Michele Grasso
CFT PAN, Warsaw

Light propagation in Numerical Relativity with BiGONLight

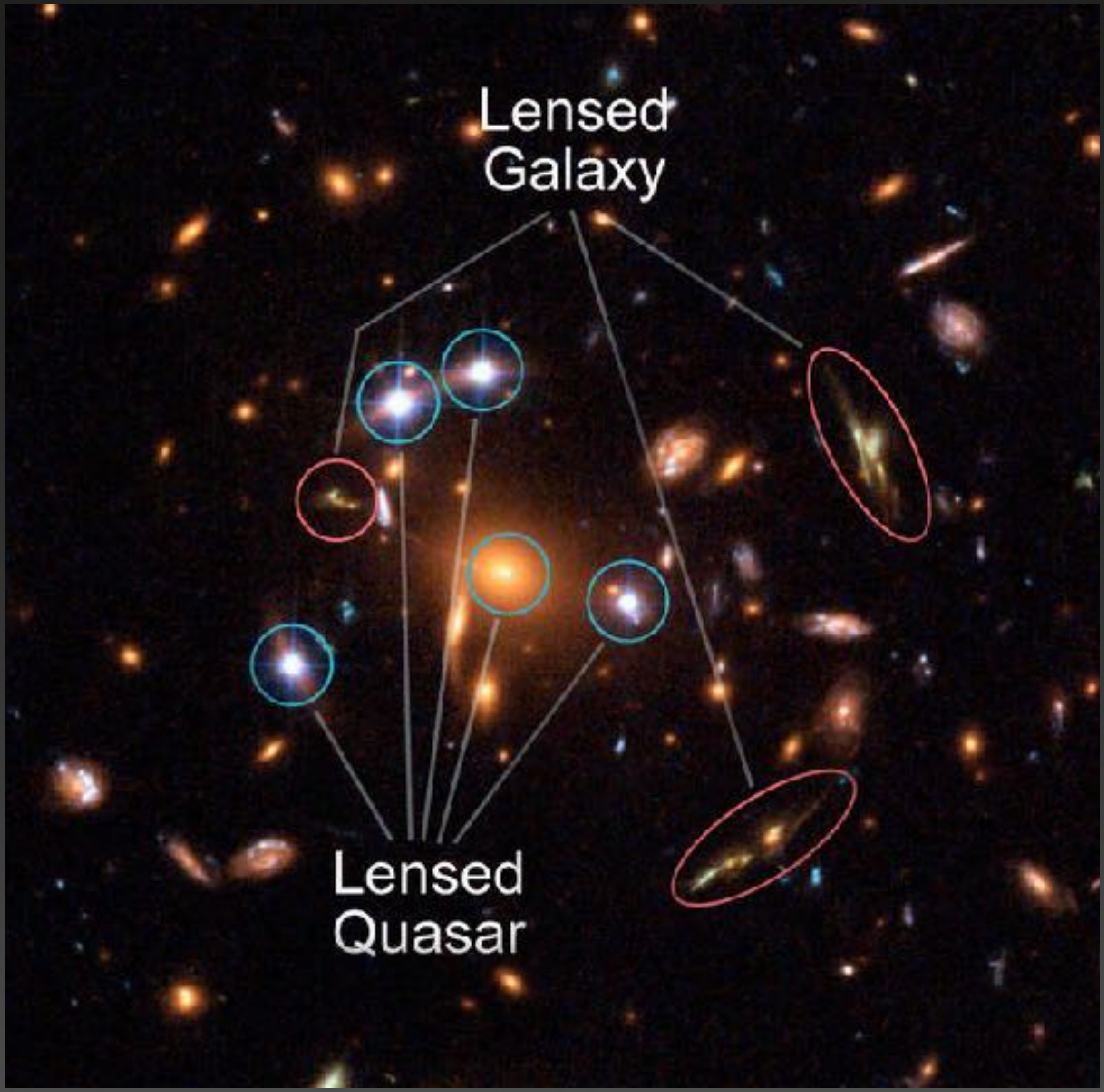
Relativity seminar

27^{th} November 2020, FWU

Supervisors:
- M. Korzyński
- E. Villa

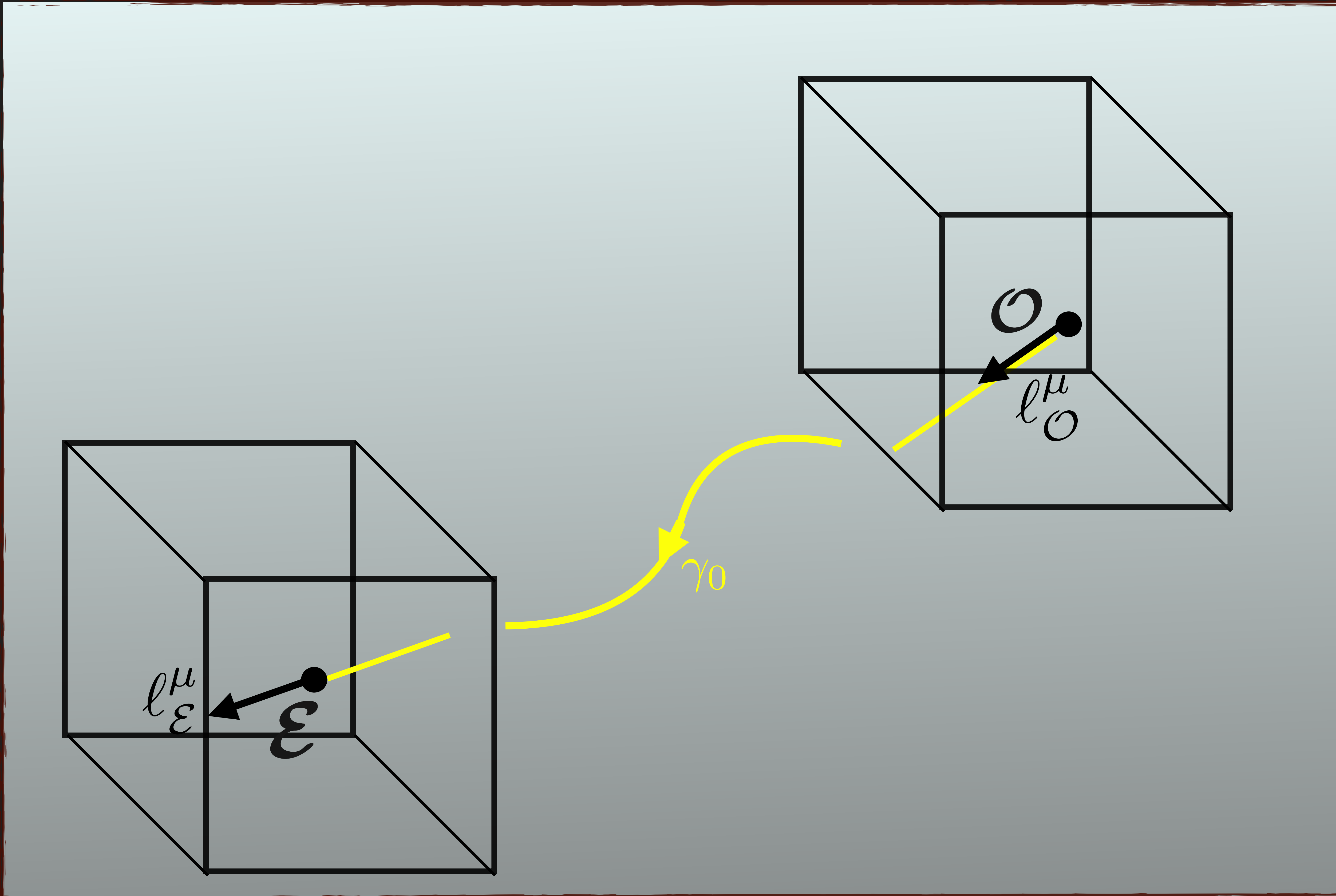
A long time ago in a galaxy far far away...

- **Optical effects**
  - ✳ image distorsion
  - ✳ apparent position
  - ✳ ...
- **Observables**
  - ✳ angular distance $\mathscr{D}_{ang}$
  - ✳ redshift z
  - ✳ position on the sky
  - ✳ ...

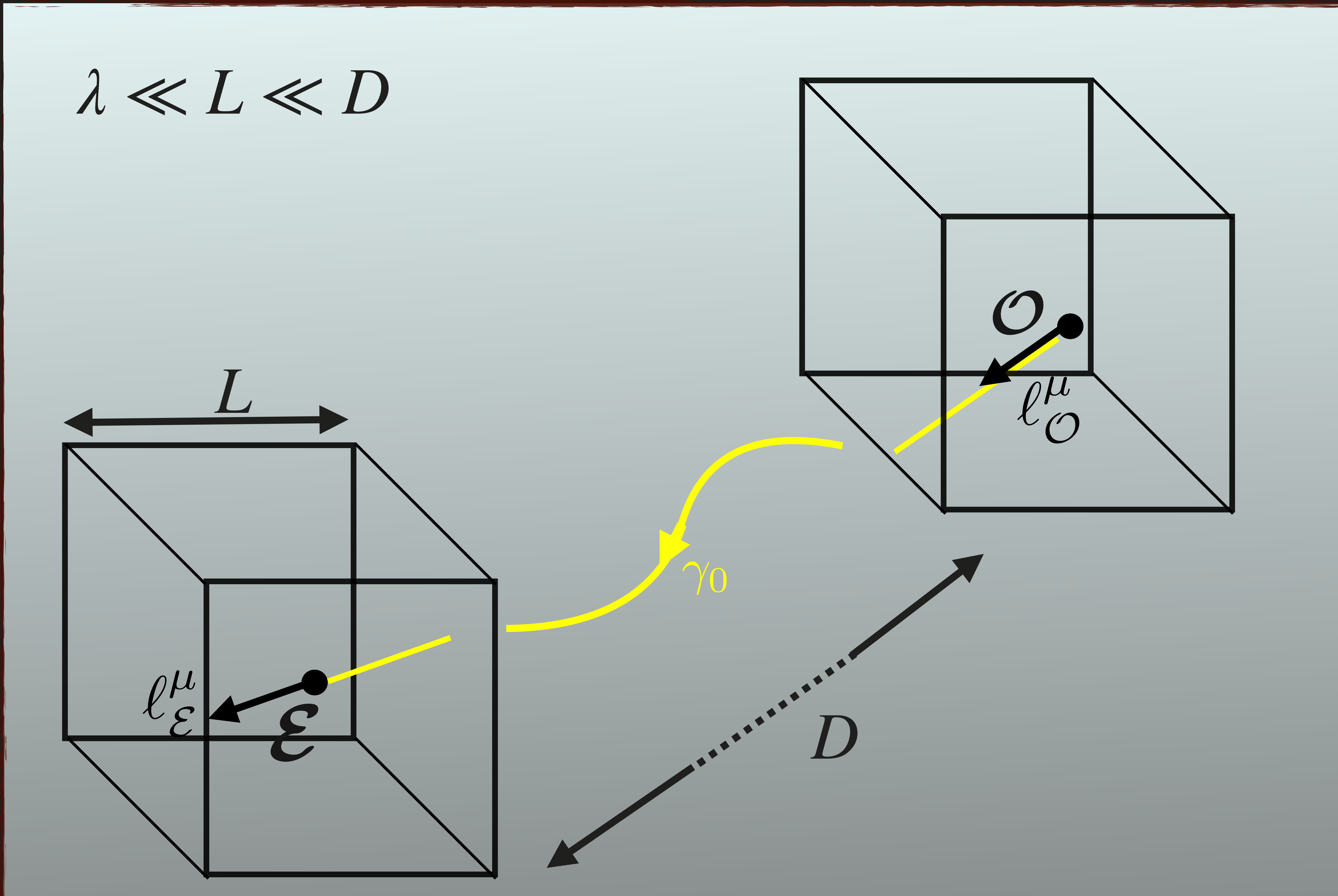


Credits: ESA, NASA, K. Sharon (Tel Aviv University) and E. Ofek (Caltech)

