Jakub Gizbert-Studnicki

in collaboration with

J. Jurkiewicz†, J. Ambjørn, A. Görlich, D. Németh, Z. Drogosz, M. Reitz, D. Coumbe, G. Czelusta

Four-dimensional CDT
the status report

Warsaw, 21st April 2023

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

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NATIONAL SCIENCE CENTRE

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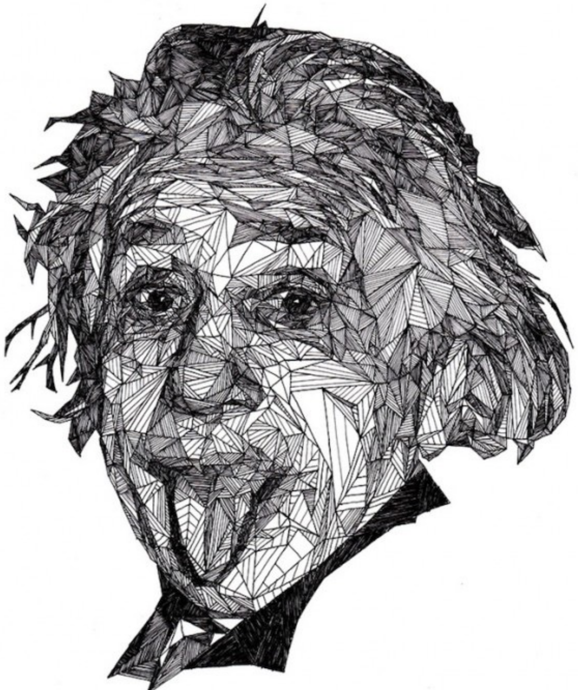
- *As early as in 1916 Einstein* pointed out that „quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation”*
- *After more than 100 years a complete, consistent quantum theory of gravity (QG) is still missing*
- *The aim: construct a fundamental theory of QG as a unitary, non-perturbative, diffeomorphism-invariant theory of dynamical geometry and study its properties in a Planckian regime*
- *We have a number of interesting but incomplete research programs*
  - *string theory*
  - *loop quantum gravity*
  - *group field theory*
  - *causal set theory*
  - *noncommutative geometry*
  - *asymptotic safety*
  - *lattice approaches*
  - *...*


A. Einstein triangulation by J. Bryan

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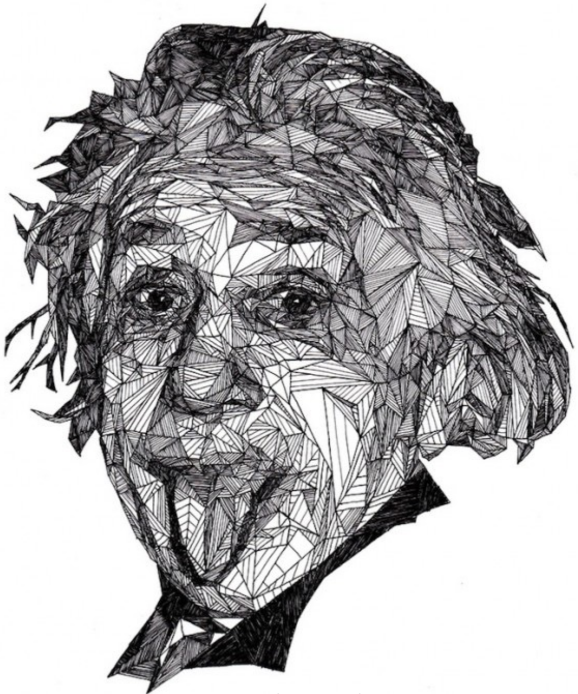

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- *GR treated as a QFT is perturbatively non-renormalizable in d > 2 dimensions**
- *But it could be renormalizable in a non-perturbative regime*
  - *asymptotic safety idea (S. Weinberg)*
  - *renormalization group flow can lead to a non-Gaussian UV fixed point*
- *Lattice formulation would allow to study a unitary, non-perturbative, background-independent and diffeomorphism-invariant quantum gravity*
  - *we need a dynamical lattice (DT)*
  - *UV fixed point should be associated with a 2nd order phase transition*
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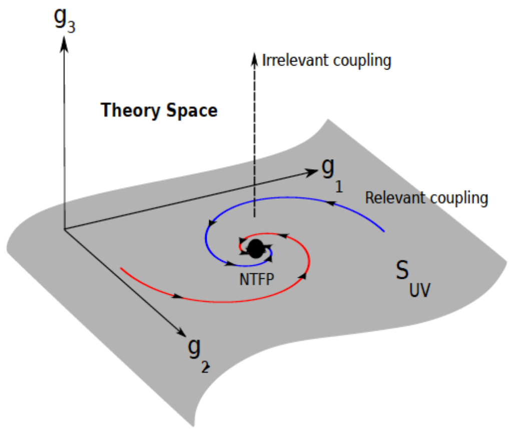

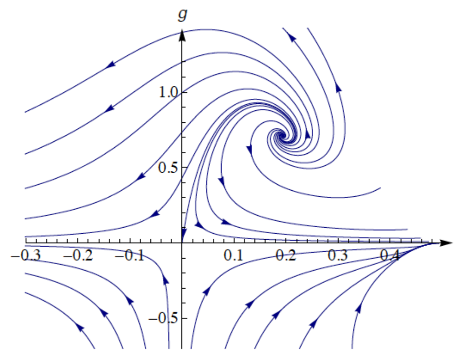

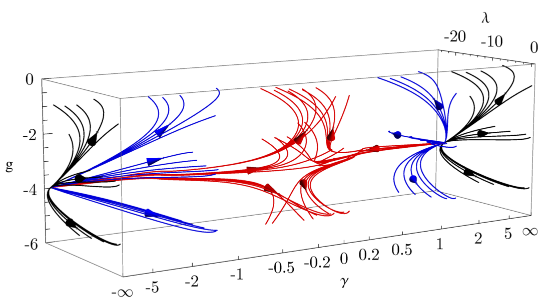

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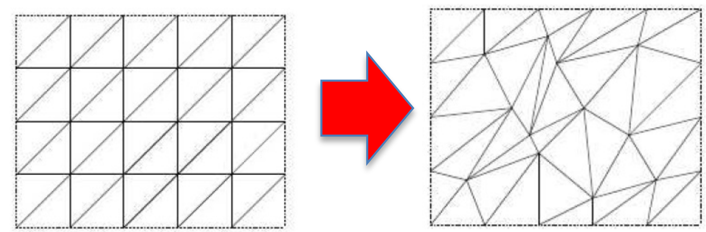

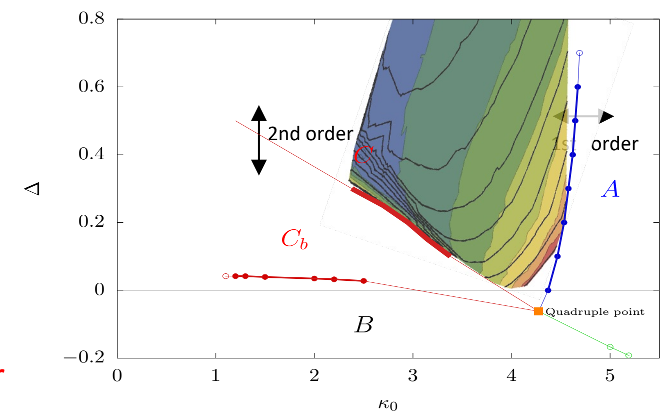

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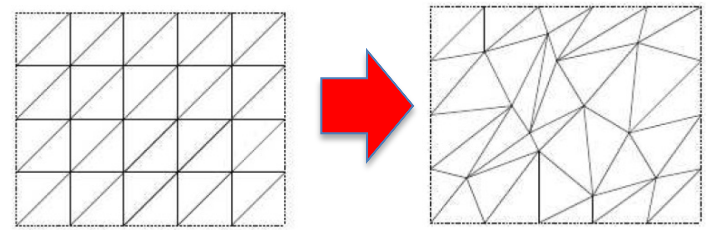

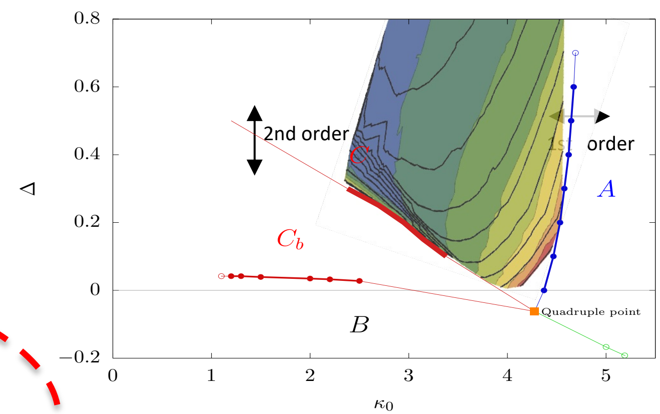

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Outline

- ✧ *Causal Dynamical Triangulations (CDT)*
- ✧ *Phase structure in spherical CDT*
- ✧ *Semi-classical phase C_{dS}*
- ✧ *Phase transitions in spherical CDT*
- ✧ *Search for a continuum limit*
- ✧ *Toroidal vs spherical topologies*
- ✧ *Conclusions*
- ✧ *Semiclassical coordinates & scalar fields in CDT (if time permits)*

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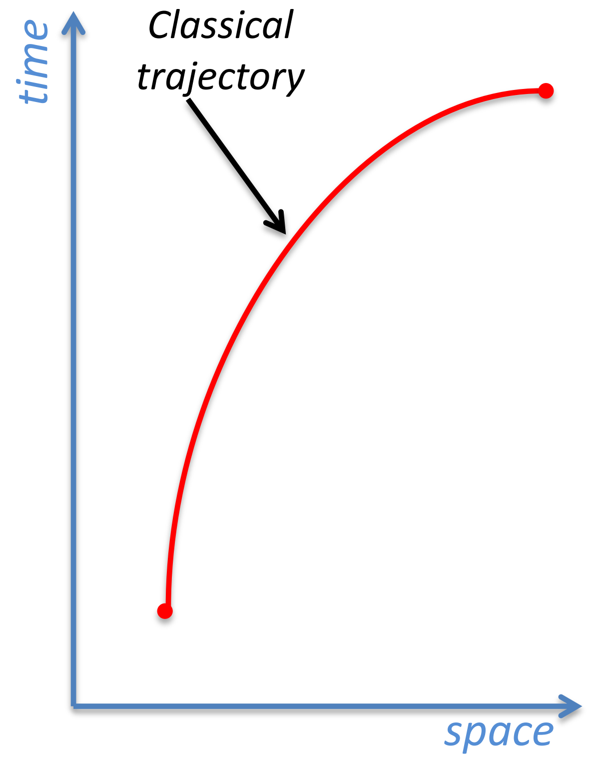

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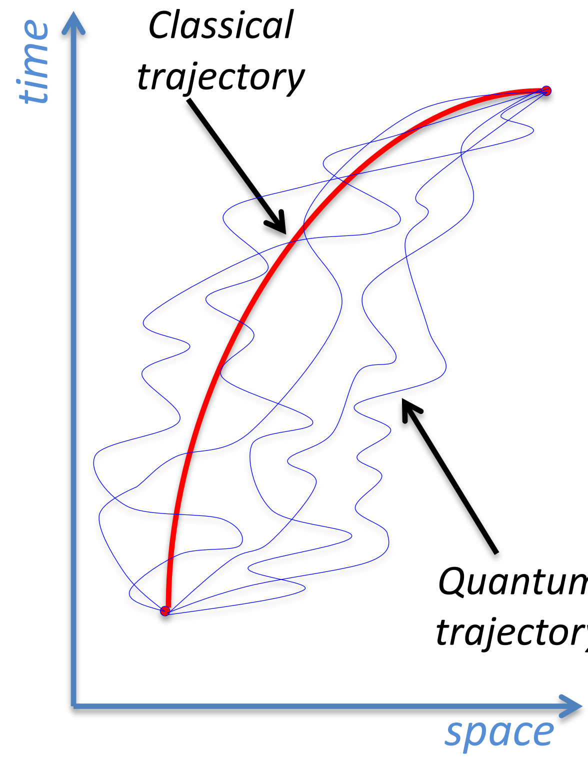

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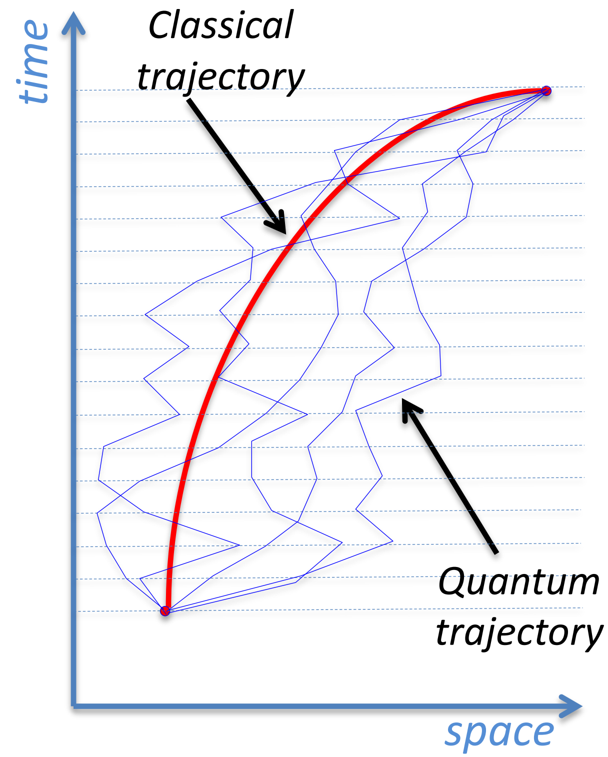

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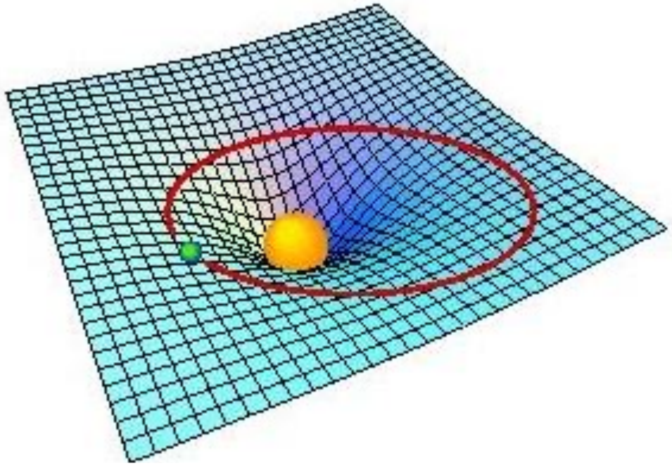

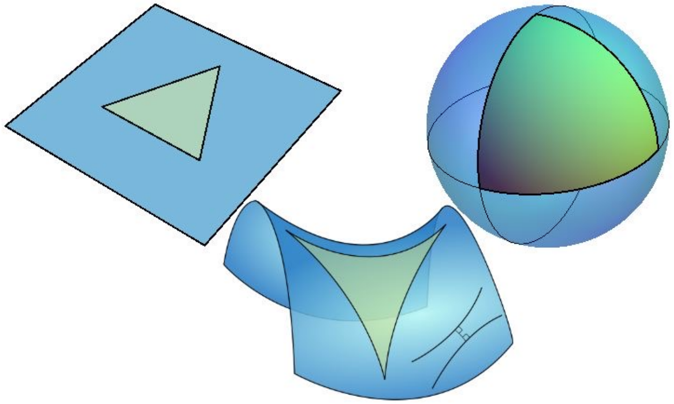

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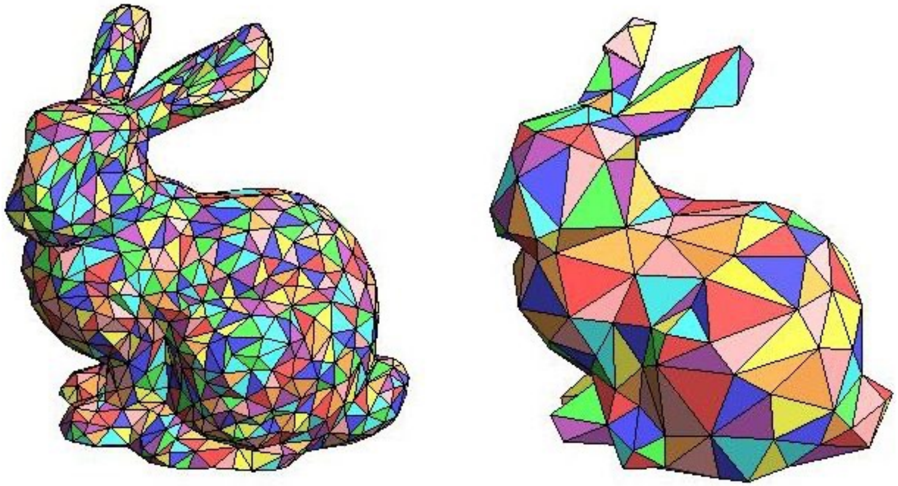

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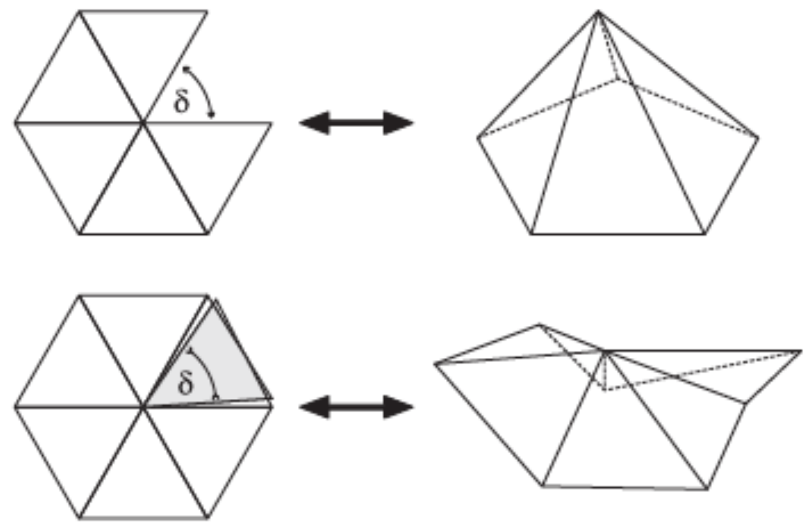

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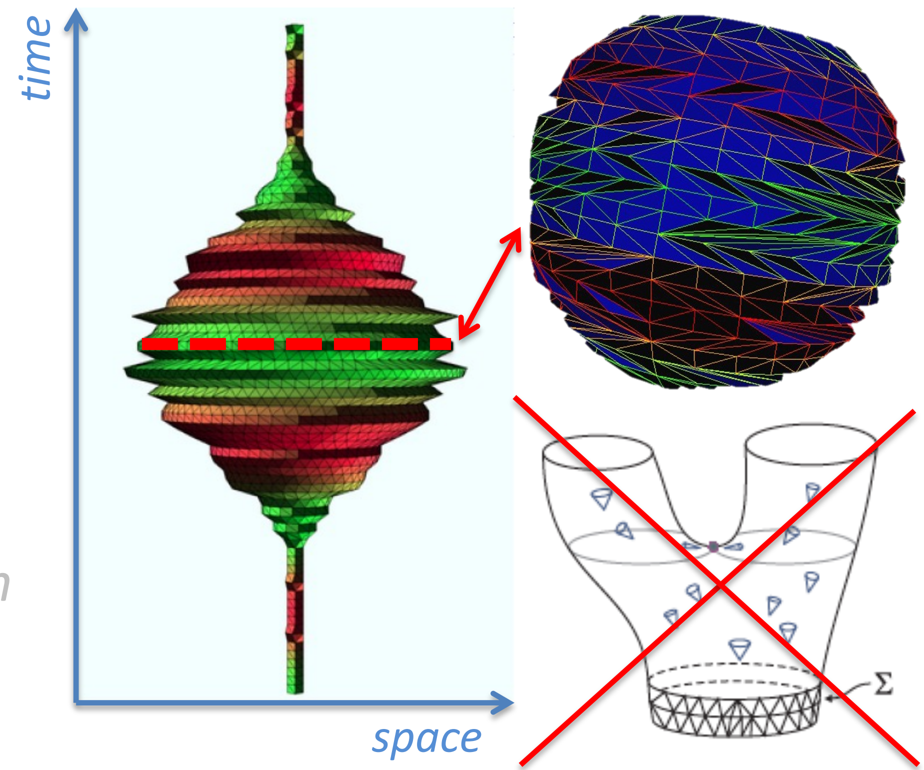

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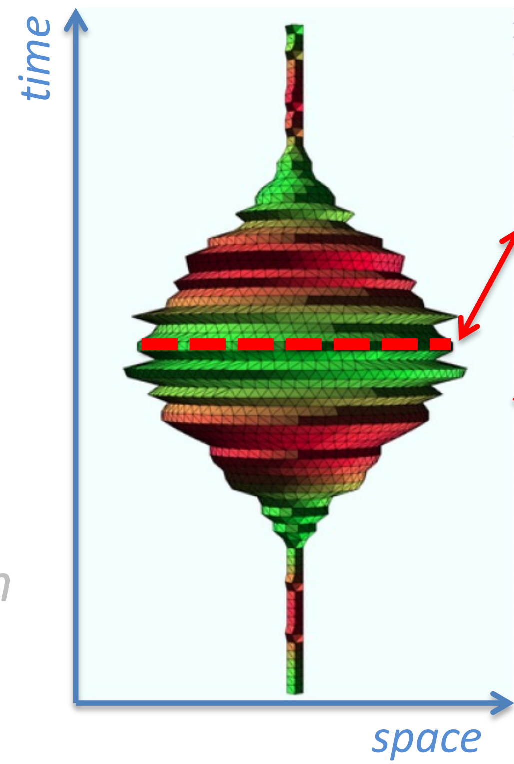

