Super Cartan geometry, LQG and applications

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Section 1

Introduction

Holst action

Asthekar's discovery:

- GR can be formulated such that (non-reduced) symplectic phase space of canonical theory is that of $\mathrm{SU}(2)$ Yang-Mills!
- Also exist in higher dimensions [Bodendorfer+Thiemann+Thurn]

In 4D: **Holst action** S^{β}_{Holst}: 1-parameter family of *deformed* actions of first-order Einstein gravity

Holst action

$$S^{\beta}_{\text{Holst}} = \frac{1}{4\kappa}\int_M \epsilon_{IJKL} e^I \wedge e^J \wedge F[\omega]^{KL} + \frac{2}{\beta} e^I \wedge e^J \wedge F[\omega]_{IJ}, \quad \beta: \text{ Immirzi}$$

$I, J, \ldots$: $\mathrm{SO}(1,3)$

- $\rightarrow$ yield **same** EOM as Einstein gravity! (metric $g_{\mu\nu} = \eta_{IJ} e^I_\mu e^J_\nu$)

Canonical decomposition

- Assume spacetime is globally hyperbolic: $M = \mathrm{R} \times \Sigma$ (Σ: $3D$ Cauchy slice)

3+1-decomposition

$$S^{\beta}_{\text{Holst}} = \frac{1}{\kappa\beta} \int_{\mathrm{R}} \mathrm{d}t \int_{\Sigma_t} \mathrm{d}^3x \, \left(E^a_i \dot{A}^i_a - \omega^i_0 G_i + N^a C_a + NC \right)$$

$i.j.\ldots$: $\mathrm{SO}(3)$ (or $\mathrm{SU}(2)$)

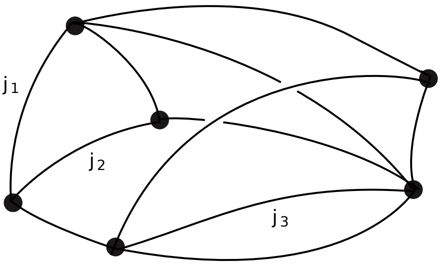
- Canonically conjugate variables [Ashtekar+Barbero+Immirzi]

$$A^i_a = \Gamma^i_a + \beta K^i_a, \qquad E^a_i = \sqrt{|\det q|} e^a_i$$

- Constraints: Gauss G_i, Diffeo C_a and Hamilton C
- $\rightarrow$ Constraints+Hamilton EOM $\Leftrightarrow$ Einstein equations

Quantum theory

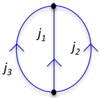
Basic building blocks: **Spin networks** [Penrose 70's]→ Spin nets associated to gauge-invariant functionals.



→ Form basis of Hilbert space [Ashtekar+Lewandowski]

$$\mathcal{H}_{\mathrm{LQG}} = \varinjlim \mathcal{H}_\gamma = L^2(\overline{\mathcal{A}}/\overline{\mathcal{G}}, \mathrm{d}\mu_{\mathrm{AL}})$$

Example:



$$\Psi[A] = \pi_{j_1}(h_{e_1}[A])^{m_1}{}_{n_1}\,\pi_{j_2}(h_{e_2}[A])^{m_2}{}_{n_2}\,\pi_{j_3}(h_{e_3}[A])^{m_3}{}_{n_3}\,\iota_{m_1 m_2 m_3}\,\iota^{n_1 n_2 n_3}$$

Why SUSY in LQG?

Motivation:

1. If a quantum theory of gravity really exists, it has to be unique!
2. Apply ideas + techniques from one approach to the other

How to achieve this?