The new BGO approach

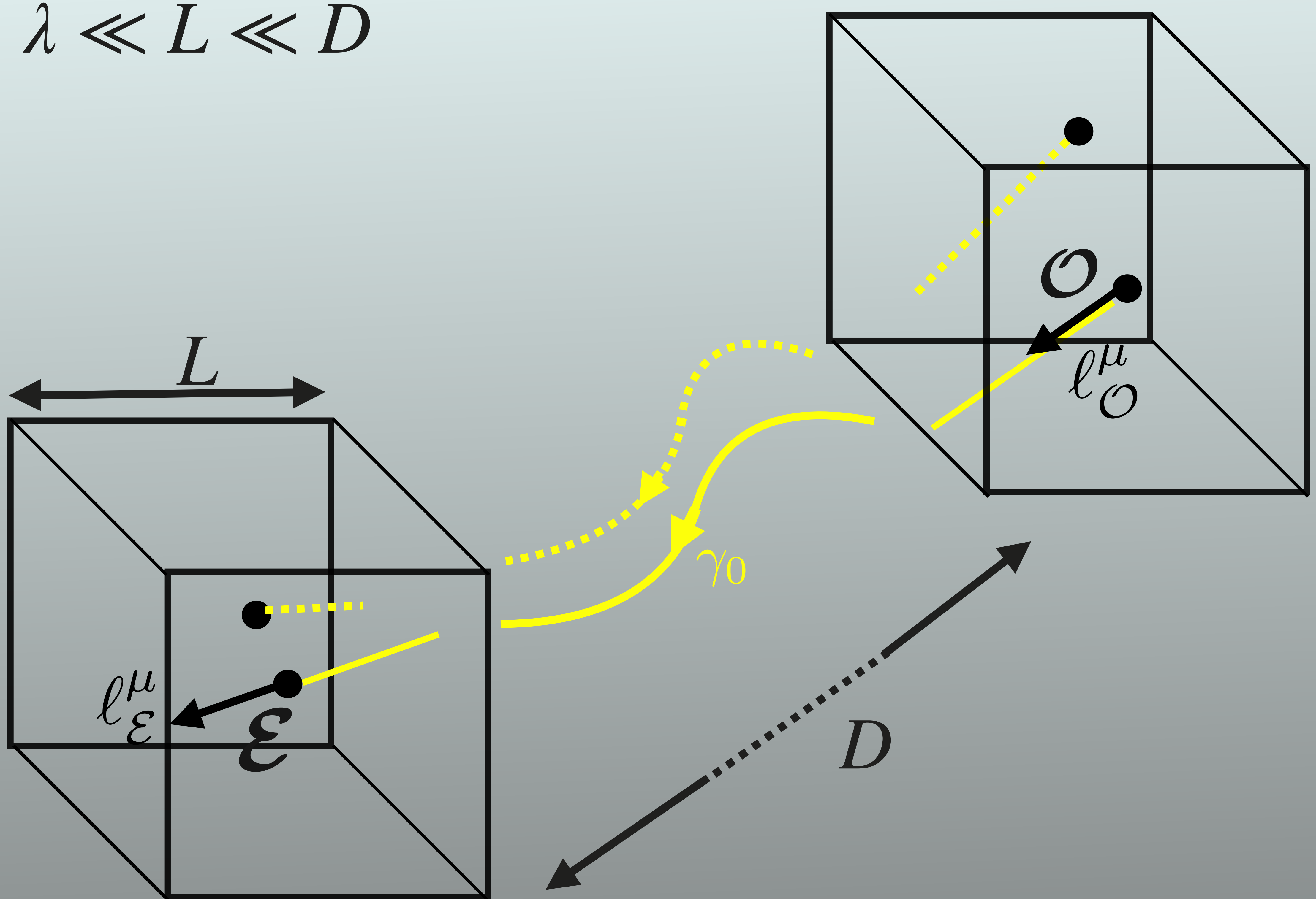


The new BGO approach

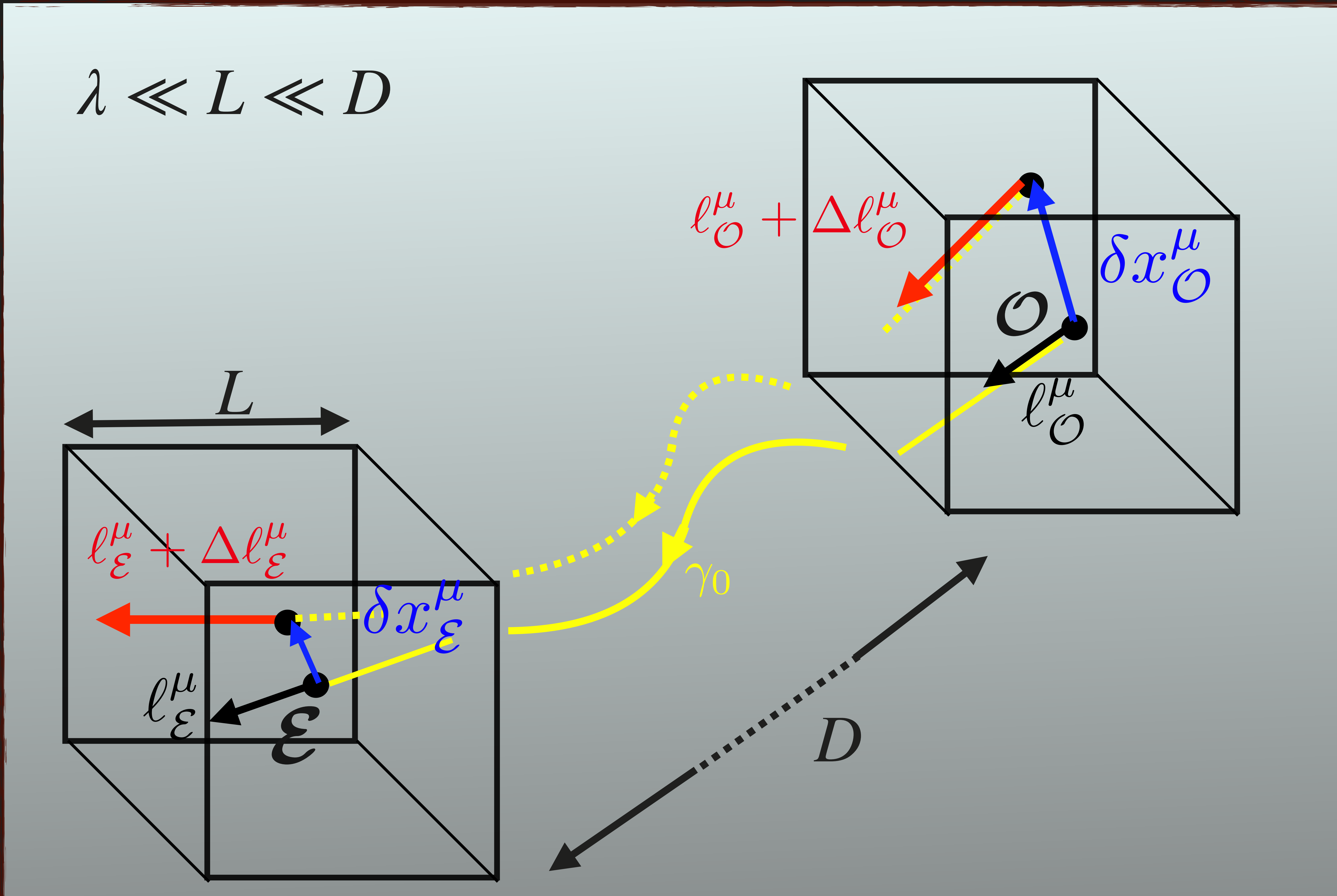


The new BGO approach

$\lambda \ll L \ll D$



The new BGO approach



The new BGO approach

- GDE

$$\begin{cases} \frac{D}{D\lambda}\delta x^\mu = \Delta\ell^\mu \\ \frac{D}{D\lambda}\Delta\ell^\mu = R^\mu{}_{\alpha\beta\nu}\delta x^\nu \end{cases}$$

↓

MG, M. Korzyński, J. Serbenta (2019), **Phys.Rev. D**

The new BGO approach

- GDE

$$\begin{cases} \frac{D}{D\lambda}\delta x^\mu = \Delta\ell^\mu \\ \frac{D}{D\lambda}\Delta\ell^\mu = R^\mu{}_{\alpha\beta\nu}\delta x^\nu \end{cases}$$

- Linear mapping

$$\begin{pmatrix} \delta x^\mu_{\mathcal{E}} \\ \Delta\ell^\nu_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x^\rho_{\mathcal{O}} \\ \Delta\ell^\sigma_{\mathcal{O}} \end{pmatrix}$$

- GDE for $\mathcal{W}$

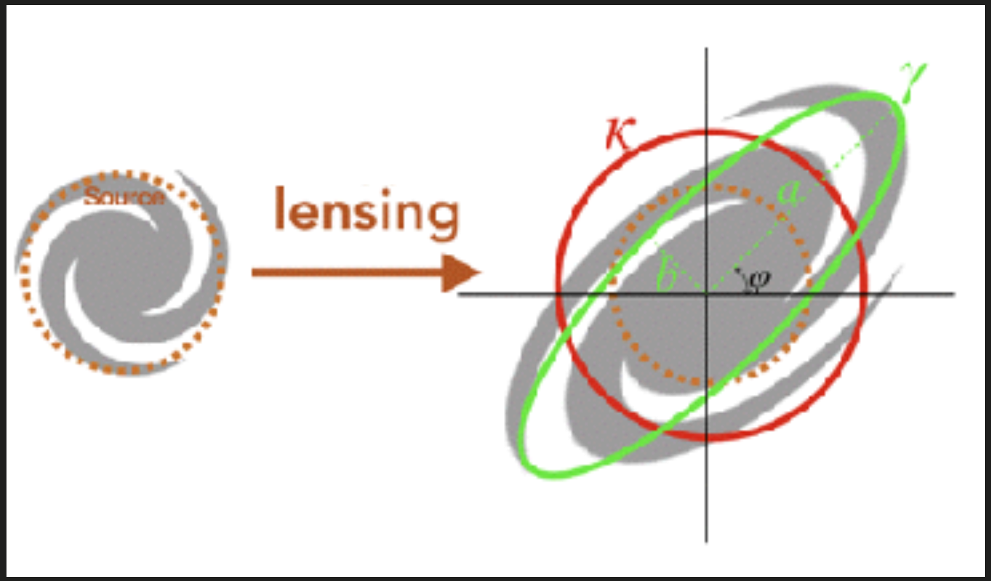
$$\frac{d}{d\lambda}\mathcal{W} = \begin{pmatrix} 0 & \delta^\mu{}_\beta \\ R^\nu{}_{\sigma\rho\alpha}\ell^\sigma\ell^\rho & 0 \end{pmatrix}\mathcal{W}$$

MG, M. Korzyński, J. Serbenta (2019), **Phys.Rev. D**

The new BGO approach

$$\mathcal{W} = \begin{pmatrix} W_{XX} & W_{XL} \\ W_{LX} & W_{LL} \end{pmatrix}$$

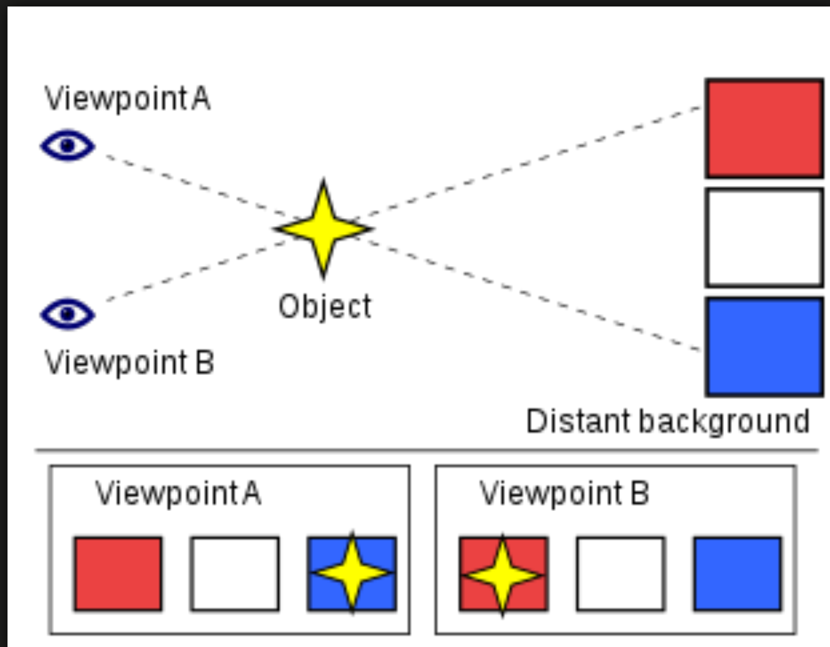
The new BGO approach

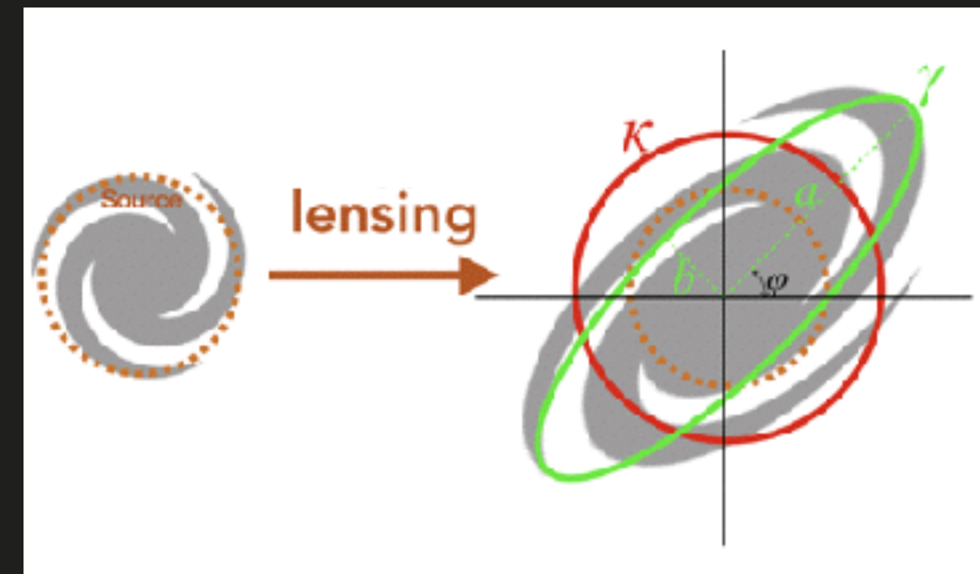


- Distorsion / Magnification

$$\mathcal{W} = \begin{pmatrix} W_{XX} & \underline{W_{XL}} \\ W_{LX} & W_{LL} \end{pmatrix}$$

The new BGO approach

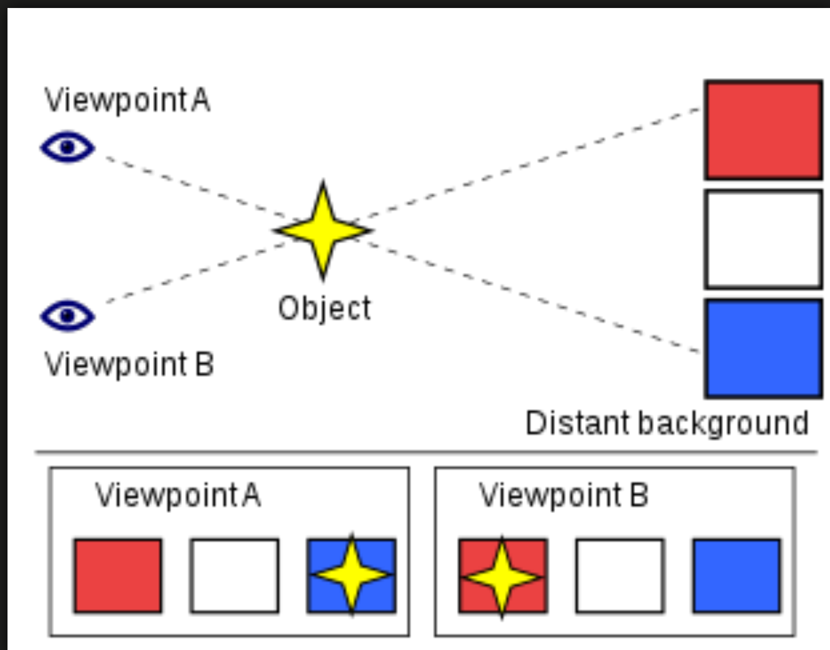


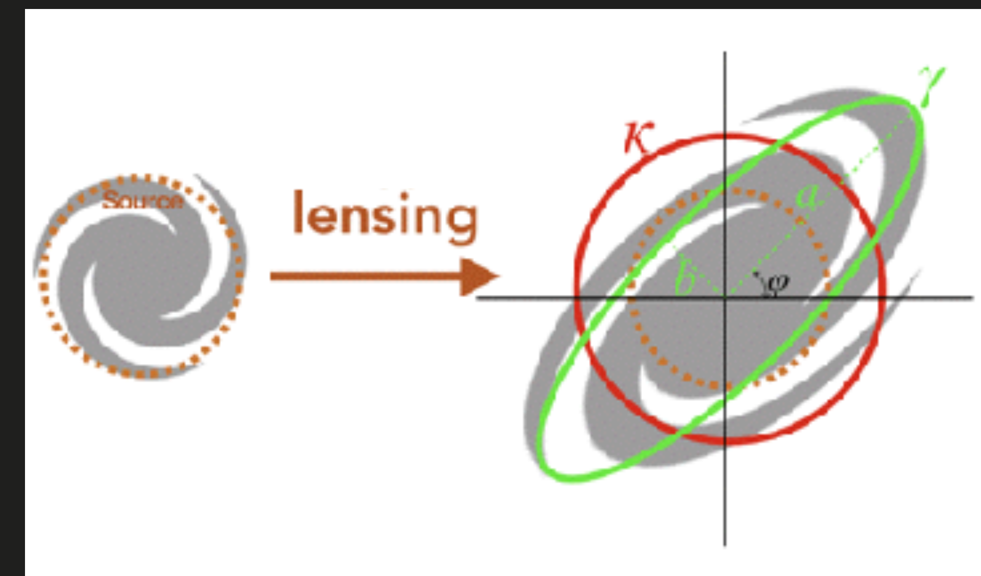


- Parallax
- Distorsion / Magnification

$$\mathcal{W} = \begin{pmatrix} W_{XX} & W_{XL} \\ W_{LX} & W_{LL} \end{pmatrix}$$

The new BGO approach





- Parallax
- Distorsion / Magnification

$$\mathcal{W} = \begin{pmatrix} \underline{W_{XX}} & \underline{W_{XL}} \\ \underline{W_{LX}} & \underline{W_{LL}} \end{pmatrix}$$

- Drifts effects
- Visual Perspective
- ...

