Gravitational Collapse of Real Scalar Fields: Fundamentals and Applications

AT

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OUTLINE

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WHAT IS GRAVITATIONAL COLLAPSE?

- In co-moving co-ordinate system, 4-velocity: $u^i_t = \delta^i_t$:

$$R(r, t) = r\, a(r, t),$$

$$\dot{R} < 0 \longrightarrow \dot{a} < 0.$$

- The time-like congruence u^a: converging:

$$\theta = D_a u^a < 0.$$

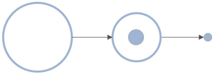

FIGURE: Evolution of apparent horizon and the collapsing cloud: black-hole.

WHAT IS A SCALAR FIELD?

- A real scalar field:

$$\phi : \mathcal{M} \to \mathbb{R}.$$

- Lagrangian of a scalar field ϕ with potential $V(\phi)$:

$$\mathcal{L}_\phi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).$$

- Stress-energy tensor:

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - -\frac{1}{2} g_{\mu\nu} (\nabla_\delta \phi \nabla^\delta \phi + V(\phi)).$$

WHY STUDY GRAVITATIONAL COLLAPSE OF A SCALAR FIELD?

1. Understanding fundamental features of space-time:
   - Cosmic Cesnsorship Conjecture[1] (CCC)[2],
   - Critical phenomena in gravitational collapse[3].
2. Applications in cosmology:
   - Dark matter[4],
   - Dark energy[5],
   - Structure formation[6].

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[1] R. Penrose. In: *Riv. Nuovo. Cimento* Num. Sp. I (1969).

[2] P. S. Joshi K. Mosani Koushiki, J. V. Trivedi, and T. Bhanja. In: *Phys. Rev. D* 108.044049 (2023).

[3] M. W. Choptuik. In: *Phys. Rev. Lett.* 70, 9 (1993).

[4] Frank Wilczek. In: *Physical Review Letters* 85 (2000), pp. 1158–1161. DOI: 10.1103/PhysRevLett.85.1158.

[5] M. Sami E. J. Copeland and S. Tsujikawa. In: *Int. Jou. Mod. Phys. D* 15.1753-1935 (2006).

[6] D. Dey P. Saha and K. Bhattacharya. In: *Phys. Rev. D* 109.104023 (2024).

General framework for gravitational collapse in spherically symmetric space-times

1. Line element:

$$ds^2 = -e^{2\nu(r,t)}dt^2 + e^{2\psi(r,t)}dr^2 + R^2(r,t)d\sigma^2.$$

2. **Type-I matter fields**: *Three space-like and one time-like eigen-vectors*: Hawking and Ellis[7], Joshi[8].
3. Weak energy condition[9]:

$$\rho \geq \ 0, \ \ \rho + p_i \geq 0.$$

4. Equation of state[10]:

$$p = \omega(r,t)\rho.$$

[7] S. W. Hawking and G. F. R. Ellis. *The large scale structure of spacetime*. 1973.

[8] P. S. Joshi. *Global Aspects in Gravitation and Cosmology (Clendron Press)*. 1993.

[9] S. D. Maharaj B. P. Brassel and R. Goswami. In: *Entropy* 23.11, 1400 (2021).

[10] DESI Collaboration et.al. In: (2024). arXiv: 2404.03002 [astro-ph.CO].

Outline

VISIBILITY OF THE SINGULARITY

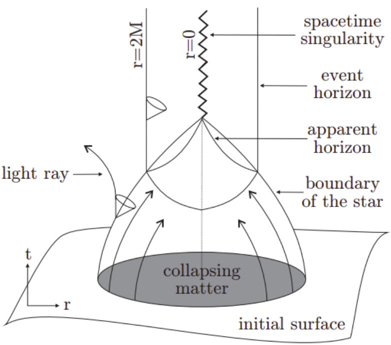

FIGURE: Formation of **Black-hole** as an end state of a spatially homogeneous dust (Oppenheimer Snyder Datt collapse)[11].

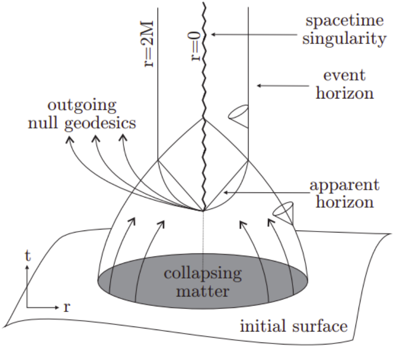

FIGURE: Formation of **Naked singularity** as an end state of a spatially inhomogeneous dust collapse (Leimatre Tolman Bondi collapse)[12].

