CREATION OF HAWKING QUANTA FAR AWAY FROM A BLACK HOLE

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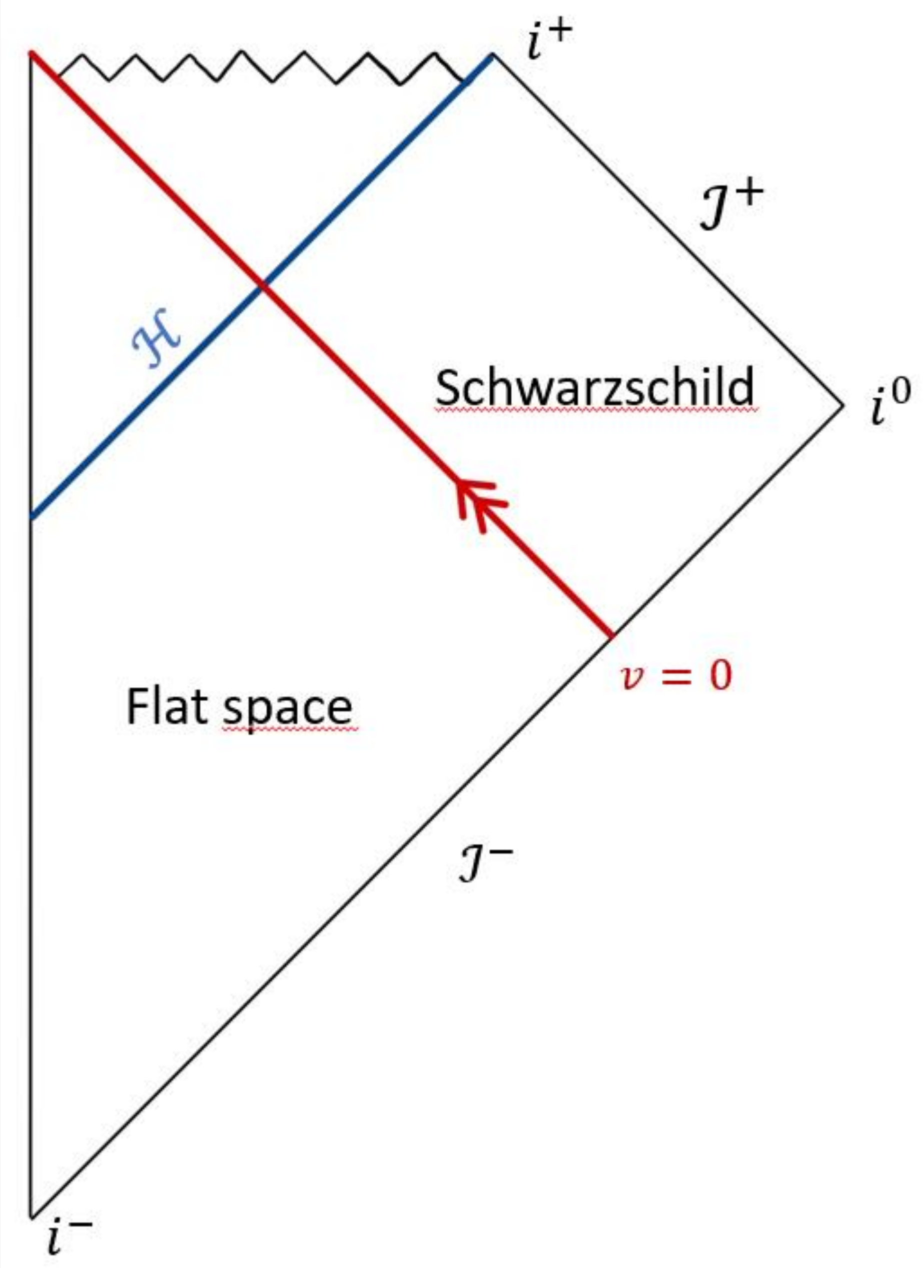
Vaidya Metric

Let us begin with a simple model of the process of black hole evaporation:

- The far past is an empty flat space
- At advanced time $v = 0$ a null shockwave with a total energy M is sent in. This infinitely thin collapsing shell of matter eventually forms a Schwarzschild black hole of mass M.
- Corresponding metric:

$$g = -\left(1 - \frac{2M}{r}\theta(v)\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

where $\theta(v)$ is the Heaviside step function.



Scalar perturbations in the Vaidya spacetime

Consider a real massless scalar field $\Phi(x)$ in the Vaidya spacetime. The Kelin-Gordon equation, $\nabla_\mu \nabla^\mu \Phi(x) = 0$, after the separation of variables:

$$\Phi(x) = \frac{1}{r}\sum_{l,m}\int_0^\infty \mathrm{d}\omega\, e^{-i\omega t}\, R_{\omega l}(r_*) Y_{lm}(\theta,\varphi)\,,$$

reduces to

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} + V(r_*)\right) R_{\omega l} = \omega^2 R_{\omega l},$$

where $t = v - r_*$ and r_* is the tortoise coordinate, $\mathrm{d}r_* = \frac{dr}{1-2M/r}$ in the Schwarzschild region $(v > 0)$, and $r_* = r$ in the flat space region $(v < 0)$. Effective potential:

$$V(r_*) = \left(1 - \frac{2M}{r}\theta(v)\right)\left[\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\theta(v)\right].$$

Scalar perturbations in the Vaidya spacetime

- Positive-frequency modes on $\mathcal{J}^-$: $p_{\omega lm}(x) \xrightarrow{x\to\mathcal{J}^-} \frac{1}{\sqrt{4\pi\omega}} \frac{e^{-i\omega(t+r)}}{r} Y_{lm}(\theta,\varphi)$

- Positive-frequency modes on $\mathcal{J}^+$: $h_{\omega lm}(x) \xrightarrow{x\to\mathcal{J}^+} \frac{1}{\sqrt{4\pi\omega}} \frac{e^{-i\omega(t-r)}}{r} Y_{lm}(\theta,\varphi)$

- We can express the field $\Phi(x)$ as a sum of these modes:

$$\Phi(x) = \int_0^\infty \mathrm{d}\omega \sum_{l,m} \left(A_{\omega lm} p_{\omega lm}(x) + A^\dagger_{\omega lm} p^*_{\omega lm}(x)\right)$$
$$= \int_0^\infty \mathrm{d}\omega \sum_{l,m} \left(B_{\omega lm} h_{\omega lm}(x) + B^\dagger_{\omega lm} h^*_{\omega lm}(x)\right) + \begin{pmatrix}\text{Part supported}\\ \text{in the BH interior}\end{pmatrix}$$

- Upon quantization, $A_{\omega lm}, B_{\omega lm}$ are anihilation operators, which allow us to formulate a definition of ”particles”.

Review of the Hawking's argument

Assume that there are no scalar particles on $\mathcal{J}^-$, i.e.:

$$|\text{state of the system}\rangle = |0\rangle, \qquad \text{s.t.} \qquad A_{\omega lm}|0\rangle = 0 \ \ \forall\, \omega, l, m.$$

The goal is to find the Bogoliubov transformation between positive-frequency modes on $\mathcal{J}^-$ and $\mathcal{J}^+$:

$$h_\omega = \int_0^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'} p_{\omega'} + \beta_{\omega\omega'} p^*_{\omega'}\right),$$

where we omitted the indices l, m since the angular momentum is conserved. The expectation value of the particle numer operator on $\mathcal{J}^+$ reads:

$$\langle N^+_\omega \rangle = \langle 0 | B^\dagger_\omega B_\omega | 0 \rangle = \int_0^\infty \mathrm{d}\omega' |\beta_{\omega\omega'}|^2 .$$

Review of the Hawking's argument

Focus on wavepackets localized near the horizon. Since there is an infinite blueshift near the horizon, such wavepackets can be relatively well localized also in the momentum space, so on their suport they can be approximated by h_ω.