The new BGO approach

BGO

Sachs

Key quantities

$$\mathcal{W} = \begin{pmatrix} W_{\mathrm{XX}}{}^{\nu}{}_{\beta} & W_{\mathrm{XL}}{}^{\mu}{}_{\beta} \\ W_{\mathrm{LX}}{}^{\nu}{}_{\alpha} & W_{\mathrm{LL}}{}^{\mu}{}_{\alpha} \end{pmatrix}$$

BGO: four 4×4 matrices

$$\mathcal{S} = \begin{pmatrix} \theta - \sigma_1 & \sigma_2 \\ \sigma_2 & \theta + \sigma_1 \end{pmatrix}$$

Deformation matrix: 2×2 matrix

The link: the Jacobi matrix $\mathcal{D}$

$$W_{\mathrm{XL}}{}^{\mu}{}_{\beta} = \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \multicolumn{2}{c}{\boxed{\mathcal{D}}} & \mathrm{x} \\ \mathrm{x} & & & \mathrm{x} \\ \mathrm{x} & x & x & \mathrm{x} \end{pmatrix}$$

$$\mathcal{S} = \mathcal{D}^{-1}\dot{\mathcal{D}}$$

The new BGO approach

BGO vs Sachs

assumptions

BGO:

- Geometric optics

Sachs:

- Geometric optics

observables

BGO:

- lensing: shear, magnification, …

Sachs:

- lensing: shear, magnification, …

The new BGO approach

BGO vs Sachs

assumptions

BGO:

- Geometric optics
- repeated observations
- complete 4D approach

Sachs:

- Geometric optics
- observation at single moment
- 2D Sachs frame

observables

BGO:

- lensing: shear, magnification, …

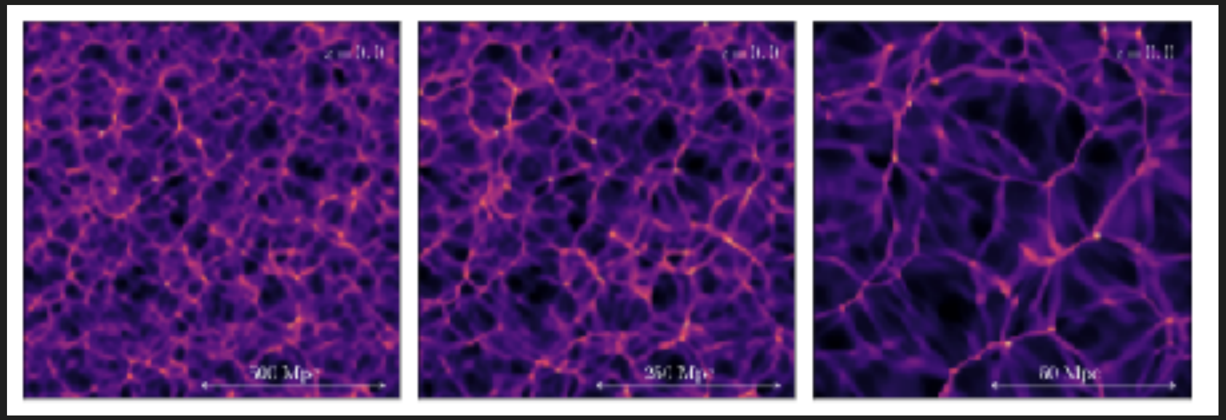
Sachs:

- lensing: shear, magnification, …

The new BGO approach

BGO	vs	Sachs
	assumptions	
• Geometric optics		• Geometric optics
• repeated observations		• observation at single moment
• complete 4D approach		• 2D Sachs frame
	observables	
• lensing: shear, magnification, …		• lensing: shear, magnification, …
• drift effects		
• parallax		

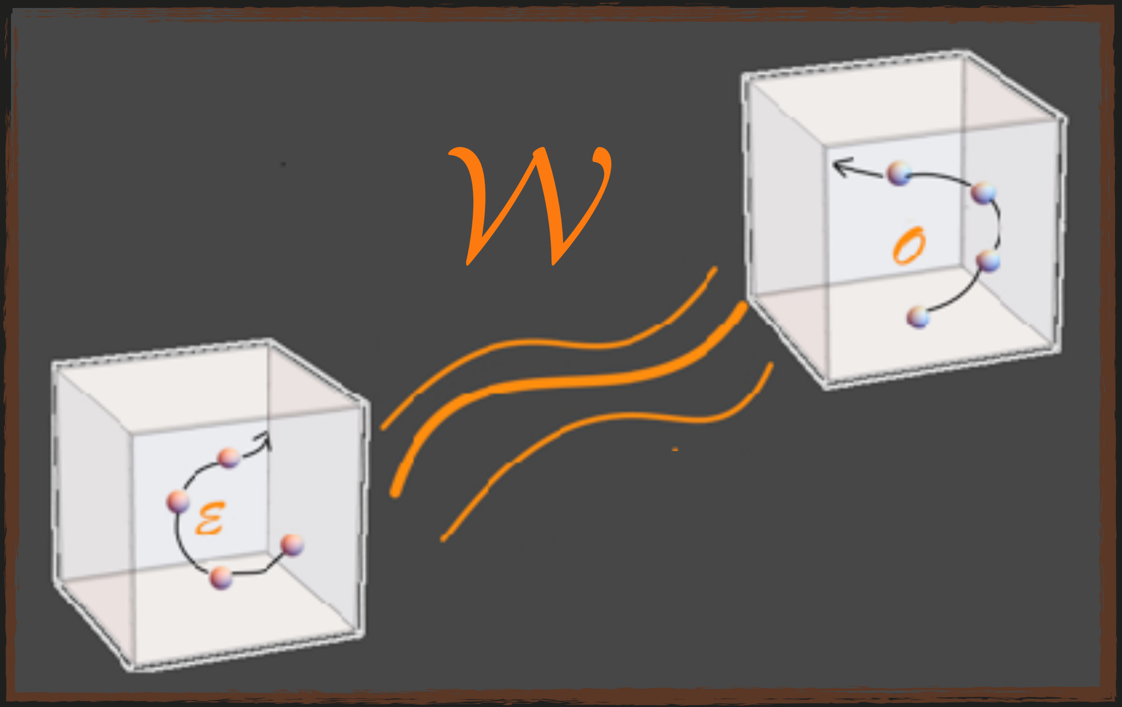
Numerical Relativity and Light propagation