[11] P. S. Joshi and D. Malafarina, International Journal of Modern Physics D **20**, 14 (2011).

[12] P. S. Joshi and D. Malafarina, International Journal of Modern Physics D **20**, 14 (2011).

- Singularity: boundary of the manifold.
- The 4-velocity or the tangent to a outgoing null geodesic for the manifold $(\mathcal{M}, g)$: $K^i = \frac{dx^i}{d\lambda}$.
- The expansion scalar of the outgoing null-congruence is:

$$\theta_l = \nabla_i K^i.$$

- There should be no trapped surfaces along K^i and the singularity would be **naked** if[13]:

$$\theta_l > 0.$$

- Singularity is **strong** if at least along one non-spacelike geodesic with the affine parameter λ, with $\lambda \to 0$ in the neighbourhood of the singularity, the following inequality should be satisfied[14 15]:

$$\lim_{\lambda \to 0} \lambda^2 R_{ij} K^i K^j > 0.$$

[13] P. S. Joshi and I. H. Dwivedi. In: *Phys. Rev. D* 47 (1993), p. 5357.

[14] C. J. S. Clarke and A. Krolak. In: *J. Geom. Phys.* 127 (1985), p. 2.

[15] F. J. Tipler. In: *Phys. Lett. A* 8 (1977), p. 64.

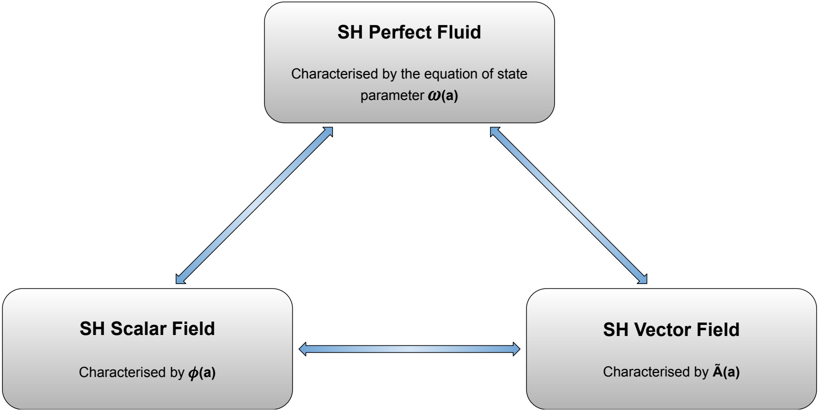

FIGURE: Equivalence between the unhindered gravitational collapse of spatially homogeneous perfect fluid, scalar field and vector field with potentials.

- Spherically symmetric spatially homogeneous cloud: Friedmann Leimatre Robertson Walker (FLRW) line-element:

$$ds^2 = -dt^2 + a(t)^2 d\sigma^2$$

where $d\sigma^2 = dx^2 + dy^2 + dz^2$.

- The stress-energy tensor in this scheme:

$$T^{\mu}_{\nu} = \mathrm{diag}\,(-\rho, p, p, p).$$

- Equation of state parameter:

$$\frac{p(a)}{\rho(a)} = \omega(a).$$

RESULTS

We find strong and naked singularities, defined by their equation of state parameter[16]:

- There will be no **trapped surfaces** in the resultant space-time if:

$$\lim_{a\to 0} \rho_0 a^2 \exp\left(\int_a^1 \frac{3\,(1+\omega(a))}{a}\right) da < 1.$$

- The singularity will be **strong** if:

$$\lim_{a\to 0} \exp\left(\int_a^1 \frac{3(1+\omega)}{a} da\right) > 0.$$

- The resultant singularity is **strong** and **naked** if:

$$0 < \lim_{a\to 0} \exp\left(\int_a^1 \frac{3(1+\omega)}{a} da\right) < \mathcal{O}(a^{-2}).$$

- We get a **strong and naked singularity** for the scalar field described by Goswami and Joshi[17].