By a general ray-tracing argument Hawking argued that the Bogoliubov coefficient satisfy

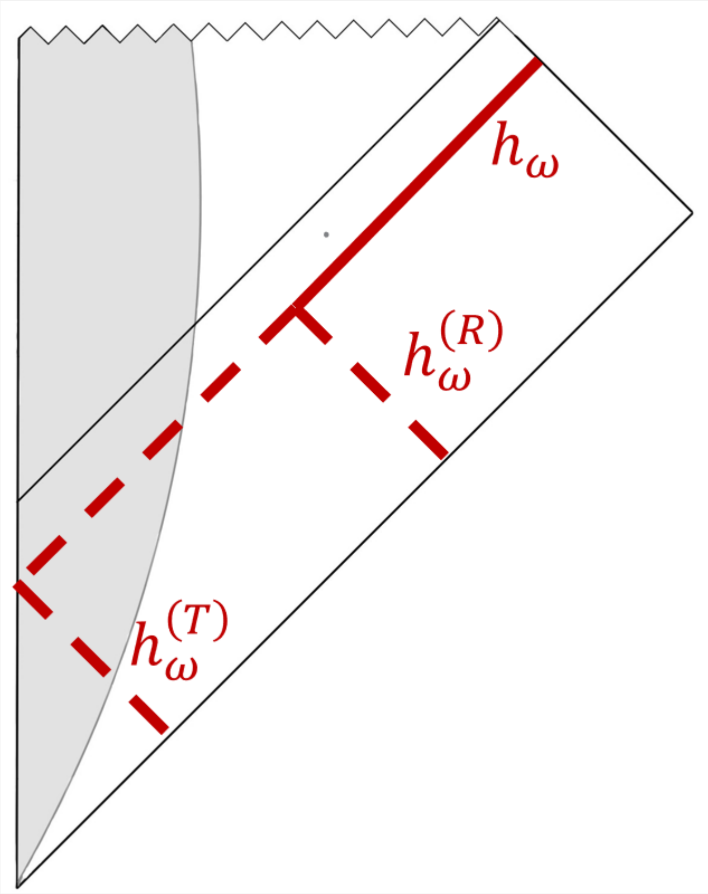
$$|\alpha_{\omega\omega'}| = e^{\pi\omega/\kappa}|\beta_{\omega\omega'}|,$$

where $\kappa = 1/4M$ is the Surface gravity of the Schwarzschild black hole. Then, completeness relation:

$$\sum_{\omega'}(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = 1,$$

implies:

$$\langle N_\omega^+\rangle = \sum_{\omega'}|\beta_{\omega\omega'}|^2 = \frac{1}{e^{2\pi\omega/k} - 1}$$



Hawking quanta far away from the horizon

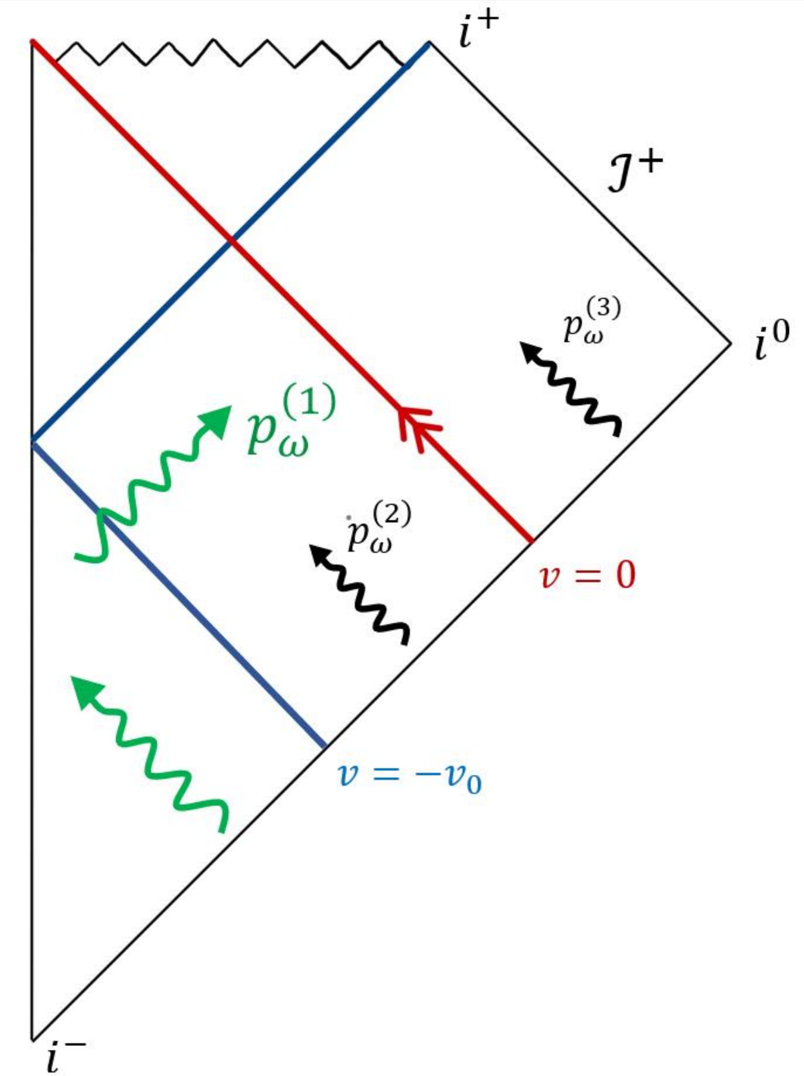
- For high frequencies $\omega \gg M^{-1}$ we can solve the radial equation without the near-horizon limit:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} + V(r_*)\right] R_{\omega l} = \omega^2 R_{\omega l} \quad \Rightarrow \quad R_{\omega l}(r) = e^{\pm i\omega r_*}\left(1 + \mathcal{O}(\omega^{-1})\right).$$

- In the case of Vaidya spacetime we can infer the relations between $h_{\omega lm}$ and $p_{\omega lm}$ from a continuity condition across the shockwave $\{v = 0\}$:

$$h_\omega\Big|_{v=0} = \int_0^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'}\, p_{\omega'} + \beta_{\omega\omega'}\, p^*_{\omega'}\right)\Big|_{v=0}$$
$$= \int_\delta^\infty \mathrm{d}\omega' \left(\alpha_{\omega\omega'}\, p^{(1)}_{\omega'} + \beta_{\omega\omega'}\, p^{(1)*}_{\omega'}\right)\Big|_{v=0} + \begin{pmatrix}\text{zero} - \text{frequency}\\ \text{part}\end{pmatrix},$$

where $p_\omega = p^{(1)}_\omega + p^{(2)}_\omega + p^{(3)}_\omega$, $p^{(1)}_\omega = p_\omega \cdot \theta(v_0 - v)$ and $\delta \to 0$.



Hawking quanta far away from the horizon

- We obtain:

$$\beta_{\omega\omega'} = \frac{1}{\pi}\left(\frac{\omega'}{\omega}\right)^{1/2} \int_{2M}^{\infty} \mathrm{d}r \, \left(\frac{r}{2M} - 1\right)^{4iM\omega} e^{2i(\omega+\omega')r}$$

and $\alpha_{\omega\omega'} = \beta_{\omega,-\omega'}$. The relation $|\alpha_{\omega\omega'}| = \exp\left(\frac{\pi\omega}{\kappa}\right)|\beta_{\omega\omega'}|$ is not satisfied!