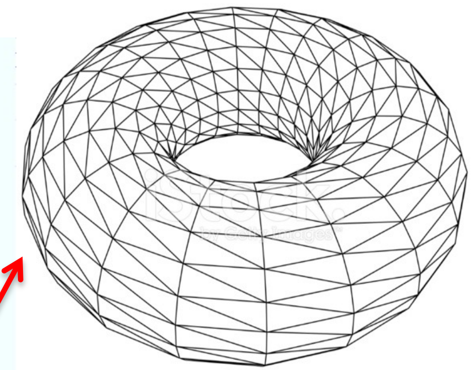

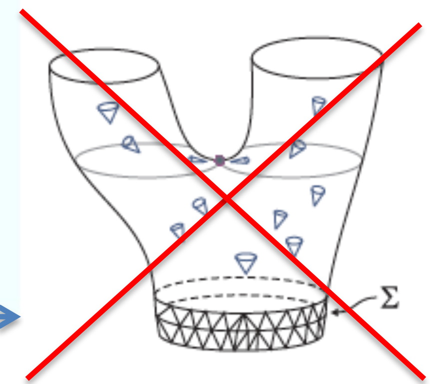

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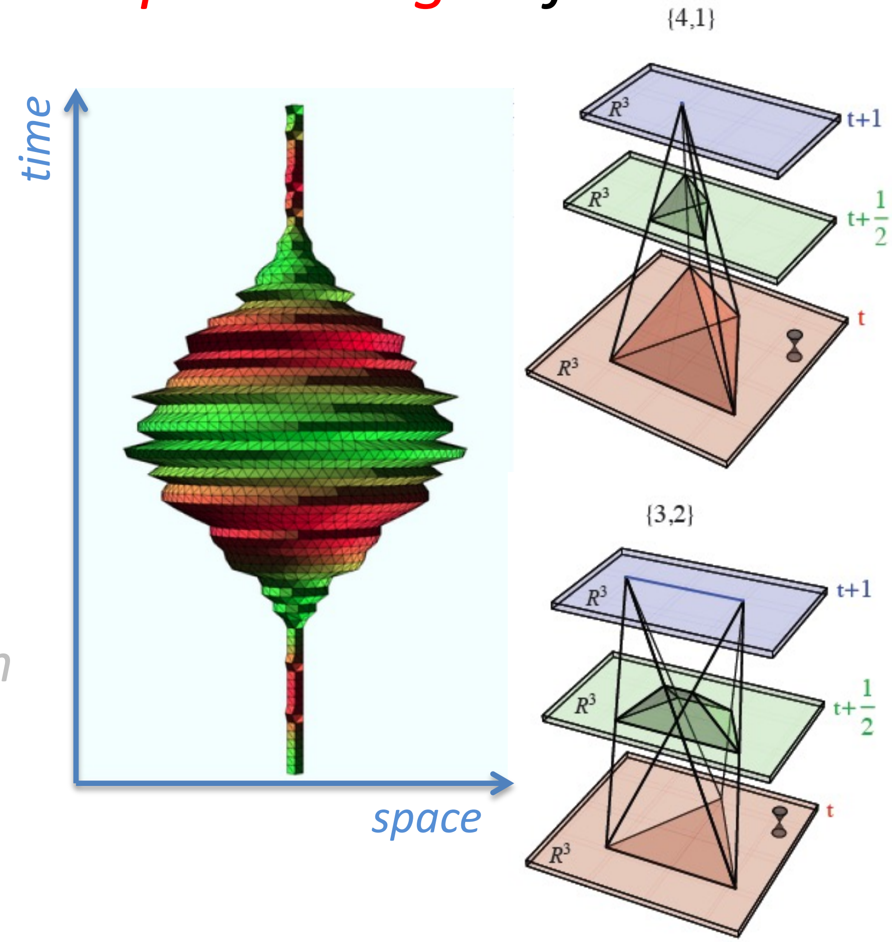

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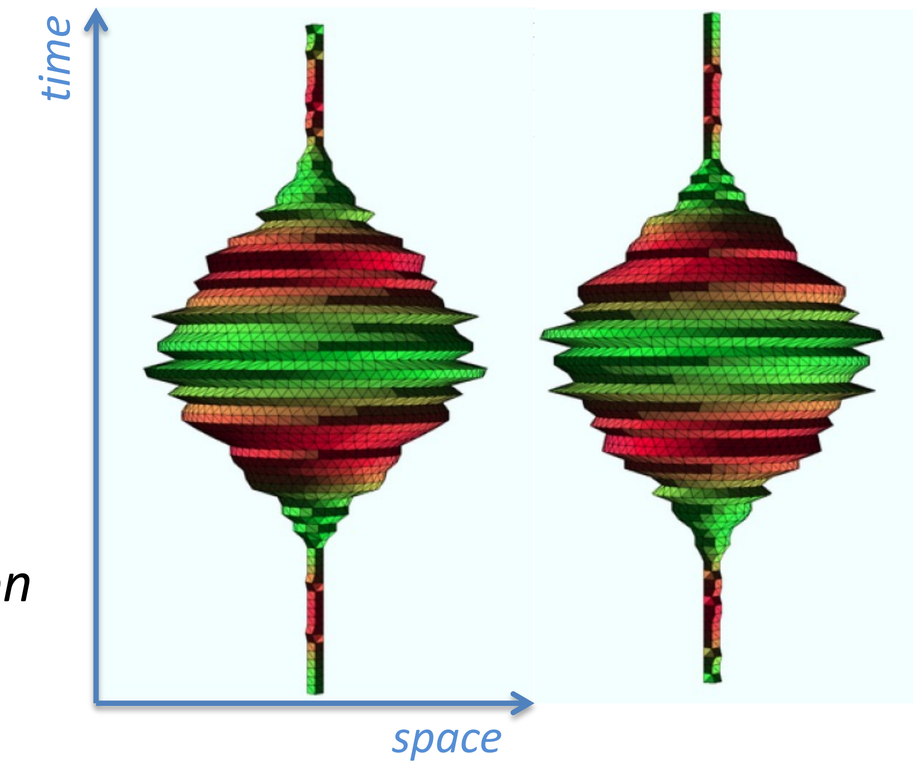

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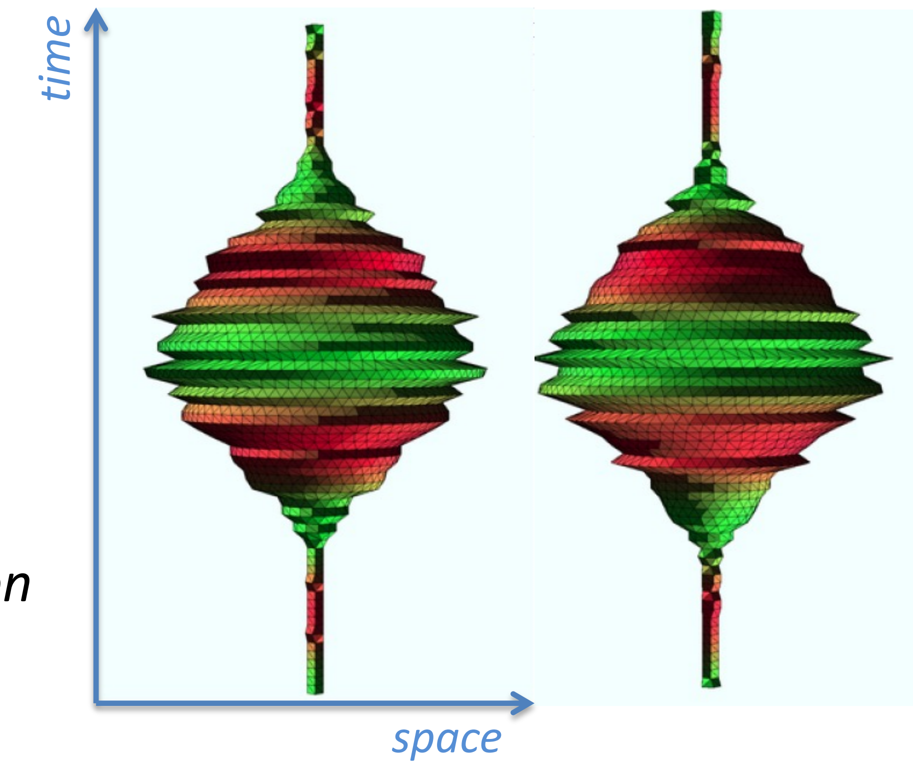

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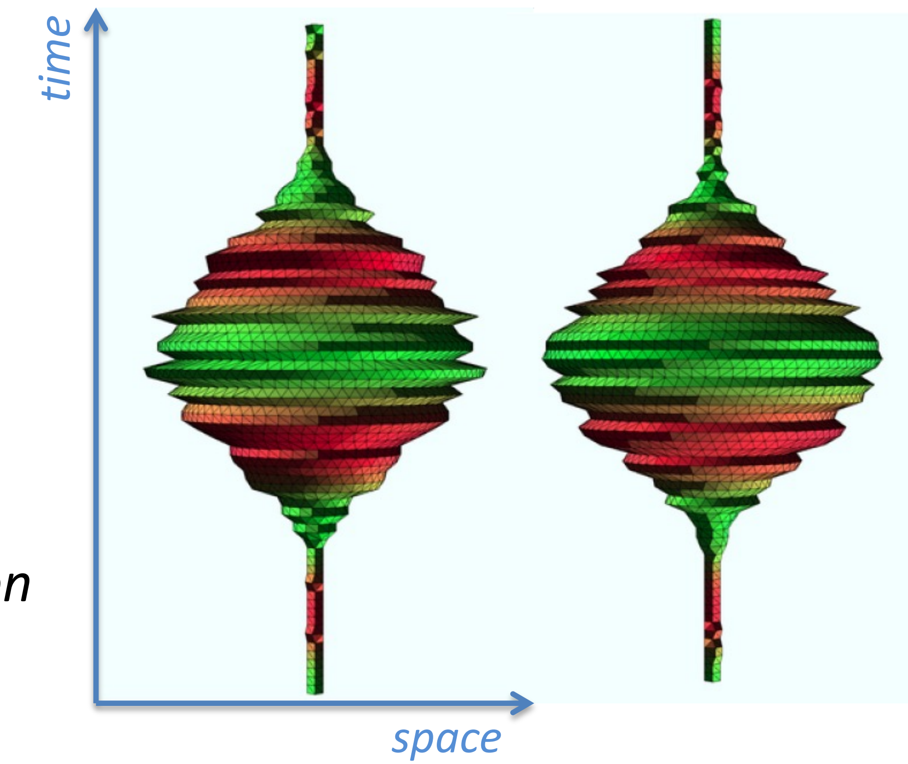

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$$S_{HE} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R-2\Lambda\right)$$

  - *CDT is formulated in a coordinate free way (Regge calculus)*
  - *Three coupling constants: k_0, K_4, Δ*
  - *After Wick's rotation: „random" geometry system*

$$Z = \int_{trajectories} D[g_{\mu\nu}]\exp(iS_{HE}[g_{\mu\nu}])$$

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vertices # 4-simplices # (4,1) 4-simplices

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$$\begin{array}{ccccccc} S_R = & -k_0 N_0 & + & K_4 N_4 & + & \Delta\left(N_4^{(4,1)}-6N_0\right) \\ & \Updownarrow & & \Updownarrow & & \Updownarrow \\ & 1/G & & \Lambda & & \alpha\ (l_t^2=\alpha l_s^2) \end{array}$$

$\alpha \rightarrow -\alpha$

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$$S_e = \ln\Omega$$

CDT

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CDT

- *4-dim CDT can be investigated using Monte Carlo techniques*
  - *The algorithm performs random walk in the space of triangulations*
  - *The walk consists of a series of local moves* (4 moves + 4 antimoves)*
  - *The moves are causal (preserve local & global topology) …*
  - *… and ergodic (any triangulation is achievable by a sequence of moves)*
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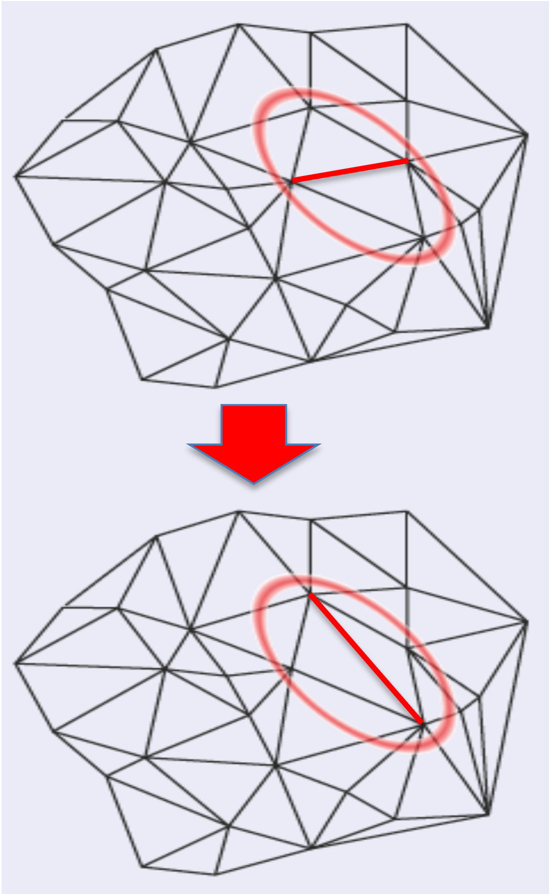

* Example of a move in 2-dim

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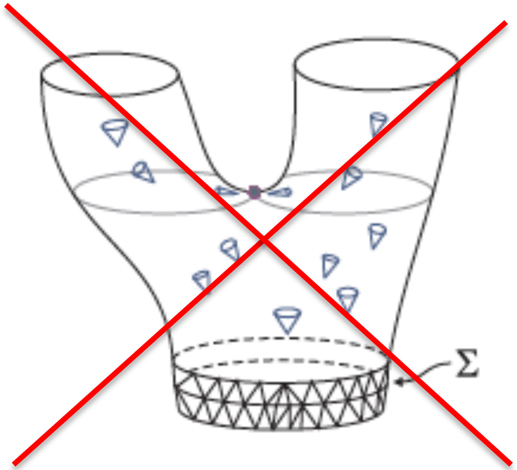

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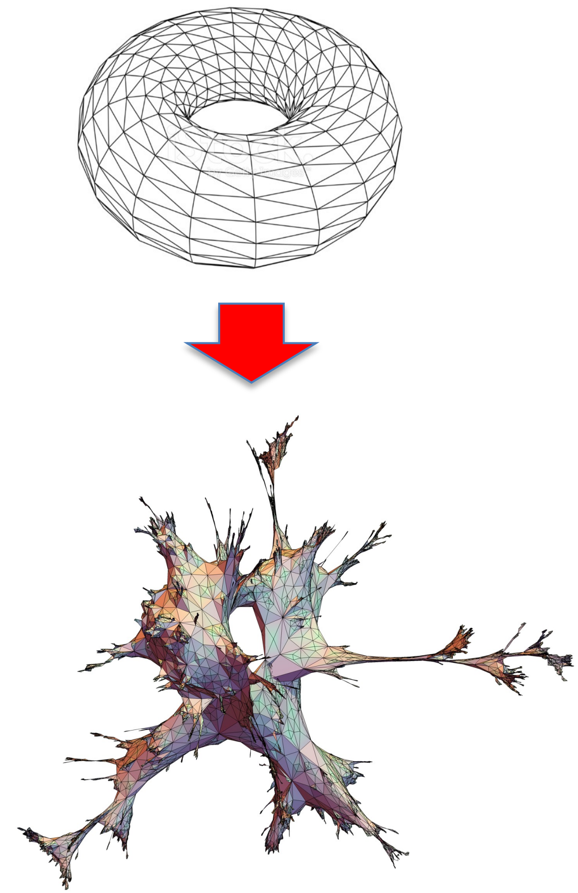

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$$P(T) = \frac{1}{Z} e^{-S_R[T]}$$

$$P(T_1) P(T_1 \xrightarrow{M} T_2) = P(T_2) P(T_2 \xrightarrow{M^*} T_1)$$

CDT

- ✧ 4-dim CDT can be investigated using Monte Carlo techniques
  - ✧ The algorithm performs random walk in the space of triangulations
  - ✧ The walk consists of a series of local moves* (4 moves + 4 antimoves)
  - ✧ The moves are causal (preserve local & global topology) …
  - ✧ … and ergodic (any triangulation is achievable by a sequence of moves)
  - ✧ Probability of a move is determined by a detailed balance condition
  - ✧ One can compute expectation values or correlators of observables

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$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

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$$\langle O \rangle = \frac{1}{Z}\sum_T O[T] e^{-S_R[T]} \approx \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} O[T^{(i)}]$$

Phase structure in S^3 CDT

lattice time lattice spacing

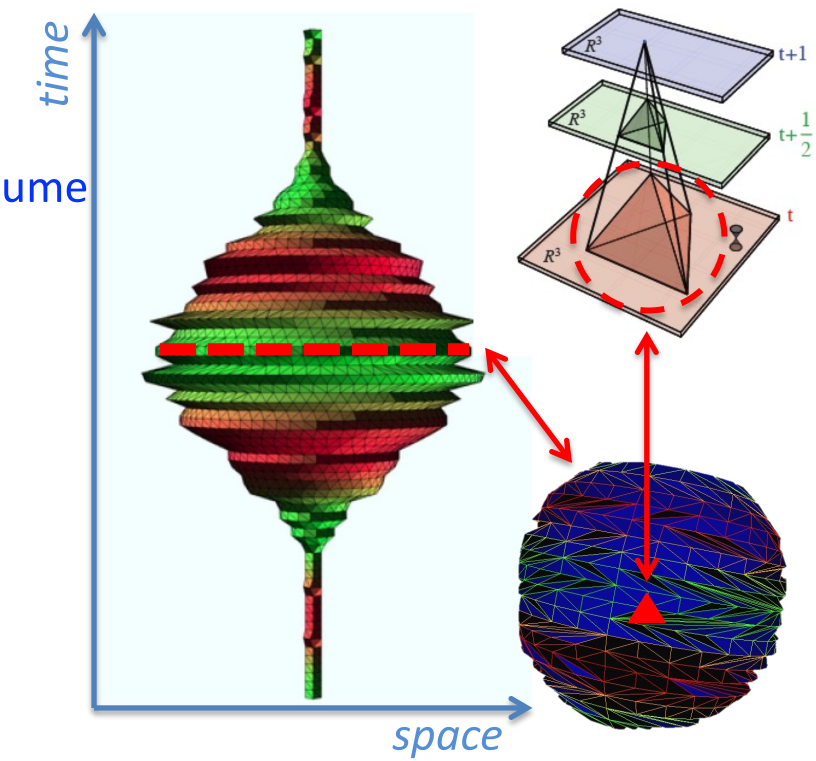
physical proper time $t_i = i \cdot l_t$

# of tetrahedra at lattice time i

- *The observable: 3-volume of spatial layers (foliation leaves of the global proper time):* $V_3(t_i) \propto n_i \equiv N_4^{(4,1)}(i)$
- *Four phases (A, B, C_{dS}, C_b) of various geometry were discovered*

physical 3-volume

- *In order to distinguish between the C_{dS} and C_b phases one can measure*
  - *the Hausdorff dimension: d_H*
  - *the Spectral dimension: d_S*



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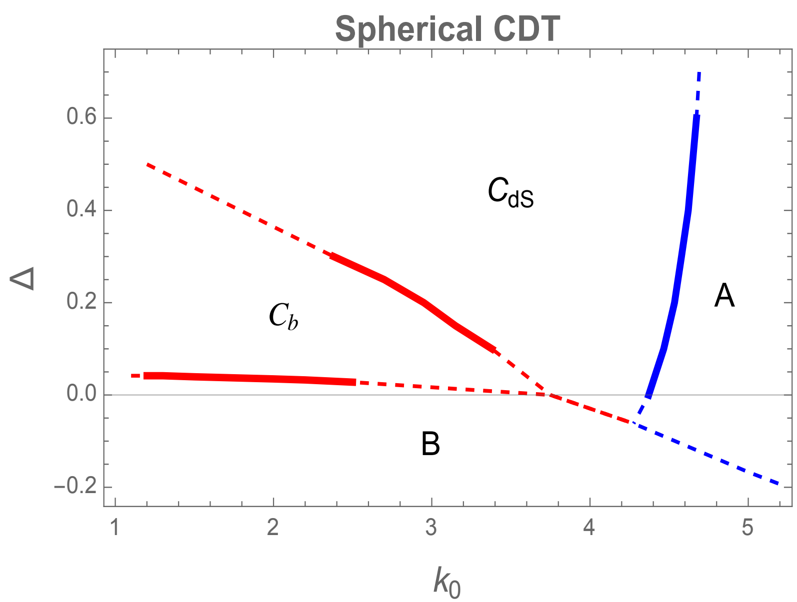

$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

$1/G \qquad \Lambda \qquad \alpha \quad (l_t^2 = \alpha l_s^2)$

We perform MC simulations with fixed lattice volume N_4

The cosmological constant K_4 is tuned to N_4

We effectively have two coupling constants: k_0 and Δ

Phase structure in S^3 CDT

- ✧ *The observable: 3-volume of spatial layers (foliation leaves of the global proper time):* $V_3(t_i) \propto n_i \equiv N_4^{(4,1)}(i)$
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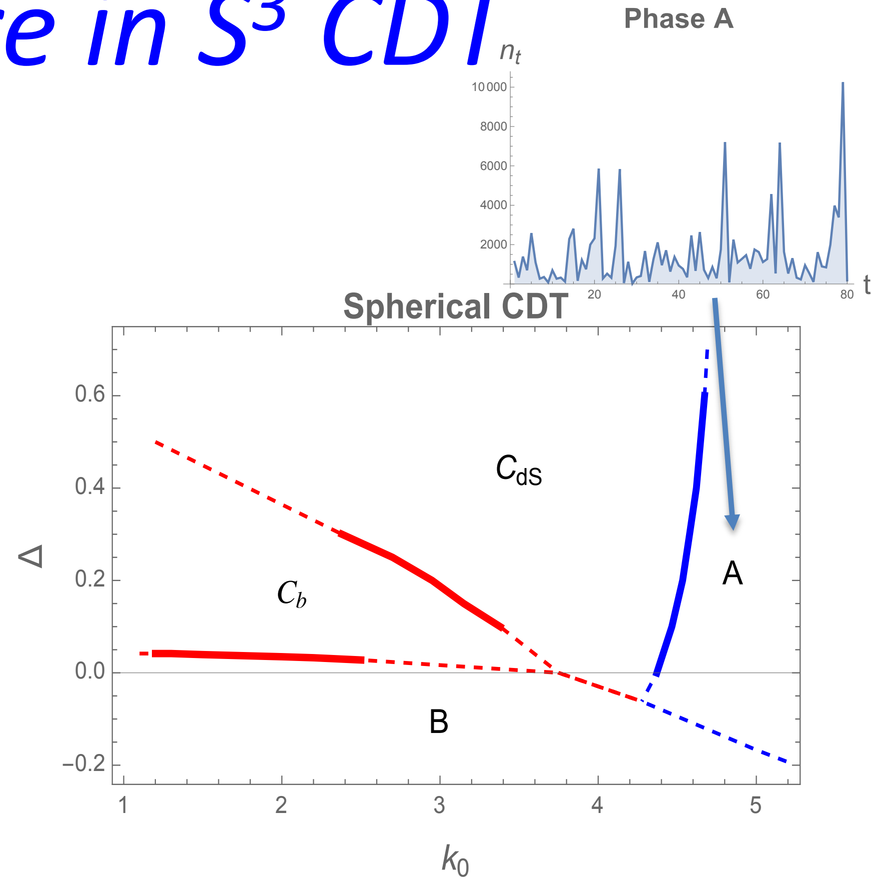

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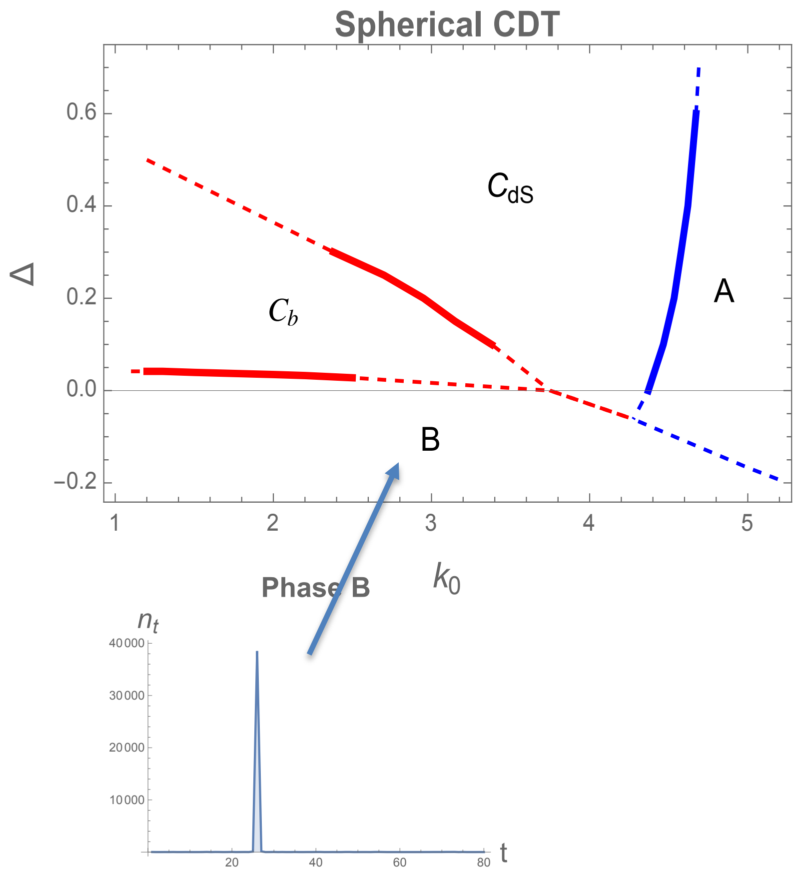