[16] P. S. Joshi K. Mosani Koushiki, J. V. Trivedi, and T. Bhanja. In: *Phys. Rev. D* 108.044049 (2023).

[17] R. Goswami and P. S. Joshi. In: *Modern Physics Letters A* 22 (2007), pp. 65–74.

EXAMPLES

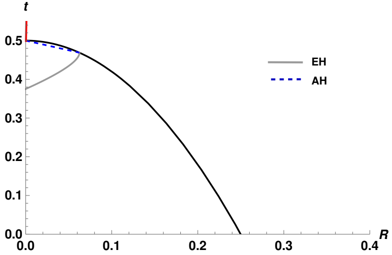

FIGURE: Massless scalar field ($V_s = 0$).

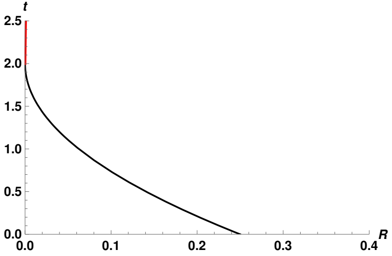

FIGURE: $V_s(\phi) \propto \exp\phi$ as described in[18]. A globally visible and strong singularity forms in finite comoving time.

R. Goswami and P. S. Joshi, Modern Physics Letters A, **22**, 01, pp. 65-74 (2007).

Choptuik's Original work[19]:

1. power-law relationship for the mass of the black hole formed near the critical threshold:
$$M \sim (p - p_*)^\gamma,$$
2. $\gamma \approx 0.37$: universal critical exponent.
3. p_*: attractor in parameter space.
4. Arbitrarily small mass black-hole.
5. The critical solution: self-similarity: *echoing.*
6. Families of scalar fields: $\gamma \to$ universal.

[19] M. W. Choptuik. In: *Phys. Rev. Lett.* 70, 9 (1993).

OUR FINDINGS

These results are from a work under preparation:

1. Universality of collapse end-states in all scalar-field families.
2. $\exists$ a dimension-less parameter that decides the visibility of the end-state.
3. The scalar field is necessarily of type-I.
4. There are *four* different outcomes:
    - dispersal and no singularity,
    - *zero*-mass black-hole,
    - a black-hole with non-zero mass,
    - a locally naked singularity with *zero* gravitational mass.

OUTLINE

- Stable equilibrium:

$$\lim_{t\to\infty} \dot{R} = \lim_{t\to\infty} \ddot{R} = 0\,.$$

- If $a_e(r) \equiv \lim_{t\to\infty} a(r,t)$ then:

$$\dot{a}_e(r) = \ddot{a}_e(r) = 0\,,$$

  where, $R(r,t) = r\ a(r,t)$.

- $a(r,t)$: smooth monotonically decreasing function of comoving time t:

$$\begin{aligned} & a(r,t) = a_e(r)\ \forall\ t \geq t_e \\ \implies \quad & \dot{a} = \ddot{a} = \dddot{a} = ... = a^{(n)} = 0\ \forall\ t \geq t_e\ . \end{aligned}$$

Line element:

$$ds^2 = -dt^2 + \frac{a^2(t)}{1-r^2} dr^2 + r^2\ a^2(t)\ d\sigma^2 .$$

1. Finding potential function $\Rightarrow$ equilibrium solution.
2. Two independent equations and three unknowns: $\phi(a)$, $V(\phi)$, and $\dot{a}(a)$.
3. Freedom to choose one free function: $\dot{a}(a)$.
4. Functional form of $\dot{a}(a)$:

$$\dot{a}(a) = \beta\ \left(f(a) - f(a_e)\right)^{\alpha} \quad \forall a \in [a_e, a_0]\,,$$

Potential class for equilibrium-condition for a spatially homogeneous scalar field in spherical symmetry[20]:

- Bound on α:

$$\alpha \geq 1 \ .$$

- $\forall\ \alpha \geq 1$ near the equilibrium $a \to a_e$

$$\begin{aligned} \partial_a \phi &\approx \frac{\pm\sqrt{2}}{\beta\ a\,(f(a) - f(a_e))^{\alpha}} \ , \\ V(a) &\approx \frac{2}{a^2} \ . \end{aligned}$$

[20] Koushiki Dipanjan Dey and Pankaj S. Joshi. In: *Phys. Rev. D* 108, 104045 (2023).