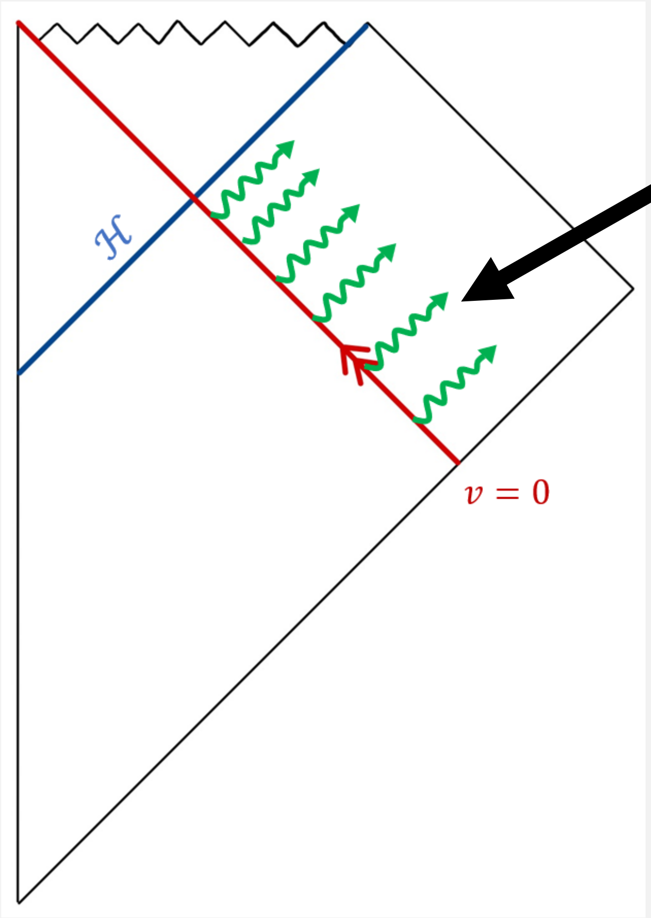
- Expectation value of the number operator is logarithmically divergent at UV, so we need to introduce a UV-cutoff $\Lambda \gg \omega$. Then:

$$\langle N_\omega^+ \rangle = \int_0^\Lambda \mathrm{d}\omega' \, |\beta_{\omega\omega'}|^2 = \frac{2M}{\pi} \frac{1}{e^{\beta_H \omega} - 1} [\log(\Lambda/\omega) + \mathcal{O}(\Lambda^0)],$$

where $\beta_H = 8\pi M$ is the inverse Hawking temperature.

- We have a non-thermal dependence $\propto \log(\Lambda/\omega)$.

Thermodynamic interpretation



N modes localized in position space. Together they form the wave h_ω.

Think of h_ω as N boxes with photon gas. Let δV be the volume of the box at position r_i. Assume that the box at position r_i has temperature

$$T_i = \frac{\hbar}{2\pi}\frac{M}{r^2}\left(1 - \frac{2M}{r}\right)^{1/2} = \frac{\hbar \alpha_i}{2\pi},$$

where α_i is the acceleration of an observer who sits at constant $r = r_i$.
Take constant density of states: $\delta\rho_i(\omega)\mathrm{d}\omega = C \cdot \delta r \cdot \mathrm{d}\omega$.

Free energy:

$$F = -\sum_i T_i \int_0^\Lambda \mathrm{d}\omega\, \delta\rho_i(\omega) \log\left(1 - e^{-\beta_i \hbar\omega}\right) = -\frac{\hbar C}{2\pi}\int_0^\infty \mathrm{d}\omega \log\left(\frac{\Lambda}{\omega}\right) \log\left(1 - e^{-\beta_H \hbar\omega}\right)$$

Thermodynamic interpretation

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Effectively, we have a thermodynamic system at temperature T_H, with density of states:

$$\rho(\omega)\mathrm{d}\omega \sim \log\left(\frac{\Lambda}{\omega}\right)\mathrm{d}\omega$$

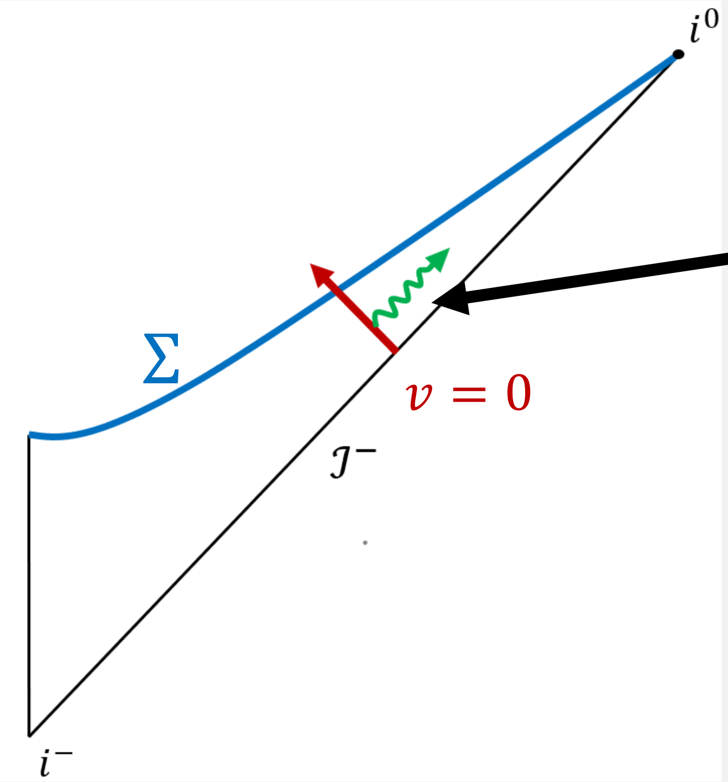
To derive this result we took the limit $N \to \infty$, with $\sum_i \delta r_i \to \int \mathrm{d}r$ and changed the order integration. Then the UV cutoff in momentum space translates into a position-space cutoff:

$$\int_{2M}^\infty \mathrm{d}r \int_0^\Lambda \mathrm{d}\omega \sim \int_0^\infty \mathrm{d}\omega \int_{2M+b}^\infty \mathrm{d}r \qquad \Rightarrow \qquad b = 2M\left(\frac{\omega}{\Lambda}\right)^2 + \mathcal{O}(\Lambda^{-3}).$$

If we fix the position-space UV-cutoff, then $\log\left(\frac{\Lambda}{\omega}\right) = \frac{1}{2}\log(2M/b)$ is independent of ω.

Entropy: $S = -\frac{\partial}{\partial T_H} F \propto \log\left(\frac{2M}{b}\right)$, resembles the formula for the entanglement entropy of a system in the 2d CFT.

Simple model of backreaction – is formation of a non-extremal black hole possible?



