Why there is (almost) nothing rather than something?

The cosmological constant problem

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The plan

- What is the cosmological constant problem?
- UV and IR sides of vacuum energy.
- Phase space regularization.
- Holography and the cosmological constant bound.

Based on L. Freidel, JKG, R. Leigh, D. Minic "The Vacuum Energy Density and Gravitational Entropy", ArxiV 2212.00901 [hep-th] and "On the Inevitable Lightness of Vacuum" ArxiV 2303.17495 [hep-th]

Why there is something rather than nothing? (1714)

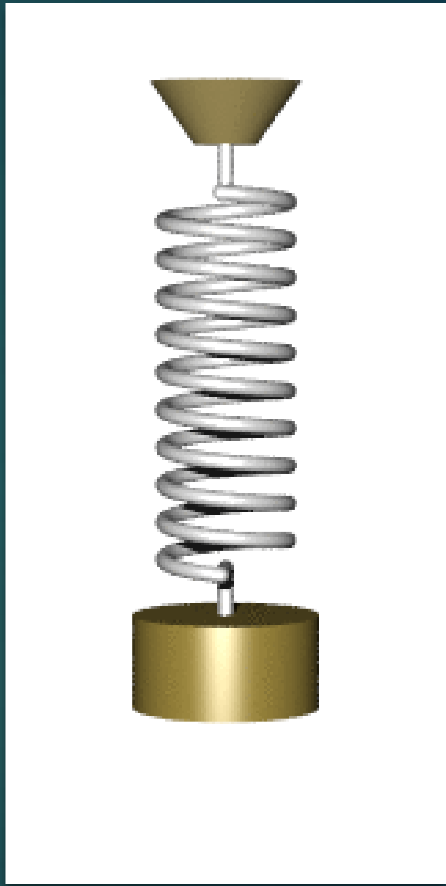
Why there is nothing rather than something? (1988)

Leibniz vs Coleman

Vacuum energy

- In quantum mechanics, the ground (vacuum) state of an oscillator of frequency ω has energy $E_0 = 1/2\, \hbar\omega$.
- Field theory describes an infinite number of oscillators (one per momentum), and the total vacuum energy density is infinite.

$$E_0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2}\omega_p \sim \lim_{\Lambda\to\infty} \frac{1}{16\pi^2}\Lambda^4$$



The cosmological constant problem

- This is not a problem if gravity is not there, because only energy differences matter, and one can always shift the excited states energies by infinite amount (using normal ordering).
- If, however, gravity is present, due to its universal nature the infinite vacuum energy produces an infinite gravitational field.
- But do the quantum fluctuations (vacuum energy) really gravitate? (Assume they do.)

The cosmological constant problem

- The gravity action contains the cosmological constant

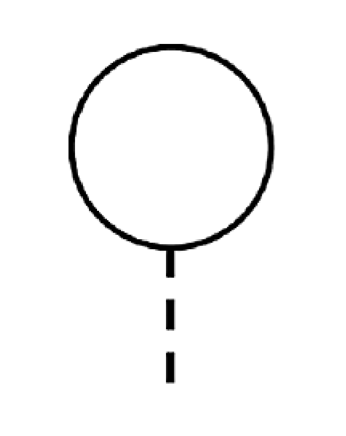
$$S \sim \int \sqrt{-g}R + \sqrt{-g}\Lambda_{cc} + \mathrm{matter}$$

- You may argue that the value of this parameter is just a constant defining the action that must be fixed observationally, but this misses the point.
- The point is that there are non-controllable contributions to the cosmological constant from matter loop diagrams.