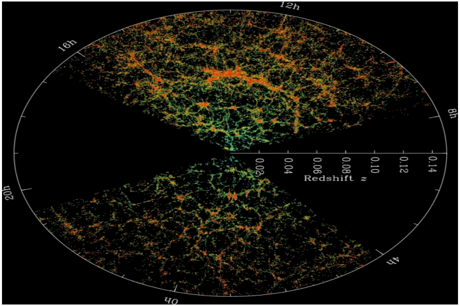

FIGURE: A plot of sky coordinates vs. distance for galaxies in the Sloan Digital Sky Survey, Source: SDSS.

- Perturbation of linear gravity regime[21].
- Top hat prescription[22]:
  1. Background: pressureless dust expanding, governed by flat FLRW.
  2. Overdense sub-universe: pressureless dust in closed FLRW geometry.
  3. Initial expansion phase, followed by a collapse.
  4. Halting of collapse: virialisation.

Why is our approach novel?

- General relativistic.
- Potential function is not presumed.
- Collapse stops automatically at assymptotic co-moving time[23].

[21] P.J.E. Peebles. *The Large-Scale Structure of the Universe*. Chapter: Linear and Nonlinear Evolution of Cosmological Perturbations. Princeton University Press, 1980.

[22] P. J. E. Peebles. In: *The Astrophysical Journal* 243 (1980), pp. 1–16.

[23] N. Ramesh S. Rajibul P. Kocherlakota and P. S. Joshi. In: *Mon. Not. Roy. Astron. Soc.* 482.1 (2019), pp. 52–64.

- Scale factor:

$$R(t) = r\ a(t). \tag{1}$$

- Expansion phase: upto the epoch of turn-around:

$$a \in (a_0, a_{max}) \begin{cases} \dot{a} > 0 \ , \\ \ddot{a} < 0 \ . \end{cases} \tag{2}$$

- At the epoch of turn-around:

$$a = a_{max} \begin{cases} \dot{a} = 0 \ , \\ \ddot{a} < 0 \ . \end{cases} \tag{3}$$

- Collapse phase: after turn-around, upto equilibrium:

$$a \in (a_{max}, a_e) \begin{cases} \dot{a} < 0 \ , \\ \ddot{a} < 0 \ . \end{cases} \tag{4}$$

Expansion phase: closed FLRW, collapse phase: geometric description? Components: pressureless dust (ordinary dark matter), weakly interacting to dark energy (seeded by a massive scalar field[24]).

1. General spherically symmetric:

$$ds^2 = -e^{2\nu(r,t)}dt^2 + \frac{R'^2(r,t)}{G(r,t)}dr^2 + R^2(r,t)d\sigma^2 \ . \tag{5}$$

2. Stress-energy tensor of the sub-universe:

$$\begin{aligned} T_{\mu\nu} &= (T_{\mu\nu})_{DM} + (T_{\mu\nu})_{\phi} \\ &= (\rho_{DM} + P_{DM})u_\mu u_\nu + P_{DM}g_{\mu\nu} + (\rho_\phi + P_\phi)u_\mu u_\nu + P_\phi g_{\mu\nu} \ . \end{aligned} \tag{6}$$

3. Isotropic fluid in its comoving frame, the non-vanishing and unique components:

$$\begin{aligned} \rho &= \rho_{DM} + \rho_\phi \ , \\ P &= P_\phi \ . \end{aligned} \tag{7}$$

[24] M. Sami E. J. Copeland and S. Tsujikawa. In: *Int. Jou. Mod. Phys. D* 11.1753-1935 (2006).

Dynamic dark-energy seeded by a scalar field potential[25]:

1. Line-element for the system: closed FLRW.
2. Equation of state of the composite system:
    - Expanding phase: *zero*.
    - Collapsing phase: *varying*.
3. Effect of dark-energy starts growing from the collapse phase.
4. Potential class: Tachyonic potential[26] [27].
5. Effective mass: imaginary.
6. Misner-Sharp mass function: real and positive for a closed FLRW space-time.

[25] D. Dey Koushiki and P. S. Joshi. In: (2024). arXiv: 2404.03901 [gr-qc].

[26] A. Sen. In: *JHEP* 07, 065 (2002).

[27] A. Sen. In: *JHEP* 04, 048 (2002).

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