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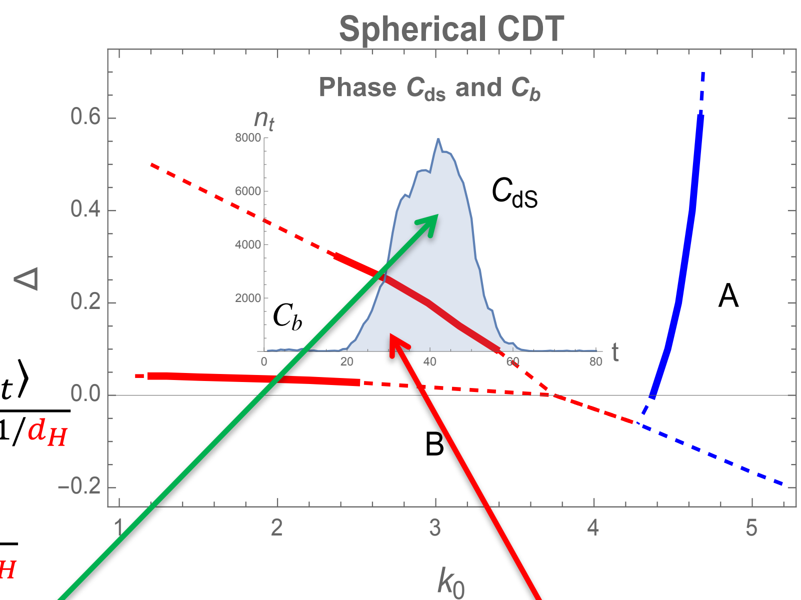

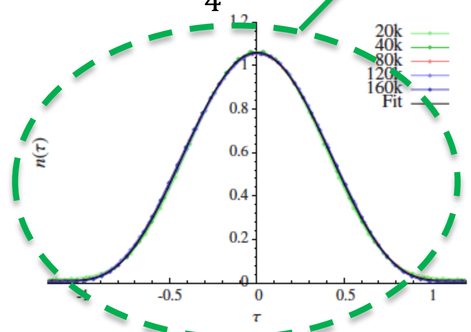

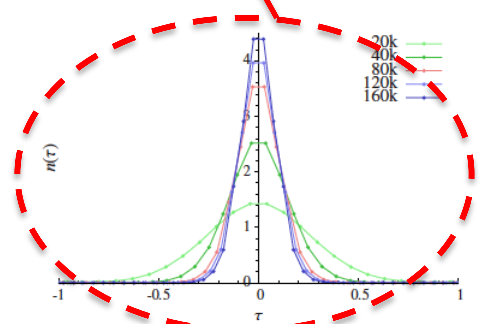

Rescaled average volume profiles $\langle n_t \rangle$ (scaling for $d_H = 4$)

Phase structure in S^3 CDT

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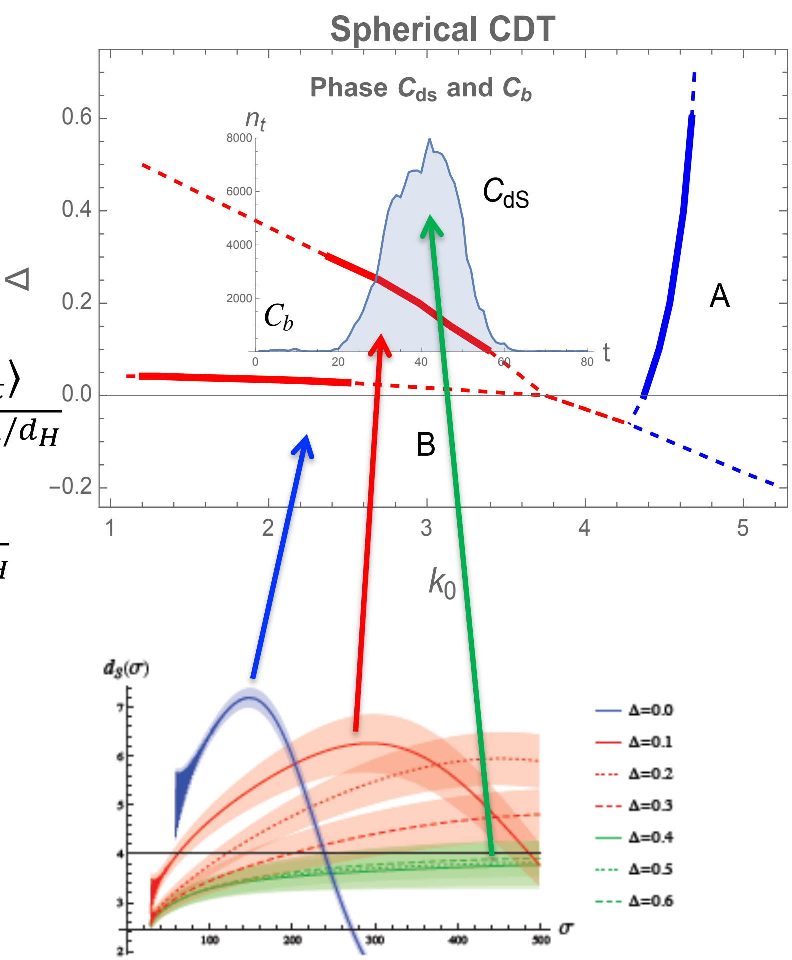
  - *the Spectral dimension: d_S* $\quad t \rightarrow \dfrac{t}{N_4^{1/d_H}}$

$$\frac{\partial}{\partial \sigma} K(x, x_0;\, \sigma) \;=\; \Delta_g K(x, x_0;\, \sigma)$$

$$P_R(\sigma) = \frac{1}{V} \int dx \sqrt{g} K(x, x;\, \sigma)$$

$$d_S(\sigma) = -2 \frac{d \log P_r(\sigma)}{d \log \sigma}$$



Semi-classical phase C_{dS}

- ✧ *Phase C_{dS} (de Sitter phase) has good semi-classical properties (IR limit)*
- ✧ *Hausdorff dim.: 4 ,*
  Spectral dim.: 2 ⟹ 4
- ✧ *Scale factor is consistent with a background geommetry of a 4-dim sphere ⟹* Euclidean de Sitter universe (positive cosmol. const.)
- ✧ *This is clasically obtained for a homogenous and isotropic metric*
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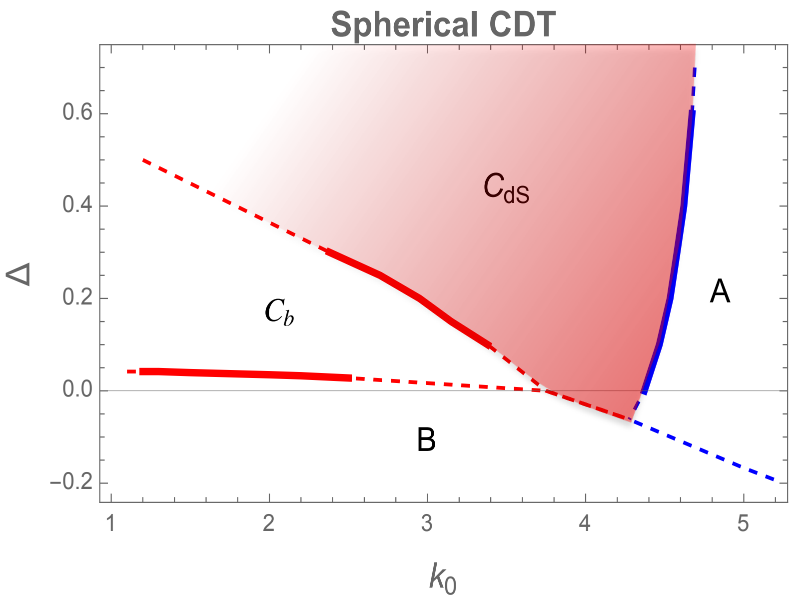

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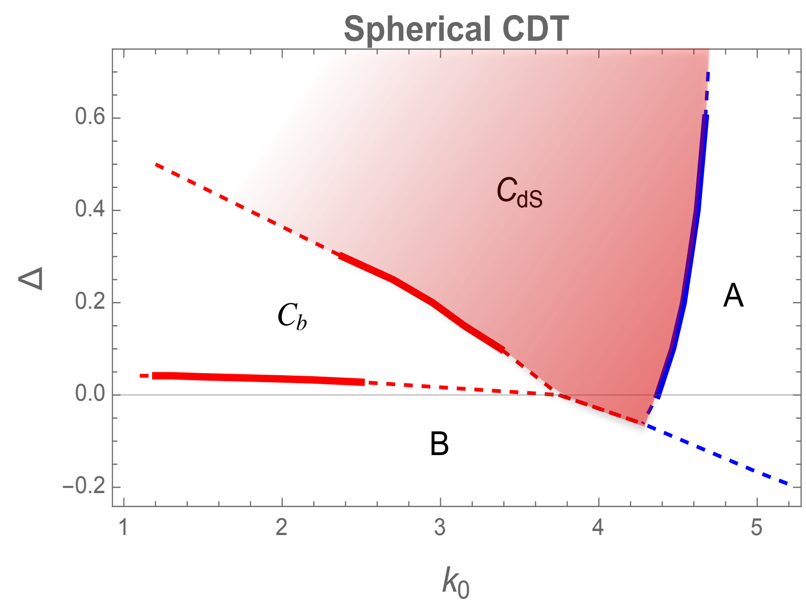

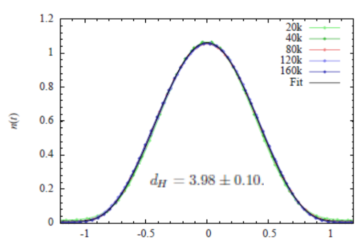

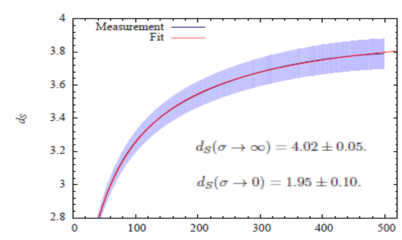

$$\bar{n}_t \to \frac{\bar{n}_t}{N_4^{1-1/d_H}} \qquad t \to \frac{t}{N_4^{1/d_H}} \qquad d_s(\sigma) = -2\frac{d \log P_R(\sigma)}{d \log \sigma}$$

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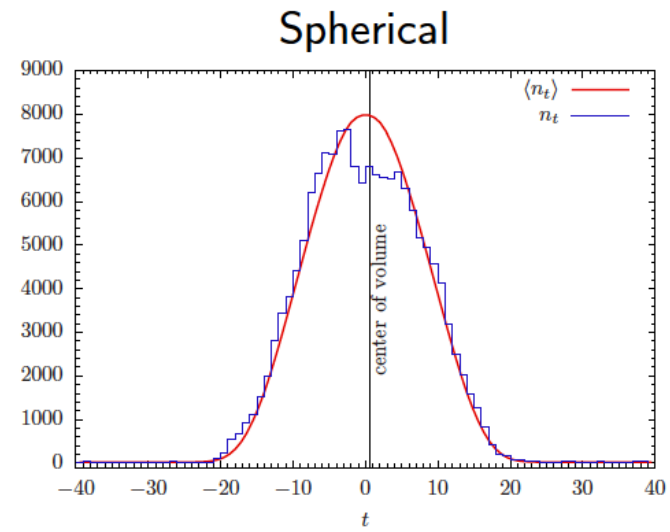

$$\langle n_i \rangle = \frac{3}{4} N_4 \frac{1}{\widetilde{\omega} N_4^{1/4}} \cos^3 \left(\frac{i}{\widetilde{\omega} N_4^{1/4}} \right)$$

$$\langle V_3(t) \rangle = \frac{3}{4} V_4 \frac{1}{\omega V_4^{1/4}} \cos^3 \left(\frac{t}{\omega V_4^{1/4}} \right)$$

$$V_3(t) \propto a^3(t)$$

Nucl. Phys. B849: 144

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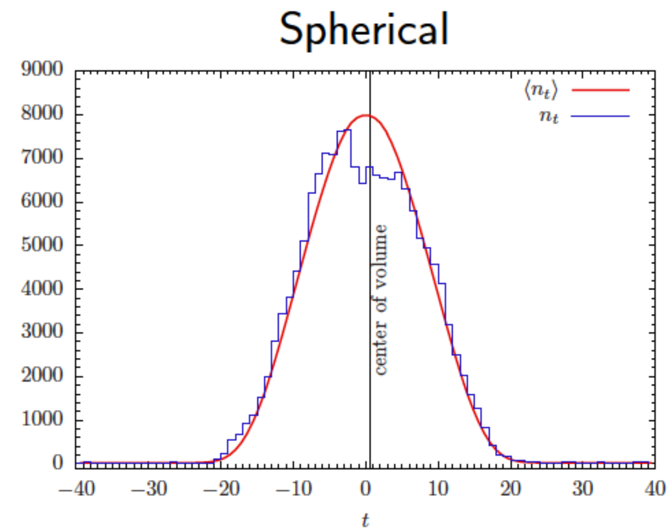

$$ds^2 = dt^2 + a^2(t)d\Omega_3^2$$

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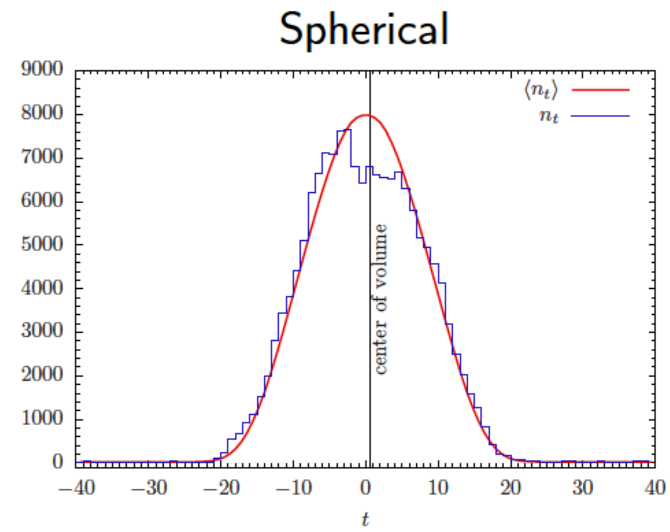

$$ds^2 = dt^2 + a^2(t)d\Omega_3^2$$

$$S_{MS} = -\frac{1}{24\pi G}\int dt\left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu\, V_3(t)^{1/3} - \lambda V_3(t)\right)$$

$$V_3(t) \propto a^3(t)$$

Nucl. Phys. B849: 144

Semi-classical phase C_{dS}

- The *effective action* for the n_i *observable* …
- … can be analyzed by looking at quantum fluctuations around the semiclassical solution
- The (inverse of) covariance matrix $P = C^{-1}$ provides information about second derivatives of the effective action
- The measured covariance matrix is consistent with MS action (with reversed overall sign) !
- The semiclassical description is obtained from "first principles" !

$$Z = \sum_T \exp(-S_R[T]) = \sum_{\{n_i\}} \sum_{T[\{n_i\}]} \exp(-S_R[T])$$

$$Z_{ef} = \sum_{\{n_i\}} \exp\left(-S_{ef}[\{n_i\}]\right)$$

$$S_{MS} = -\frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu\, V_3(t)^{1/3} - \lambda V_3(t) \right)$$

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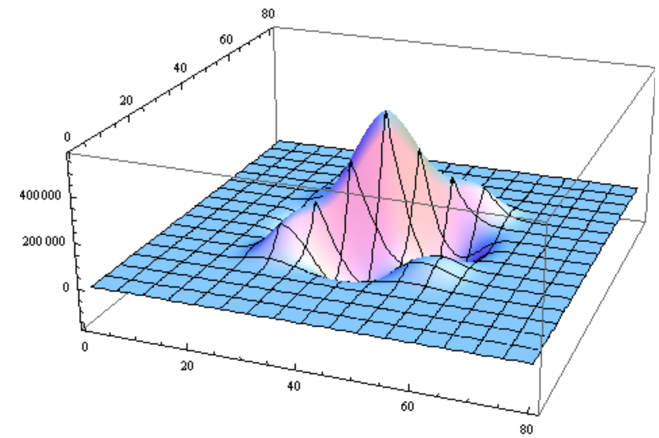
$$n_i = \langle n_i \rangle + \delta n_i \qquad C_{ij} = \langle \delta n_i \delta n_j \rangle$$



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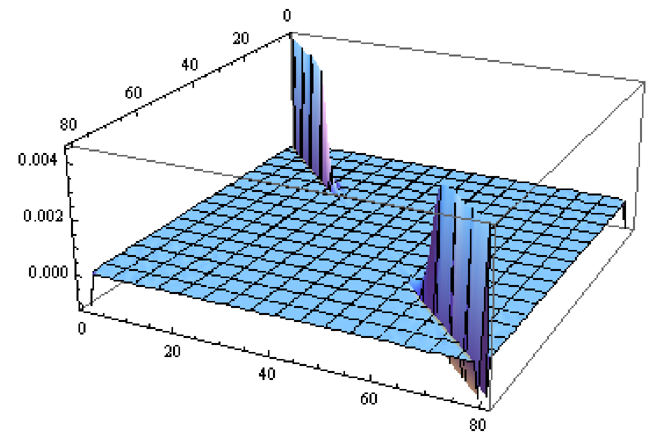

$$C_{ij}^{-1} = \left. \frac{\partial^2 S_{ef}[n]}{\delta n_i \delta n_j} \right|_{n=\langle n \rangle}$$

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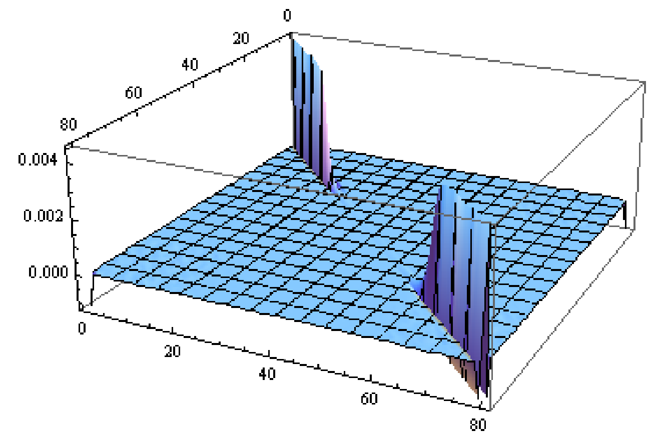

$$S_{ef} = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(n_{i+1} - n_i)^2}{(n_{i+1} + n_i)} + \tilde{\mu}\, n_i^{1/3} - \tilde{\lambda}\, n_i \right)$$

$$S_{MS} = \frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu\, V_3(t)^{1/3} - \lambda V_3(t) \right)$$

Agrees with Hartle–Hawking „noboundary" proposal !

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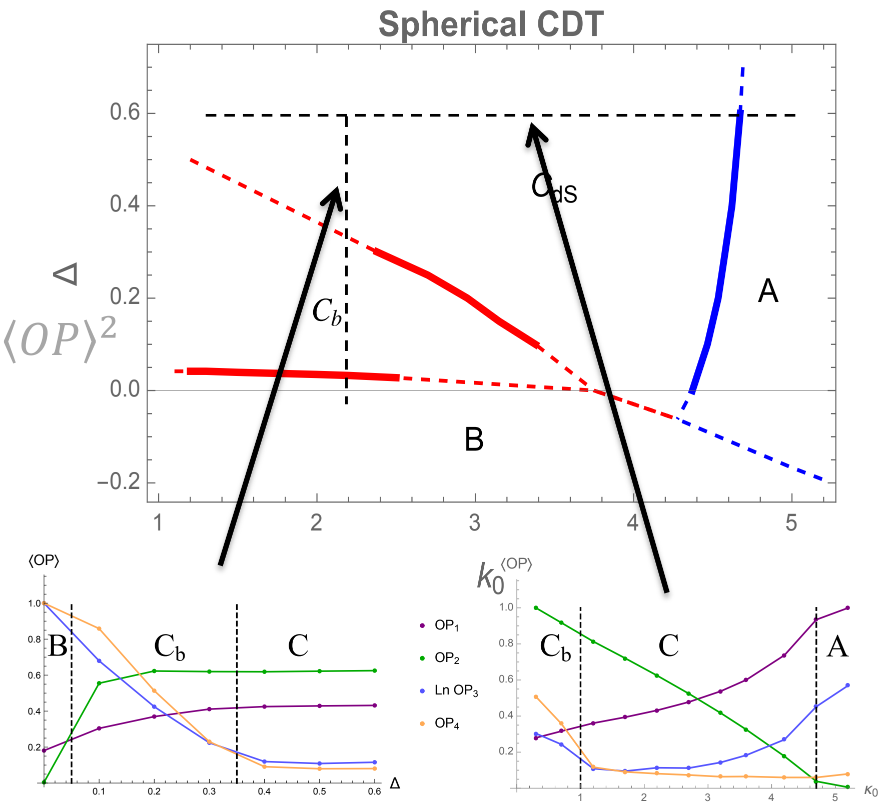
Phase transitions in S^3 CDT

- To analyze phase transitions one needs to define a suitable order parameter OP (e.g. N_0 / N_4 , ...)
- (Pseudo)critical point is signaled by max. of susceptibility $\chi_{OP} = \langle OP^2 \rangle - \langle OP \rangle^2$
- Two-states jumping of OP (double peak structure of measured histograms) may signal a 1st order transition
- But one must be careful and check $N_4 \to \infty$ limit
- B-C_b transition is 2-nd order
- The C_{ds}-C_b trans. is also 2-nd order
- Common points: UV limit candidates



$OP_1 = N_0/N_4$

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$OP_3 = \sum_t (n_{t+1} - nt)^2$

$OP_4 = \max_v O(v)$

OP	Phase A	Phase B	Phase C_{ds}	Phase C_b
OP_1	large	small	medium	medium
OP_2	small	small	large	large
OP_3	medium	large	small	medium
OP_4	small	large	small	large