- Essential ingredient: **Supersymmetry**
- **Supergravity theories** as low-energy limits of certain superstring theories
- $\rightarrow$ Apply LQG methods to supergravity

Supersymmetry

Coleman-Mandula: most general Lie algebra of symmetries of the S-matrix has form

$$\mathfrak{iso}(\mathbb{R}^{1,3}) \oplus \text{internal sym.}$$

$\rightarrow$ Only way around this through new form of symmetry:

Haag-Łopuszański-Sohnius theorem

Have to consider **super Lie algebras**, i.e. $\mathbb{Z}_2$-graded algebras $(\mathfrak{g}, [\cdot,\cdot])$ of the form

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \quad \text{with} \quad [\cdot,\cdot] : \text{(anti) commutator on } \mathfrak{g}_0 \ (\mathfrak{g}_1)$$

such that $[\mathfrak{g}_i, \mathfrak{g}_j] \subseteq \mathfrak{g}_{i+j}$ (+ graded Jacobi identity)

Supersymmetry

super Poincaré/super anti-de Sitter $\mathfrak{osp}(1|4)$

$$\mathfrak{iso}(\mathbb{R}^{1,3|4}) = \underbrace{\mathbb{R}^{1,3} \rtimes \mathfrak{so}(1,3)}_{\mathfrak{g}_0} \oplus \underbrace{S_{\mathbb{R}}}_{\mathfrak{g}_1}$$

Generators: P_I, M_{IJ}, Q_α (Majorana spinor)

$$[P_I, Q_\alpha] = 0 - \frac{1}{2L} Q_\beta (\gamma_I)^\beta{}_\alpha$$

$$[M_{IJ}, Q_\alpha] = \frac{1}{2} Q_\beta (\gamma_{IJ})^\beta{}_\alpha$$

$$\{Q_\alpha, Q_\beta\} = \frac{1}{2}(C\gamma_I)_{\alpha\beta} P^I + \frac{1}{4L}(C\gamma^{IJ})_{\alpha\beta} M_{IJ}$$

Further generators $Q^r_\alpha \to \mathcal{N}$**-extended SUSY**!

$$AdS_4 := \{x \in \mathbb{R}^5 | -(x^0)^2 + (x^1)^2 + \ldots + (x^3)^2 - (x^4)^2 = -L^2\}$$

Supersymmetry

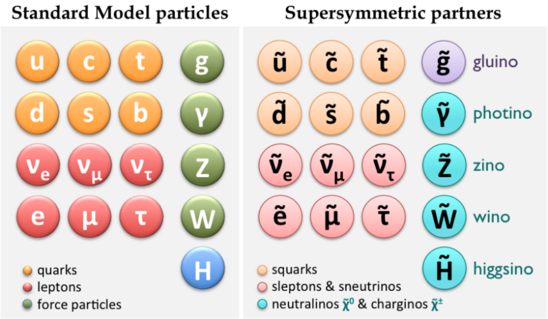


Figure: ATLAS Silicon IFIC

Application to gravity

Most ambitious use of this kind of symmetry:

local supersymmetry $\Rightarrow$ **supergravity** (SUGRA)

For $\mathcal{N} = 1$: contains gravitational field and a spin-$\frac{3}{2}$ fermion (*gravitino*)

LQG and supergravity

What has been done?

- Canonical SUGRA adapted to LQG [Jacobson ‘88]
- Hidden $\mathfrak{osp}(1|2)$-symmetry in constraint algebra, construction of $\mathfrak{osp}(1|2)$-valued connection [Fülöp ‘94, Gambini+Obregon+Pullin ‘96]
- Quantization [Gambini+Obregon+Pullin ‘96, Ling+Smolin ‘99]

But:

- Considerations remain formal
- Origin of hidden symmetry unclear
- Generalizations?

- Canonical theory for higher D and quantization [Bodendorfer +Thiemann+Thurn ‘11]

But:

- SUSY not manifest
- Formalism/constraints quite complicated

LQG and supergravity

My goals:

Follow [L+S]: Keep SUSY manifest as much as possible!