H. Macpherson et. al., arXiv:1807.01711

Cosmic web

from Numerical Relativity

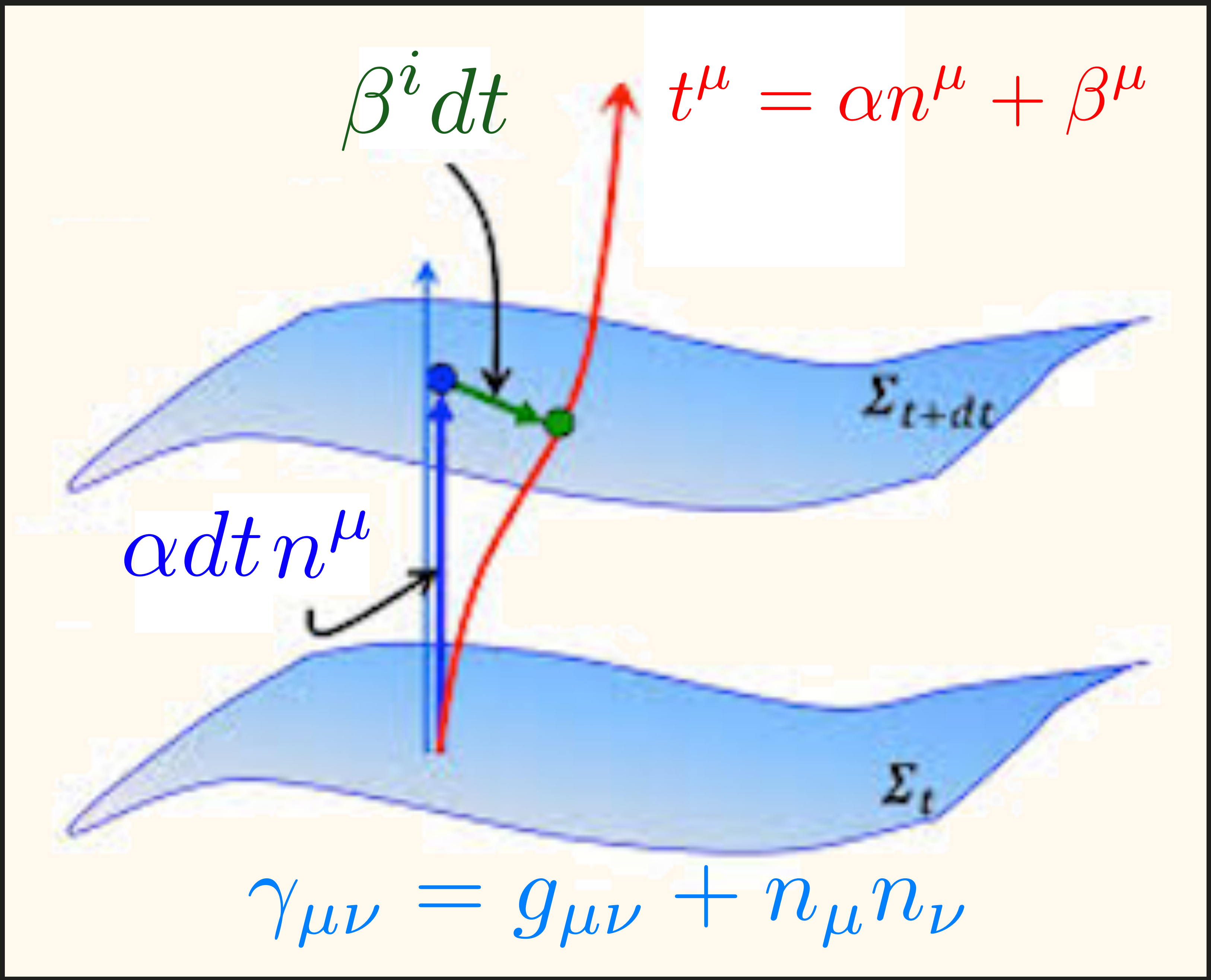


To Do

einstein toolkit

Light Propagation
in Numerical Relativity

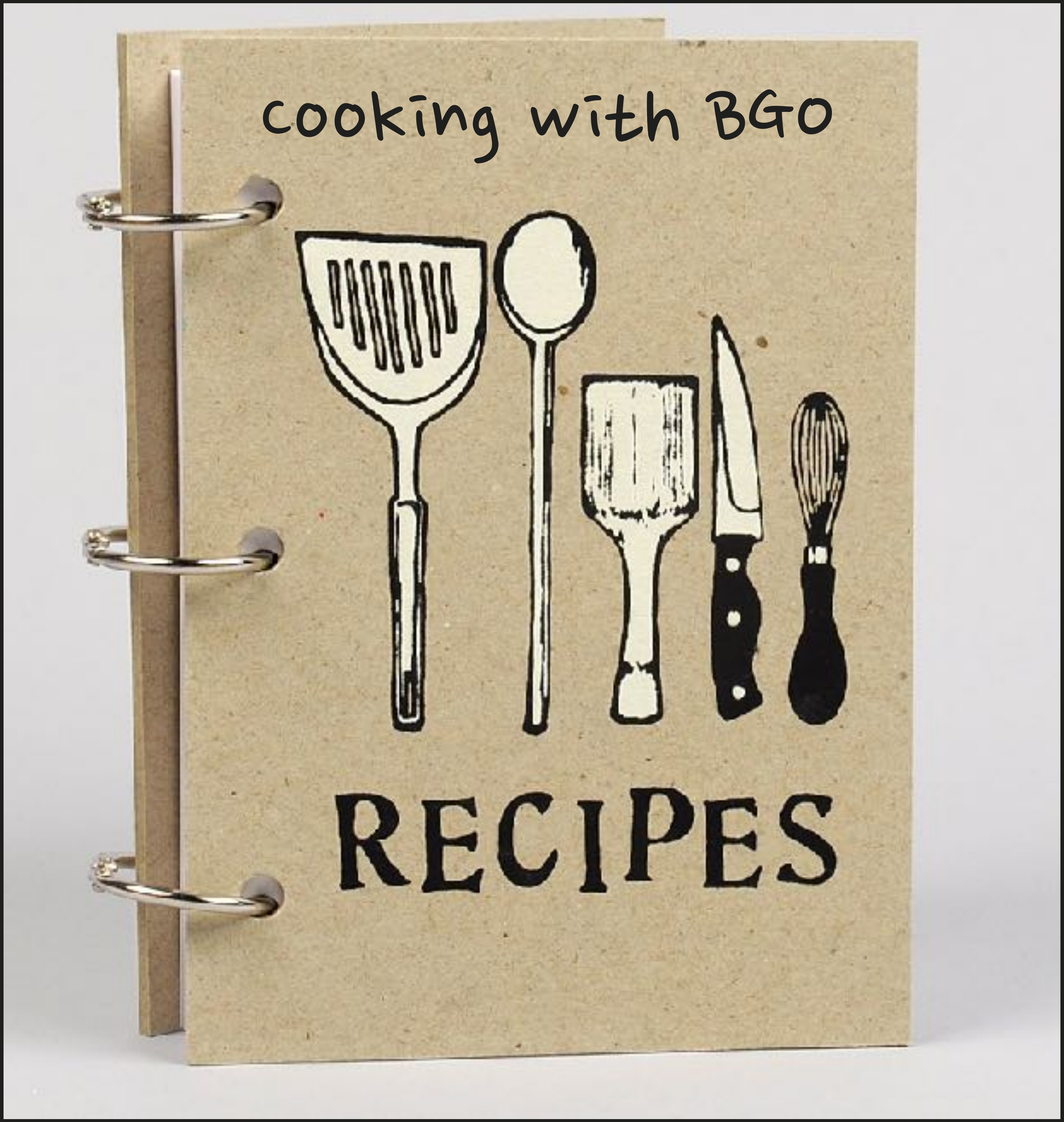
3+1 Foliation of Spacetime



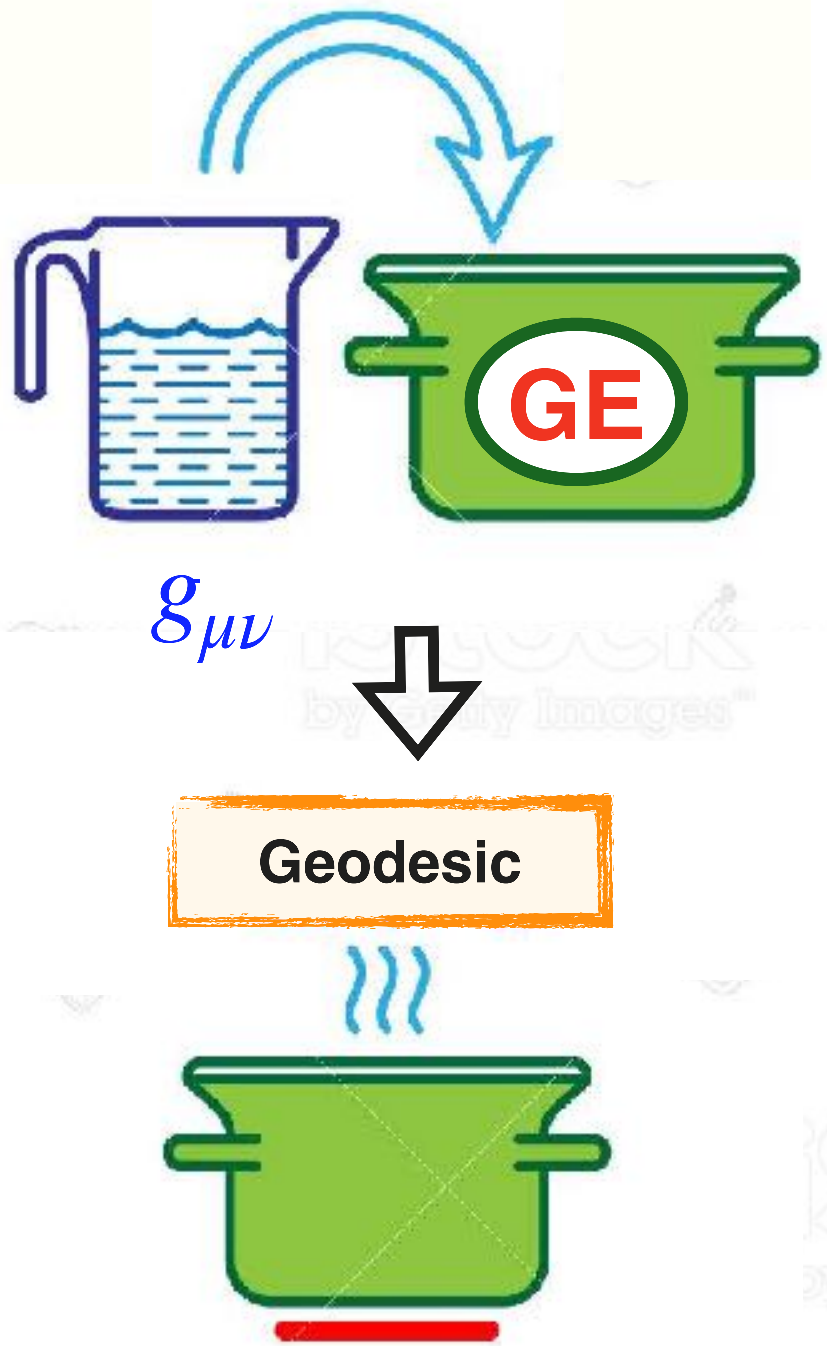
$$(\mathcal{M}, g_{\mu\nu})$$

$$(\alpha, \beta^i, \gamma_{ij}, K_{ij})$$

Recipe for cooking observables using BGO



Geodesic equation



$$\ell^\nu \nabla_\nu \ell^\mu = 0$$

$$\ell^\nu = E(n^\mu + V^\mu)$$

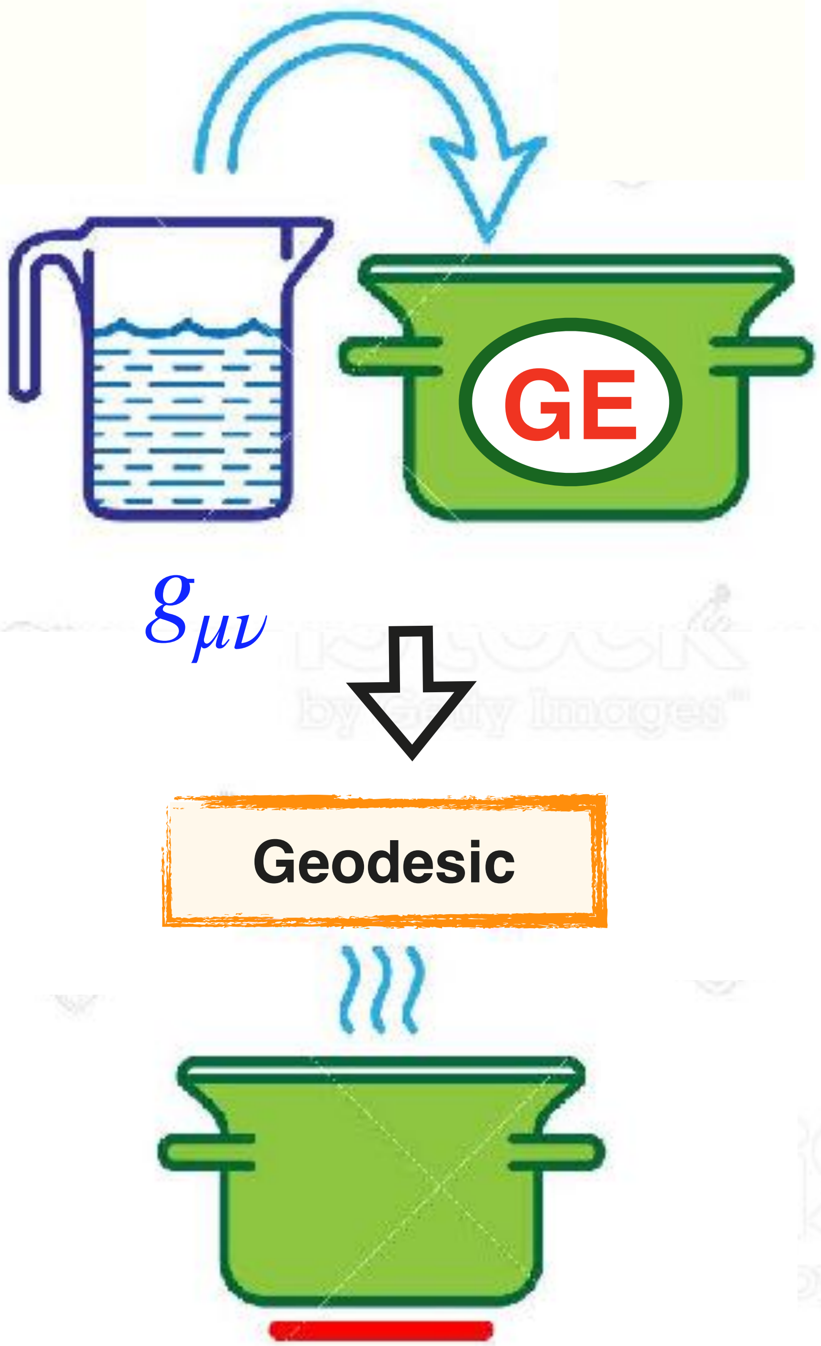
$$\frac{d\lambda}{dt} = \frac{\alpha}{E}$$

$$\frac{dE}{dt} = E\left(\alpha K_{jk} V^i V^k - V^j \partial_j \alpha\right)$$

$$\frac{dX^i}{dt} = \alpha V^i - \beta^i$$

$$\frac{dV^i}{dt} = \alpha V^j [V^i(\partial_j \log \alpha - K_{jk} V^k) + 2K^i{}_j - {}^{(3)}\Gamma^i_{jk} V^k] - \gamma^{ij}\partial_j \alpha - V^j \partial_j \beta^i$$

Geodesic equation



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Vincent, F. H., Gourgoulhon, E., & Novak, J. (2012). CQG 29(24), 245005.

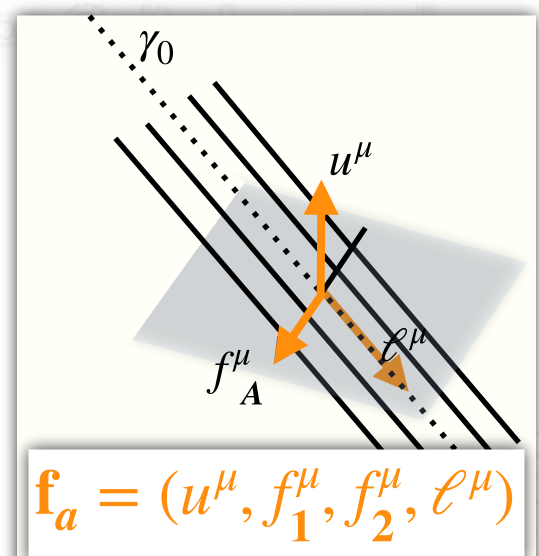
Parallel transported frame

$$\ell^\nu \nabla_\nu {}^{(4)}\mathrm{e}^\mu = 0$$

$${}^{(4)}\mathrm{e}^\mu = C n^\mu + e^\mu$$

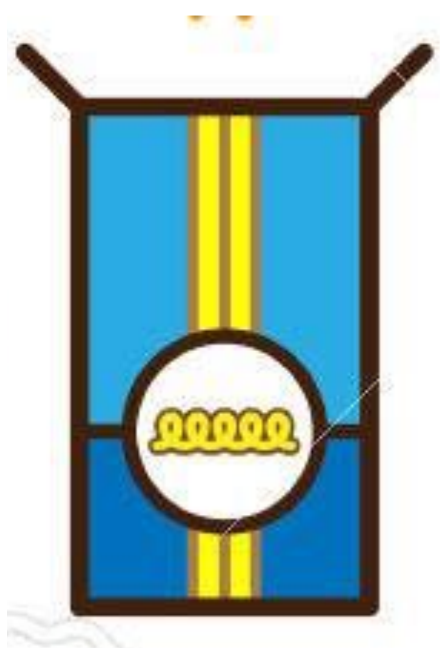
$$\alpha^{-1}\frac{dC}{dt} + e^i\partial_i \log\alpha - K_{ij}V^i e^j = 0$$

$$\alpha^{-1}\left(\frac{de^i}{dt} + e^j\partial_j\beta^i\right) = K^i{}_j e^j - {}^{(3)}\Gamma^i{}_{jk}V^j e^k - C\gamma^{ij}\partial_j\log\alpha + CK^i{}_j V^j$$



$$u^\mu u_\mu = -1 \; ; \; g_{\mu\nu} f^\mu_A f^{\nu\, B} = \delta^B_A \; ; \; \ell^\mu \ell_\mu = 0 \; ; \; \ell^\mu u_\mu = Q > 0$$

Curvature



$$R^{\boldsymbol{\mu}}{}_{\sigma\rho\boldsymbol{\nu}}\ell^{\sigma}\ell^{\rho}$$

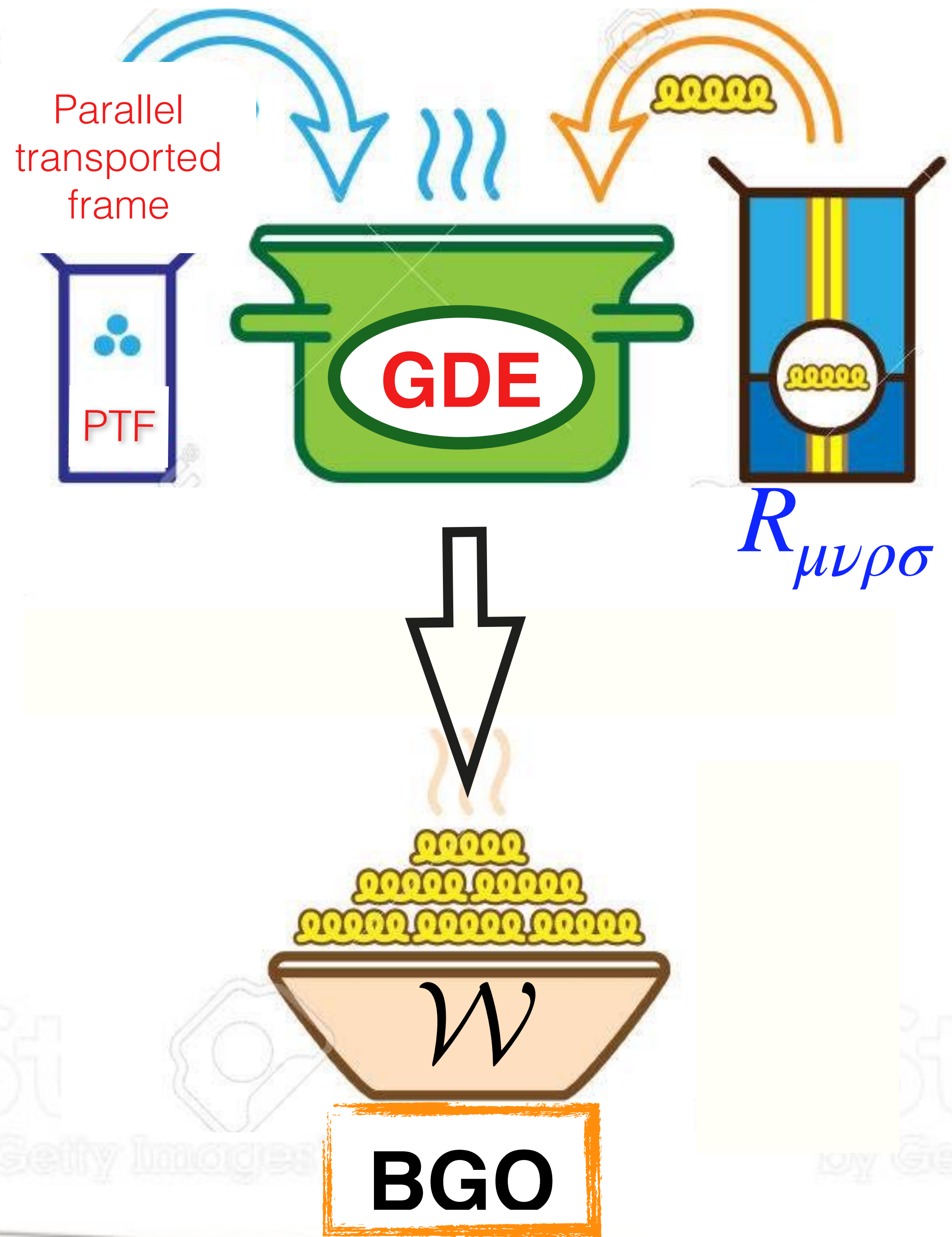
$$\mathscr{G}^{\mu}{}_{\alpha\beta\nu} = R^{\mu}{}_{\alpha\beta\nu} + K^{\mu}{}_{\beta}K_{\alpha\nu} - K^{\mu}{}_{\alpha}K_{\beta\nu}$$

