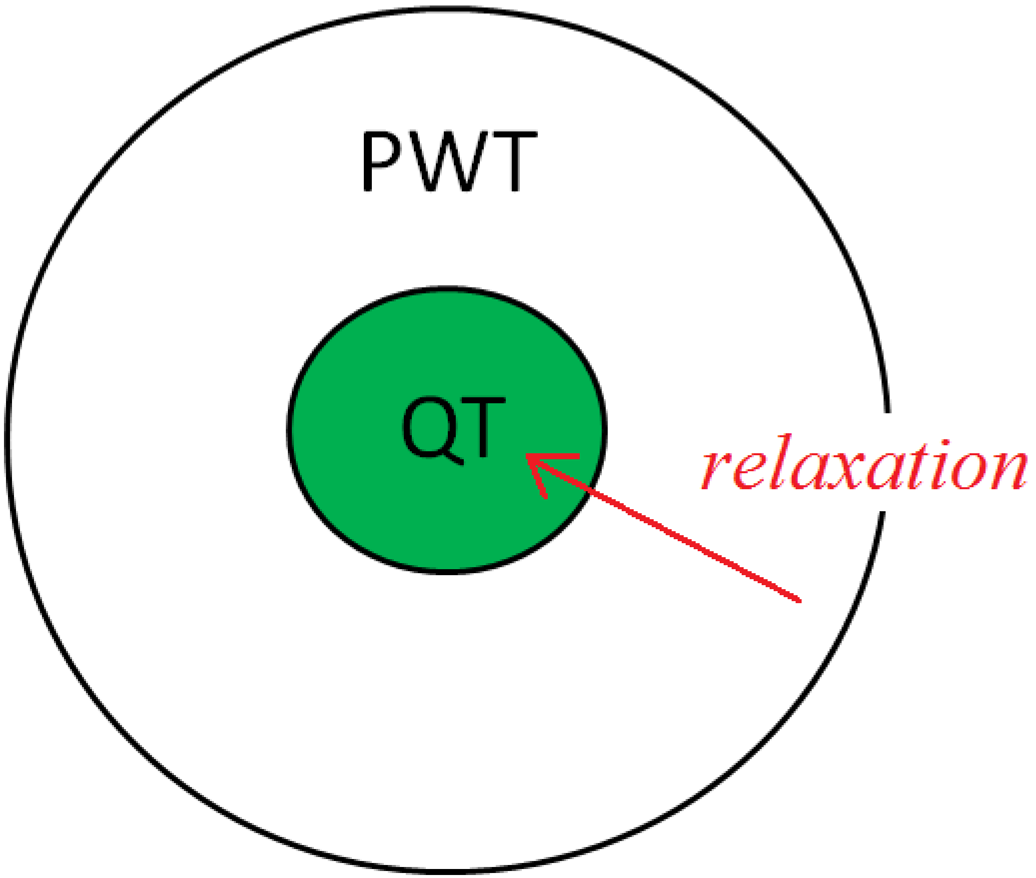
CMB anisotropies are ultimately generated by early quantum noise (inflationary vacuum)



Signatures of quantum relaxation in the CMB?