- Origin of $\mathfrak{osp}(1|2)$-symmetry? $\rightarrow$ Understand the geometric origins with a view towards generalizations:
  - Immirzi parameters
  - Higher $N > 1$
  - Boundary theory ($\rightarrow$ **black holes**)
- Mathematically rigorous formulation, both classically and in quantum theory:
  - Construction of super spin-nets ($\rightarrow$ *super parallel transport*)
  - Structure of Hilbert spaces $\leftrightarrow$ relation to standard quantization of fermions in LQG

In this talk

Classical theory:

- Supergravity via Cartan geometry
- Holst-MacDowell-Mansouri SUGRA action for any β
- Extension to $\mathcal{N} > 1$!
- Boundary theory ($\rightarrow$ black holes!)
- Special properties of self-dual theory

Quantum theory:

- Super spin networks, super area operator
- Applications: black holes, cosmology

Section 2

Gravity as Cartan geometry

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Klein geometry

F. Klein (Erlanger Programm): "Classify geometry of space via group symmetries".

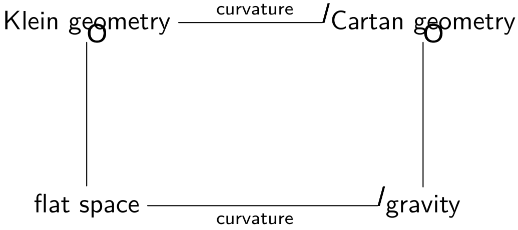
Example: **Minkowski spacetime** $\mathsf{M} = (\mathsf{R}^{1,3}, \eta)$

- Isometry group $\mathrm{ISO}(\mathsf{R}^{1,3}) = \mathsf{R}^{1,3}$ n $\mathrm{SO}_0(1,3)$
- Fix event $p \in \mathsf{M}$: $G_p = \mathrm{SO}_0(1,3)$ (isotropy subgroup)

Klein geometry

$$\mathrm{ISO}(\mathsf{R}^{1,3})/\mathrm{SO}_0(1,3) \cong \mathsf{M}$$

Cartan geometry



Cartan geometry

Gauge field with values in Poincaré algebra

$$A_\mu = \mathrm{pr}_{\mathsf{R}^{1,3}} \circ A_\mu + \mathrm{pr}_{\mathfrak{so}(1,3)} \circ A_\mu =: e_\mu + \omega_\mu$$

- e: co-frame, ω: Lorentz-connection

Cartan geometry

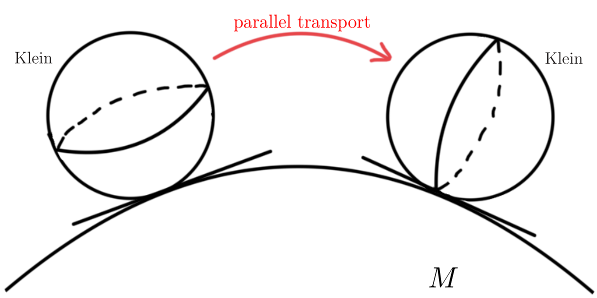
Gauge field with values in Poincaré algebra

$$A_\mu = \mathrm{pr}_{\mathbb{R}^{1,3}} \circ A_\mu + \mathrm{pr}_{\mathfrak{so}(1,3)} \circ A_\mu =: e_\mu + \omega_\mu$$

- e: co-frame, ω: Lorentz-connection

Cartan condition

$\Rightarrow$ e induces **metric** on M: $\quad g_{\mu\nu} = \eta_{IJ} e^I_\mu e^J_\nu$



Gravity as Cartan geometry

Can use Cartan connection A to formulate **YM-type action**:

- Need non-trivial cosmological constant Λ
- $\rightarrow$ Klein geometry: (Anti-)de Sitter space $(A)dS_4$

MacDowell-Mansouri action [M+M '77]

$$S_{\mathrm{MM}}[A] = \frac{3}{\kappa\Lambda} \int_M \langle F[A] \wedge F[A] \rangle$$

- $F[A]$: field strength (curvature) induced by A
- $\langle \cdot, \cdot \rangle$: inner product on (anti-)de Sitter algebra ("generalized trace")
- $\rightarrow$ yields 1^{st}-order gravity + *boundary terms*

Gravity as Cartan geometry

Can use Cartan connection A to formulate **YM-type action**:

- need non-trivial cosmological constant Λ
- $\rightarrow$ Klein geometry: (anti-)de Sitter space $(A)dS_4$