$$\mathscr{C}^{\mu}{}_{\alpha\nu} = D_{\nu}K^{\mu}{}_{\alpha} - D_{\alpha}K^{\mu}{}_{\nu}$$

$$\mathscr{R}^{\mu}{}_{\nu} = \frac{1}{\alpha}\mathscr{L}_{\alpha\mathbf{n}}K^{\mu}{}_{\nu} + \frac{1}{\alpha}\gamma^{\mu\sigma}D_{\sigma}D_{\nu}\alpha + K^{\mu}{}_{\sigma}K^{\sigma}{}_{\nu}$$

$$\begin{aligned} R_{\boldsymbol{\rho}\ell\ell\boldsymbol{\sigma}} = E^2 \Big[& \mathscr{R}_{\beta\alpha}\left(\Phi_{\boldsymbol{\rho}}F^{\beta}{}_{\boldsymbol{\sigma}}V^{\alpha} + \Phi_{\boldsymbol{\sigma}}F^{\beta}{}_{\boldsymbol{\rho}}V^{\alpha} - \Phi_{\boldsymbol{\rho}}\Phi_{\boldsymbol{\sigma}}V^{\beta}V^{\alpha} - F^{\alpha}{}_{\boldsymbol{\rho}}F^{\beta}{}_{\boldsymbol{\sigma}}\right) \\ & + \mathscr{C}_{\nu\beta\alpha}\left(\Phi_{\boldsymbol{\rho}}F^{\nu}{}_{\boldsymbol{\sigma}}V^{\alpha}V^{\beta} + \Phi_{\boldsymbol{\sigma}}F^{\nu}{}_{\boldsymbol{\rho}}V^{\alpha}V^{\beta} - F^{\alpha}{}_{\boldsymbol{\rho}}F^{\nu}{}_{\boldsymbol{\sigma}}V^{\beta} - F^{\alpha}{}_{\boldsymbol{\sigma}}F^{\nu}{}_{\boldsymbol{\rho}}V^{\beta}\right) \\ & + \mathscr{G}_{\mu\alpha\beta\nu}F^{\mu}{}_{\boldsymbol{\rho}}V^{\alpha}V^{\beta}F^{\nu}{}_{\boldsymbol{\sigma}}\Big] \end{aligned}$$

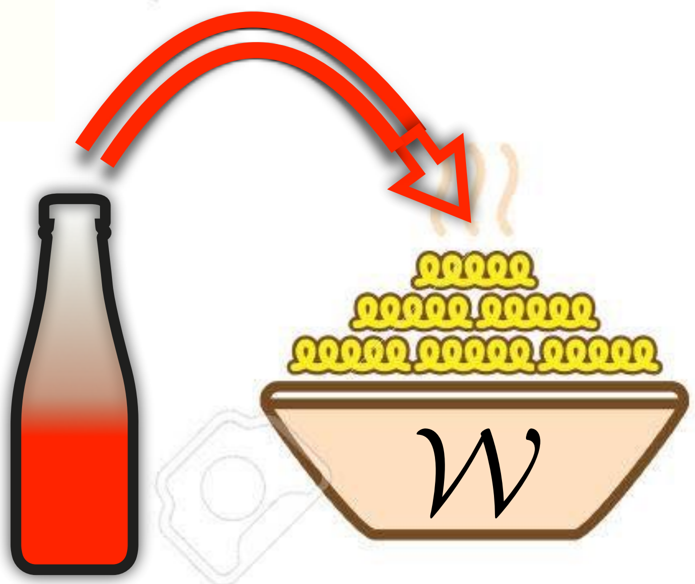
GDE for BGO



$$\frac{d}{dt}\boldsymbol{\mathcal{W}} = \frac{\alpha}{E}\begin{pmatrix} 0 & \delta^{\mu}{}_{\beta} \\ R^{\nu}{}_{\sigma\rho\alpha}\ell^{\sigma}\ell^{\rho} & 0 \end{pmatrix}\boldsymbol{\mathcal{W}}$$

$$\boldsymbol{\mathcal{W}} = \begin{pmatrix} \mathcal{W}_{\rm XX}{}^{\nu}{}_{\beta} & \mathcal{W}_{\rm XL}{}^{\mu}{}_{\beta} \\ \mathcal{W}_{\rm LX}{}^{\nu}{}_{\alpha} & \mathcal{W}_{\rm LL}{}^{\mu}{}_{\alpha} \end{pmatrix}$$

Observer and source motion



u^μ **and** a^μ

$$u^\mu_{\mathcal{O}} \quad a^\mu_{\mathcal{O}}$$

$$u^\mu_{\mathcal{E}} \quad a^\mu_{\mathcal{E}}$$

Cosmological observables

- $$D_{\text{ang}} = (\ell_{\mathcal{O}\,\mu}\, u^{\mu}_{\mathcal{O}}) \left| \det W_{\text{XL}}{}^{A}{}_{B} \right|^{\frac{1}{2}}$$
- $$D_{\text{par}} = (\ell_{\mathcal{O}\,\mu}\, u^{\mu}_{\mathcal{O}}) \left(\frac{\left| \det W_{\text{XL}}{}^{A}{}_{B} \right|}{\left| \det W_{\text{XX}}{}^{A}{}_{B} \right|} \right)^{\frac{1}{2}}$$
- $$\frac{\delta \log(1+z)}{\delta t_{\mathcal{O}}} = \Xi - \left(u_{\mathcal{O}}, \frac{u_{\mathcal{E}}}{1+z} \right) \boldsymbol{U} \begin{pmatrix} u_{\mathcal{O}} \\ \frac{u_{\mathcal{E}}}{1+z} \end{pmatrix}$$
- . . .

BiGONLight: Bi-local **G**eodesic **O**perators framework for **N**umerical **Light** propagation

MG in preparation

BiGONLight.m

ADM[], CovD[],
Christoffel[], Riemann[],
BGOEquations[],
SolveBGO[],
PTransportedFrame[],
. . .

input:
METRIC TENSOR
+
$\mathscr{E}$ and $\mathscr{O}$ MOTION

→

BiGONLight:
BGO formalism in 3+1 splitting

→

output:
OBSERVABLES

Code Tests 1:

Angular distance D_{ang} and redshift z in ΛCDM model:

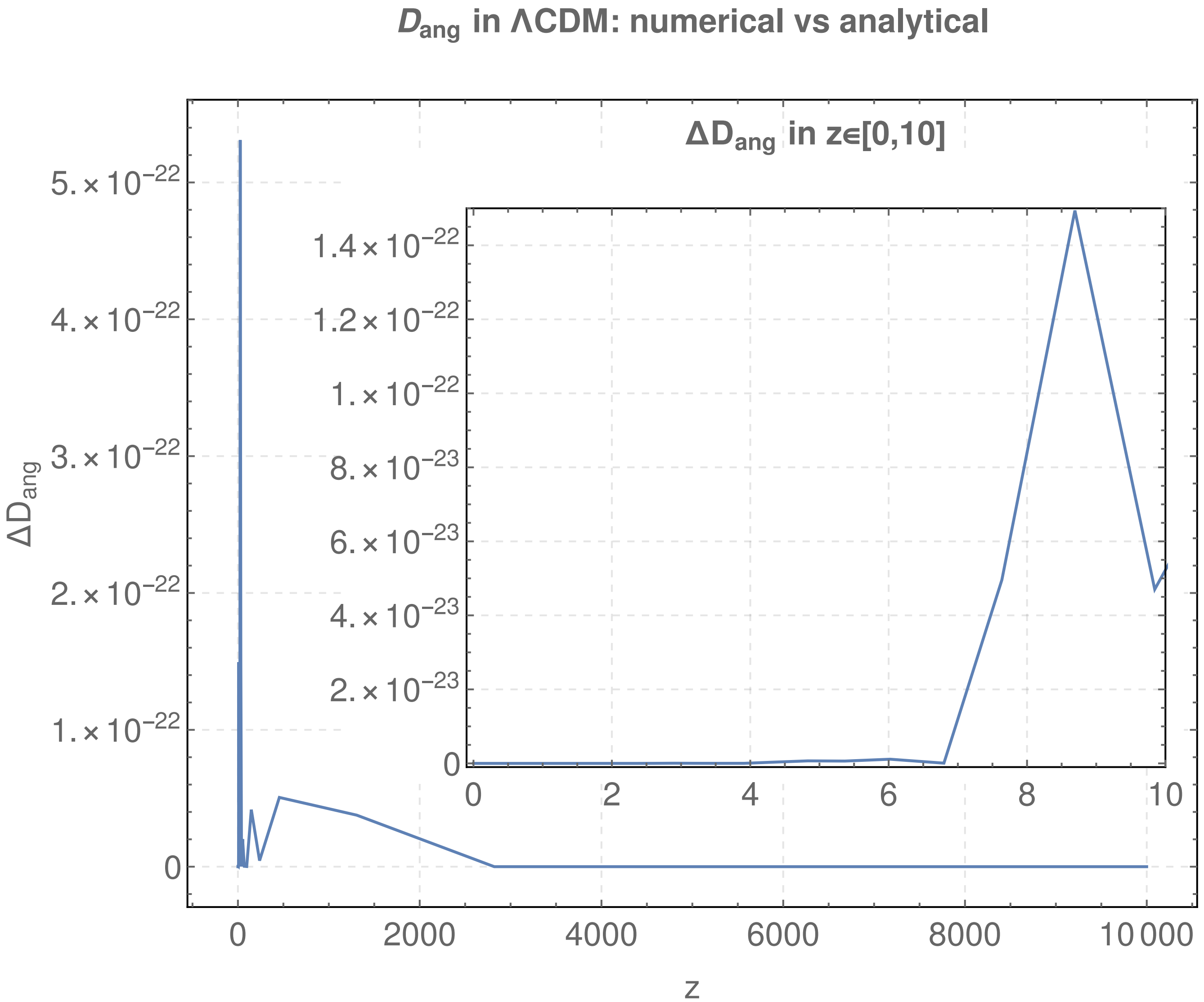
numerical with BiGONLight
vs
analytical expression

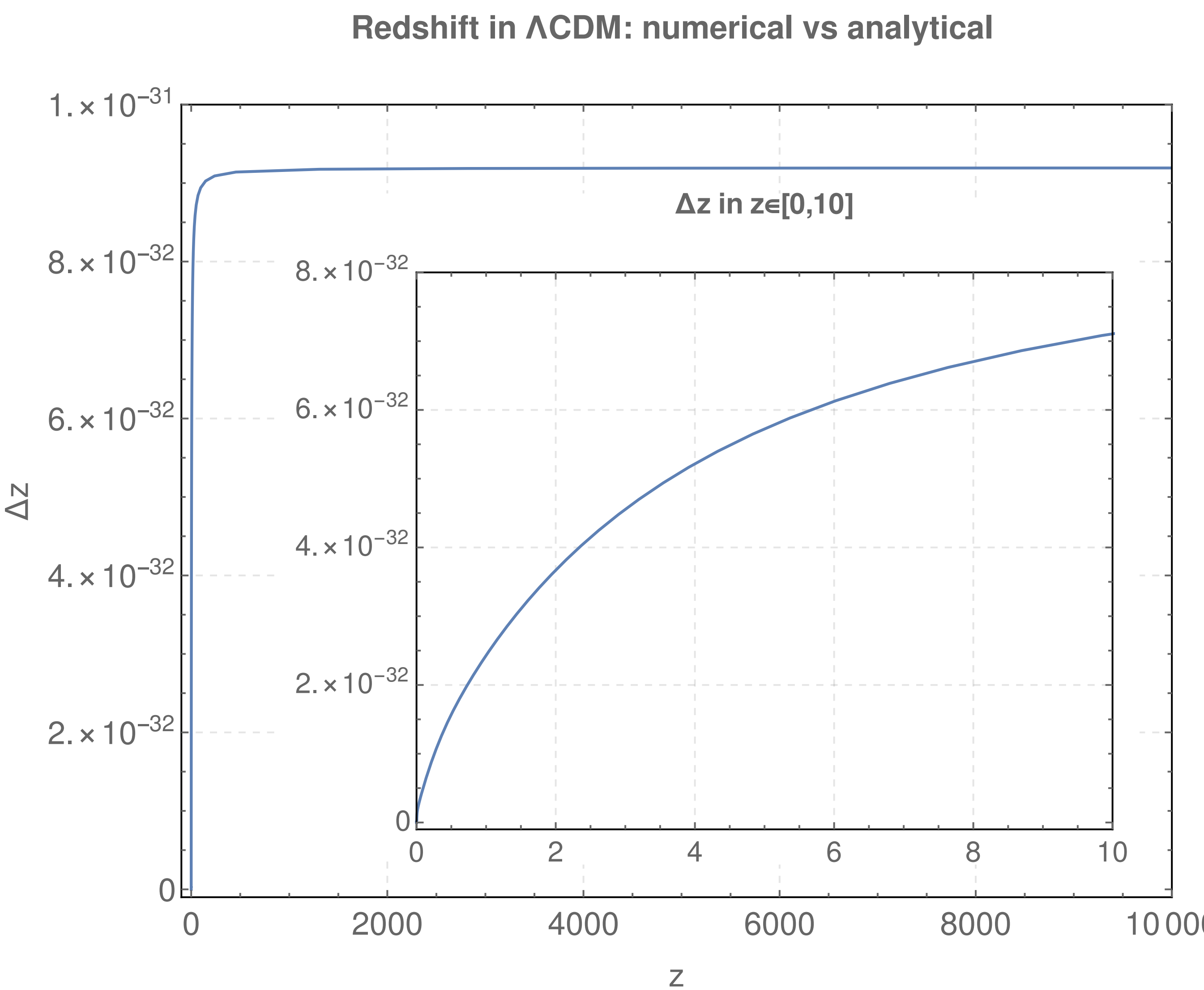
$$\text{variation} = \frac{\text{numerical} - \text{analytical}}{\text{analytical}}$$