Dynamical part of the spacetime. A little bit of Hawking radiation is created here, at early times

Energy of the shockwave is decreased:

$$M \to M - \delta U$$

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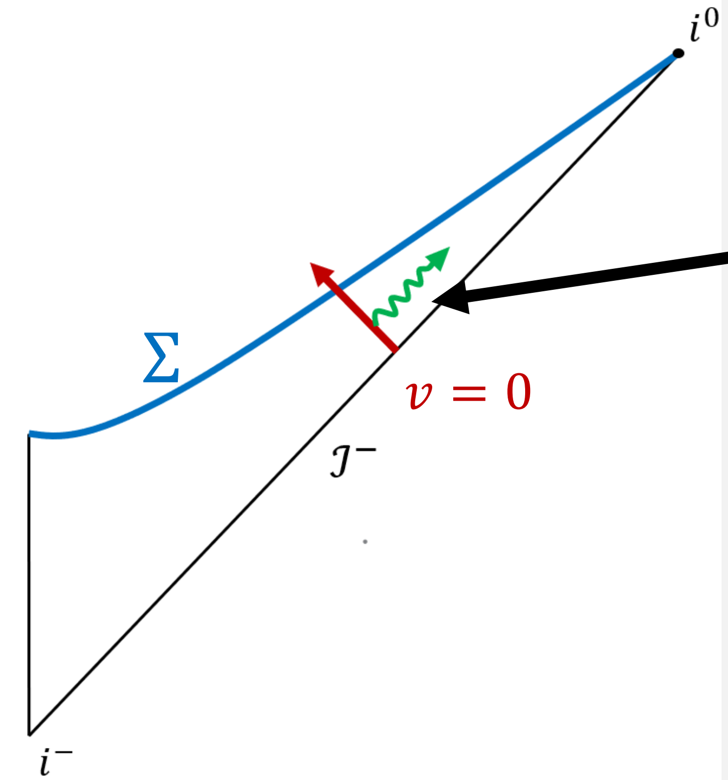

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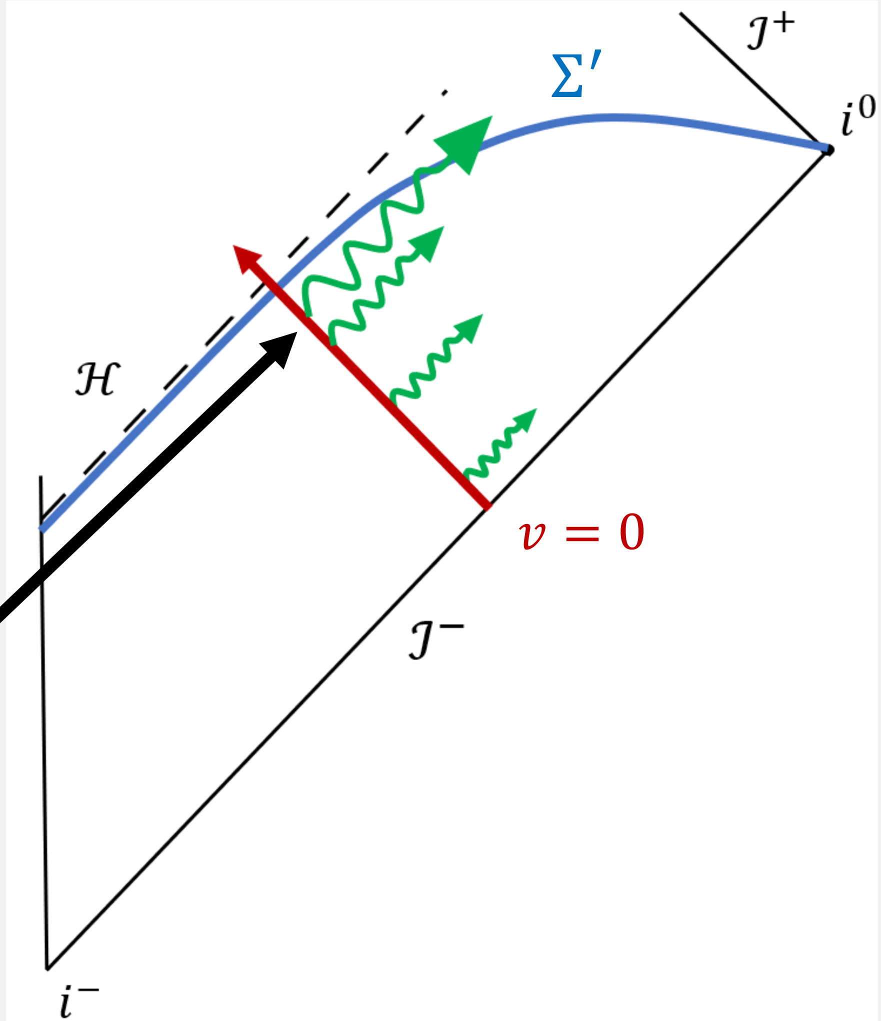

Large number of Hawking quanta is created at later times:

$$U_{Hawking} \sim M$$

Semi-classical horizon is shifted to $r'_H = 2(M - U_{Hawking})$

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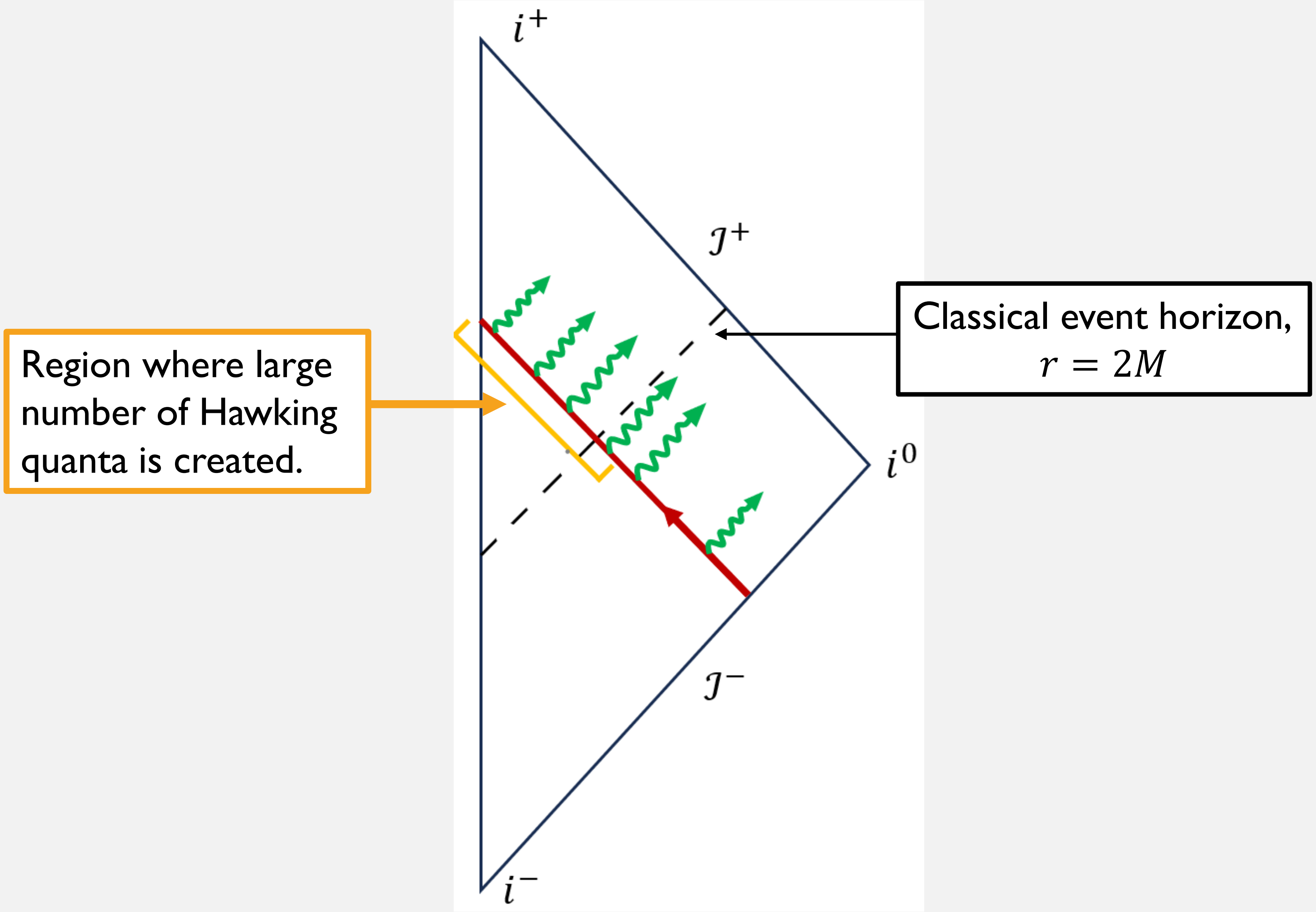
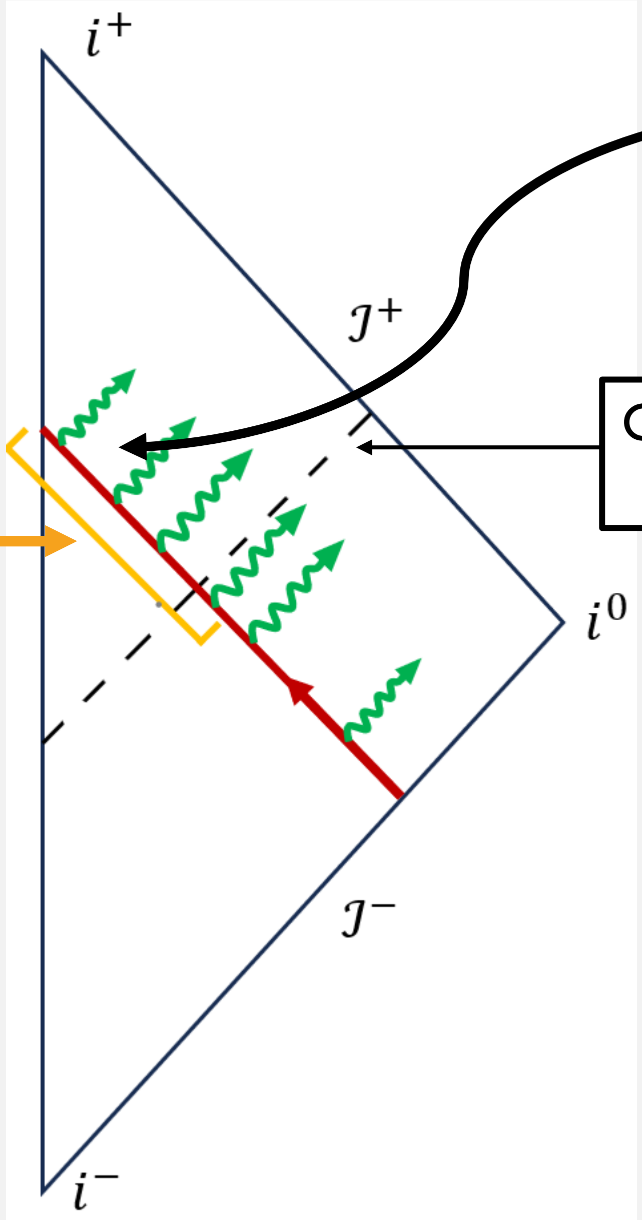