Rough calculation*

- In the leading order the matter-linearized gravity (graviton) coupling is

$$S_{int} \sim \int h_{\mu\nu} T^{\mu\nu}$$



- Computing the tadpole diagram, we get

$$\Delta\mathcal{L} \sim h_{\mu\nu} \times \int \frac{d^4p}{(2\pi)^4} \frac{2p^\mu p^\nu - \eta^{\mu\nu}\left(p^2 - m^2\right)}{p^2 - m^2 + i\epsilon}$$
$$\sim h_{\mu\nu} \times \eta^{\mu\nu} \frac{1}{64\pi^2} \Lambda^4$$

- Notice that that loop contribution to cosmological constant is proportional to the regularized volume of momentum space.

*J. F. Donoghue, Phys. Rev. D **104**, 045005

The lesson

- This rough calculation shows that in order to get the cosmological constant vacuum energy contribution, we must compute the loop diagram, multiply it by $\sqrt{-g}$ and add the result to the action.

Theory vs observation/comments

- The measured value of cosmological constant is 10^{-120} in Planck units.
- Depending on what is your favorite cut-off scale* the parameter Λ is between 10^{-17} (Standard Model scale) and 1 (Quantum Gravity scale). Whichever we choose, the discrepancy is huge.
- The cosmological constant problem, or the vacuum energy problem, is also associated with the enormous hierarchy of scales between the observed vacuum energy scale and the naive quantum gravity scale set by the Planck energy.

* String theory is UV finite, but this does not solve the problem, because in the string loop calculation the cutoff scale Λ is just replaced by the string scale (and the mass of the lightest string state), again many orders of magnitudes off the desired result. Besides, superstrings are incompatible with positive cosmological constant.

An idea

- The cosmological constant problem clearly needs a new idea.
- The crucial observation is that it is about not only UV, but also IR.
- To see this let us revisit the computation from slightly different perspective. Instead of computing the loop diagram with no external legs, we can start with a particle moving on a circle S^1, the circle amplitude

$$Z_{S^1} = \int_0^\infty \frac{d\tau}{2\tau} \mathrm{Tr}\, e^{i\hat{\mathscr{H}}\tau} \sim \rho V_4$$

Polchinski, String theory, ch. 7

Equivalence of loops and circles

- Consider the vacuum partition function

$$Z_{vac} = \int D\phi e^{-\int \frac{1}{2}\phi(-\partial^2+m^2)\phi} \sim \left(\det\left(-\partial^2 + m^2\right)\right)^{-1/2} = e^{-\frac{1}{2}\mathrm{Tr}\log\left(-\partial^2+m^2\right)}$$

- In momentum space trace is an integral over momenta, while

$$-\frac{1}{2}\log\left(k^2+m^2\right) = \int \frac{\mathrm{d}\tau}{2\tau} e^{-(k^2+m^2)\tau/2}$$

- so that $Z_{vac} = \exp(Z_{S^1})$

$$Z_{S^1} = V_4 \int \frac{d^4k}{(2\pi)^4}\frac{d\tau}{2\tau} e^{-(k^2+m^2)\tau/2} \sim V_4 \int \frac{d^3k}{(2\pi)^3}\omega_p$$

UV & IR

$$\delta^4(p) = \int d^4q e^{-ipq}$$

$$\delta^4(0) = \int d^4q e^{-i0q} = V_q$$

- The circle amplitude is

$$
\begin{aligned}
Z_{S^1} &= \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} \langle p_\mu | e^{i\hat{\mathscr{H}}(\hat{p})\tau} | p_\mu \rangle \\
&= \delta^{(4)}(0) \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau} \\
&= V_q \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau} \\
&= \int_0^\infty \frac{d\tau}{2\tau} \int \frac{d^4q d^4p}{(2\pi)^4} e^{i\mathscr{H}(p)\tau}
\end{aligned}
$$

$\Leftarrow$ divergent IR volume contribution

$\Leftarrow$ phase space integration

Phase space integral

- To proceed, we leave the τ integration to the end and consider the Wick rotated integral

$$Z(\tau) = \int \frac{d^4q d^4p}{(2\pi\hbar)^4} e^{-p_\mu^2 \tau/2} = \mathrm{Tr}\, e^{-\hat{p}_\mu^2 \tau/2}$$