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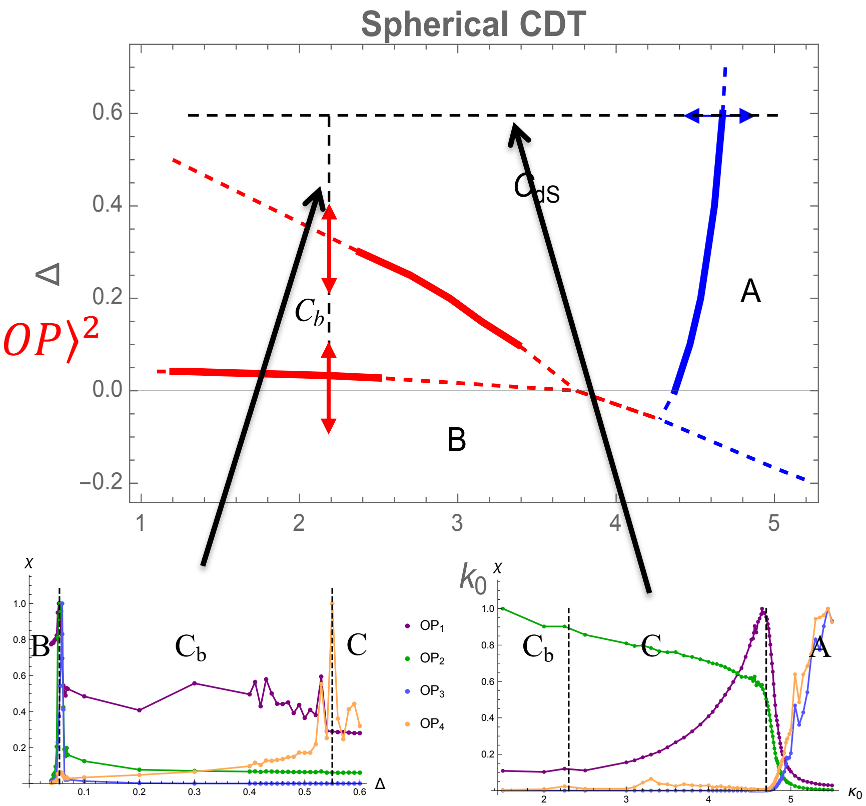

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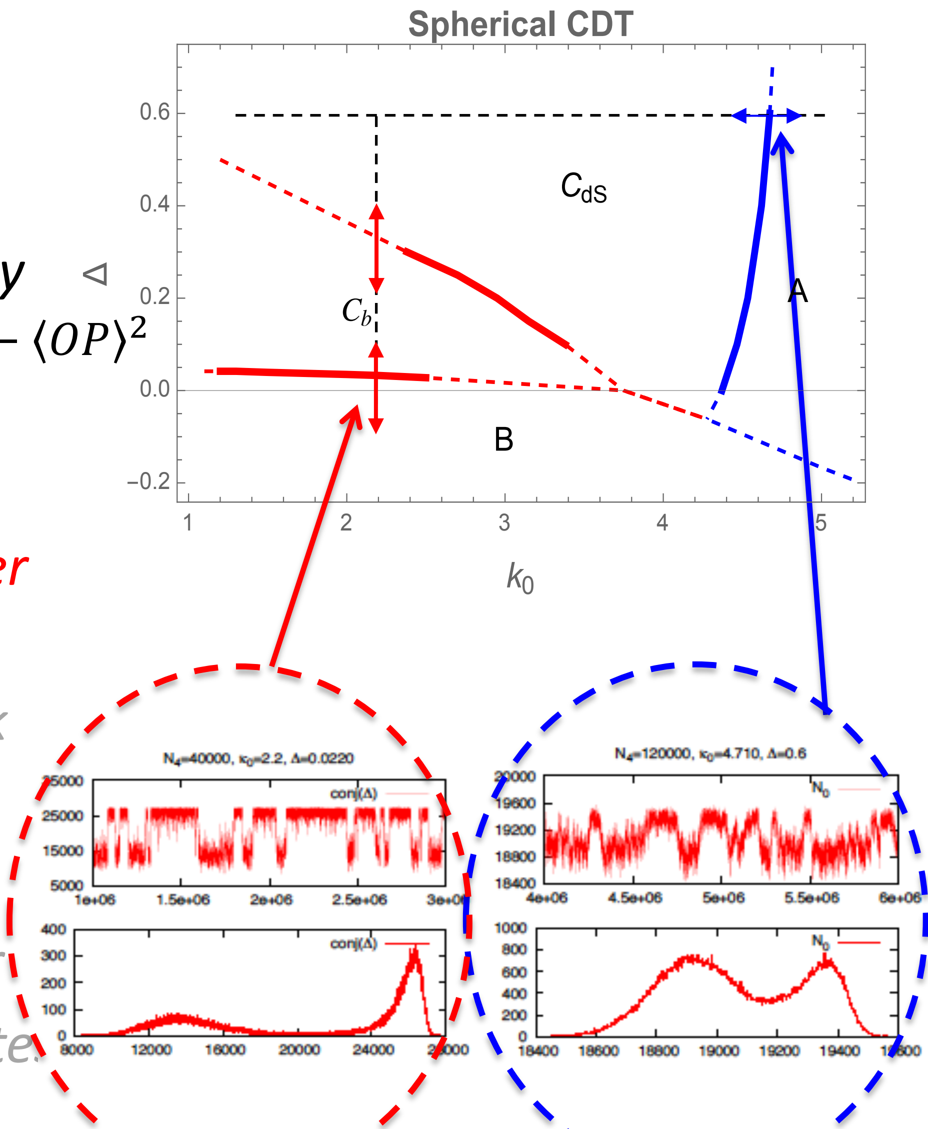

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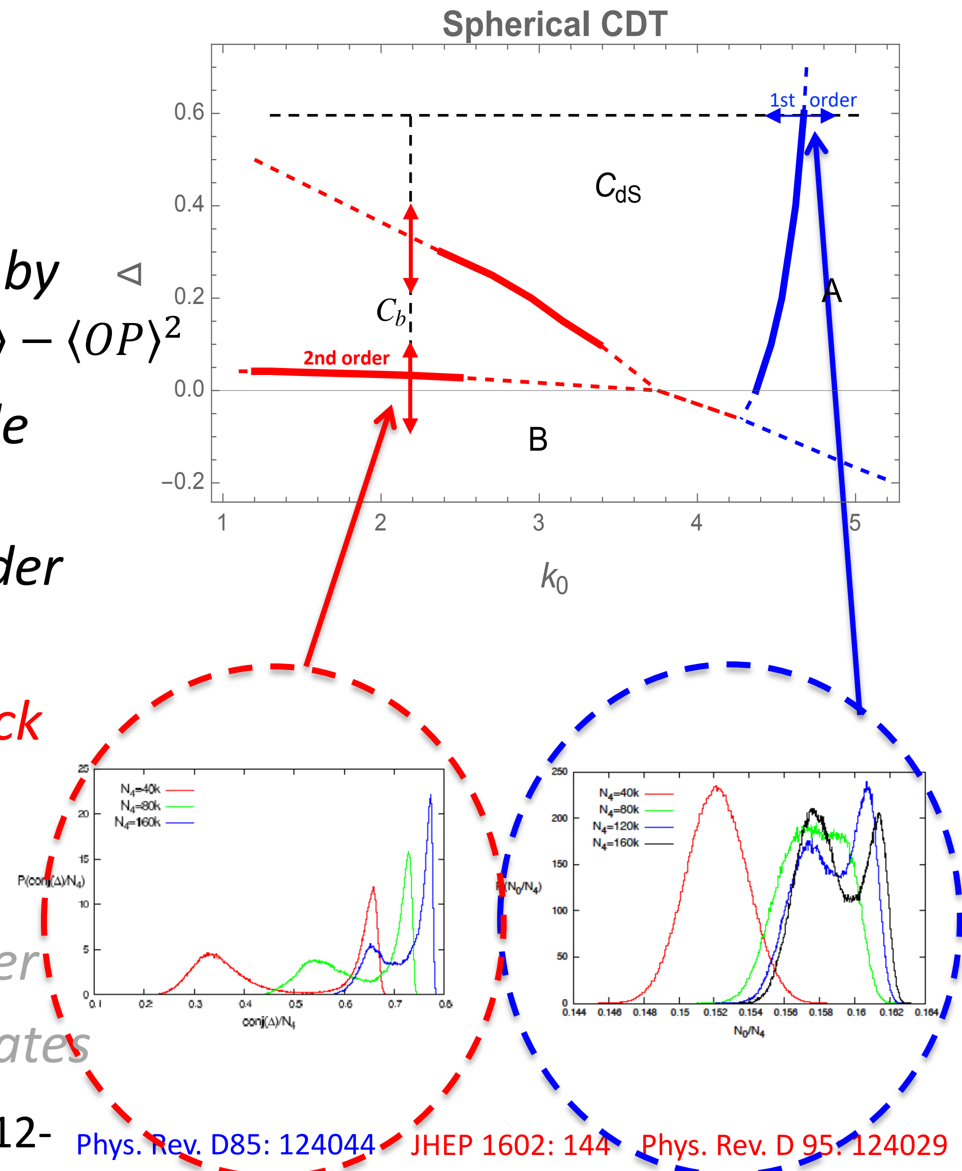

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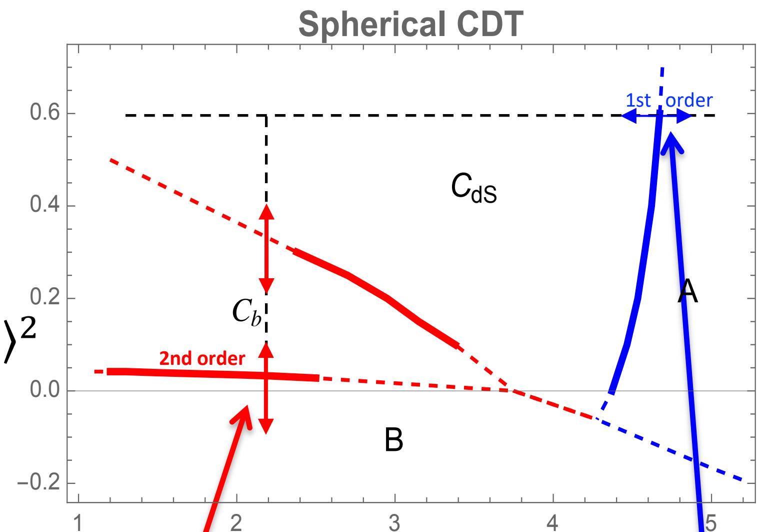

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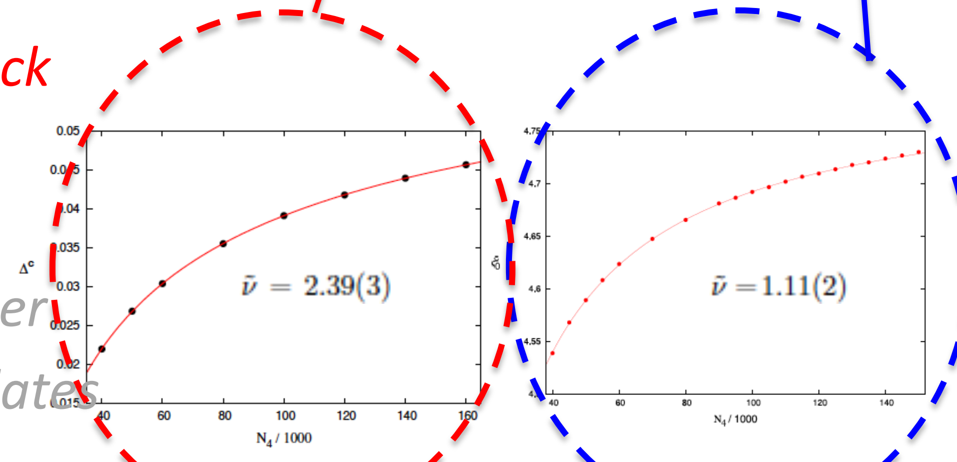

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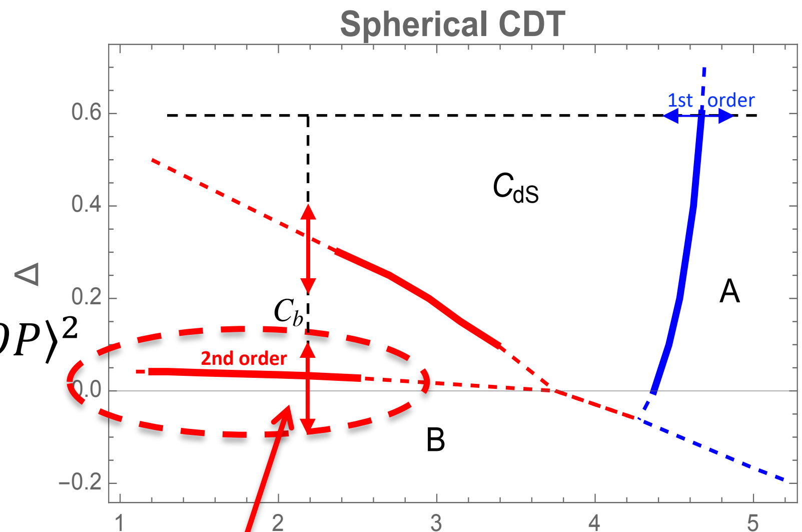

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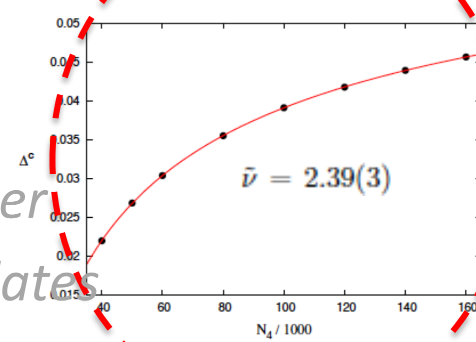

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Phase transitions in S^3 CDT

- ✧ *To analyze phase transitions one needs to define a suitable order parameter OP (e.g. N_0 / N_4 , ...)*
- ✧ *(Pseudo)critical point is signaled by max. of susceptibility* $\chi_{OP} = \langle OP^2 \rangle - \langle OP \rangle^2$
- ✧ *Two-states jumping of OP (double peak structure of measured histograms) may signal a 1st order transition*
- ✧ *But one must be careful and check $N_4 \to \infty$ limit*
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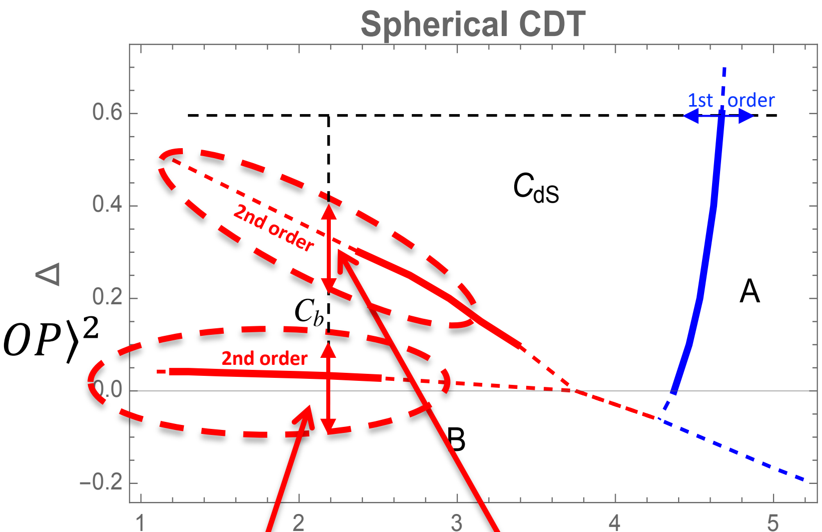
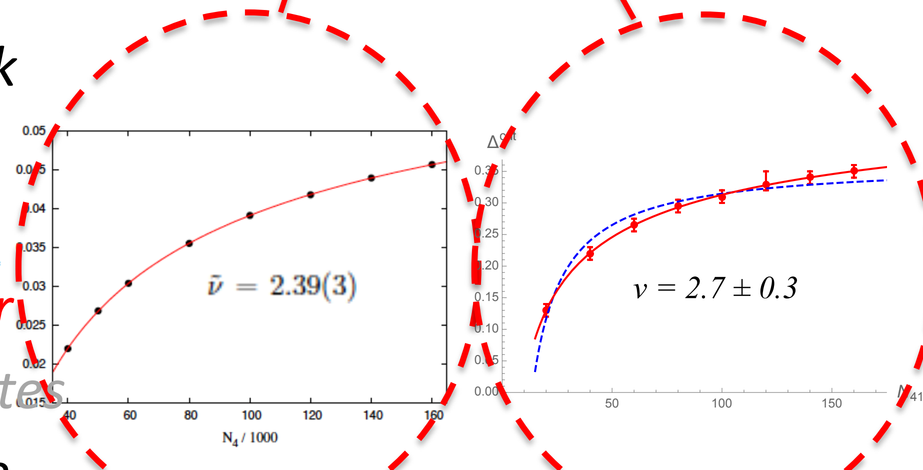
- ✧ *Common points: UV limit candidates*



$$\Delta^c(N_4) = \Delta^c(\infty) - \alpha N_4^{-1/\nu}$$



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Phys. Rev. D85: 124044 JHEP 1602: 144 Phys. Rev. D 95: 124029

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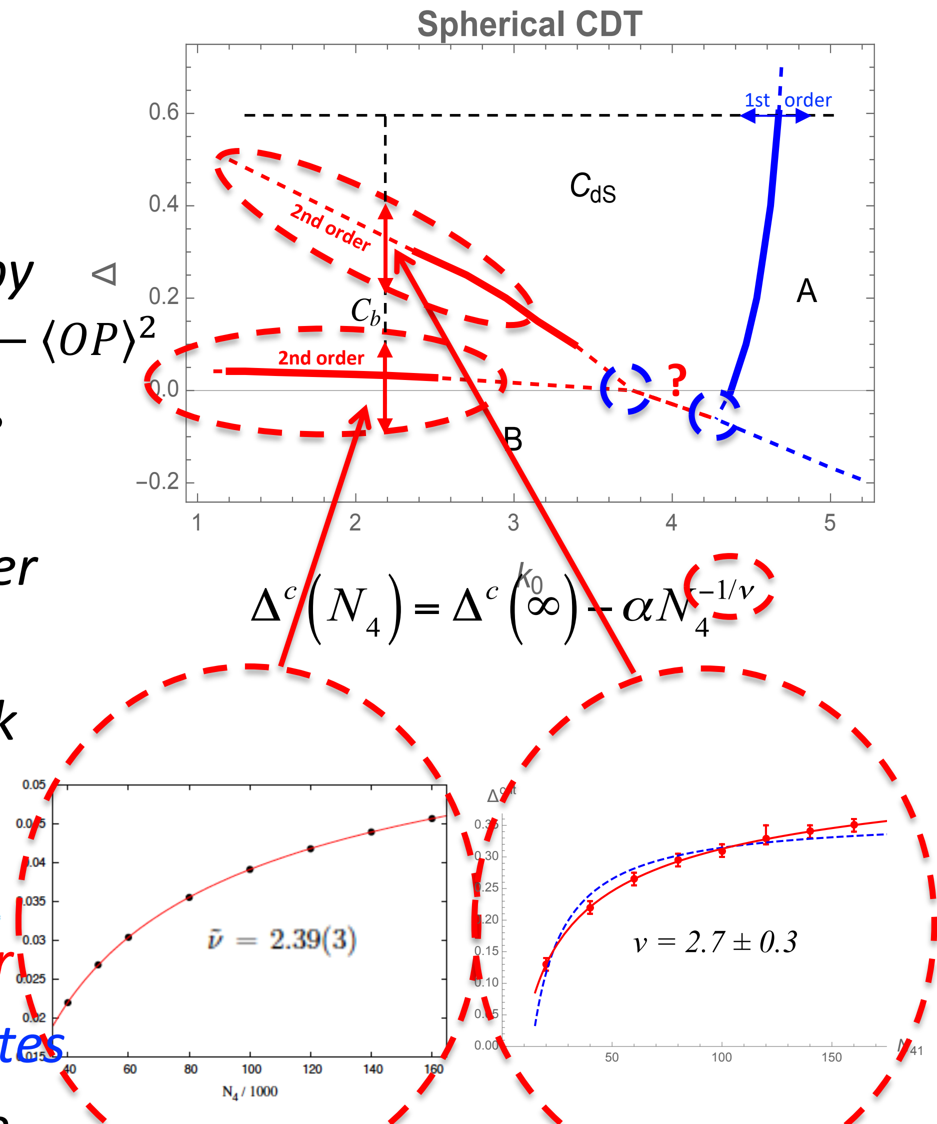

Phys. Rev. D85: 124044 JHEP 1602: 144 Phys. Rev. D 95: 124029

Search for a continuum limit

- ✧ *We observe a semiclassical behaviour inside phase C_{dS}*
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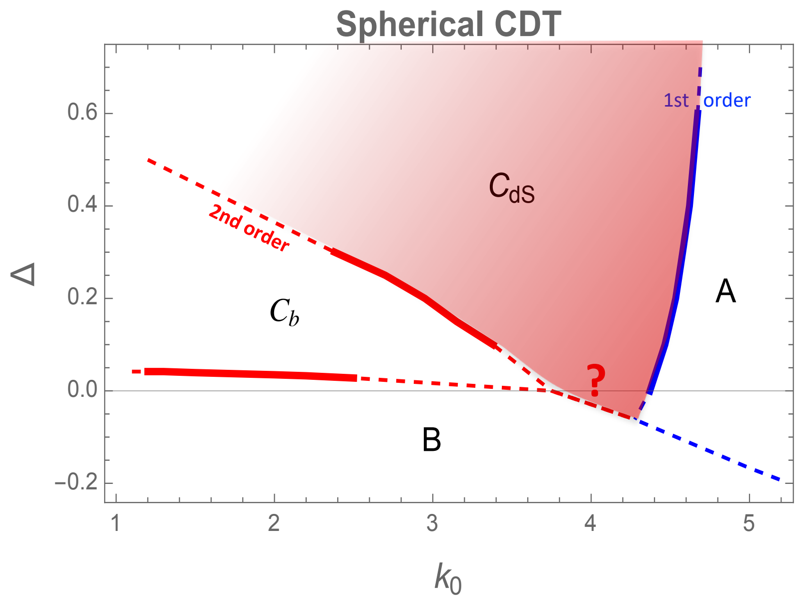

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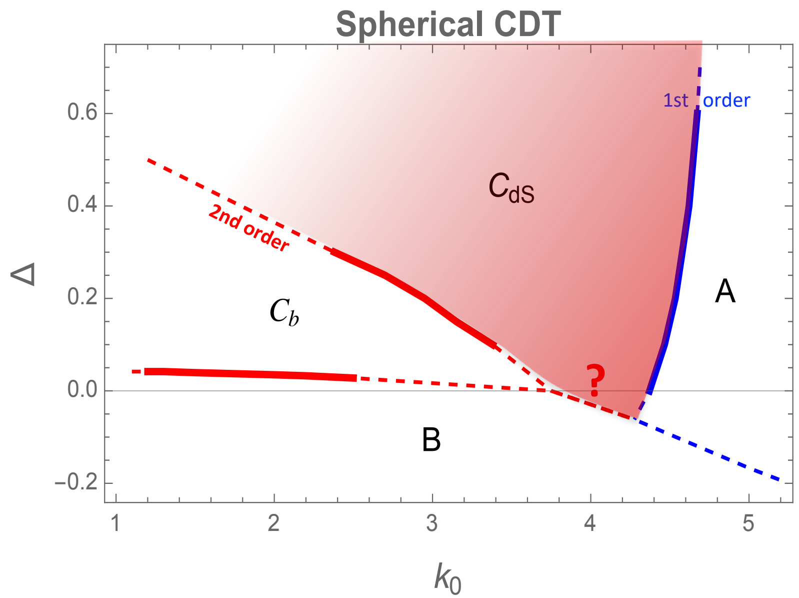

$$\langle n_i \rangle = N_4^{3/4} \frac{3}{4\tilde{\omega}} \cos^3\left(\frac{i}{\tilde{\omega} N_4^{1/4}}\right)$$

universal f-ctions

$$\langle \delta n_i \delta n_j \rangle = \tilde{\Gamma} N_4 \; \mathrm{F}\left(\frac{i}{\tilde{\omega} N_4^{1/4}}, \frac{j}{\tilde{\omega} N_4^{1/4}}\right)$$

fluct. amplitude

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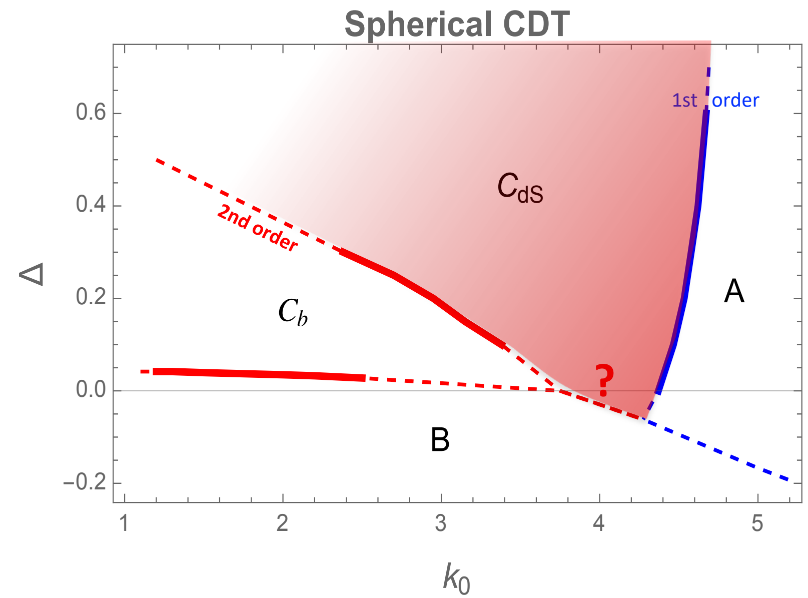