Code Tests 1: the ΛCDM model

$$ds^2 = a^2\left\{-d\eta^2 + dq_1^2 + dq_2^2 + dq_3^2\right\}$$

$$\Delta D_{\rm ang} = \frac{D_{\rm ang}^{\rm BGO} - D_{\rm ang}^{an}}{D_{\rm ang}^{an}} \qquad \Delta z = \frac{z^{\rm BGO} - z^{an}}{z^{an}}$$





$D_{\rm ang} \sim 10^{-22}$

$z \sim 10^{-31}$

Code Tests 2:

Angular distance D_{ang} and redshift z in Szekeres model:

$$ds^2 = a^2 \left\{ -d\eta^2 + \left[A(\boldsymbol{q}) + F(\eta, q_1)\right]^2 dq_1^2 + dq_2^2 + dq_3^2 \right\}$$

$$A(\boldsymbol{q}) = 1 + \beta_+(q_1) B \left[q_2^2 + q_3^2\right] \quad \& \quad F(\eta, q_1) = \beta(q_1)\mathscr{D}(\eta)$$

N. Meures, M. Bruni (2011), **Phys.Rev. D**

numerical with **BiGONLight**

vs

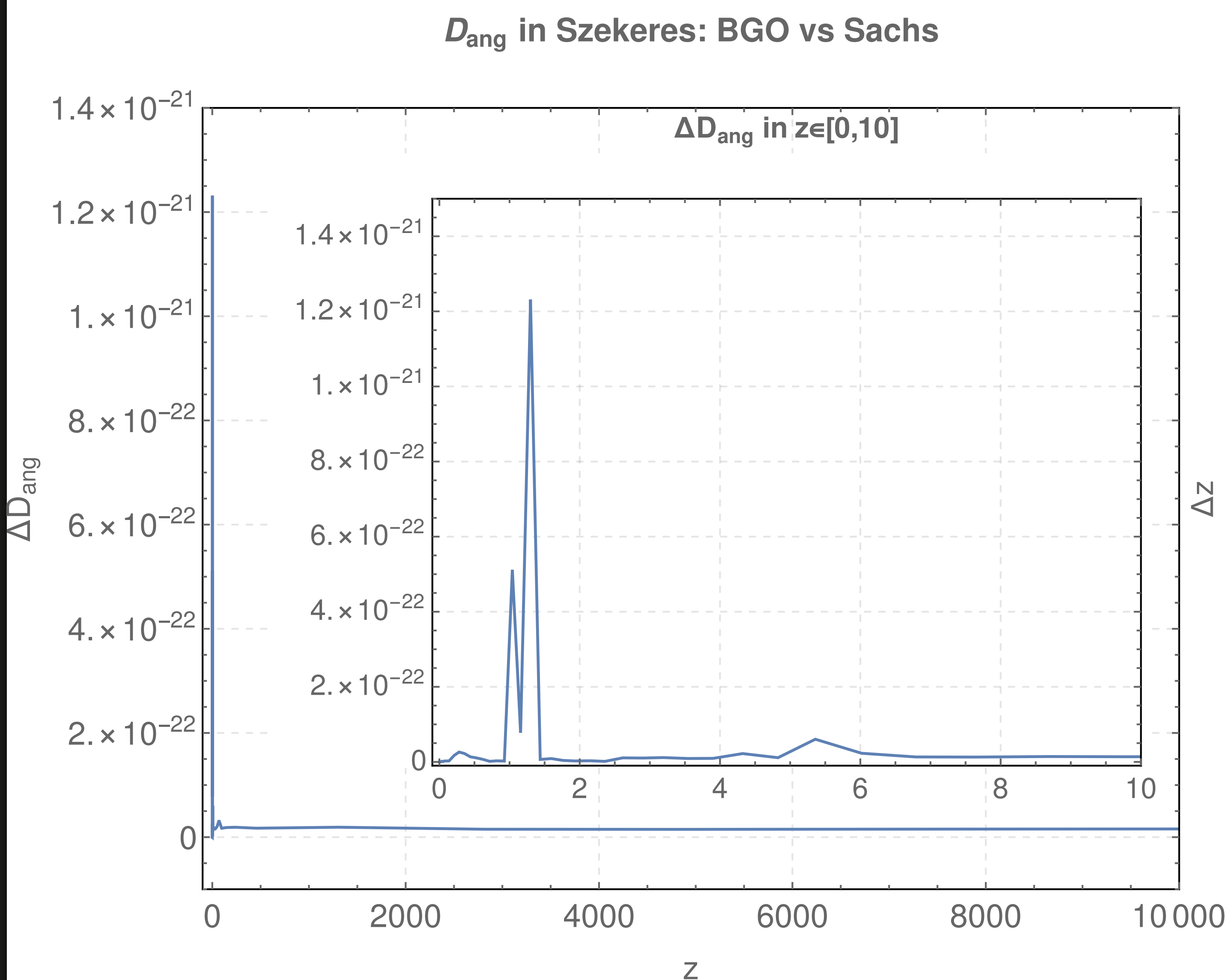
numerical using **Sachs**

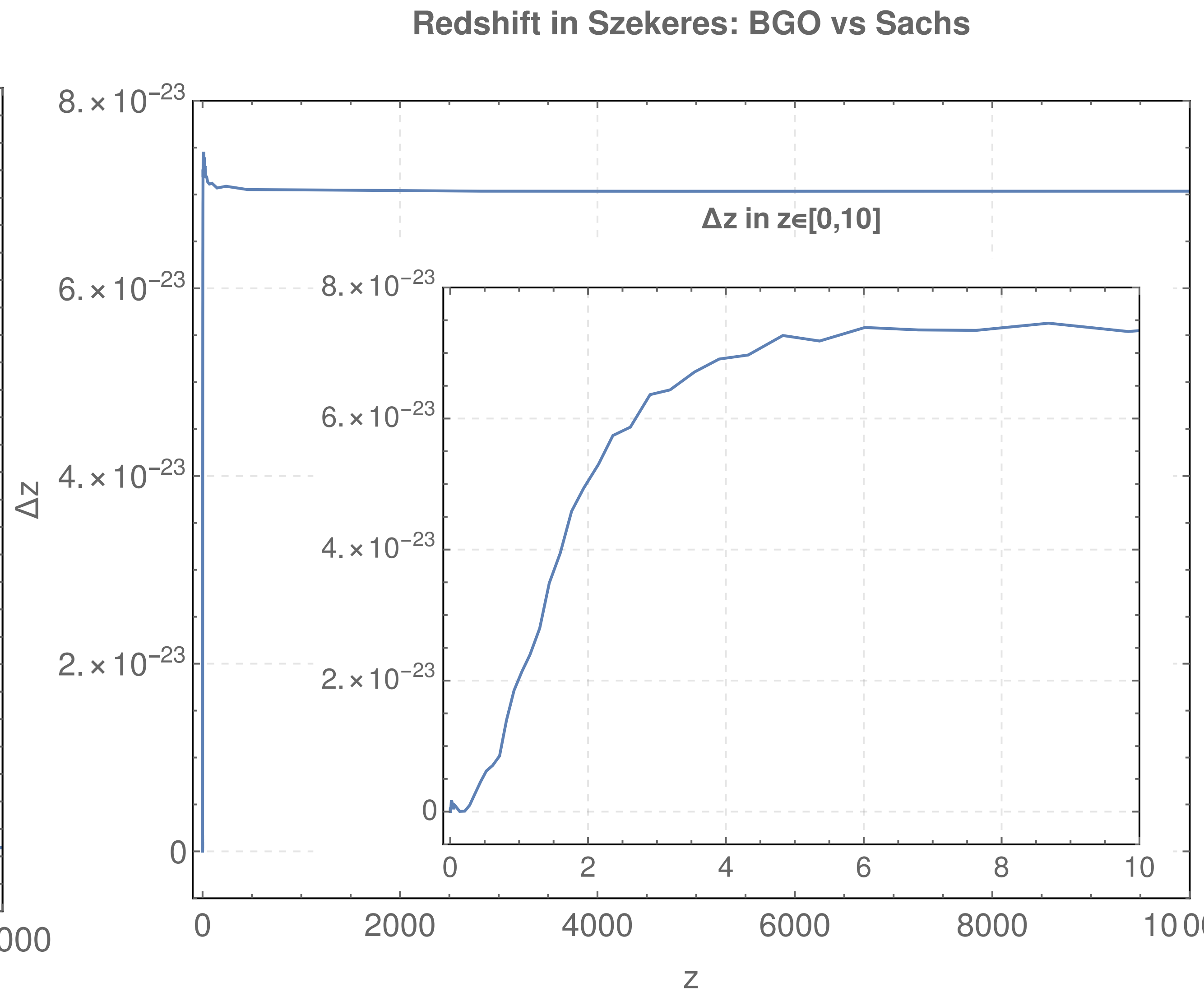
$$\text{variation} = \frac{\text{BGO} - \text{Sachs}}{\text{Sachs}}$$

Code Tests 2: Szekers metric

$$ds^2 = a^2\{-d\eta^2 + [A(\boldsymbol{q}) + F(\eta, q_1)]^2\, dq_1^2 + dq_2^2 + dq_3^2\}$$

$$\Delta D_{\rm ang} = \frac{D_{\rm ang}^{\rm BGO} - D_{\rm ang}^{\rm Sachs}}{D_{\rm ang}^{\rm Sachs}} \qquad \Delta z = \frac{z^{\rm BGO} - z^{\rm Sachs}}{z^{\rm Sachs}}$$





$D_{\rm ang} \sim 10^{-21}$ $\qquad z \sim 10^{-23}$

Code Tests 3: Redshift drift in ΛCDM model

Redshift drift, i.e. how cosmic redshift changes over time

general formula with the BGO

$$\frac{\delta \log(1+z)}{\delta t_{\mathcal{O}}} = \Xi - \left(u_{\mathcal{O}}, \frac{u_{\mathcal{E}}}{1+z}\right) \boldsymbol{U} \begin{pmatrix} u_{\mathcal{O}} \\ \frac{u_{\mathcal{E}}}{1+z} \end{pmatrix}$$