- In the next step we split the integral over phase space into a sum over integrals in a finite cell, via $p \to \varepsilon\tilde{x} \quad , \quad q \to \lambda x$. The dimensionful scales λ, ε are arbitrary here.

$$Z(\tau) = \prod_{j=1}^{4} \left[\frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dq_j \int_{-\infty}^{\infty} dp_j \, e^{-p_j^2 \tau/2} \right]$$

$$= \left[\frac{\lambda\varepsilon}{2\pi\hbar} \sum_{k,\tilde{k}\in\mathbb{Z}} \int_0^1 dx \int_0^1 d\tilde{x}\, e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

Modular polarization

- These manipulations are just rearrangements of the integral, but the resulting expression has an interpretation of the trace done in another basis, a so-called modular polarization. It is unitarily equivalent (via so called Zak transform) to the momentum basis.
- The phase space decomposes into modular cells of size

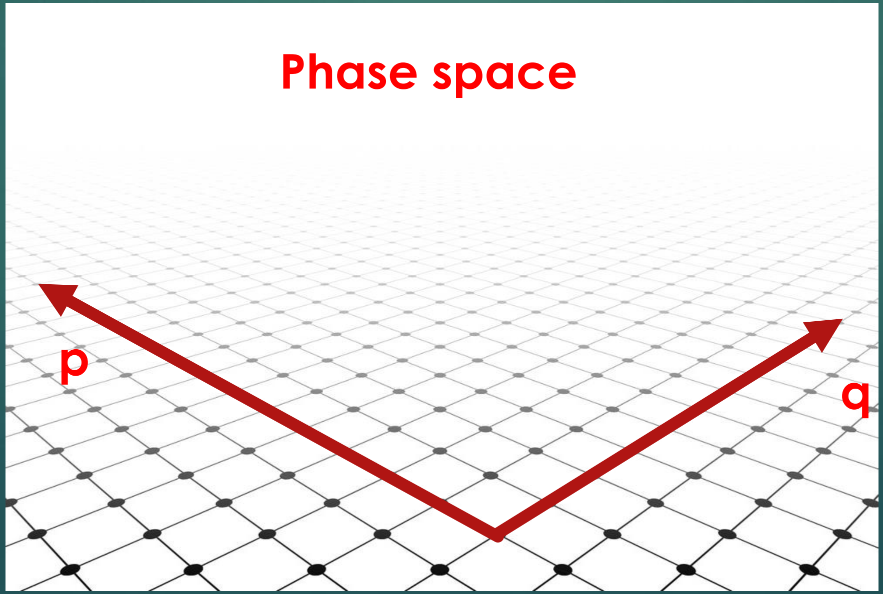
$$\boxed{\varepsilon\lambda = 2\pi\hbar}$$

- We call this quantum area constraint. It should be stressed that apart the area constraints the scales ε and λ are arbitrary; nothing forces us to identify them with the Planck scales.
- The sums can then be interpreted as counting such modular cells.

* Y. Aharonov, D. Rohrlich, "Quantum Paradoxes", Wiley 2005

The phase space

- Instead of the minimal length, area, volume … we have here the minimal phase space cell of size $\hbar$, the notion that lies at heart of quantum mechanics. The exact size of the cell is fixed contextually, relative to the actual physical situation. For example, in the double slit experiment λ is the distance between the slits



Regularization

- The expression $Z(\tau)$ is divergent and must be regularized

$$Z(\tau)_{m.r.} := \left[\sum_{k=0}^{N_q-1} \sum_{\tilde{k}=0}^{N_p-1} \int_0^1 dx d\tilde{x} e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

- Here N_q and N_p are finite integers. Knowing them and λ, ε we can determine the size of spacetime and momentum space

$$L = N_q \lambda \,, \quad M = N_p \varepsilon$$

- Also, we know that the total number of degrees of freedom is

$$\boxed{N = (N_q N_p)^4}$$

Regularization

- Further,

$$\boxed{\left(\frac{LM}{2\pi\hbar}\right)^4 = N}$$

- This result is unusual: if we regard N as fixed, then the cutoffs on space and momentum are not separately arbitrary but are inversely related. This clearly can be interpreted as a UV/IR mixing phenomenon. In EFT, there is no such relation, because there is no notion of finite N.