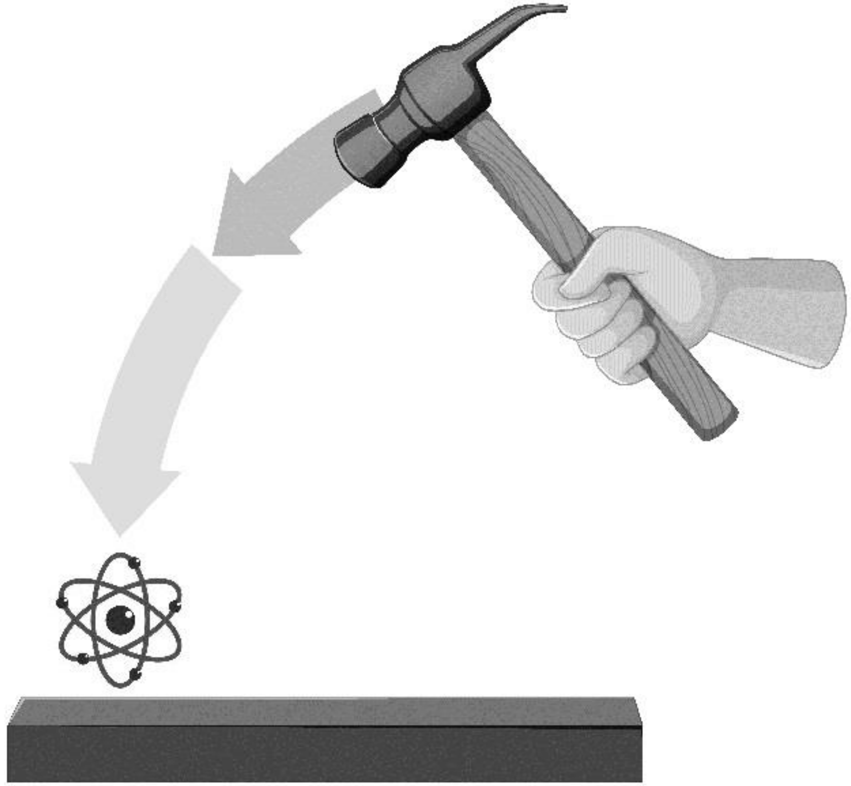
(AV, PRD 2010; Colin and AV, PRD 2013; Colin and AV, PRD 2015; Vitenti, Peter and AV, PRD 2019)

Trapped in quantum equilibrium



Is there a way to escape?

Can we *create* nonequilibrium (from equilibrium)?



Apparently not: $\rho = |\psi|^2$ is preserved by the dynamics.

At least in non-gravitational physics…

Quantum gravity may change the game

Pilot-wave quantum gravity

(Vink 1992, Horiguchi 1994, Shtanov 1996, Pinto-Neto 2021)

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0 \qquad \Psi = \Psi[g_{ij}]$$

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i} \qquad \Psi = |\Psi|\, e^{iS}$$

(canonical momentum = phase gradient)

-- dynamics of a single system

-- how can we construct the theory of a quantum equilibrium ensemble?

(where $\left|\Psi[g_{ij}]\right|^2$ is not normalisable)

Theory for a *general* ensemble

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

(dynamics of a single system)

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i}$$

arbitrary ensemble with distribution P will evolve via

$$\frac{\partial P}{\partial t} + \int d^3x\ \frac{\delta}{\delta g_{ij}}\left(P\frac{\partial g_{ij}}{\partial t}\right) = 0$$

(by construction)

We claim: this theory *has no equilibrium state*

Illustrate with model of quantum cosmology ($\Psi(a,\phi)$)

$$\frac{1}{m_{\mathrm{P}}^2}\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial\Psi}{\partial a}\right)-\frac{1}{a^2}\frac{\partial^2\Psi}{\partial\phi^2}+2a^4\mathcal{V}(\phi)\Psi=0$$

Polar decomp. $\Psi=|\Psi|\,e^{iS}$ implies 'pseudo-continuity eqn.'

$$\frac{\partial}{\partial a}\left(a^2\,|\Psi|^2\,\dot{a}\right)+\frac{\partial}{\partial\phi}\left(a^2\,|\Psi|^2\,\dot{\phi}\right)=0$$

Non-normalisable 'density' $a^2\,|\Psi|^2$ and de Broglie velocities

$$\dot{a}=-\frac{1}{m_{\mathrm{P}}^2}\frac{1}{a}\frac{\partial S}{\partial a}\ ,\quad \dot{\phi}=\frac{1}{a^3}\frac{\partial S}{\partial\phi}$$

(can. mom. = phase gradient)

General probability density $P(a,\phi,t)$ evolves by

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial a}(P\dot{a})+\frac{\partial}{\partial\phi}\left(P\dot{\phi}\right)=0$$

Beware of naïve Schrödinger interpretation

(Dürr and Struyve 2020; also Vink 1992)