Simple model of backreaction – is formation of a non-extremal black hole possible?

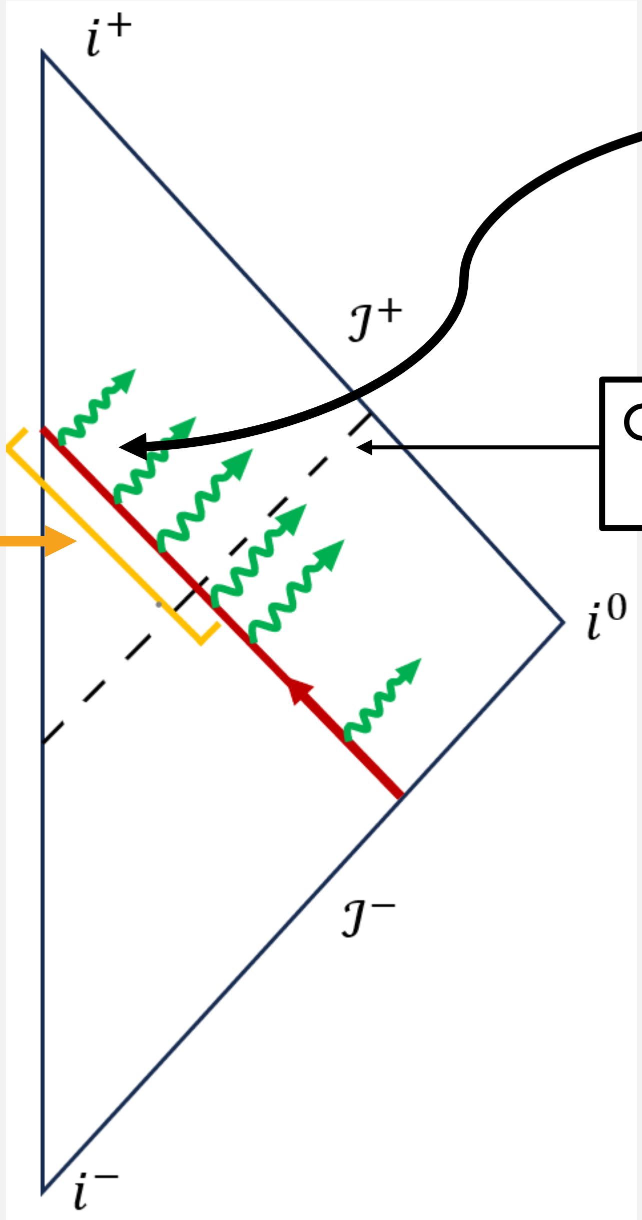


Energy of the shockwave gradually decreases. When the shockwave reaches $r = 0$, its energy vanishes. No singularity is formed.

Classical event horizon, $r = 2M$

Region where large number of Hawking quanta is created.

Simple model of backreaction – is formation of a non-extremal black hole possible?



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Because of backreaction, no event horizon is formed and there is no sigularity. We are left with an ordinary scattering problem in an asymptotically flat spacetime!

Simple model of backreaction – is formation of a non-extremal black hole possible?

Divide a surface $v = \mathrm{const.} > 0$ into small compartments of fixed affine length δr, and assume that the compartment at $r = r_i$ is filled with an ideal gas at Unruh temperature T_i.

Free energy of the system at position r_i:

$$\delta U_i = \delta r_i \cdot \int_0^\infty \mathrm{d}\omega \, \rho(\omega) \, \frac{\omega}{\exp(\hbar\omega/T_i) - 1}$$

Primitive model of backreaction:

$$M(r) = M(\infty) + \int_\infty^r \mathrm{d}r' \int_0^\infty \mathrm{d}\omega \, \rho(\omega) \, \frac{\omega}{\exp(\hbar\omega/T_i) - 1}.$$

For $\rho(\omega) = c_0 \cdot \omega$, with suitable c_0 we can make the whole black hole evaporate

$$M(r = 0) = 0,$$

and recover the Bekenstein-Hawking formula for the black hole entropy from the standard thermodynamic formula:

$$S = -\frac{\partial F}{\partial T_H} = 4\pi M^2.$$

Hawking radiation of an extremal Reissner-Nordstrom black hole

Consider an extremal Reissner-Nordstrom-Vaidya metric:

$$g = -\left(1 - \frac{M}{r}\theta(v)\right)^2 \mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2.$$

From a simple generalization of the procedure of gluing scalar modes (in the WKB approximation) across the shockwave $v = 0$ one can calculate Bogoliubov coefficients between:

$$h_\omega = \frac{\theta(r - r_H)}{\sqrt{4\pi\omega}} \frac{e^{-i\omega v + 2i\omega r_*}}{r}, \qquad \text{with} \qquad \mathrm{d}r_* = \frac{\mathrm{d}r}{(1 - M/r)^2} \qquad \text{for} \qquad v > 0$$

and

$$p_\omega = \frac{1}{\sqrt{4\pi\omega}} \frac{e^{-i\omega v}}{r} \qquad \text{on } \mathcal{J}^-.$$