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fluct. amplitude

$$S_{ef} = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(n_{i+1} - n_i)^2}{(n_{i+1} + n_i)} + \tilde{\mu}\, n_i^{1/3} - \tilde{\lambda}\, n_i \right)$$

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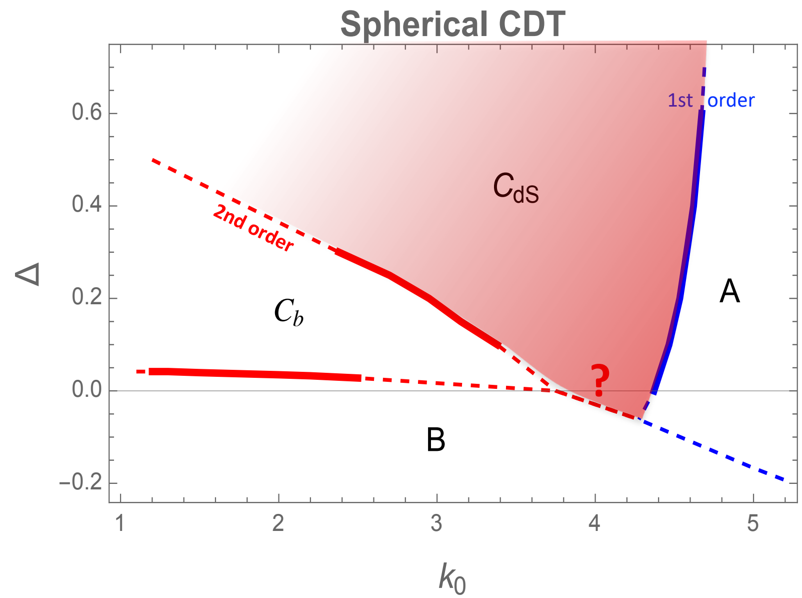

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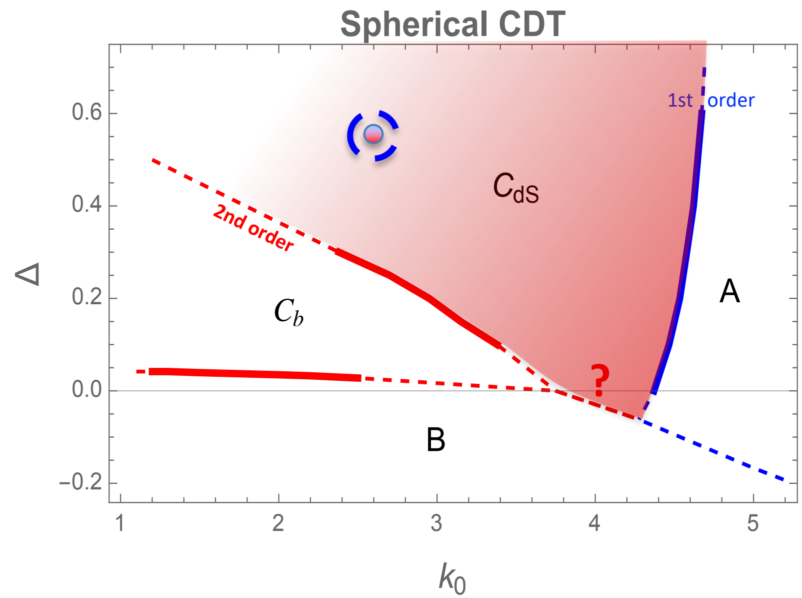

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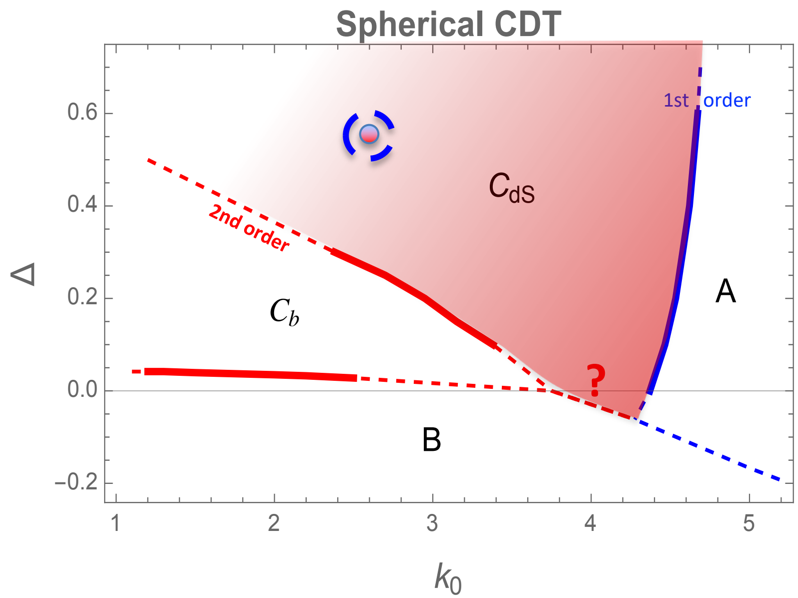

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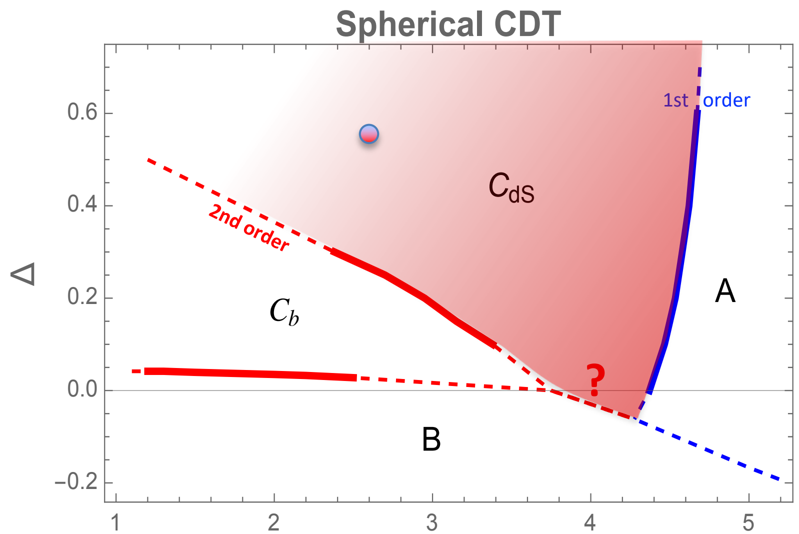

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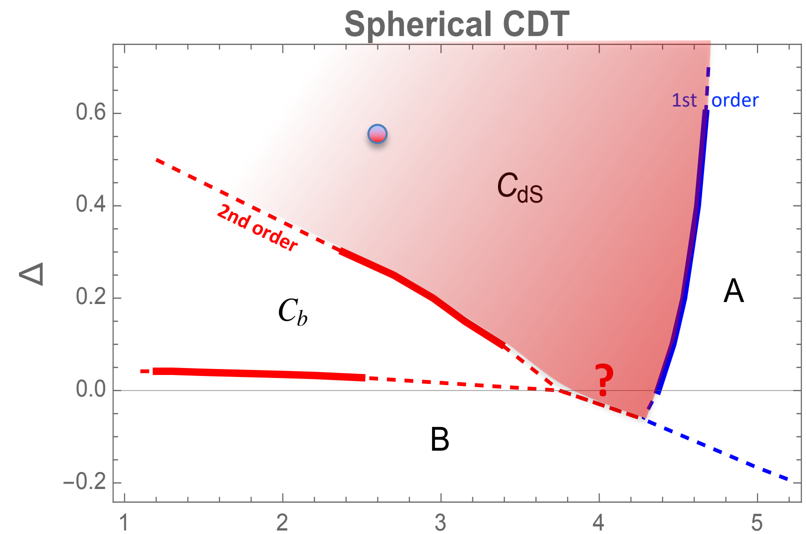

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μ is related to ω

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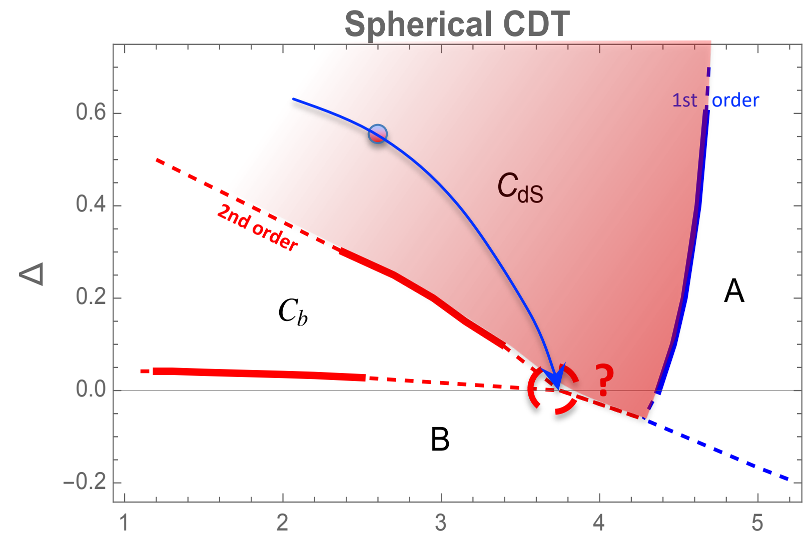

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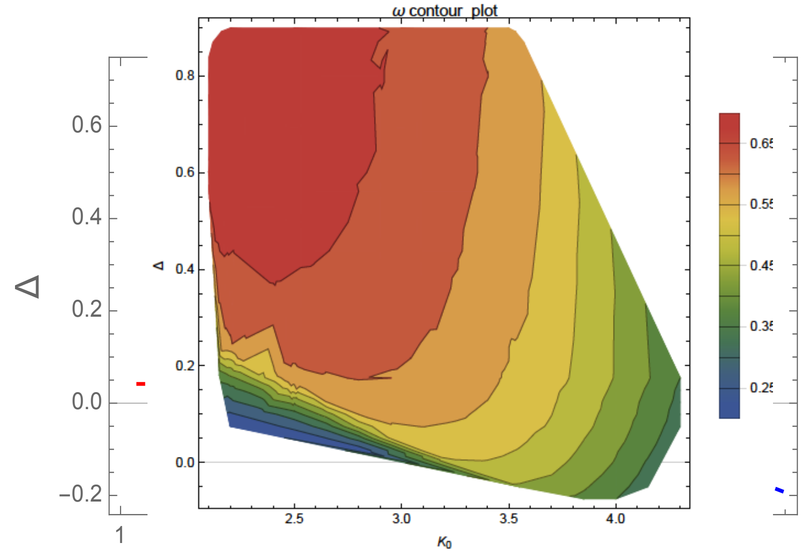

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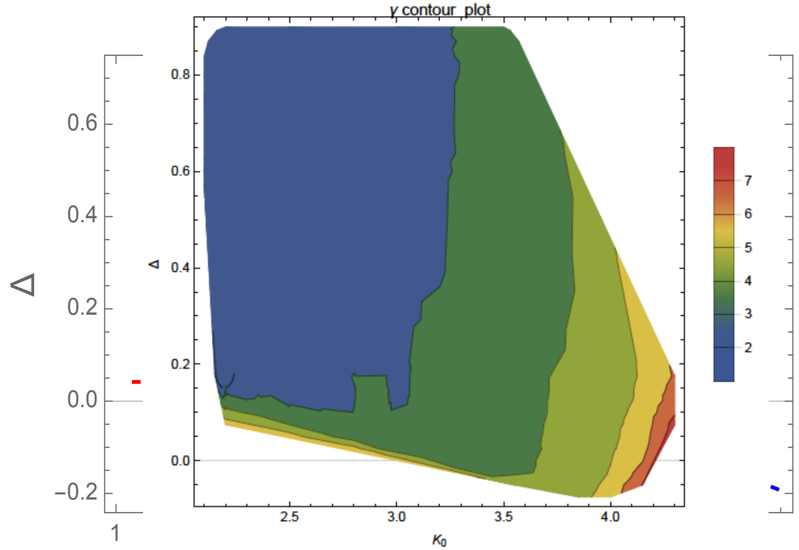

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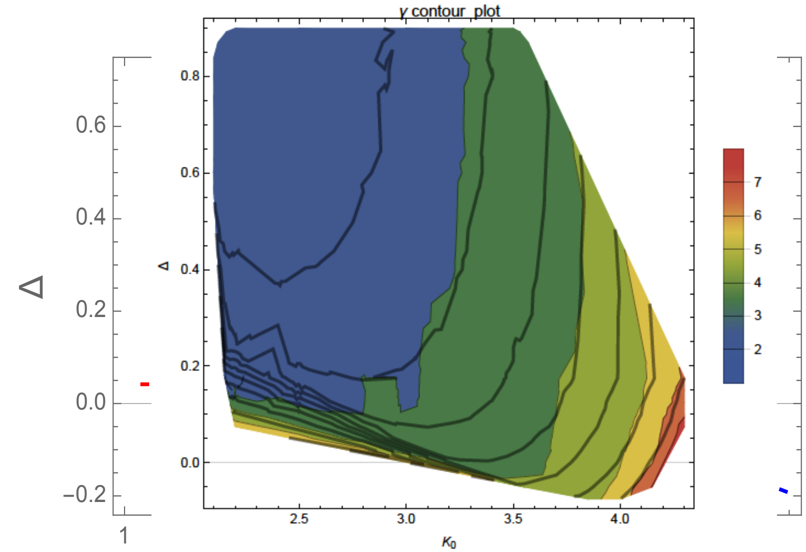

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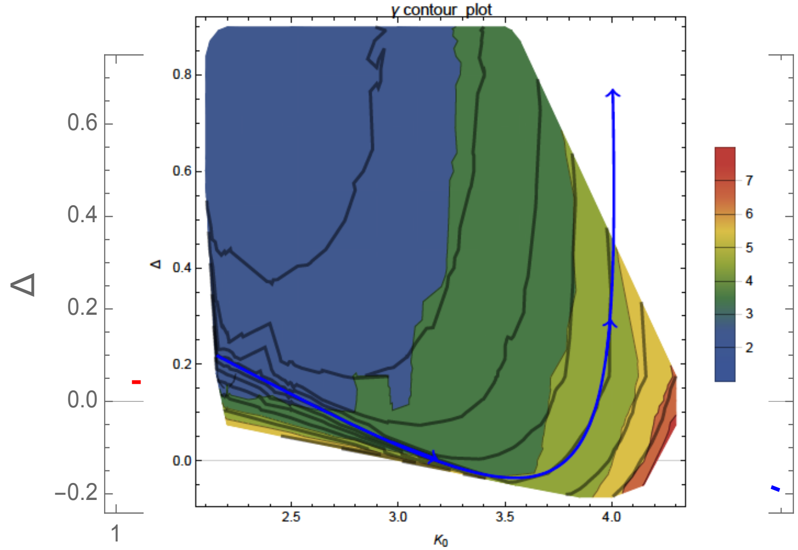

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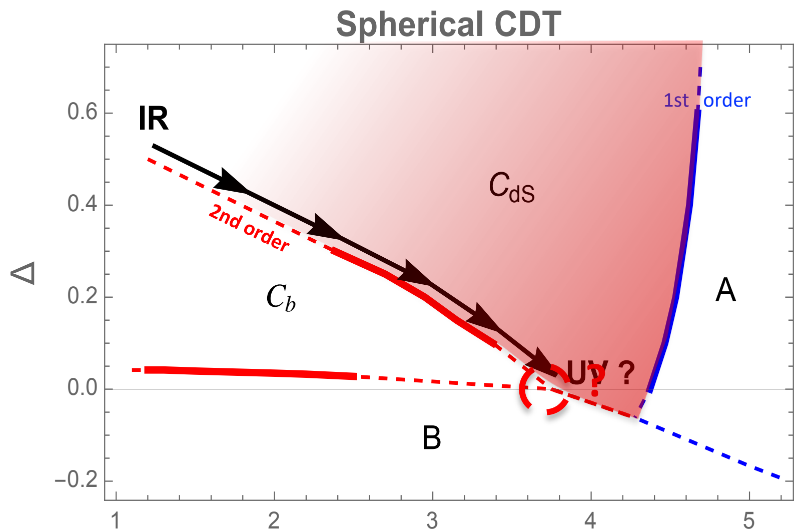

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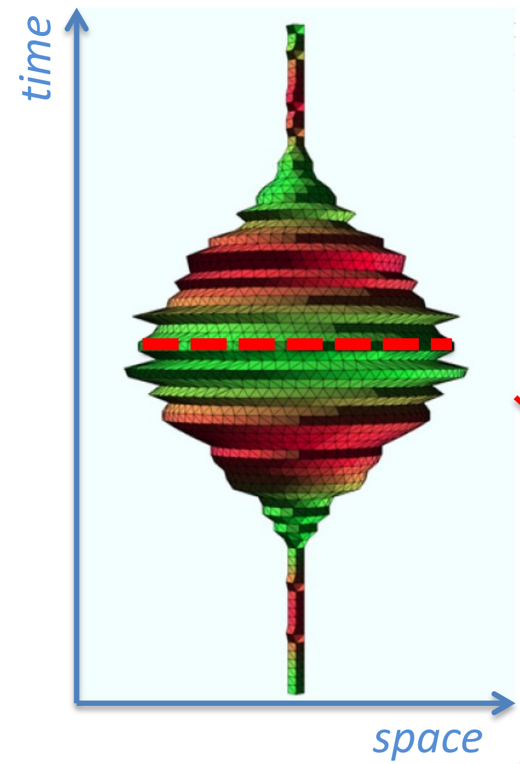
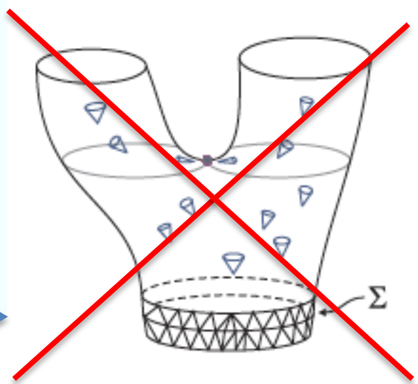
Toroidal vs spherical topologies

✧ *In CDT the topology of spatial slices is fixed and it is not allowed to change in time (causality condition).*





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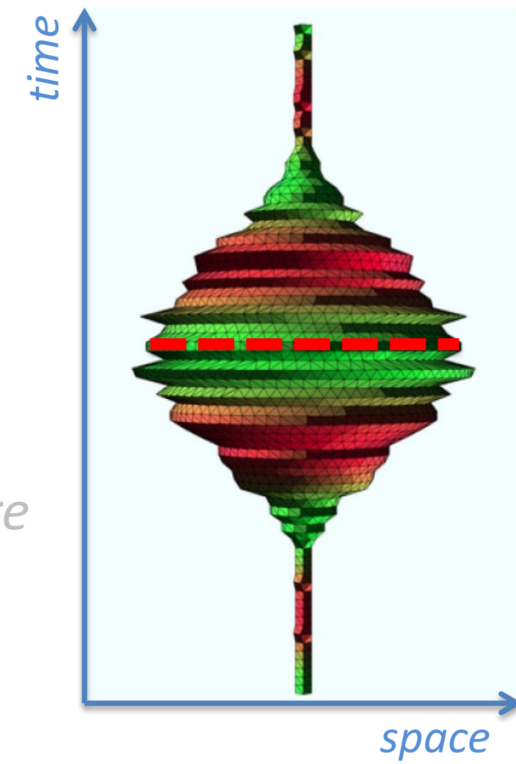

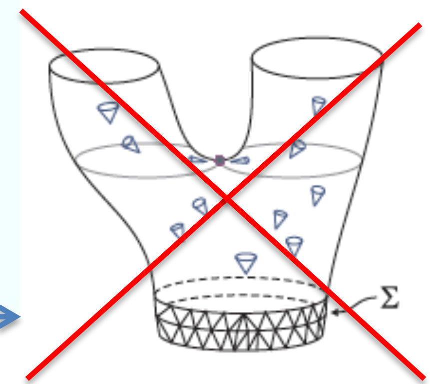

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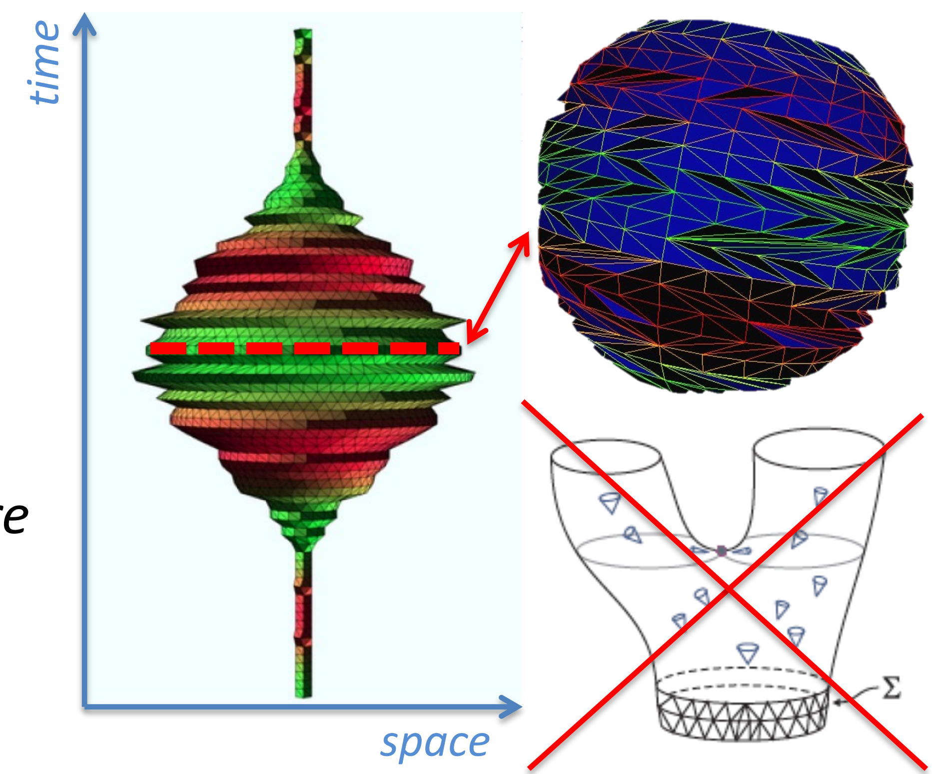