Since P and $a^2 |\Psi|^2$ obey the *same* equations

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial a}(P\dot{a}) + \frac{\partial}{\partial \phi}\left(P\dot{\phi}\right) = 0$$

$$\frac{\partial}{\partial a}\left(a^2 |\Psi|^2 \dot{a}\right) + \frac{\partial}{\partial \phi}\left(a^2 |\Psi|^2 \dot{\phi}\right) = 0$$ (no *t* in Ψ)

might think that have a state of 'quantum equilibrium'

$$P = a^2 |\Psi|^2$$

BUT: left-hand side is normalisable (by construction),

right-hand side is not

No Born rule in quantum gravity

We *must have* $P \neq a^2 \left|\Psi\right|^2$ *always*

Formally: coarse-grained *H*-function

$$\bar{H}(t) = \int\int da d\phi \ \bar{P} \ln(\bar{P}/\overline{a^2 \left|\Psi\right|^2})$$

has *no lower bound* (usually bounded below by zero).

'Equilibrium' can never be reached

Cf. recent numerical simulations (Kandhadai and AV)

Quantum gravity is in perpetual nonequilibrium

Implications

Born rule emerges only in semiclassical regime (quantum relaxation on a classical spacetime)

time-dependent Schrödinger equation $i\frac{\partial\psi}{\partial t} = \hat{H}\psi$

$|\psi|^2$ can be a probability (after relaxation)

How? …

Semiclassical approximation

Solution to Wheeler-DeWitt equation

$$\Psi[g_{ij}, \phi] \approx \Psi_{\mathrm{WKB}}[g_{ij}]\psi[\phi, g_{ij}]$$

(approximately classical spacetime background)

Effective time-dep. wave fn. for the quantum field:

$$\psi_{\mathrm{eff}}[\phi, t] = \psi[\phi, g_{ij}(t)]$$

$$\frac{\partial \psi_{\mathrm{eff}}}{\partial t} = \int d^3x \, \frac{\delta \psi_{\mathrm{eff}}}{\delta g_{ij}} \dot{g}_{ij}$$

Effective Schrödinger equation (e.g. Kiefer and Singh 1991)

$$i \frac{\partial \psi[\phi, t]}{\partial t} = \hat{H}_{\mathrm{eff}} \psi[\phi, t]$$

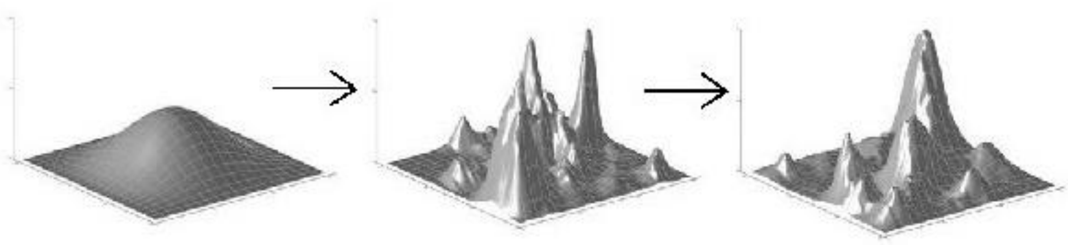
(with a normalisable $|\psi|^2$)

Semiclassical quantum relaxation

Once we have an effective Schrödinger equation

$$i\frac{\partial\psi[\phi,t]}{\partial t}=\hat{H}_{\rm eff}\psi[\phi,t]$$

conventional quantum relaxation can take place.



Semiclassical universe begins out of equilibrium,

relaxes to the Born rule only afterwards (cf. AV 1991)

Are we again stuck forever in quantum equilibrium?

Perhaps not …

Gravity changes the game?

Tiny quantum-gravitational corrections to the Schrödinger equation can make the Born rule unstable ($\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$).

Higher order semiclassical expansion: find small *non-Hermitian terms* in the effective Hamiltonian.

Inconsistent with standard QM (breaks unitarity).

Consistent with pilot-wave theory: non-Hermitian terms generate a small instability of the Born rule.