Then:

$$\beta_{\omega\omega'} = -\frac{4iM}{\pi} e^{4iM(\omega+\omega')} \left(\frac{-\omega}{\omega + \omega'}\right)^{\frac{1}{2}+4iM\omega} K_{1+4iM\omega}\left(8M\sqrt{\omega(\omega + \omega')}\right)$$

$$\langle N_\omega^+ \rangle = \int_0^\infty \mathrm{d}\omega \, |\beta_{\omega\omega'}|^2 < \infty$$

Kerr-Vaidya black hole

Simplest model of formation of a Kerr black hole:

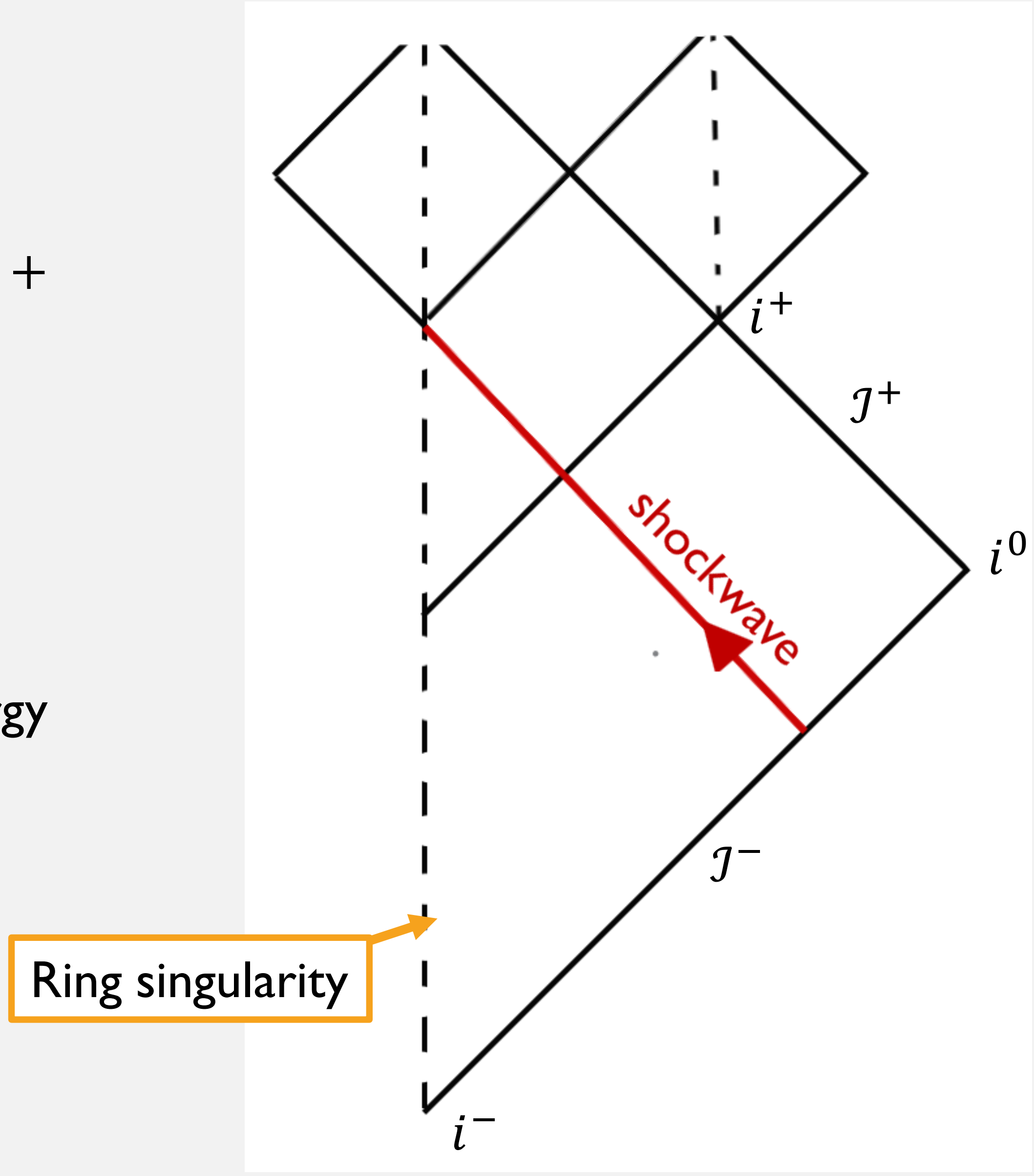
$$g = -\frac{\Delta}{\Sigma}(\mathrm{d}v - a\sin^2\theta\,\mathrm{d}\phi)^2 + 2\mathrm{d}r(\mathrm{d}v - a\sin^2\theta\,\mathrm{d}\phi) + \\ + \Sigma \mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Sigma}\left((r^2+a^2)\mathrm{d}\phi - a\mathrm{d}v\right)^2$$

where $\Delta = r^2 - 2M(v)r + a^2$ with $M(v) = M \cdot \theta(v)$.

Problems with Kerr-Vaidya:

- Corresponding stress tensor does not satisfy the Null Energy Condition.
- For $M = 0$ we have a singularity at $r = 0$ and $\theta = \pi/2$.

Nevertheless, for $M(v) = M \cdot \theta(v)$ we can solve equations of motion in the WKB approximation in separately in regions $v < 0$ and $v > 0$.



Two approximations

Klein-Gordon equation in Kerr spacetime, after a separation of variables:

$$\Phi(t,r,\theta,\varphi) = e^{-i\omega t} e^{im\varphi} S_{lm\omega}(\theta) R_{\omega lm}(r),$$

reduces to a Heun equation:

$$\left[\frac{\mathrm{d}}{\mathrm{d}r}\Delta\frac{\mathrm{d}}{\mathrm{d}r} + \frac{(2Mr_+ - am)^2}{(r-r_+)(r_+-r_-)} - \frac{(2Mr_-\omega - am)^2}{(r-r_-)(r_+-r_-)} + \left(r^2 + 2M(r+2M)\right)\omega^2\right] R(r) = K_{\omega lm} R(r).$$

To get some analytical expressions, we need to do approximations:

1. WKB approximation: $\omega \gg 1/M$ ⟶ $R_{\omega lm}(r) = \frac{1}{r}\exp(i\omega r_*)$, $\mathrm{d}r_* = \frac{\mathrm{d}r}{\Delta(r)}$

2. Low-energy approximation: $\omega r \ll 1$ or $r \gg M$.

$\omega r \ll 1$ ↙ Hypergeometric equation ↘

$r \gg M$ ↘ Bessel equation ↙

Can be glued together at such r that both conditions $\omega r \ll 1$ and $r \gg M$ are satisfied.