Holst-MacDowell-Mansouri action [Wise '09, KE '21]

$$S_{\text{H-MM}}[A] = \frac{3}{\kappa\Lambda}\int_M \langle F[A] \wedge F[A] \rangle_\beta$$

- $F[A]$: field strength (curvature) induced by A
- $\langle \cdot, \cdot \rangle_\beta$: β-deformed inner product on (anti-)de Sitter algebra induced by

$$\mathcal{P}_\beta := \frac{\mathbb{1} + i\beta\gamma_5}{2\beta}, \quad \beta : \textit{ Immirzi}$$

Section 3

Supergravity & boundary theory

Supergravity

AdS Supergravity as **super Cartan geometry** modeled on super Klein geometry $(\mathrm{OSp}(1|4), \mathrm{Spin}(1,3))$ [D'Auria-Fré-Regge '80, D'Auria-Fré '80, Castellani-D'Auria-Fré '91, KE '21]

Super Cartan connection

$$\mathcal{A}_\mu : M \to \mathfrak{osp}(1|4)$$

Decomposition

$$\mathcal{A} = \underbrace{\mathrm{pr}_{\mathfrak{g}_1} \circ \mathcal{A}}_{\psi} + \underbrace{\mathrm{pr}_{\mathbb{R}^{1,3}} \circ \mathcal{A}}_{e} + \underbrace{\mathrm{pr}_{\mathfrak{spin}(1,3)} \circ \mathcal{A}}_{\omega}$$

- $\psi = \psi^\alpha Q_\alpha$: (spin-3/2) *Rarita-Schwinger field*

Supergravity and LQG

Holst action for (extended) Poincaré SUGRA [Tsuda '00, Kaul '07]
via MM ($N = 1$, β as θ-ambiguity) [Obregon+Ortega-Cruz+Sabido '12]
as constrained BF ($N = 1$) [Kowalski-Glikman+Durka+Szczachor '10]

Here: via Holst projection [KE '21]

- $\mathfrak{osp}(1|4) \cong \mathfrak{sp}(4) \oplus S_{\mathrm{R}}$
- S_{R} Clifford module $\rightarrow$ can naturally extend $\mathcal{P}_\beta$

Definition [KE '21]

$$\mathbf{P}_\beta := \mathbf{0} \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta : \mathfrak{osp}(1|4) \rightarrow \mathfrak{osp}(1|4)$$

with

$$\mathcal{P}_\beta := \frac{\mathbb{1} + i\beta\gamma_5}{2\beta}$$

$\rightarrow$ induces inner product on $\mathfrak{osp}(1|4)$:

$$\langle \cdot, \cdot \rangle_\beta := \mathrm{str}(\,\cdot\,\mathbf{P}_\beta\,\cdot\,)$$

Supergravity and LQG

super Holst-MacDowell-Mansouri action [KE '21]

$$S_{\text{sH-MM}}[\mathcal{A}] = \int_M \langle F[\mathcal{A}] \wedge F[\mathcal{A}] \rangle_\beta$$

Curvature

- $F[\mathcal{A}]^I = \Theta^{(\omega)I} - \frac{1}{4}\bar{\psi} \wedge \gamma^I \psi$
- $F[\mathcal{A}]^{IJ} = R[\omega]^{IJ} + \frac{1}{L^2}\Sigma^{IJ} - \frac{1}{4L}\bar{\psi} \wedge \gamma^{IJ}\psi$
- $F[\mathcal{A}]^\alpha = D^{(\omega)}\psi^\alpha - \frac{1}{2L}e^I(\gamma_I)^\alpha{}_\beta \wedge \psi^\beta$

- $\rightarrow$ yields Holst action of $D = 4$, $\mathcal{N} = 1$ AdS-SUGRA + *bdy terms*
- **Can show:** Boundary theory **unique** by SUSY-invariance! [Andrianopoli+D'Auria '14, KE '21]

Extended SUGRA and boundary theory

What about **extended** SUSY? $\to$ Consider $\mathcal{N}=2$ [KE '21]

Super Cartan connection

$$\mathcal{A} = e^I P_I + \frac{1}{2}\omega^{IJ} M_{IJ} + \Psi^{\alpha}_r Q^r_{\alpha} + \hat{A} T$$