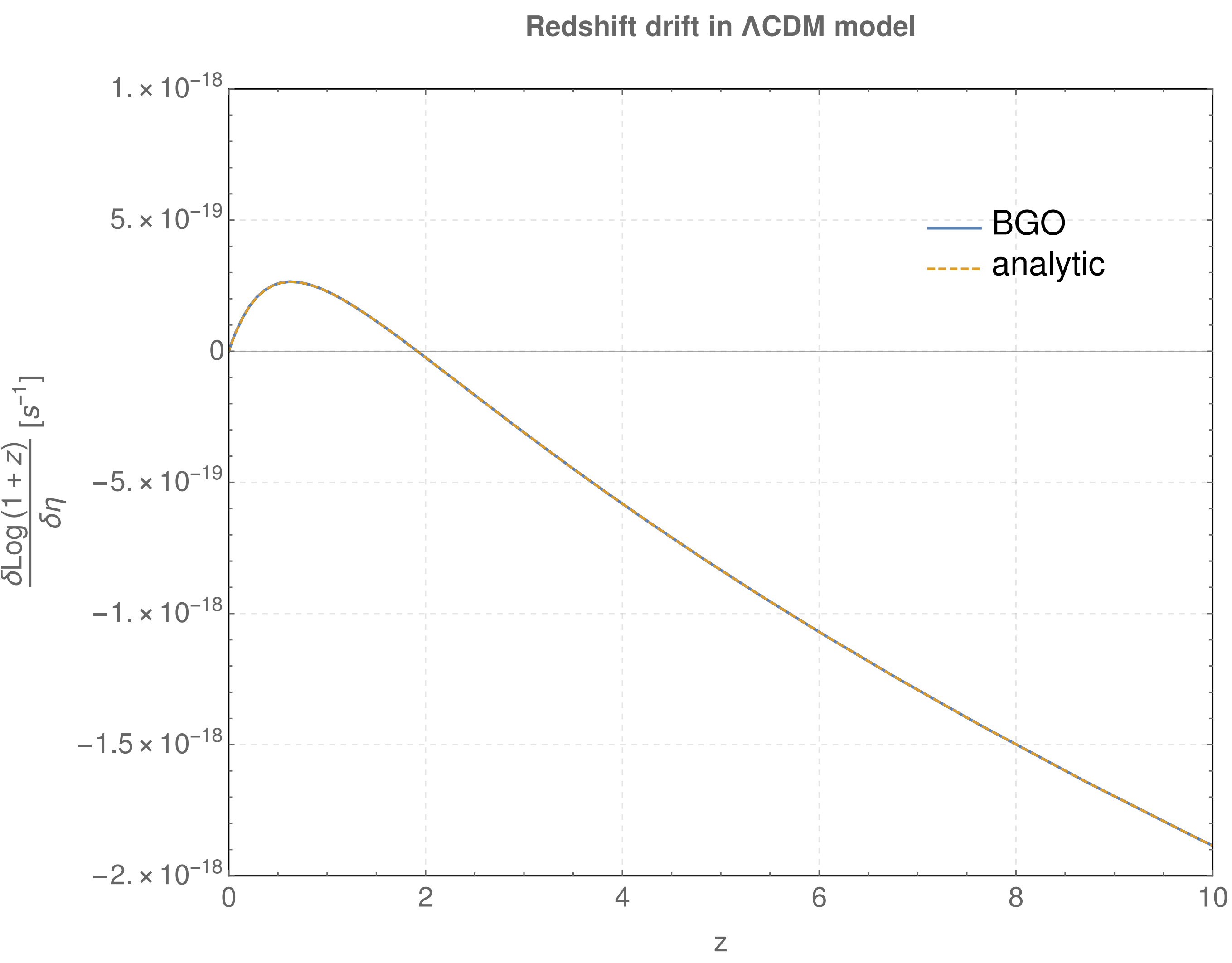
in ΛCDM reduces to

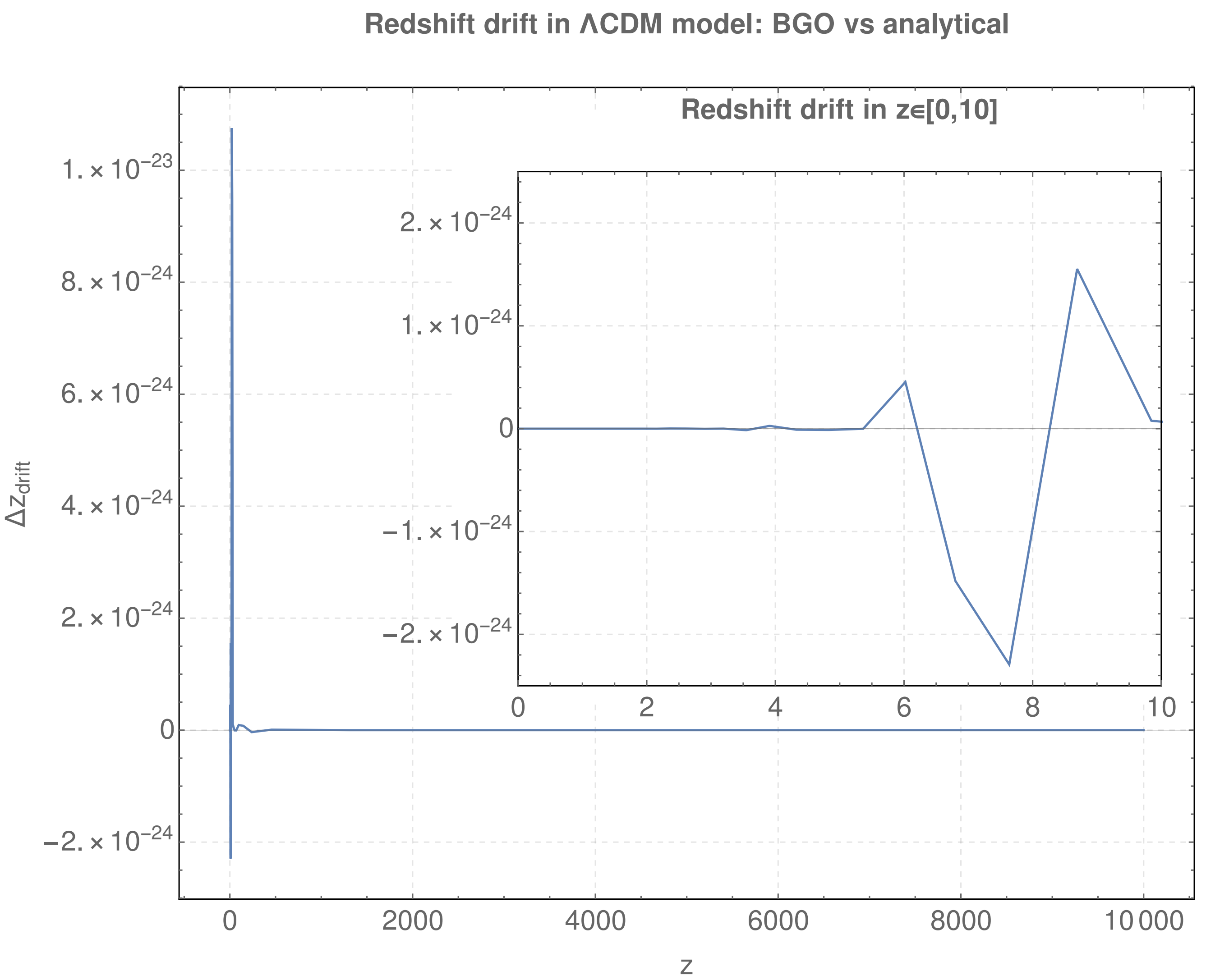
$$\frac{\delta \log(1+z)}{\delta t_{\mathcal{O}}} = H_0 - \frac{H(z)}{1+z}$$

Code Tests 3: Redshift drift in ΛCDM model

$$\frac{\delta \log(1+z)}{\delta t_{\mathcal{O}}}$$

$$\Delta z_{\text{drift}} = \frac{\text{numerical} - \text{analytical}}{\text{analytical}}$$





$$\sim 10^{-24}$$

APPLICATION: isolating non linearities in light propagation in inhomogeneous cosmology

Effects of inhomogeneties on light propagation:

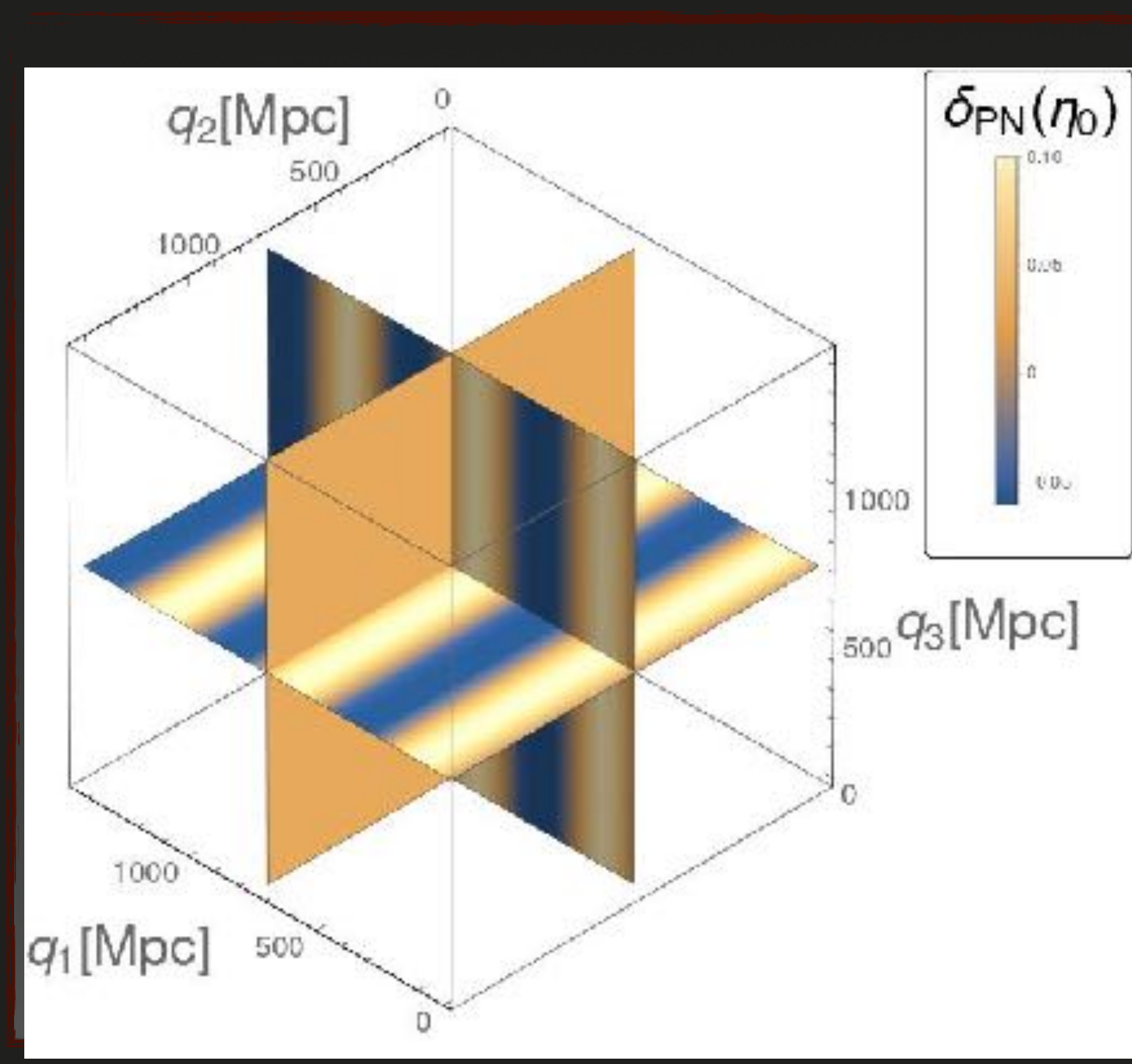
- don’t explain the distance-redshift relation

- negligible bias on cosmological parameters

APPLICATION: isolating non linearities in light propagation in inhomogeneous cosmology

Our toy-model: plane-parallel model

Λ + irrotational dust uniformly distributed along parallel sheets orthogonal to the q_1 axis



$$\phi_0(q_1)$$

APPLICATION: isolating non linearities in light propagation in inhomogeneous cosmology

$$ds^2 = a^2(\eta)\left\{-c^2 d\eta^2 + \gamma_{11}(\eta, q_1)dq_1^2 + \gamma_{22}(\eta, q_1)dq_2^2 + \gamma_{33}(\eta, q_1)dq_3^2\right\}$$

$$\gamma_{11}(\eta, q_1) = \gamma_{11}^{\mathrm{N}}(\eta, q_1) + \frac{1}{c^2}\gamma_{11}^{\mathrm{PN}}(\eta, q_1) \qquad \gamma_{22}(\eta, q_1) = 1 + \frac{1}{c^2}\gamma_{22}^{\mathrm{PN}}(\eta, q_1) \qquad \gamma_{33}(\eta, q_1) = 1 + \frac{1}{c^2}\gamma_{33}^{\mathrm{PN}}(\eta, q_1)$$

E. Villa, S. Matarrese, & D. Maino (2011). ***JCAP***.

Comparison of cosmological observables in three cases:

(I) **Newtonian** dynamics + **exact** relativistic light propagation

(II) **PN** dynamics + **exact** relativistic light propagation

(III) **Linear standard PT** for both

MG, E. Villa, M. Korzyński & S. Matarrese, in preparation

Set up

- Gravitational potential $\phi_0 = \mathscr{A} \sin(\frac{2\pi}{k} q_1)$

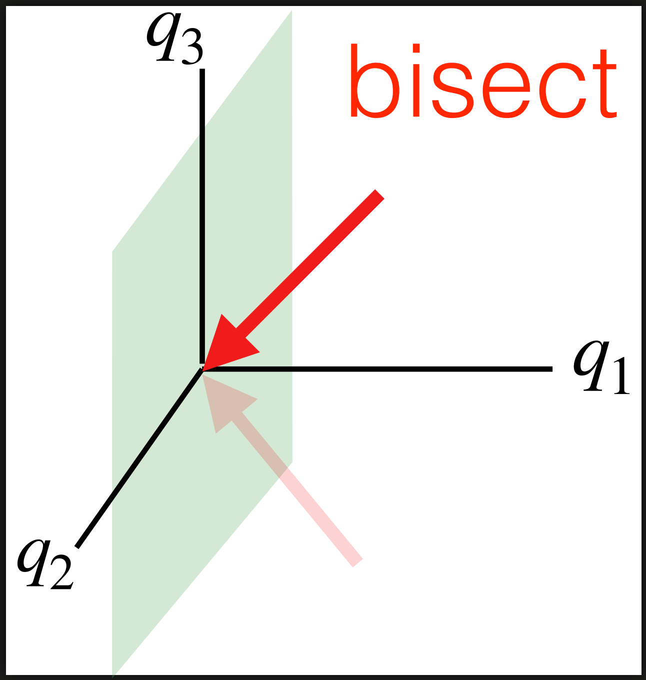
$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

The scales of perturbations:

- $k = 500\,\mathrm{Mpc}$ and $\mathscr{A}_{k=500\,\mathrm{Mpc}}$ such that $\delta_{max} = 0.1$ today
- $k = 100\,\mathrm{Mpc}$ and $\mathscr{A}_{k=100\,\mathrm{Mpc}}$ such that $\delta_{max} = 1$ today

Set up

- Gravitational potential $\phi_0 = \mathscr{A}\sin(\frac{2\pi}{k}q_1)$
- Light rays on the bisect to the planes
- Observer in $\delta_{in} = 0$



$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

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Redshift:

Linear vs Newtonian and post-Newtonian vs Newtonian

$$\Delta z = \frac{z^{Lin} - z^{N}}{z^{N}}$$

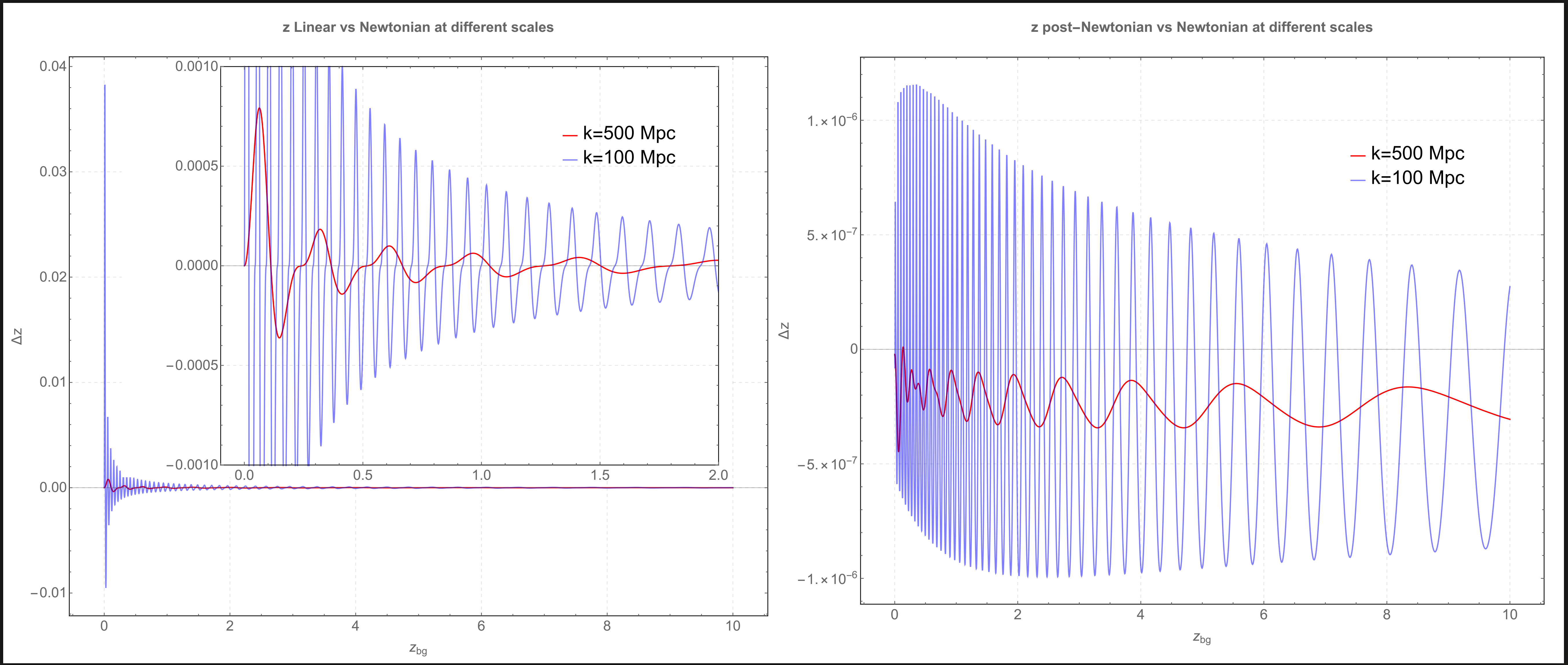
$$\Delta z = \frac{z^{PN} - z^{N}}{z^{N}}$$