High-energy quanta in Kerr

$$p_\omega^{(1)} \sim \frac{1}{r} e^{-i\omega v + 2i\omega r_*^<}, \qquad h_\omega \sim \frac{1}{r} e^{-i\omega v + 2i\omega r_*^>},$$

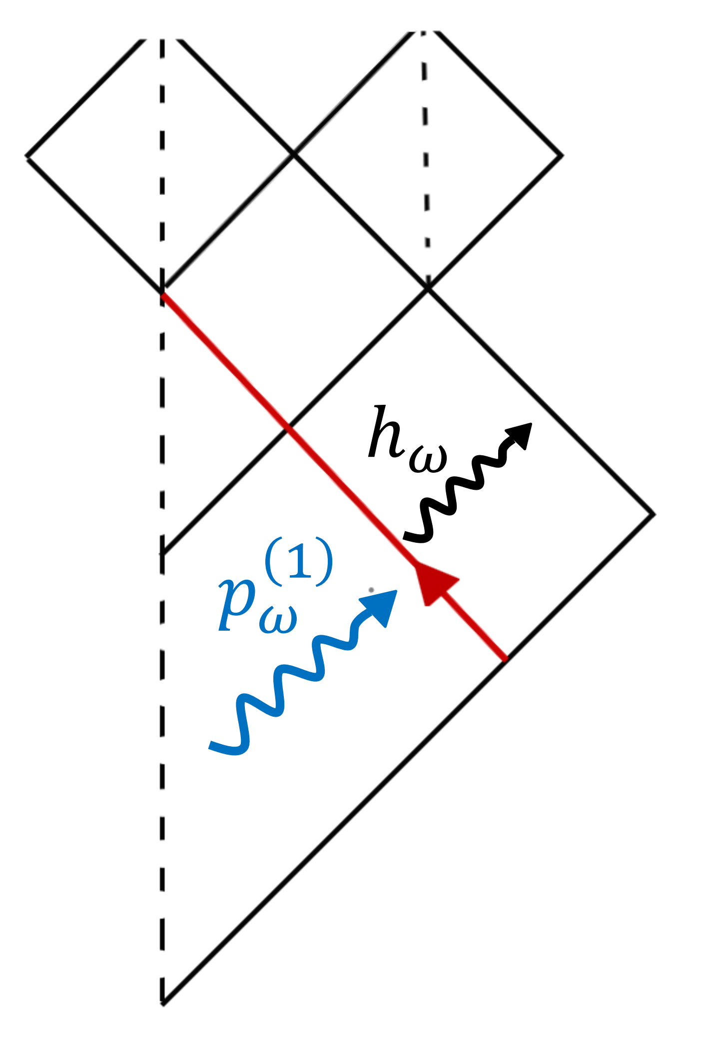
Since $p_\omega^{(1)} \sim \frac{1}{r} e^{-i\omega v + 2i\omega y}$ for certain coordinate y, the Bogoliubiv coefficients are given by Fourier coefficients of $r \cdot h_\omega|_{\{v=0\}}$ treated as a function of y.

$$\beta_{\omega\omega'} = \frac{1}{2\pi}\left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}} \int_{r_+}^{\infty} \mathrm{d}r \left(1 + \sqrt{\frac{r^2 - K_{\omega lm}/\omega^2}{r^2 + a^2}}\right) \left(\frac{r - r_+}{r - r_-}\right)^{-\frac{im\Omega_+}{\kappa_+}} e^{im\arctan\left(\frac{r}{a}\right)} \exp\left[i\omega'\left(r + \int_{r_+}^{r} \mathrm{d}r' \sqrt{\frac{r'^2 - K_{\omega lm}/\omega^2}{r'^2 + a^2}}\right)\right] \times$$
$$\times \exp\left[i\omega'\left(r + \frac{1}{2\kappa_+}\log\left(\frac{r}{r_+} - 1\right) + \frac{1}{2\kappa_-}\log\left(\frac{r}{r_-} - 1\right) + \int_{r_+}^{r} \mathrm{d}r' \frac{\sqrt{r'^4 + a^2 r'(r' + 2M) - \Delta(r') K_{\omega lm}/\omega^2}}{(r' - r_+)(r' - r_-)}\right)\right],$$

Contribution from the near-horizon region (which gives the logarithmically divergent part of the particle number on $\mathcal{J}^+$):

$$\beta_{\omega\omega'} \sim \left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}} \left(2ir_+(\omega + \omega')\right)^{1 + \frac{i\omega}{\kappa_+} - \frac{im\Omega_+}{2\kappa_+}} \Gamma\left(1 + \frac{i\omega}{\kappa_+} - \frac{im\Omega_+}{2\kappa_+}\right)$$

$$\langle N_\omega^+ \rangle \sim \log\left(\frac{\Lambda}{\omega}\right)\left[\exp\left(\frac{2\pi}{\kappa_+}\left(\omega - \frac{m\Omega_+}{2}\right)\right) - 1\right]^{-1}$$



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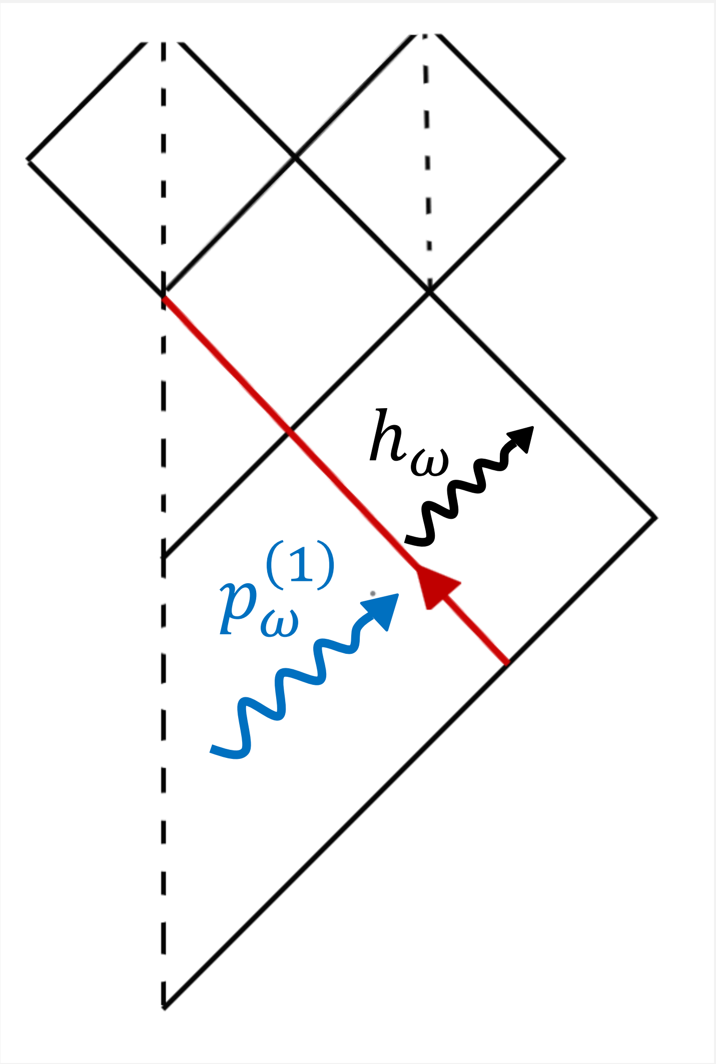
Contribution from the angular momentum does not agree with the standard results!?



Hawking radiation of low-energy quanta

For low-energy modes p_ω, h_ω we cannot identify $h_\omega = \sum_{\omega'}(\alpha_{\omega\omega'} p_{\omega'} + \beta_{\omega\omega'}\bar{p}_{\omega'})$ with an invertible integral transform.