$\hat{A}$: $\mathrm{U}(1)$ gauge field (*graviphoton*)

super Holst-MacDowell-Mansouri action ($\mathcal{N}=2$) [KE '21]

$$S^{N=2}_{\text{sH-MM}}[\mathcal{A}] = \int_M \mathrm{str}(F[\mathcal{A}] \wedge \mathbf{P}_\beta F[\mathcal{A}])$$

$$\mathbf{P}_\beta := \mathbf{0} \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta \oplus \frac{1}{2\beta}(1+\beta\star)$$

$\Rightarrow$ β literally has interpretation as θ-**ambiguity**!

Section 4

The chiral theory

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Chiral Theory

In general: $\mathbf{P}_\beta$ destroys manifest SUSY-invariance!

$\rightarrow$ This changes in **chiral theory ($\beta = \mp i$)**

Holst projection [KE '21]

$$\mathbf{P}_{-i}: \mathfrak{osp}(1|4)_{\mathbb{C}} \rightarrow \mathfrak{osp}(1|2)_{\mathbb{C}}$$

$$M_{IJ} \mapsto T_i^+ = \frac{1}{2}(J_i + iK_i)$$

$$Q_\alpha \mapsto Q_A$$

Q_A: left-handed Weyl-fermion

Chiral Theory

Chiral Holst-MM action [KE '20, KE+HS '21]

$$S_{\text{sH-MM}}[\mathcal{A}] = \int_M \mathrm{str}(F[\mathcal{A}] \wedge \mathbf{P}_{-i} F[\mathcal{A}])$$

- Manifestly $\mathrm{OSp}(1|2)_{\mathbb{C}}$-gauge invariant (similar for $\mathcal{N} = 2$!)
- SUSY partially becomes true gauge symmetry!

→ Geometric explanation of [Fülöp '94, Gambini+Obregon+Pullin '96, Ling+Smolin '99]

Chiral Theory

Super Ashtekar connection [KE+HS '21]

$$\mathcal{A}^+ := \mathbf{P}_{-i}\mathcal{A} = A^{+i}T_i^+ + \psi_r^A Q_A^r + \frac{1}{2}\hat{A}_{rs}T^{rs}$$

Chiral action

$$S_{\text{sH-MM}}^{\beta=-i} = \frac{i}{\kappa}\int_M \langle \mathcal{E} \wedge F(\mathcal{A}^+)\rangle + \frac{1}{4L^2}\langle \mathcal{E} \wedge \mathcal{E}\rangle + \underbrace{S_{\text{CS}}^{\mathrm{OSp}(\mathcal{N}|2)}(\mathcal{A}^+)}_{\text{boundary term}}$$

$\mathcal{E}$: *super electric field*

Coupling bulk ↔ boundary

$$F(\underset{\Longleftarrow}{\mathcal{A}^+}) = -\frac{1}{2L^2}\underset{\Longleftarrow}{\mathcal{E}}$$

Super Ashtekar connection

$\mathcal{A}^+$ natural candidate to quantize SUGRA à la LQG

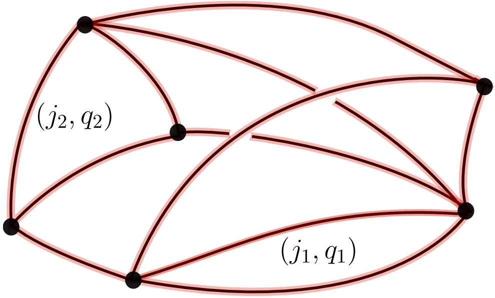
- Contains both gravity and matter d.o.f. $\rightarrow$ unified description, more fundamental way of quantizing fermions
- Substantially simplifies constraints
- **Boundary theory** described by **super Chern-Simons theory** $\rightarrow$ natural candidate to study inner boundaries in LQG ($\rightarrow$ BPS states, black holes)
- $\leftrightarrow$ Boundary theories in string theory [Mikhaylov + Witten '14]
- **Issues**: Non-compactness gauge group, reality conditions

Quantum theory: Overview

What do we need for quantum theory?