Vacuum energy

- From regularized $Z(\tau)_{m.r.}$ we get the vacuum energy density

$$\rho(\tau)_{m.r.} = \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \frac{1}{N_q} \sum_{k=0}^{N_q-1} \int_0^1 d\tilde{x} e^{-(\tilde{x}+k)^2 \varepsilon^2 \tau/2} \right]^4$$

- Assuming that the τ integration does not change things substantially, we get a bound

$$\rho_{m.r.} \leq \hbar \left[\frac{\varepsilon N_p}{2\pi\hbar} \right]^4$$

- This can be rewritten as

$$\rho_{m.r.} \leq \hbar \left[\frac{M}{2\pi\hbar} \right]^4$$

- and if M is identified with a large mass scale such as Planck mass, then the usual conundrum pertains. ***But there is nothing here that makes it necessary/natural.***

Vacuum energy

- The bound can be also written as

$$\rho_{m.r.} \leq \hbar \frac{N}{V_q} \,, \quad V_q = L^4$$

- It relates the vacuum energy density times space-time volume, ρV_q, to N.
- This is remarkable. Moreover, we know from the theory of geometric quantization that (in the limit of small $\hbar$) N is the dimension of the Hilbert space of the system, and therefore its entropy.

Entropy and N

- The gravitational entropy sales as an area

$$S_{grav} \sim \ell_{Pl}^{-2}\,\text{Area} \sim \left(\frac{\ell}{\ell_{Pl}}\right)^2$$

- The holographic principle states that matter entropy N cannot exceed de Sitter gravitational entropy which gives the vacuum energy bound

$$\rho_{m.r.} \leq \hbar \frac{N}{V_q} \lesssim \frac{\hbar}{\ell^2 \ell_{Pl}^2}\,, \qquad V_q = \ell^4$$

- which gives the value of the vacuum energy contribution to cosmological constant

$$\boxed{\Lambda_{cc} \sim \rho G \sim \frac{1}{\ell^2}}$$

The cosmological constant

$$\boxed{\Lambda_{cc} \leq \rho G \sim \frac{1}{\ell^2}}$$

- The scale ℓ is the size of the system. In our universe it is the cosmic horizon size, and since our universe is essentially de Sitter, it equals de Sitter horizon. Now everything fits together perfectly, because *N* equals de Sitter entropy.
- The cosmological constant is small because the universe is large. This is almost tautological: a nearly empty universe, corresponding to a small *N*, would have an extremely large cosmological constant and therefore be of Planckian size.
- Why the universe is large? It is large, because it is stable against fluctuations. If we have N degrees of freedom, the statistical fluctuations are of order of $N^{1/2}$ and are (relatively) small if N is large.

Conclusions: what has just happened?

- **The steps:**

1. We started with the standard vacuum energy formula and the semiclassical relation between vacuum energy and cosmological constant.
2. We noticed that vacuum energy formula is defined on phase space; we regularize it assuming that there is a finite number of the elementary phase space cells in the system of interest.
3. We identify the number of cells with entropy using the holographic principle.
4. De Sitter entropy of our universe provides us with the upper bound on the vacuum energy contribution to cosmological constant. This bound agrees with the observed cosmological constant value.

Conclusion

- In a large universe the vacuum energy contribution to the cosmological constant must be small.
- The universe is large because it contains a lot of degrees of freedom.
- It must have a lot of degrees of freedom in order to be.