To find $\beta_{\omega\omega'}$ calculate the symplectic product between h_ω and $\bar{p}_\omega$:

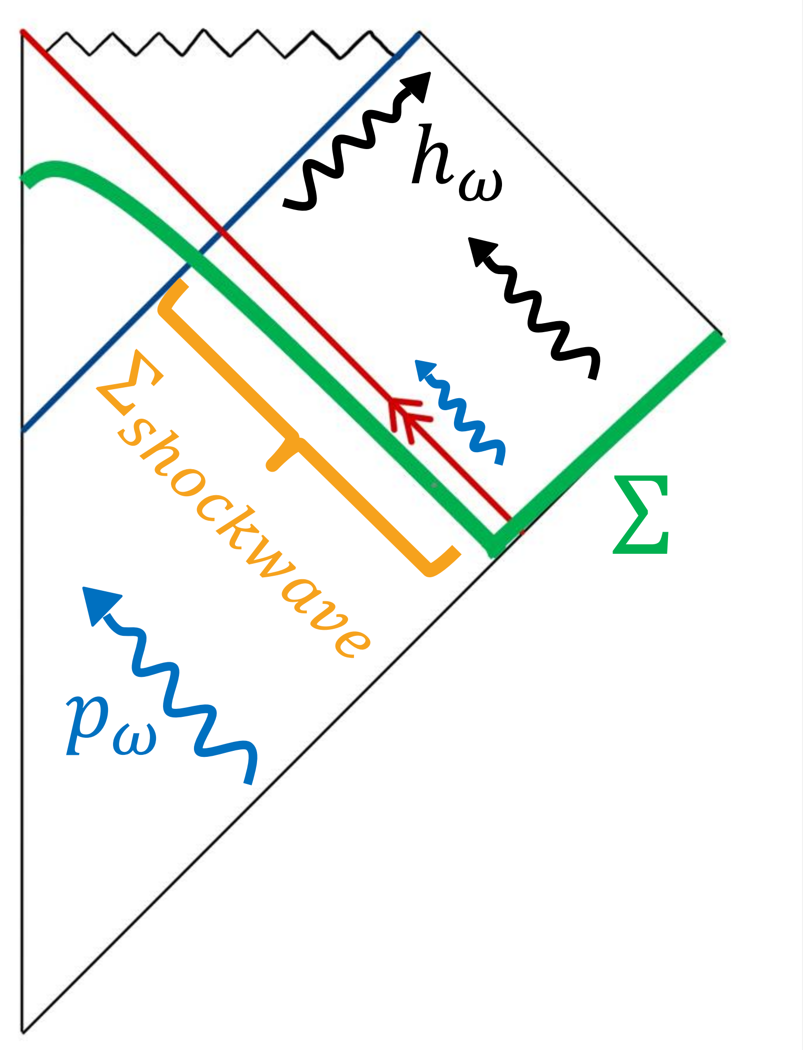
$$\beta_{\omega\omega'} = -(\bar{p}_{\omega'}|h_\omega), \qquad -i\int_\Sigma \mathrm{d}\sigma^\mu(x)\,\bar{\alpha}(x)\,\overleftrightarrow{\partial}_\mu\,\beta(x)$$

Contribution to $\beta_{\omega\omega'}$ from $\Sigma_{shoskwave}$:

$$\beta^{shockwave}_{\omega\omega'} = i\int_{2M}^{\infty} \mathrm{d}r\, r^2 \left(\frac{(-i)^{l+1}}{2\sqrt{2}} \frac{1}{\sqrt{r}} H^{(2)}_{l+\frac{1}{2}}(\omega' r)\, e^{i\omega' r_\star} \overleftrightarrow{\partial}_r\, e^{i\omega r_*} R_h(r) \right),$$

$$R_h(\omega r \ll 1) = \frac{\sqrt{2\omega}}{2\Gamma\left(\frac{3}{2}+l\right)} \left(\frac{\omega r}{2}\right)^l \left(1 - \frac{2M}{r}\right)^{2iM\omega} i^{l+1}\;{}_2F_1\left(-l + 2iM\omega, -l + 2iM\omega; -2l; \frac{2M}{r}\right)$$

Note that $\beta^{shockwave}_{\omega\omega'} \sim \omega^{l+\frac{1}{2}}$ - the „grayody factor" from the standard analysis is already encoded in $\beta^{shockwave}_{\omega\omega'}$.



Hawking radiation of gravitons

Quantize perturbations $h_{\mu\nu}$ of an asymptotically flat black hole metric $\bar{g}_{\mu\nu}$.
On $\mathcal{J}^+$ we have:

$$g = \eta + \frac{2M}{r}\mathrm{d}u^2 + r\, C_{AB}\mathrm{d}\theta^A\mathrm{d}\theta^B + \cdots$$

$$C_{AB} = \lim_{r\to\infty}\frac{\kappa}{r}h_{AB}, \qquad h_{\mu\nu} = \sum_{\alpha=\pm}\int \widetilde{\mathrm{d}k}\,\left(\varepsilon^{\alpha}_{\mu\nu}{}^*(k)b_\alpha(k)u_k(x) + \varepsilon^{\alpha}_{\mu\nu}(k)b^\dagger_\alpha(k)\bar{u}_k(x)\right),$$

These quantities are directly related to the Newman-Penrose coefficient $\Psi_4 = -W_{\mu\nu\rho\sigma}n^\mu\overline{m}^\nu n^\rho\overline{m}^\sigma$:

$$\Psi_4 = \frac{1}{2r}\mathcal{N}_{AB}\overline{m}^A\overline{m}^B, \qquad \mathcal{N}_{AB} = \partial_u^2 C_{AB},$$

Hence, for the Kerr background $\bar{g}_{\mu\nu}$, $u_k(t,r,\theta,\varphi)$ solve the Teukolsky equation – we can separate variables and label the modes by ω, l, m:

$$u_{\omega,l,m}(x) = e^{-i\omega t}S^{s=2}_{lm\omega}(\theta)e^{-im\varphi}R_{\omega l}(r),$$

$$\left[\Delta^{-s}\frac{\mathrm{d}}{\mathrm{d}r}\Delta^{s+1}\frac{\mathrm{d}}{\mathrm{d}r} - V(r)\right]R(r) = 0$$

Can soft hair of black holes modify the spectrum of Hawking quanta?

Supertranslated Schwarzschild black hole metric:

$$g = (1 + \mathcal{L}_\xi)\bar{g}, \qquad \bar{g} = -\left(1 - \frac{2M}{r}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\gamma_{AB}\mathrm{d}\theta^A\mathrm{d}\theta^B,$$

$$\xi_f = f\partial_v + \frac{1}{r}D^A f\,\partial_A - \frac{1}{2}D^2 f\,\partial_r$$

h_ω solves linearized Einstein equations in backgroud $\bar{g}$ → $h'_\omega = (1 + \mathcal{L}_\xi)h_\omega$ solves linearized Einstein equations in backgroud $g = (1 + \mathcal{L}_\xi)\bar{g}$

In the WKB approximation, Bogoliubov coefficients read:

$$\beta_{\omega l m}^{\omega' l' m'} = \int \mathrm{vol}_{S^2} \int_{r_H}^{\infty} \mathrm{d}r\; 4\pi\omega' e^{2i\omega' r}\, Y^*_{l'm'}(\theta^A)\, h_{\omega l m}\left(x - \xi_f(x)\right)$$

The only term dependent on the supertranslation.

Can soft hair of black holes modify the spectrum of Hawking quanta?

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Influence of supertranslations on the Hawking spectrum was investigated in [JHEP02(2021)038, JHEP01(2019)089], where it was argued that supertranslations do not modify $|\beta_{\omega\omega'}|$ at all.

However, because of the r-dependent part of ξ_f, we in our case $|\beta_{\omega\omega'}|$ is slightly modified.

But the divergent part of $\int_0^\infty \mathrm{d}\omega' |\beta_{\omega\omega'}|^2$ is not changed.

Can soft hair of black holes modify the spectrum of Hawking quanta?

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Can we find a Generalized Gibbs Ensable-like spectrum?

$$\langle N_{\omega,q_i} \rangle \sim \left(e^{\beta\omega + \sum_i \lambda_i q_i} - 1\right)^{-1}$$