- Holonomies (parallel transport map) ✓
- Hilbert spaces (✓)
- Spin network states ✓

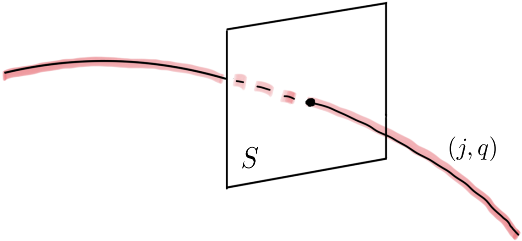
- Hilbert spaces gain **Krein structure** $\leftrightarrow$ standard LQG with fermions [Thiemann '98, Lewandowski+Zhang '21]
- $\rightarrow$ Gauge-invariant states: **super spin-networks**



Quantum theory: Overview

Super area operator:

$$\mathrm{gAr}_S = \int_S \|\mathcal{E}\|$$



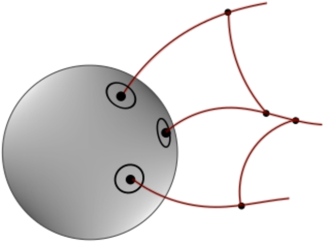
Action super area operator ($\mathcal{N} = 1$) [Ling+Smolin '00, KE+HS '22]

$$\widehat{\mathrm{gAr}}_S \,|\Psi\rangle = -8\pi i l_{\mathrm{Pl}}^2 \sqrt{j\left(j + \frac{1}{2}\right)}\,|\Psi\rangle$$

$j \in \mathbb{C}$: Principal series of $\mathrm{OSp}(1|2)_{\mathbb{C}}$

SUSY black holes in LQG

Holst-MM in the presence of inner boundary: [KE '21, KE+HS '22]



- Geometric theory induces super-CS on inner boundary
- For $\mathcal{N}=1$: $\quad G = \mathrm{(U)OSp}(1|2)$

$\mathrm{UOSp}(1|2)$ state counting

$$N = \frac{1}{2\pi}\int_0^{\pi} \mathrm{d}\theta\, \sin^2(2\theta)\left[4 - n + \sum_{i=1}^{p} n_i d_{j_i}\frac{\tan(d_{j_i}\theta)}{\tan\theta}\right]\prod_{l=1}^{p}\left(\frac{\cos(d_{j_l}\theta)}{\cos\theta}\right)^{n_l}$$

$\rightarrow$ *analytic continuation*

Entropy

$$S = \ln N = \frac{a}{4} + \ldots \quad \rightarrow \text{[Bekenstein '73, Hawking '75]}$$

Section 5

Summary & Outlook

Summary & Outlook

Classical theory:

- Gauge theoretic description of $\mathcal{N} = 1, 2$, $D = 4$ Holst-SUGRA + **unique** boundary terms compatible with SUSY
- $\Rightarrow$ Chiral SUGRA the has structure of **super YM** $\rightarrow$ *super Ashtekar connection*
- Boundary theory: **Super Chern-Simons theory**

Summary & Outlook

Quantum theory:

- Rigorous construction: **super spin-nets**, **super area operator** ...
- Intriguing structure of state space $\leftrightarrow$ strong similaritites with standard quantization of fermions in LQG
- Application to quantum SUSY black holes [KE '21, KE+HS '22]
- Application to symmetry reduced model (*supercosmology*) [KE '21]

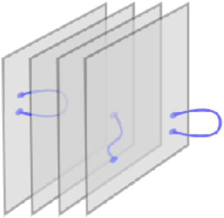
Difficulties:

- Implementation continuum limit (functional measure)
- Solution reality conditions?