Non-Hermitian corrections

Higher orders in the semiclassical expansion, find

$$i\frac{\partial\psi}{\partial t} = \left(\hat{H}_\phi + \hat{H}_a + i\hat{H}_b\right)\psi$$

$\hat{H}_\phi$ is the field Hamiltonian

$\hat{H}_a$ is small and Hermitian

$i\hat{H}_b$ is small and non-Hermitian

(Kiefer and Singh 1991; Brizuela, Kiefer and Krämer 2016)

[Existence of non-Hermitian terms is controversial]

Explicitly (Kiefer and Singh 1991)

Semiclassical expansion:

$$\Psi = \exp i\left(\mu S_0 + S_1 + \mu^{-1} S_2 + \cdots\right) \qquad \mu = c^2/32\pi G$$

Corrected Schrödinger equation:

$$i\frac{\partial\psi^{(1)}}{\partial t} = \int d^3x\, N\left(\hat{\mathcal{H}}_\phi + \hat{\mathcal{H}}_a + i\hat{\mathcal{H}}_b\right)\psi^{(1)}$$

$$\hat{\mathcal{H}}_a = \frac{1}{8\mu}\frac{1}{\sqrt{g}R}\hat{\mathcal{H}}_\phi^2 \qquad \hat{\mathcal{H}}_b = \frac{1}{8\mu}\frac{\delta}{\delta\tau}\left(\frac{\hat{\mathcal{H}}_\phi}{\sqrt{g}R}\right)$$

$$\delta/\delta\tau = \dot{g}_{ij}\delta/\delta g_{ij}$$

Non-Hermitian pilot-wave theory

Semiclassical expansion of Wheeler-DeWitt equation:

$$i\frac{\partial\psi}{\partial t} = (\hat{H}_1 + i\hat{H}_2)\psi$$

Semiclassical expansion of de Broglie equation:

$$v = \frac{j_1}{|\psi|^2}$$ (standard current j_1 associated with $\hat{H}_1$)

Thus $\frac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0$ *but* $\frac{\partial\,|\psi|^2}{\partial t} + \partial_q \cdot (|\psi|^2\, v) = s$

with effective 'source' $s = 2\,\mathrm{Re}\left(\psi^* \hat{H}_2 \psi\right)$

Mismatch between the two continuity equations

$\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$ on a timescale $\tau_{\mathrm{noneq}} \sim \frac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$

Could such effects be observed?

Cosmology and the CMB

Kiefer et al. (2016): for scalar perturbations on de Sitter space, find a corrected Schrödinger equation

$$i\frac{\partial \psi_{\mathbf{k}}^{(1)}}{\partial t} = \hat{H}_{\mathbf{k}}\psi_{\mathbf{k}}^{(1)} - \frac{\bar{k}^3}{2m_{\mathrm{P}}^2 H^2}\frac{1}{\psi_{\mathbf{k}}^{(0)}}\left[\frac{1}{a^3}(\hat{H}_{\mathbf{k}})^2\psi_{\mathbf{k}}^{(0)} + i\frac{\partial}{\partial t}\left(\frac{1}{a^3}\hat{H}_{\mathbf{k}}\right)\psi_{\mathbf{k}}^{(0)}\right]\psi_{\mathbf{k}}^{(1)}$$

Kiefer et al. consider only the Hermitian term.
Find tiny correction to power spectrum of order 10^{-10}
(far too small to see)

AV 2021: non-Hermitian correction generates quantum nonequilibrium during inflation, similar tiny correction to power spectrum (far too small to see)

Evaporating black holes

Field mode Hamiltonian $\hat{H}_{\mathbf{k}}$ has a non-Hermitian correction $i\hat{H}_2$ with

$$\hat{H}_2 \simeq -\frac{1}{12}\kappa \left(\frac{m_{\mathrm{P}}}{M}\right)^4 \hat{H}_{\mathbf{k}}$$

(Kiefer, Müller and Singh 1994).