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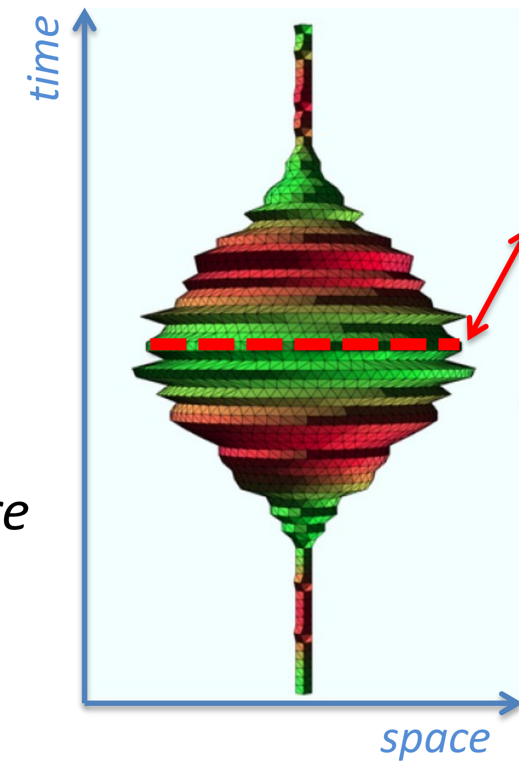

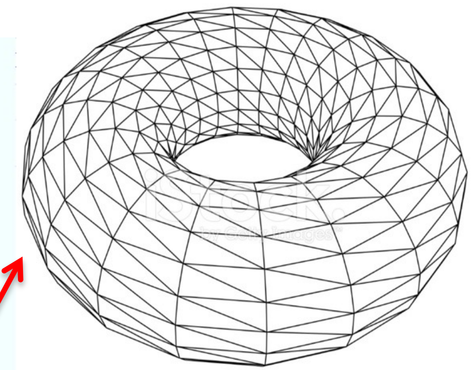

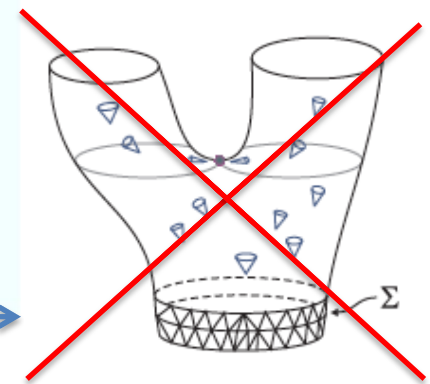

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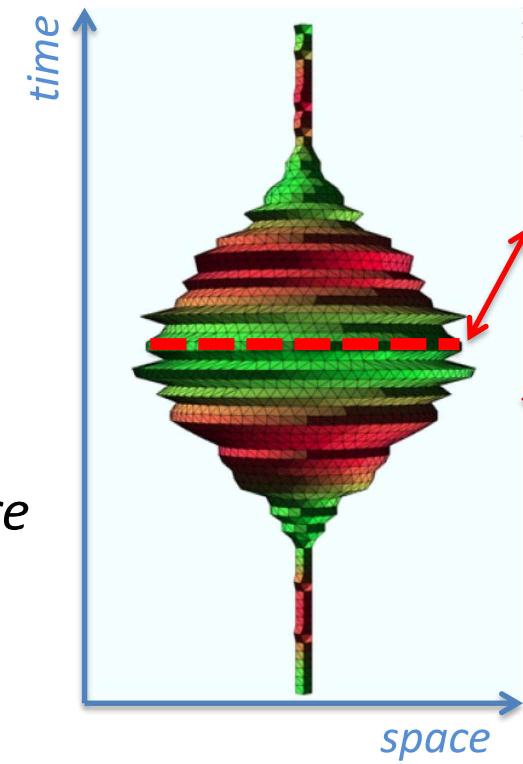

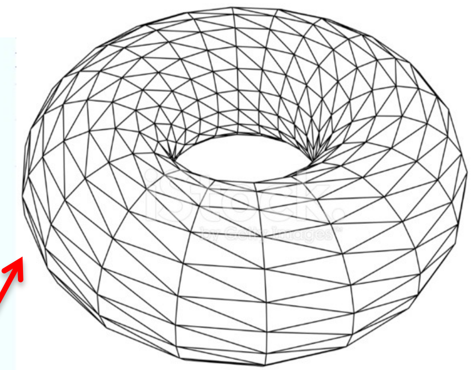

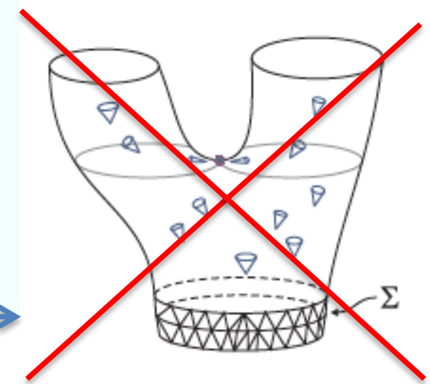

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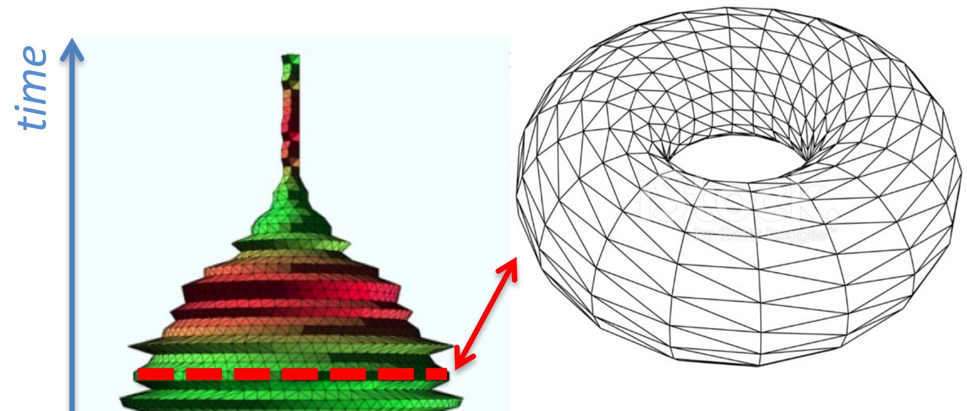

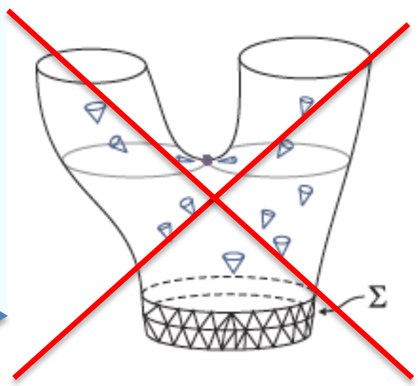

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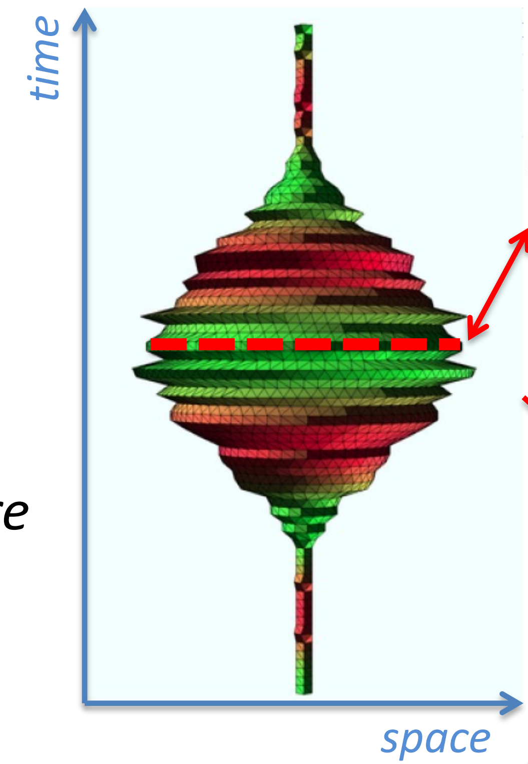

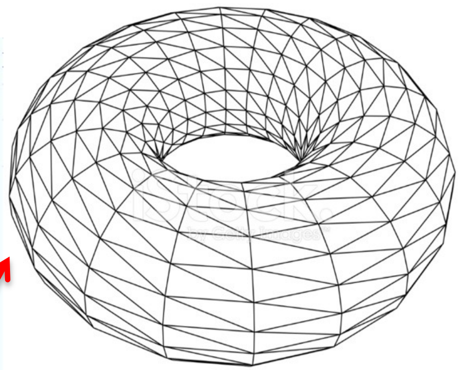

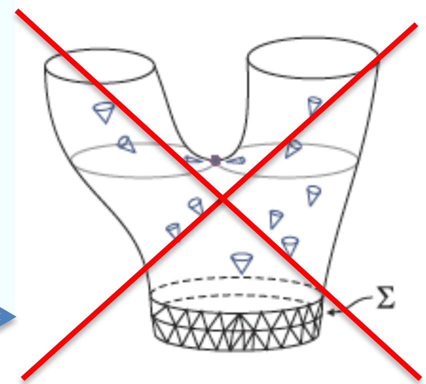

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Class.Quant.Grav. 36: 224001
JHEP 2005: 030
JHEP 2204: 103

Toroidal vs spherical topologies

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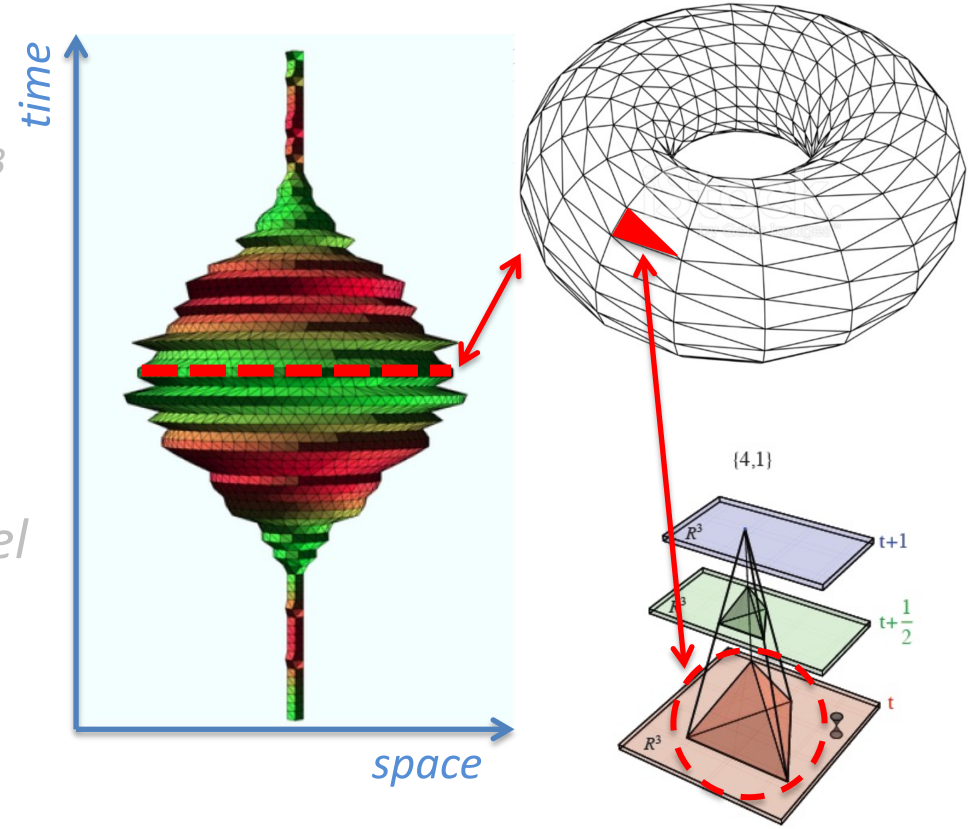

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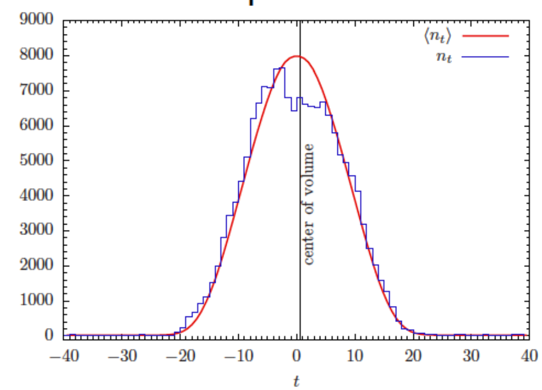

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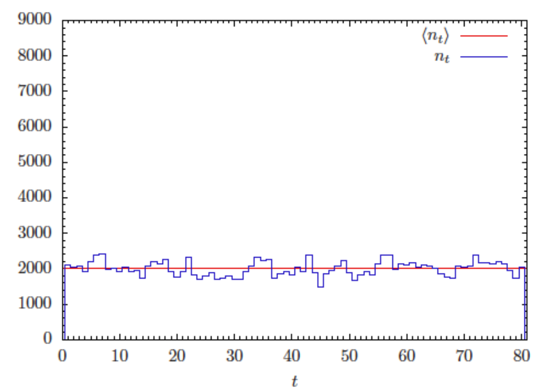

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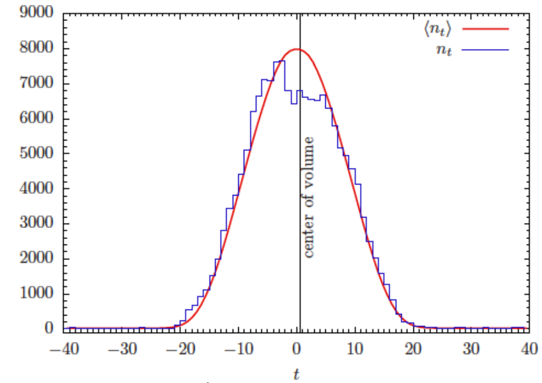

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$$ds^2 = dt^2 + a^2(t)d\Omega_3^2 \Rightarrow V_3(t) \propto a^3(t)$$

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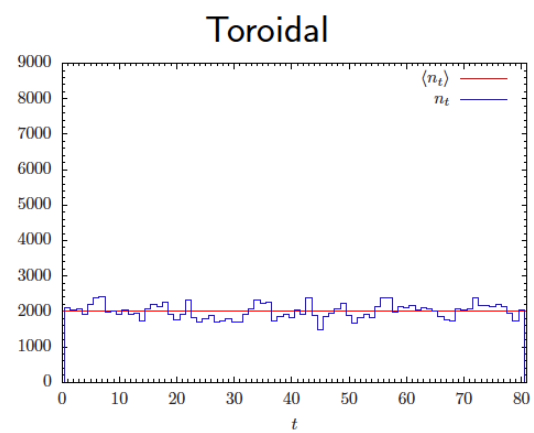

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Toroidal vs spherical topologies

- **Semiclassical backround** inside phase C (T^3 vs S^3 spatial topology)
  - The *effective action* can be measured in CDT using the *covariance matrix* data
    - in toroidal case: $n_i \propto N_4/T$ and one can combine a collection of data measured for various N_4 and T
    - one can also use non-standard boundary conditions to force n_i in some range
    - due to a lack of semiclassical potential it is easier to observe a quantum correction term

$$n_t = \bar{n}_t + \delta n_t \qquad C_{tt'} \equiv \langle \delta n_t \delta n_{t'} \rangle$$

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$$\gamma \approx -3/2$$

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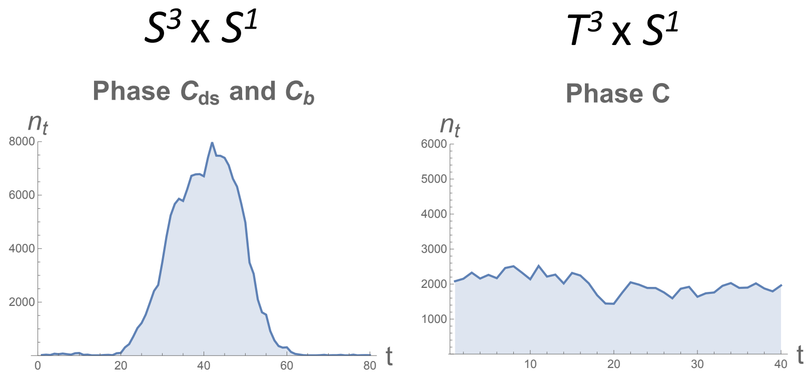

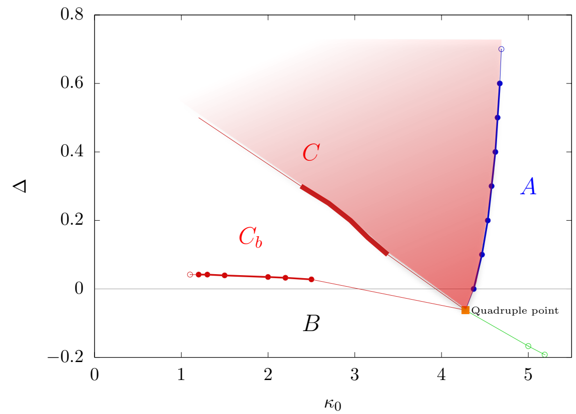

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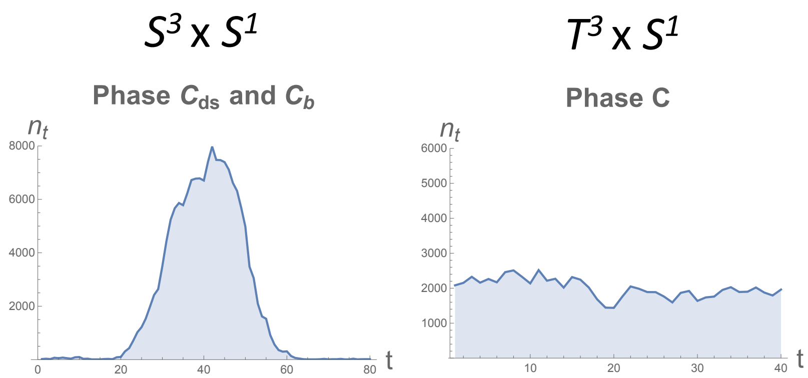

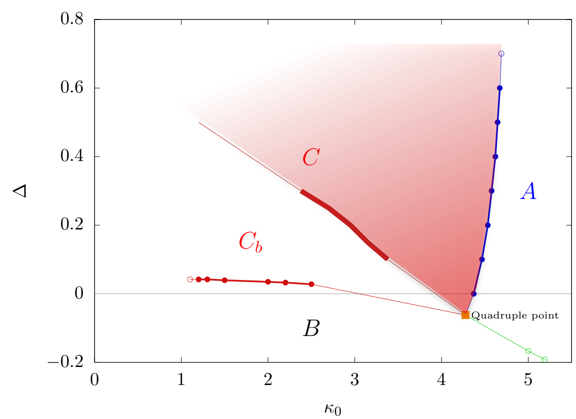

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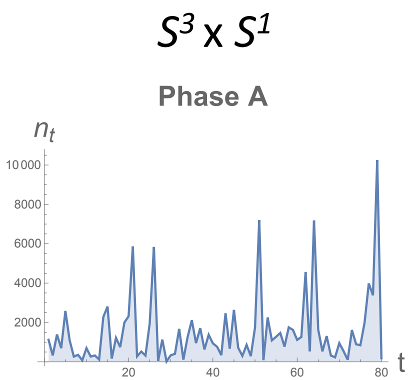

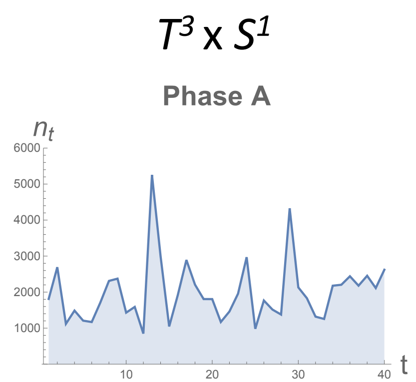

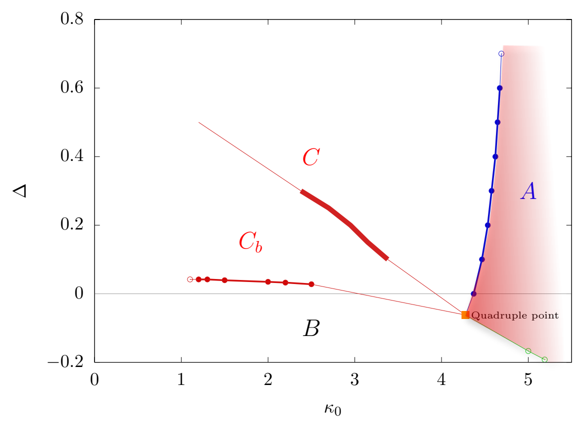

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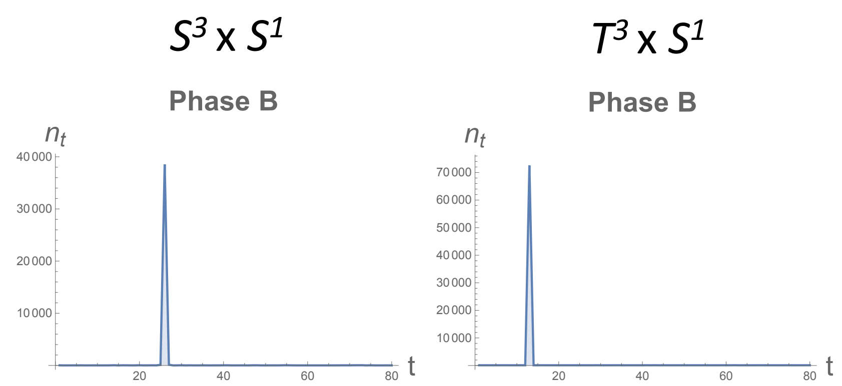

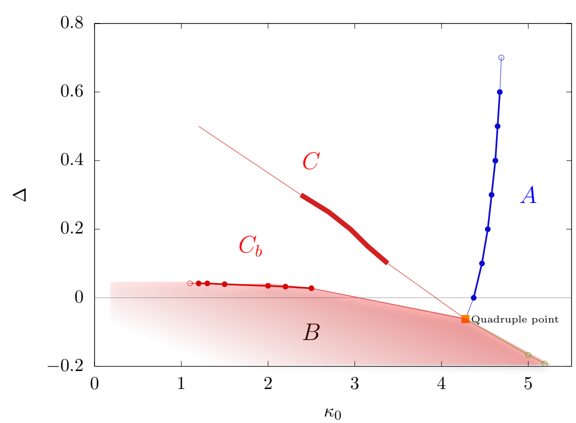

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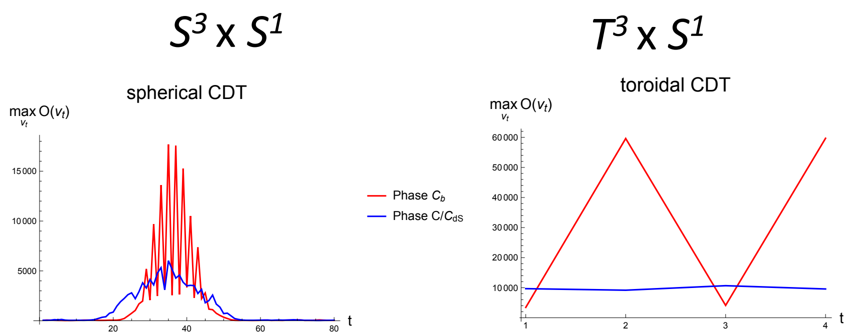