Preliminary Results: Redshift

Redshift: $k = 500\,\mathrm{Mpc}$ (red), $k = 100\,\mathrm{Mpc}$ (blue)

Lin vs N

PN vs N



~ 0.038 and ~ 0.0008

$\sim 10^{-6}$ and $\sim 10^{-7}$

Preliminary Results

Angular diameter distance:

Linear and post-Newtonian

vs

Newtonian

and

Linear

vs

post-Newtonian

$$\Delta D_{\text{ang}} = \frac{D_{\text{ang}}^{Lin} - D_{\text{ang}}^{N}}{D_{\text{ang}}^{N}}$$

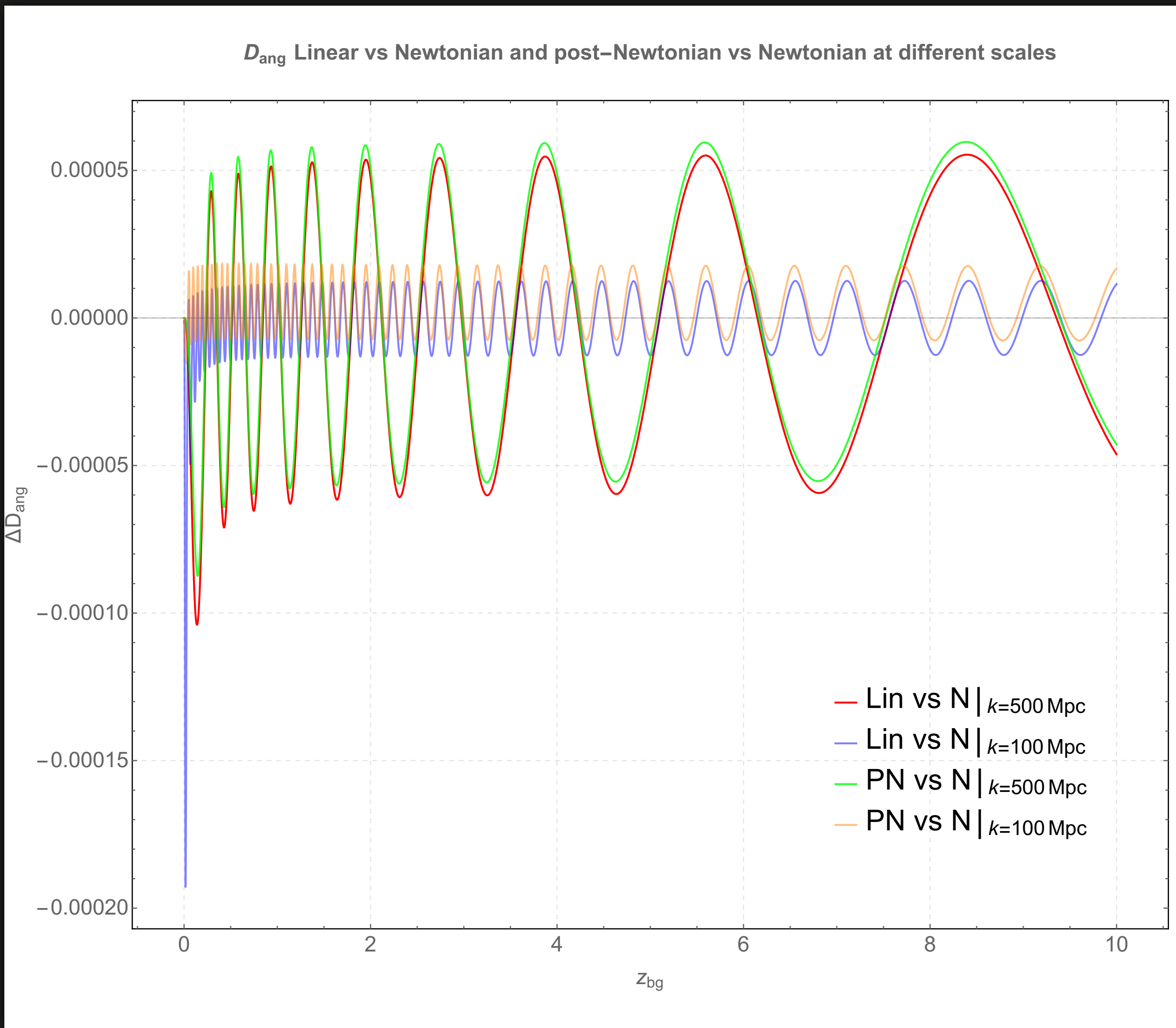
$$\Delta D_{\text{ang}} = \frac{D_{\text{ang}}^{PN} - D_{\text{ang}}^{N}}{D_{\text{ang}}^{N}}$$

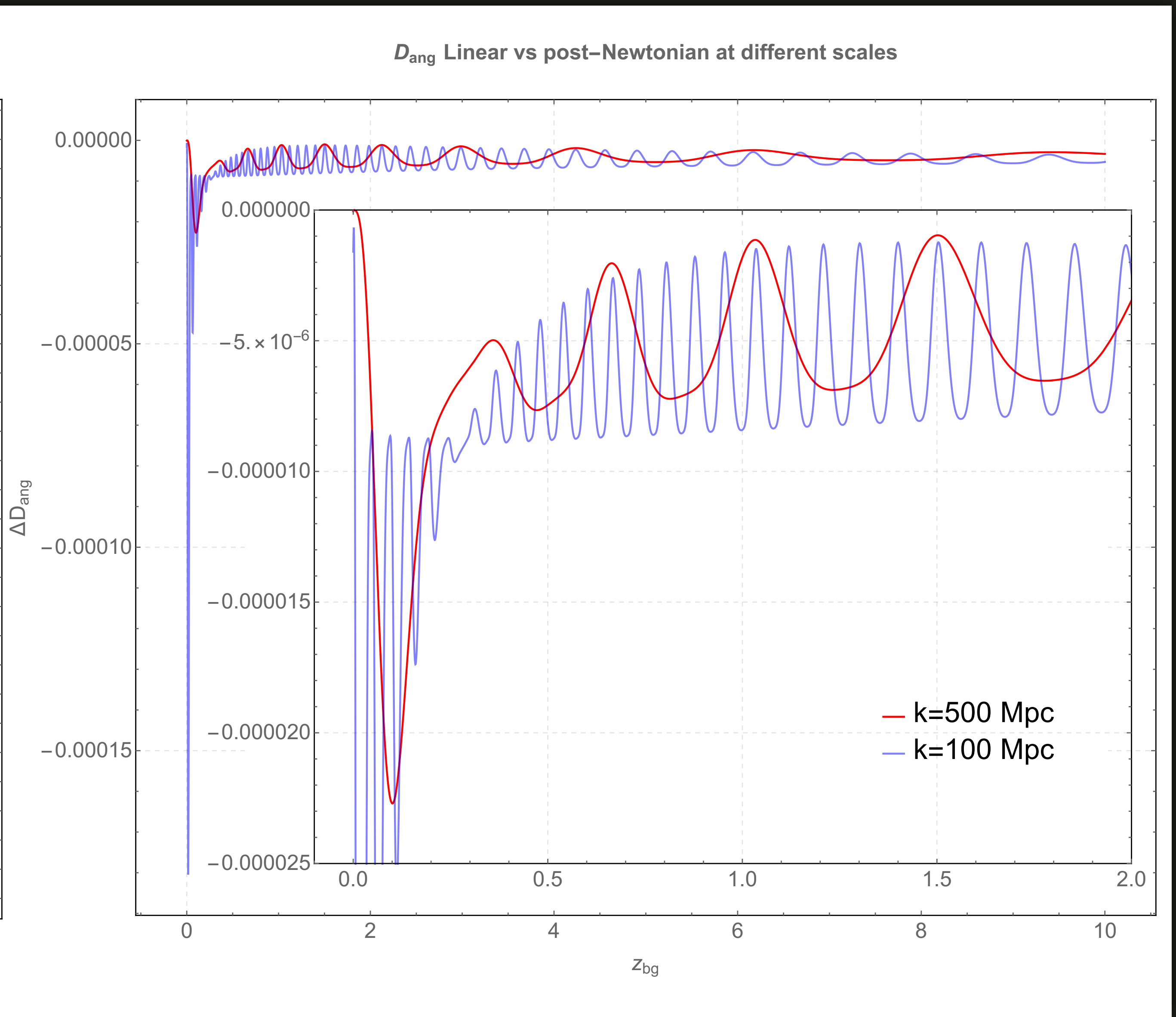
$$\Delta D_{\text{ang}} = \frac{D_{\text{ang}}^{Lin} - D_{\text{ang}}^{PN}}{D_{\text{ang}}^{PN}}$$

Preliminary Results: Angular diameter distance

Lin vs N & PN vs N

Lin vs PN





$k = 500\,\mathrm{Mpc}{:}\ \sim 5\times10^{-5}$

$k = 100\,\mathrm{Mpc}{:}\ \sim 2\times10^{-5}$

both k $\sim 3\times10^{-6}$

Preliminary Results: summary

1. Redshift: Newtonian approximation is the leading order for non-linearities. Post-Newtonian small corrections $\sim 10^{-7} - 10^{-6}$

2. Angular diameter distance:

   - $\Delta D_{\rm ang}(Lin,\, PN) \sim 10^{-6}$

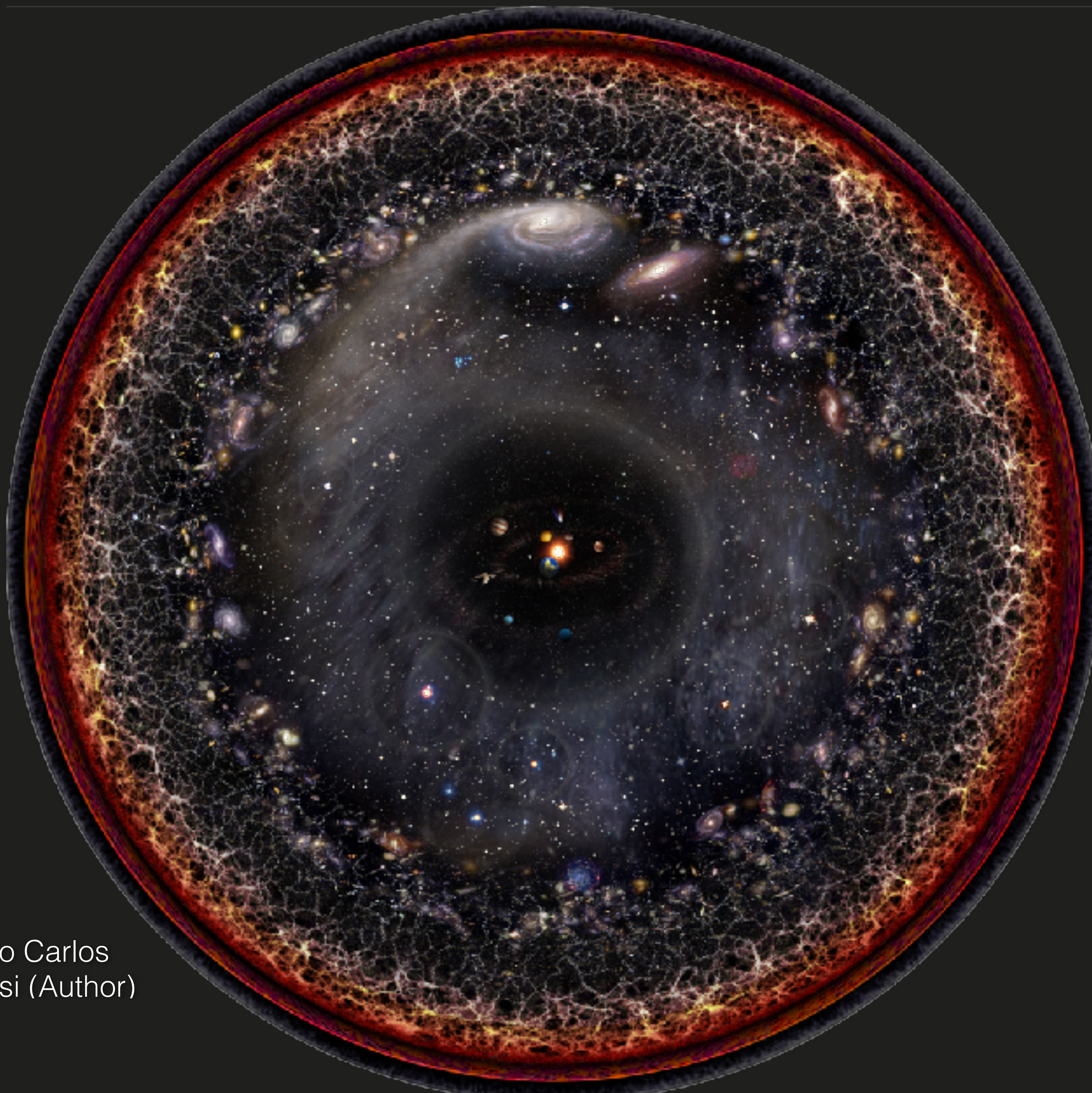
   - $\Delta D_{\rm ang}(Lin,\, N)$ & $\Delta D_{\rm ang}(PN,\, N) \sim 10^{-5}$

Work in progress

1. Geodesic directions w.r.t. the planes
2. Varying perturbation scales and superposition
3. Varying observer position
4. Different gauge choices
5. Other observables: redshift and position drifts

Take-home messages

- BGOs as unified framework for **all** the observables and **all** scales
- BiGONLight.m valid numerical tool for **high precision** light propagation



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