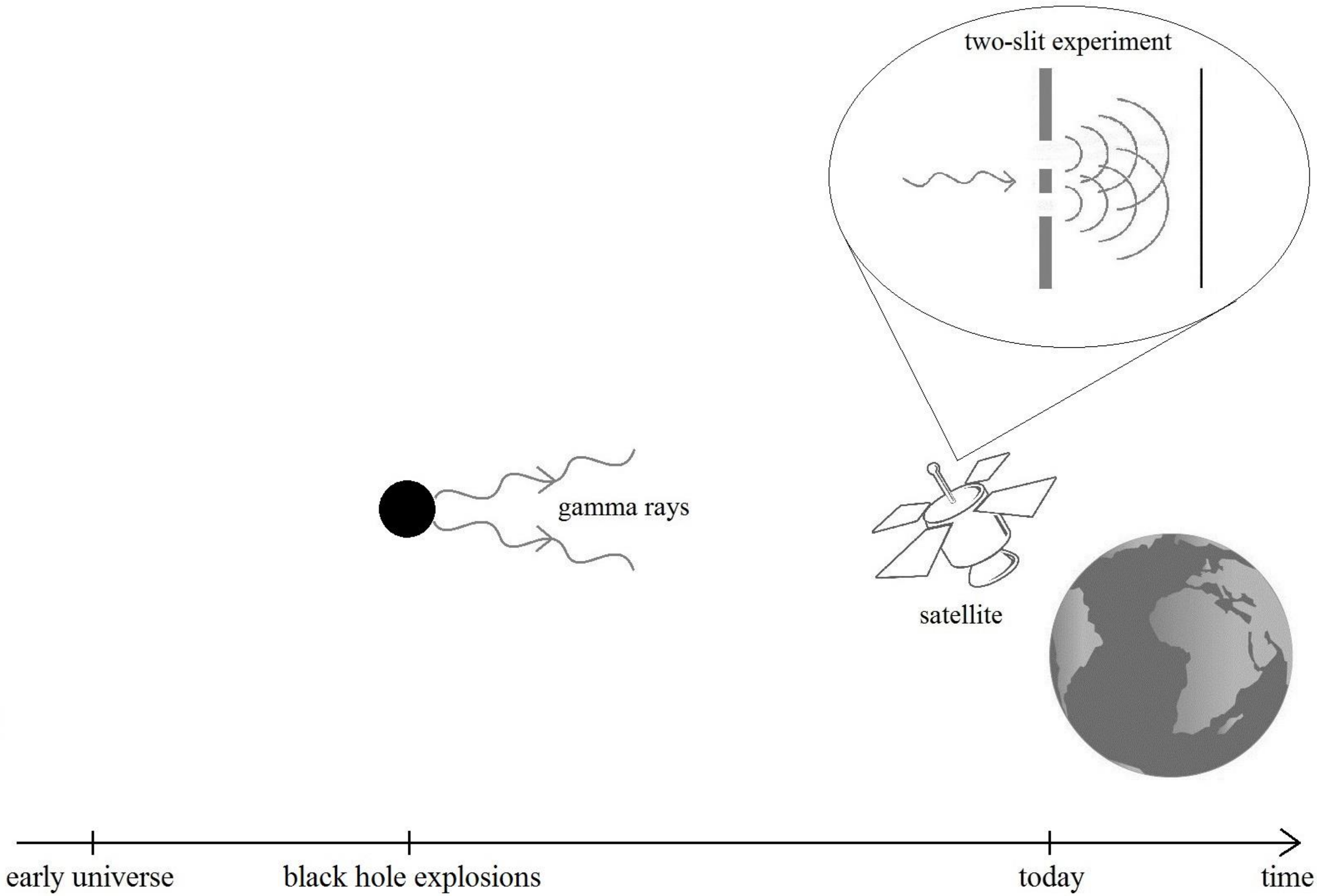
Significant when mass *M* approaches Planck mass.

$$\tau_{\mathrm{noneq}} \sim \frac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$$

$$\tau_{\mathrm{noneq}} \sim \frac{48\pi}{\kappa} t_{\mathrm{P}} \left(\frac{M}{m_{\mathrm{P}}}\right)^5$$

Final burst of Hawking radiation breaks the Born rule

Exploding primordial black holes



QUICK³ satellite mission (launch in 2025?) has an interferometer to test the Born rule in space

Conclusions

-- quantum gravity is fundamentally a nonequilibrium theory (no Born rule)