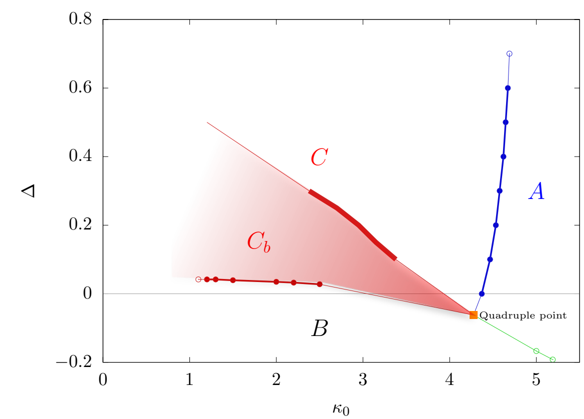

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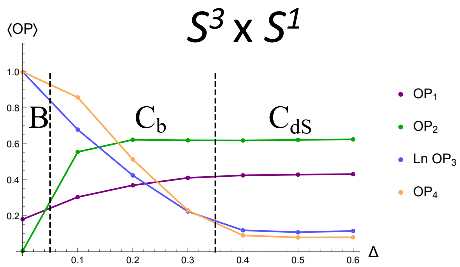
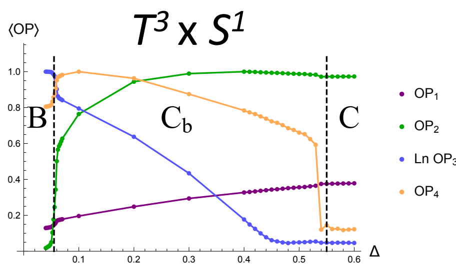
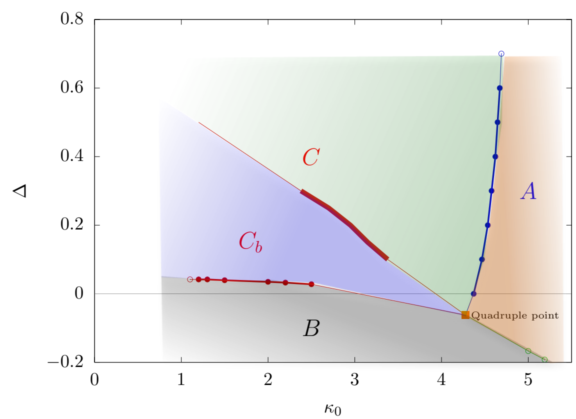




$$OP_1 = N_0/N_4 \qquad OP_3 = \sum_t (n_{t+1} - nt)^2$$

$$OP_2 = N_4^{(3,2)}/N_4^{(4,1)} \qquad OP_4 = \max_v O(v)$$



JHEP 1806: 111

Toroidal vs spherical topologies

✧ The phase structure of CDT (T^3 vs S^3 spatial topology)

- ✧ In toroidal CDT there exists a semiclasical phase C
- ✧ What about other phases ?
  - ✧ MC results show that all phases previously observed in the S^3 spatial topology also exist in the T^3 case
- ✧ Position of the phase transitions …
  - ✧ we analyse order parameters similar to the spherical case
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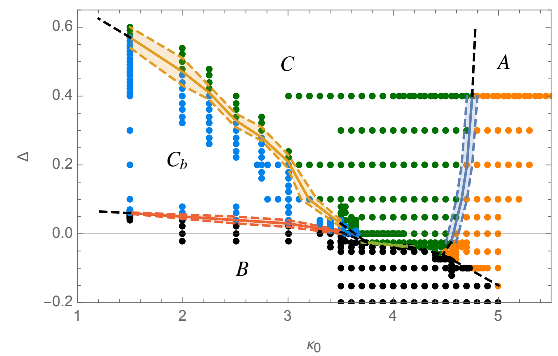
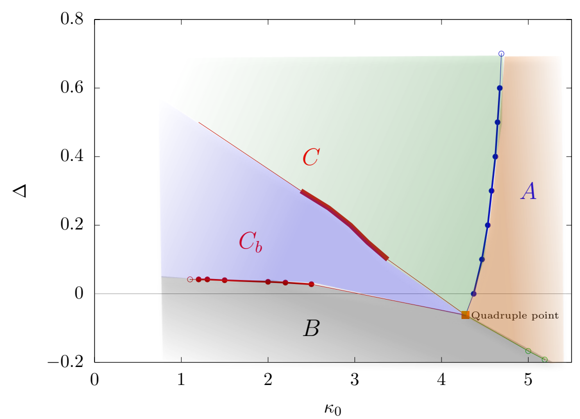
T^3 x S^1



S^3 x S^1



JHEP 1806: 111

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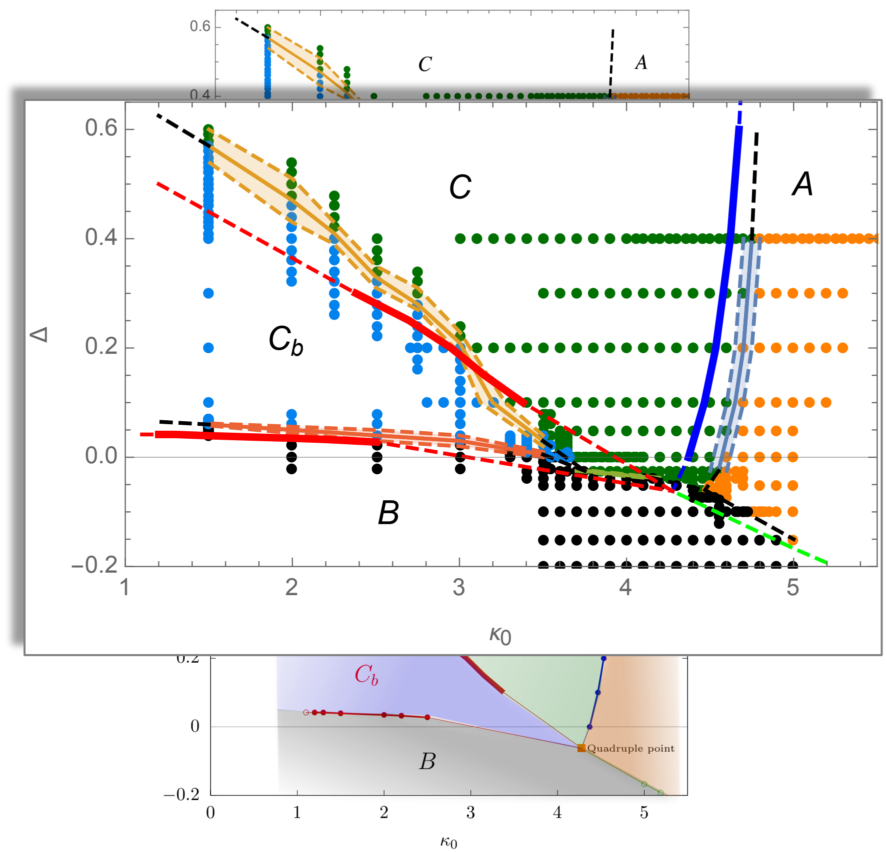

JHEP 1806: 111

Toroidal vs spherical topologies

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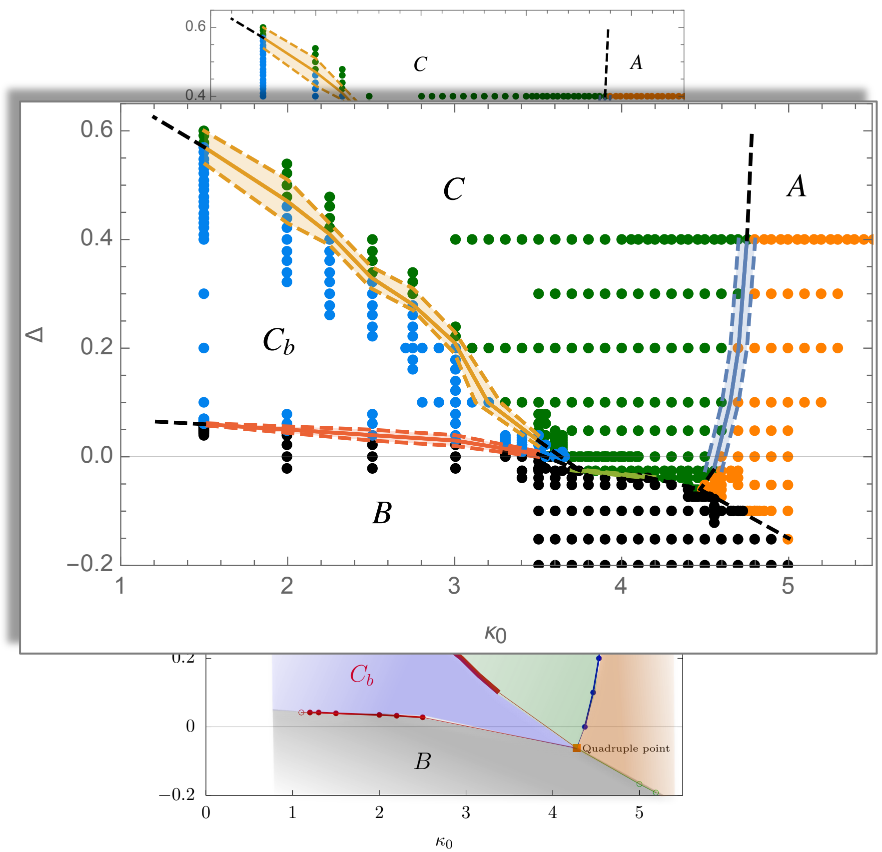
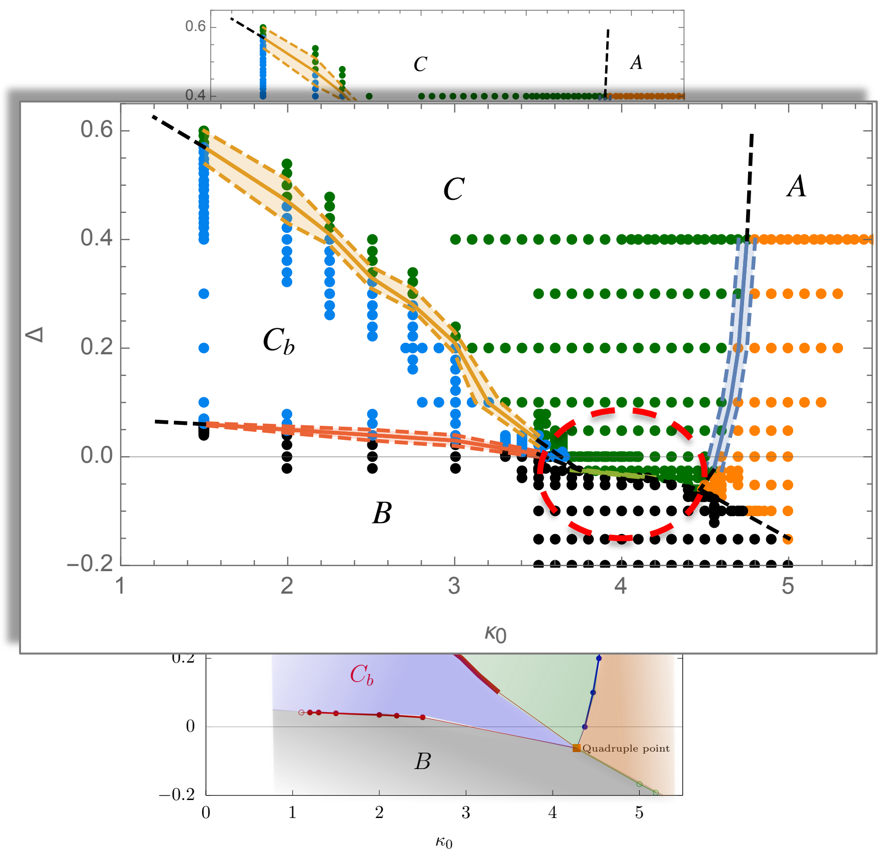


Toroidal vs spherical topologies

✧ Phase transitions (T^3 vs S^3 spatial topology)

- ✧ In T^3 CDT we have investigated the region where all phases meet
  - ✧ we observe a direct B-C transition
  - ✧ the B-C transition was classified to be 1[st] order: visible hysteresis, order parameters on both sides do not converge with increased lattice volume N_4
  - ✧ but with some untypical properties: hysteresis shrinks with $N_4 \to \infty$, nontrivial scaling exponents
  - ✧ it is possible that the endpoinds are higher order (work in progress)

$T^3 \times S^1$



JHEP 1907: 166
JHEP 2204: 103

Toroidal vs spherical topologies

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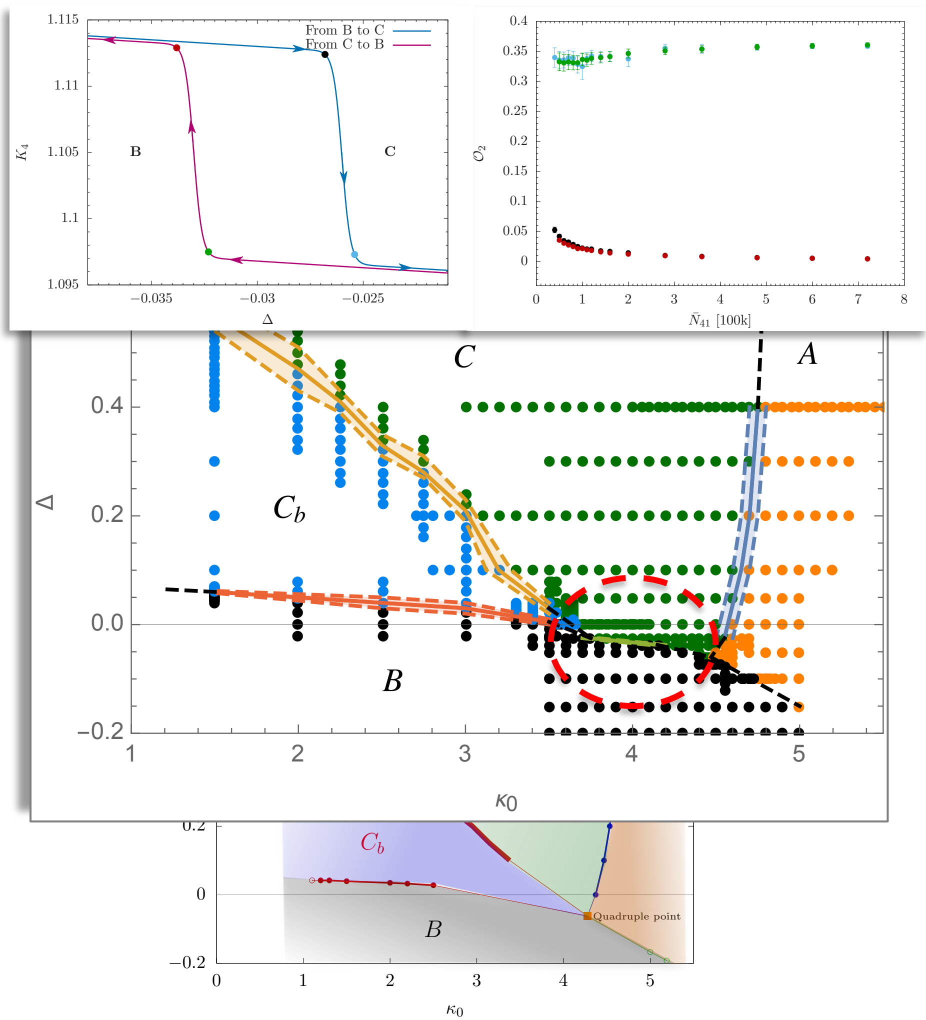

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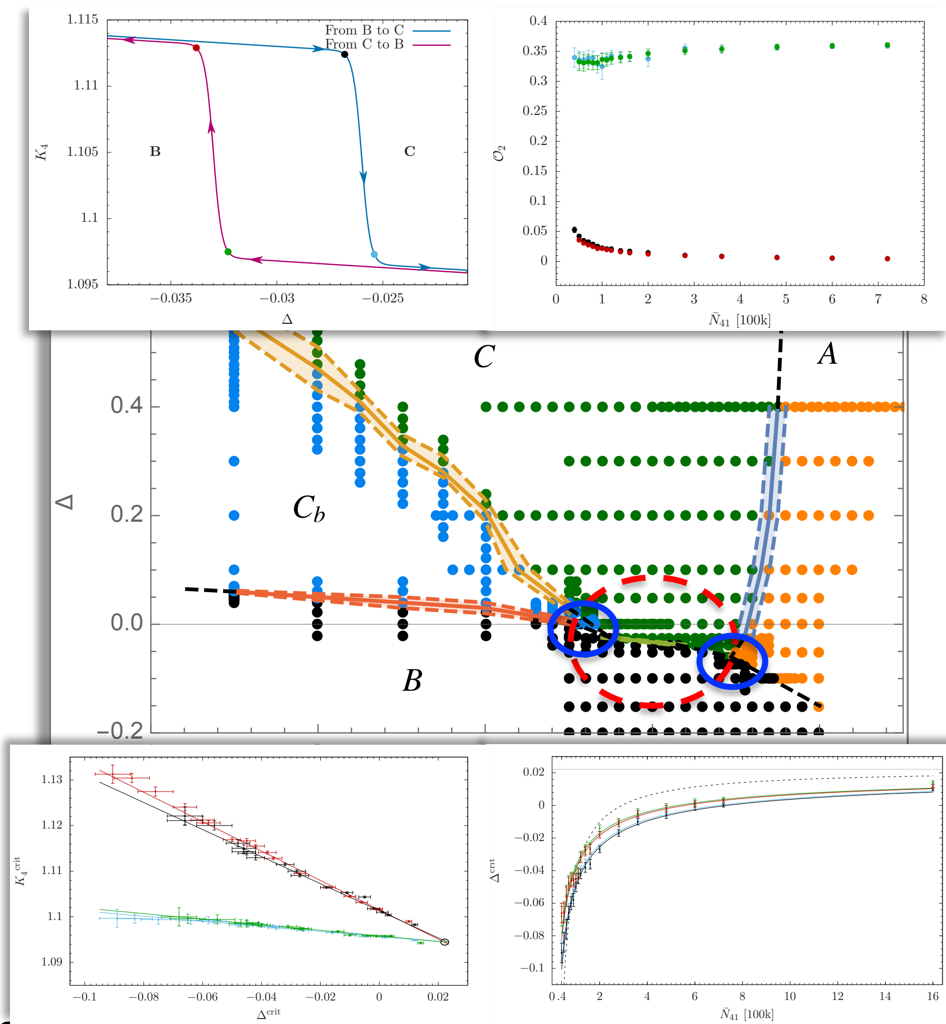

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JHEP 1907: 166
JHEP 2204: 103

Toroidal vs spherical topologies

✧ Phase transitions (T^3 vs S^3 spatial topology)

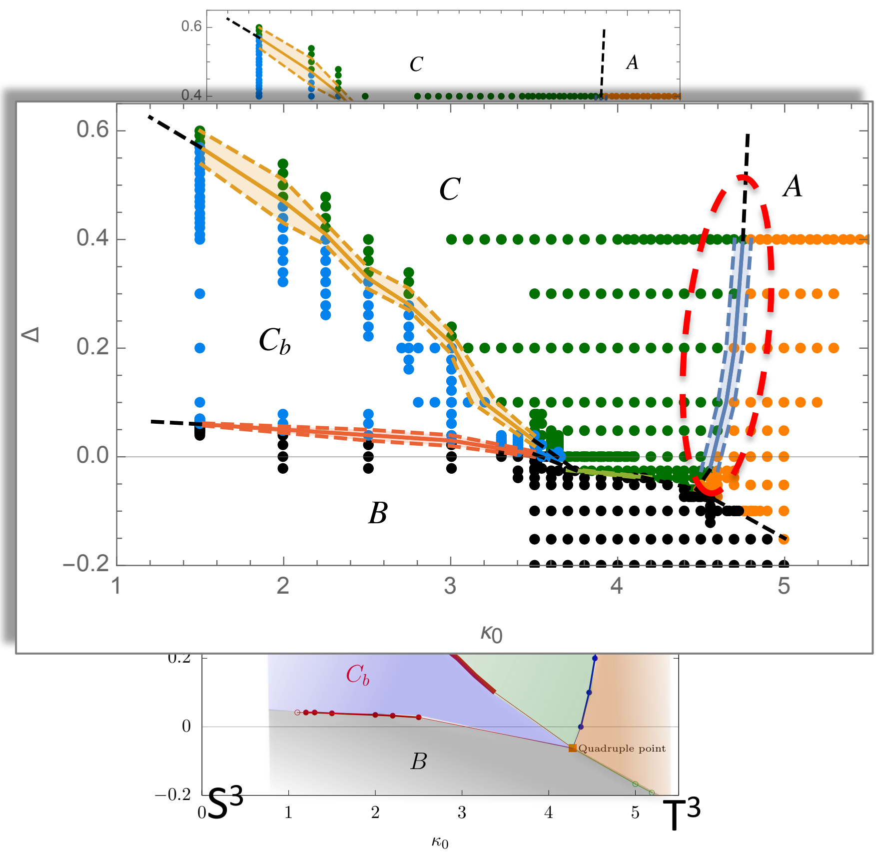
✧ In T^3 CDT the A-C transition was confirmed to be 1st order (as in S^3)

- ✧ critical scaling exponent: $\nu \approx 1$

$$K_0^c\,(N_{41}) = K_0^c(\infty) - \alpha N_{41}^{-1/\nu}$$

- ✧ but the transition is smoother than for the S^3 case: no metastable state jumping visible in Monte Carlo history

T^3 x S^1



Class.Quant.Grav. 36: 224001

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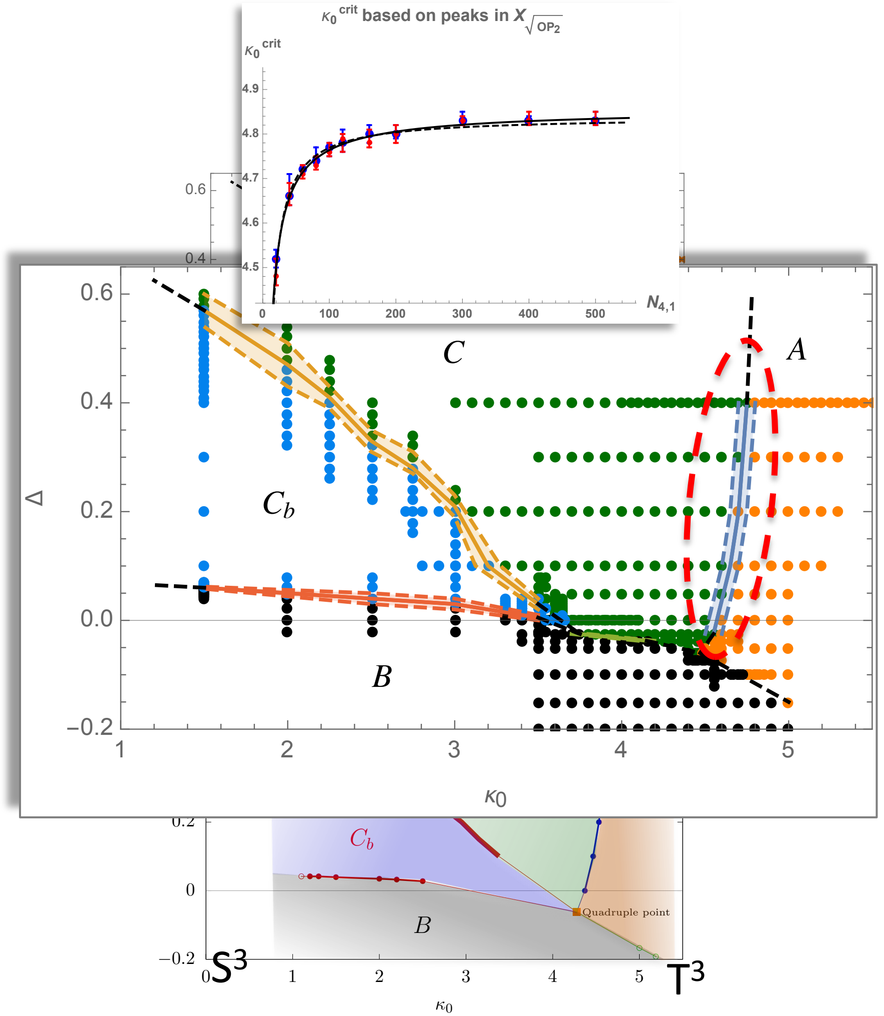