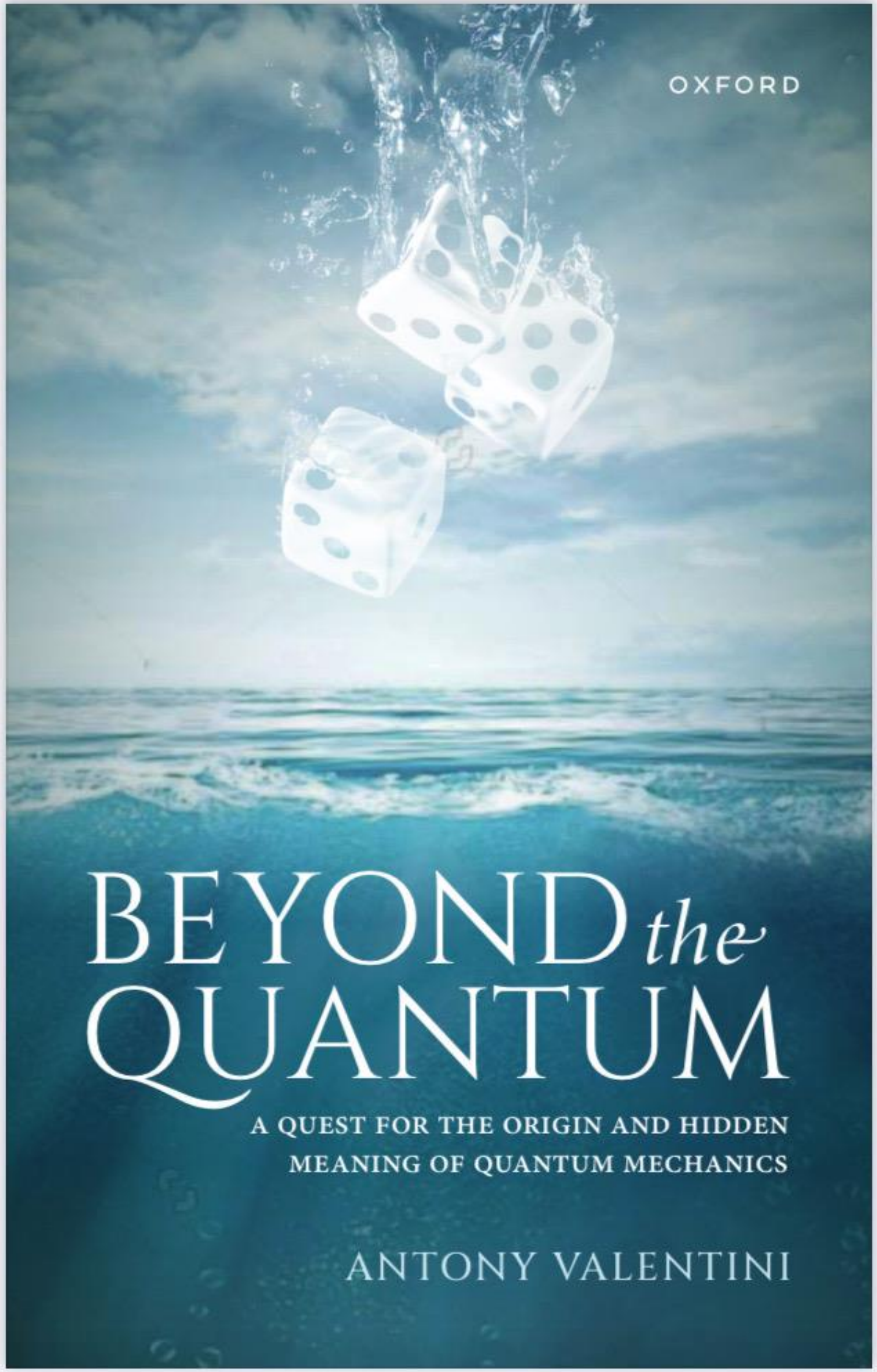
-- in the semiclassical regime we find relaxation to the usual Born rule

-- corrections to the semiclassical regime yield a small instability of the Born rule (though controversial)

-- latter effects are very small (except in final burst of an evaporating black hole)

Advertisement

Popular book (out in summer 2025)


OXFORD
BEYOND the QUANTUM
A QUEST FOR THE ORIGIN AND HIDDEN MEANING OF QUANTUM MECHANICS
ANTONY VALENTINI