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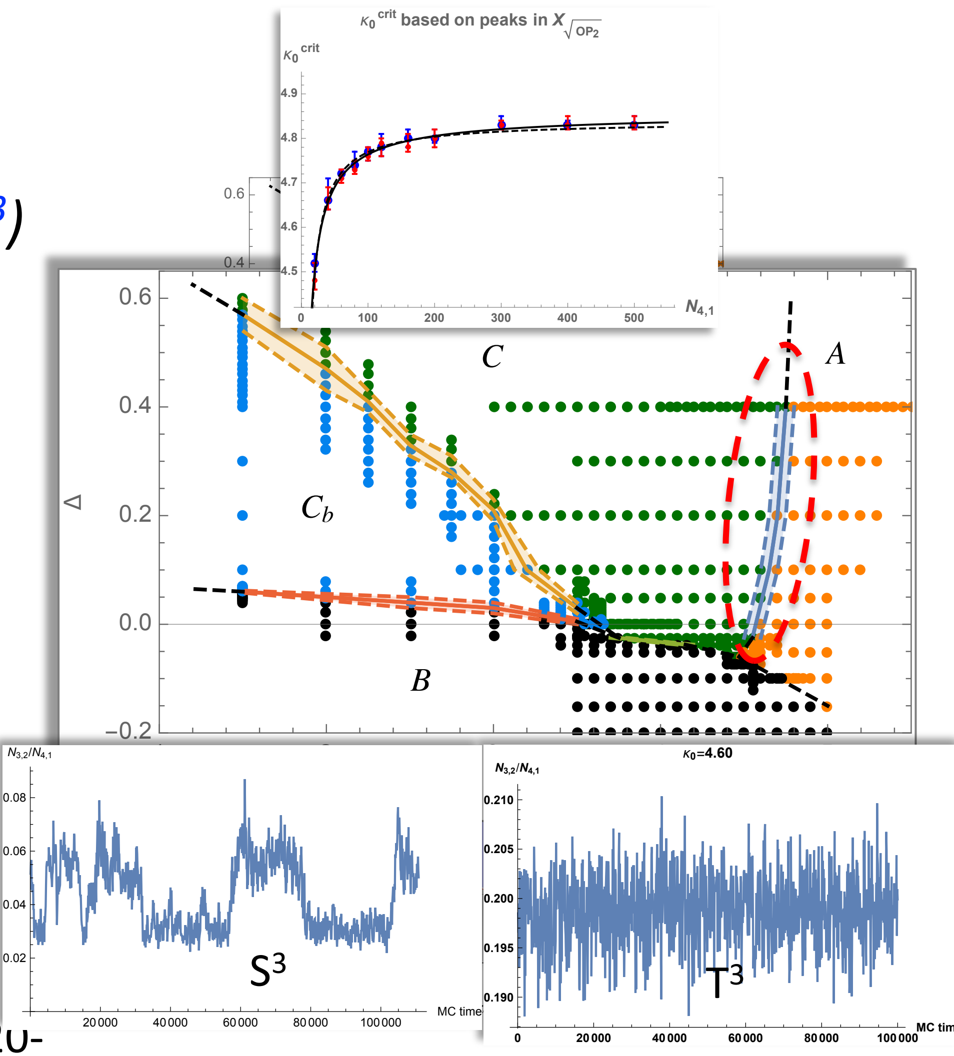

Class.Quant.Grav. 36: 224001

Toroidal vs spherical topologies

✧ Phase transitions (T^3 vs S^3 spatial topology)

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- ✧ *critical scaling exponent: ν=2.6 ≠ 1*

  $$\Delta^c\,(N_{41}) = \Delta^c\,(\infty) - \alpha N_{41}^{-1/\nu}$$

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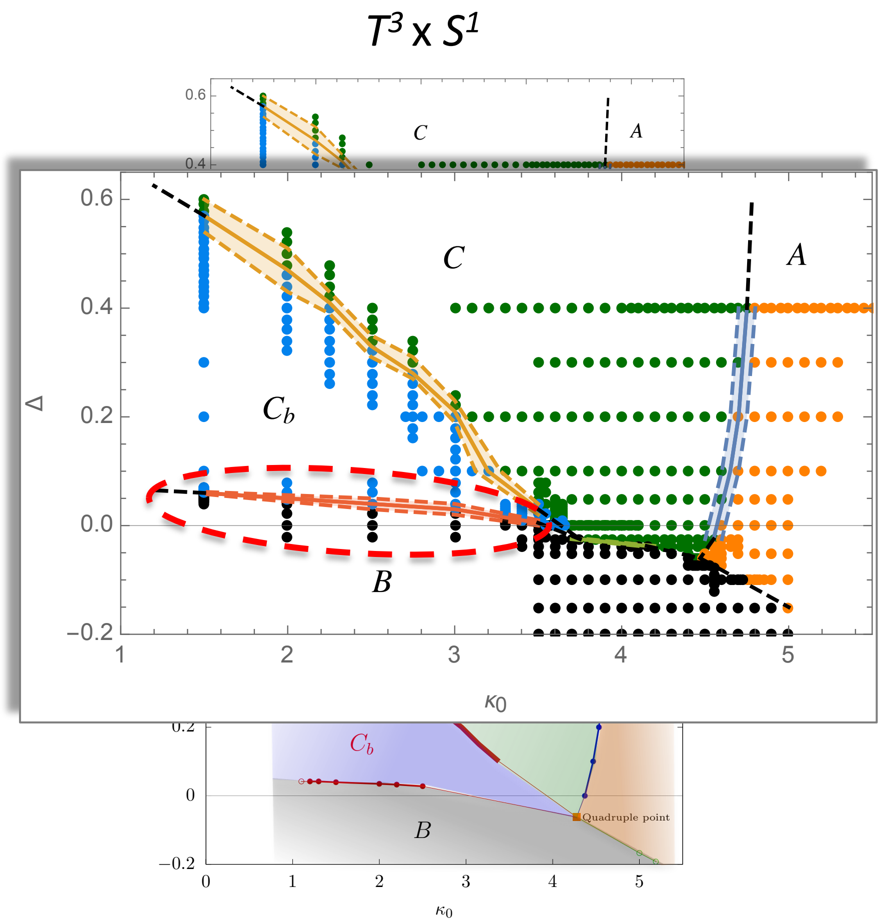
- ✧ *Binder cumulants are approaching zero with $N_4 \to \infty$*



JHEP 2005: 030

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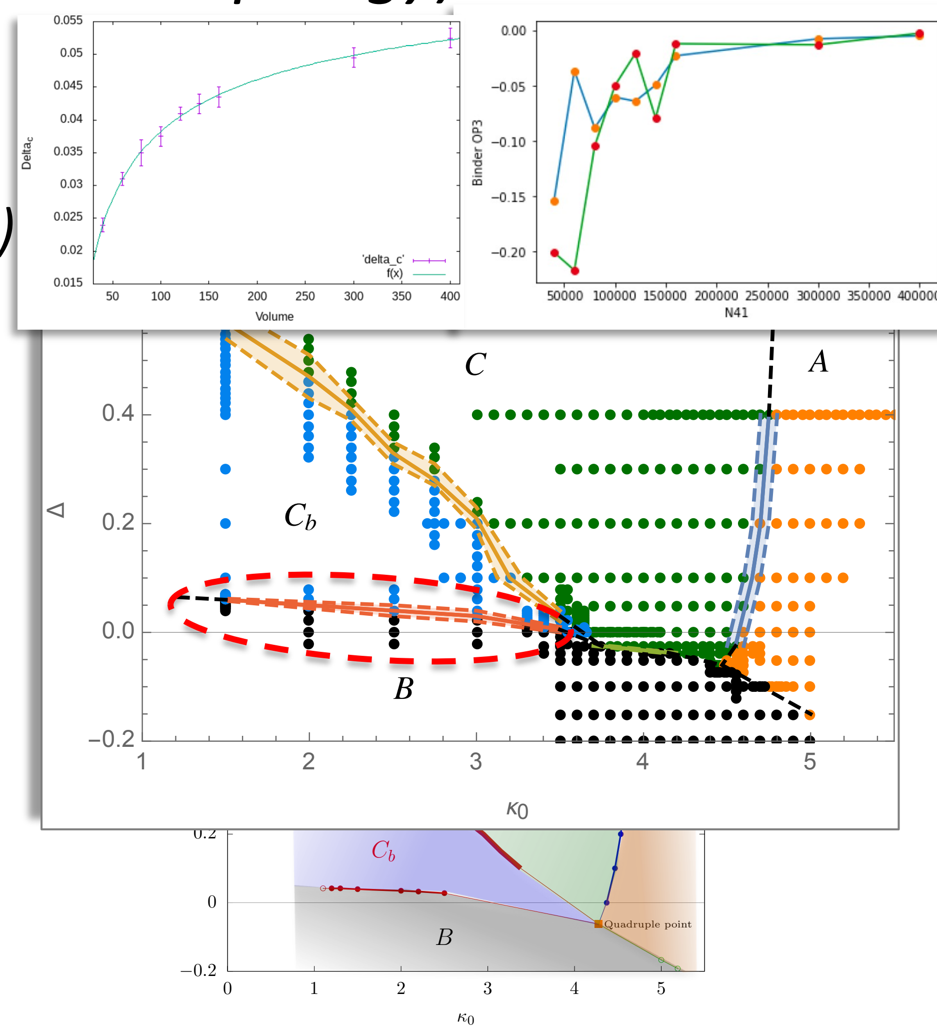


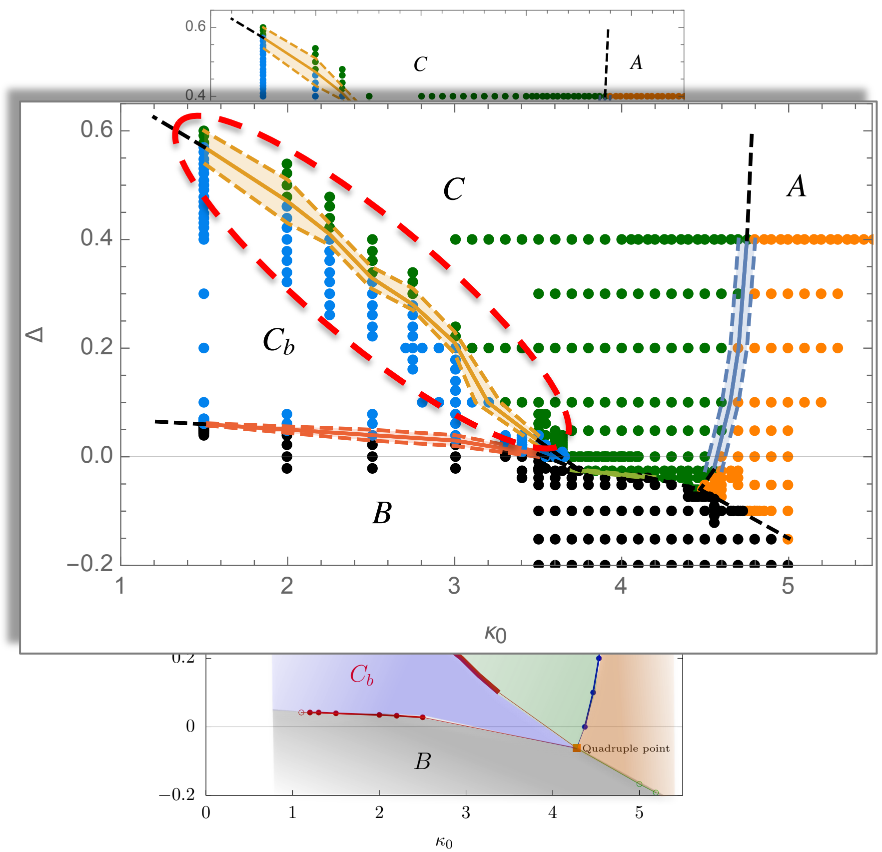
JHEP 2005: 030

Toroidal vs spherical topologies

✧ Phase transitions (T^3 vs S^3 spatial topology)

- ✧ In T^3 CDT the C-C_b transition is most likely 1st order
  - ✧ one observes strong hysteresis in the transition region which makes precise phase transition studies very difficult
  - ✧ it suggests that the transition is now 1st order transition (C-C_b transition was found to be 2nd order in S^3)
  - ✧ but one cannot exclude that it is an algotythmic issue due to much stronger finite size effects in T^3 vs S^3

$T^3 \times S^1$



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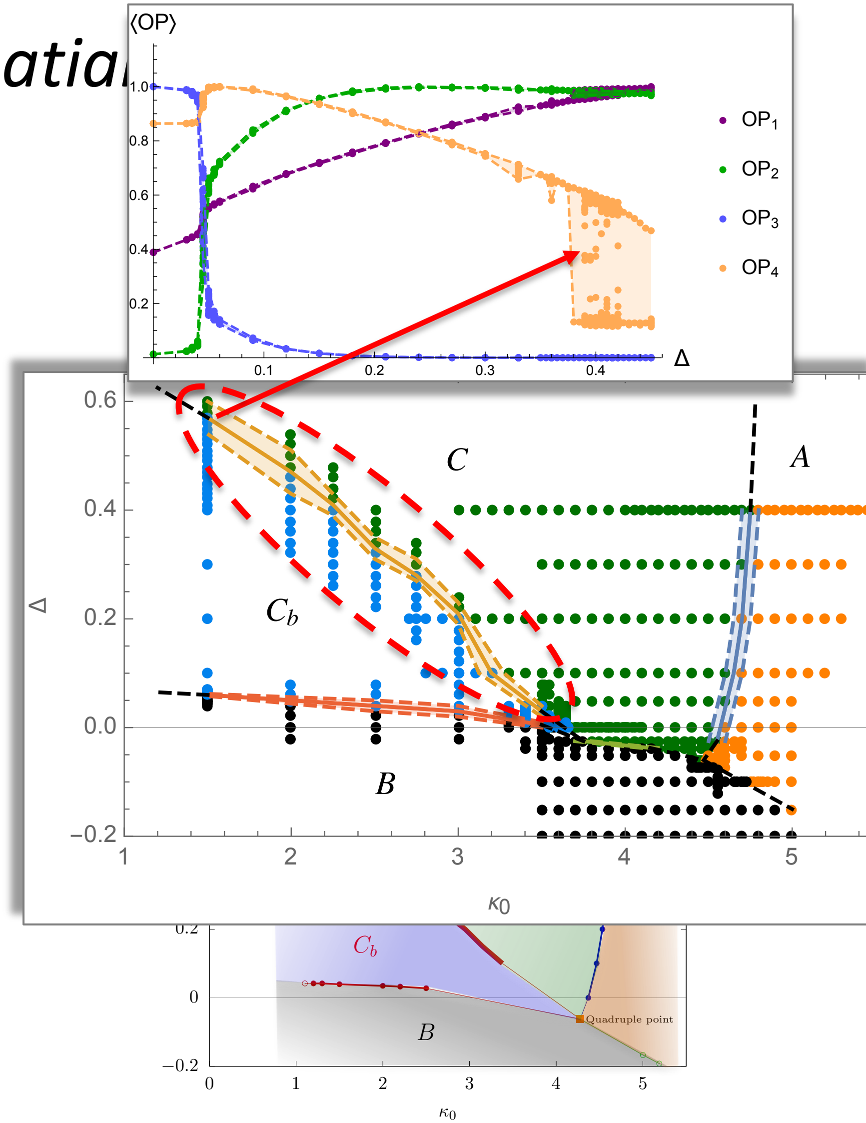


JHEP 2005: 030

Toroidal vs spherical topologies

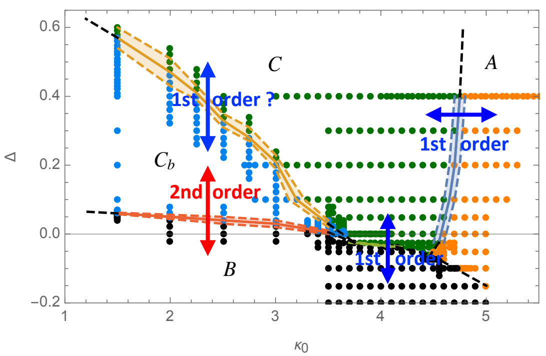
✧ *Phase transitions (T^3 vs S^3 spatial topology)*

✧ *Phase transitions summary:*

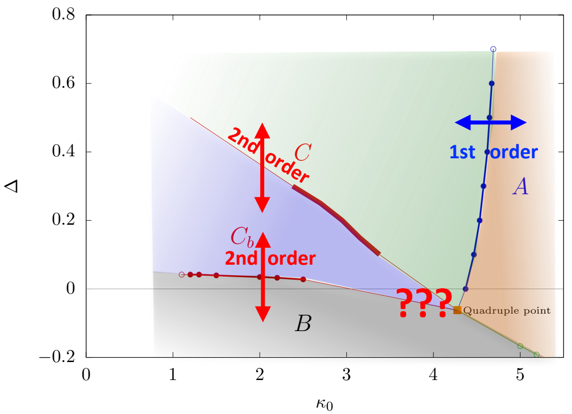
Transition	Spherical CDT	Toroidal CDT
A - C_{dS}	1^{st} order	1^{st} order
B - C_{dS}	???	1^{st} order
C_{dS} - C_b	2^{nd} order	1^{st} order (?)
B - C_b	2^{nd} order	2^{nd} order

✧ *Conjecture: phase transitions involving a change of effetive topology are 1^{st} order transitions*

T^3 x S^1



S^3 x S^1



JHEP 1907: 166
Class.Quant.Grav. 36: 224001
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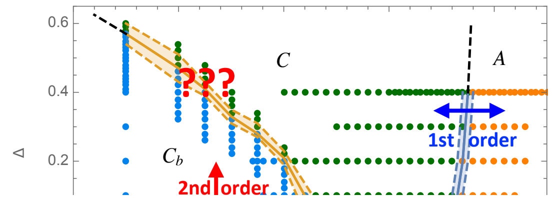
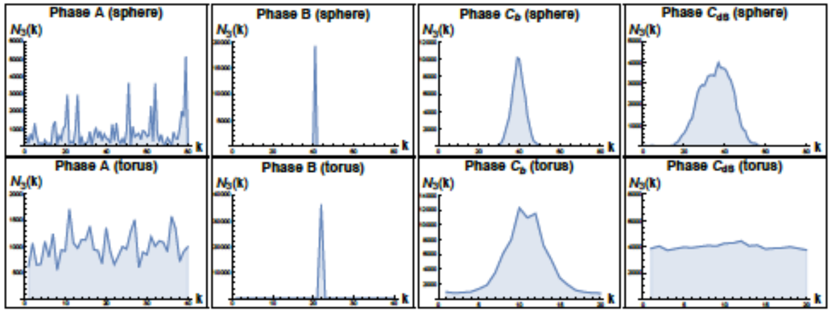
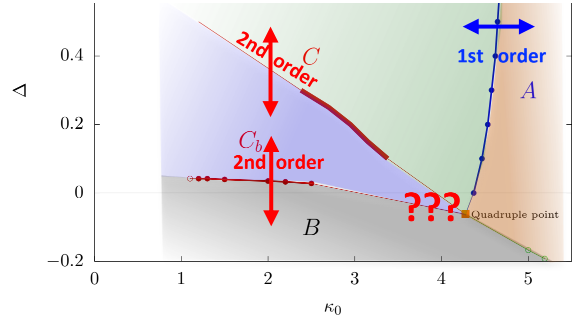
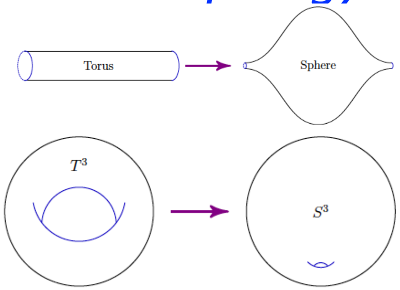
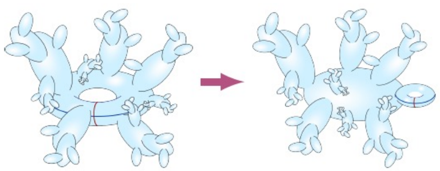






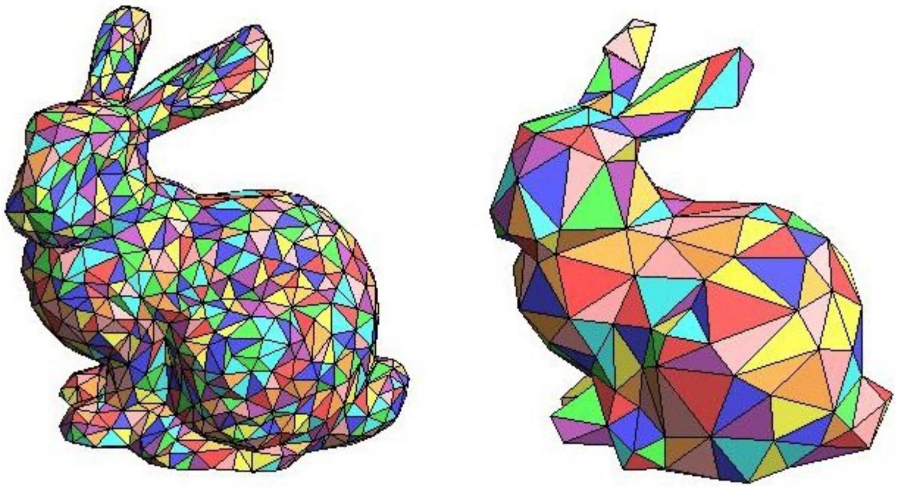




JHEP 2204: 103

Conclusions

- ✧ *CDT is very well suited to test the asymptotic safety conjecture*
- ✧ *It is also a promising candidate for describing Quantum Gravity in a fully non-perturbative way if asymptotic safety scenario is valid*
- ✧ *CDT has rich phase structure (incl. the semi-classical phase C_{dS})*
- ✧ *The spatial volume fluctuations inside the C_{dS} phase are very well described by the MS action which enables one to define the RG flow*
- ✧ *There exist 2nd order phase transitions + 1st order transitions with potentially higher order endpoints (perspective UV limits)*
- ✧ *Most CDT results seem to be universal, independent of the spatial topology chosen (at least for the toroidal vs spherical cases), however the order of the phase transitions may depend on the topology (important in the search for a continuum limit !)*



Thank You !

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI

NATIONAL SCIENCE CENTRE

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