$\rightarrow$ *Resolvable in symmetry reduced model!*

Outlook:

- Generalization to $\mathcal{N} \geq 3$ (*in particular:* $\mathcal{N} = 4, 8$)
- Limit $L \to \infty$ (vanishing cosmological constant) [Concha+Ravera+Rodríguez '19]
- (Charged) supersymmetric BHs $\leftrightarrow$ entropy calculations in string theory [Strominger+Vafa '96, Cardoso et al. '96, KE+HS '22]
- $\leftrightarrow$ Boundaries in string theory and **OSp**-super Chern-Simons theory [Mikhaylov+Witten '14]



References

[1] K. Eder, J. Math. Phys. **62** (2021), 063506 doi:10.1063/5.0044343 [arXiv:2101.00924 [math.DG]].

[2] K. Eder and H. Sahlmann, Phys. Rev. D **103** (2021) no.4, 046010 doi:10.1103/ PhysRevD.103.046010 [arXiv:2011.00108 [gr-qc]].

[3] K. Eder and H. Sahlmann, JHEP **07** (2021), 071 doi:10.1007/ JHEP07(2021)071 [arXiv:2104.02011 [gr-qc]].

[4] K. Eder and H. Sahlmann, JHEP **21** (2020), 064 doi:10.1007/JHEP03(2021)064 [arXiv:2010.15629 [gr-qc]].

[5] K. Eder, "Super Cartan geometry and the super Ashtekar connection," arXiv:2010.09630 [gr-qc] (submitted).

[6] K. Eder and H. Sahlmann, "Towards black hole entropy in chiral loop quantum supergravity," arXiv:2204.01661 [gr-qc] (submitted).

Quantum theory: Overview

- Quantization: study $\mathcal{A}^+$ and associated holonomies
- turns out $\rightarrow$ need additional **parametrizing supermanifold** $\mathcal{S}$ to incorporate **anticommutative** nature of fermionic fields

Super holonomies [KE '20+'21]

$$h_e[\mathcal{A}^+] = h_e[A^+] \cdot \mathcal{P}\exp\left(-\oint_e \mathrm{Ad}_{h_e[A^+]^{-1}}\psi^{(\tilde{s})}\right) : \mathcal{S} \rightarrow \mathcal{G}$$

- $\mathcal{A}^+ = A^+ + \psi$ and $e : [0,1] \rightarrow M \subset \mathcal{M}$
- $h_e[A^+]$: bosonic holonomy associated to A^+

Quantum theory: Overview

- $\mathcal{A}_{S,\gamma}$: set of generalized super connections

$$\begin{array}{ccc} \mathcal{A}_{S',\gamma'} & \longrightarrow & \mathcal{A}_{S',\gamma} \\ \Big| & & \Big| \\ \mathcal{A}_{S,\gamma'} & \longrightarrow & \mathcal{A}_{S,\gamma} \end{array}$$

- for fixed graph γ induces functor $\mathcal{A}_\gamma : \mathcal{S} \to \mathcal{A}_{S,\gamma}$
  $\to$ **Molotkov-Sachse-type smf.**
- $\to$ can study cylindrical functions and invariant measures on $\mathcal{A}_\gamma$
- covariance under reparametrization requires **Berezin-type integral** for fermionic d.o.f.
- $\to$ induces **Krein structure** on state space $\leftrightarrow$ strong similarities to quantization of fermions in standard LQG

Symmetry reduction

- supersymmetric minisuperspace models [D'Eath + Hughes '88+'92, D'Eath + Hawking + Obregon '93]
- hybrid homogeneous isotropic ansatz for bosonic and fermionic d.o.f.
- in general: fermions not compatible with isotropy
- **But**: in (chiral) LQC can exploit enlarged gauge symmetry!
- $\rightarrow$ natural interpretation in terms of homogeneous isotropic super connection [KE '20, KE+HS '20]

Symmetry reduced connection

$$\mathcal{A}^{+} = c\,\mathring{e}^{i}\,T_i^{+} + \psi_A \mathring{e}^{AA'}\,Q_{A'}$$

$\mathring{e}^{i}$: fiducial co-triad

- **Also**: can show that this is the most general ansatz consistent with reality conditions (contorsion tensor isotropic)

Loop quantum cosmology

- Construction of state space via super holonomies $h_e[\mathcal{A}^+]$ along straight edges of a fiducial cell
- $\Rightarrow$ motivates state space of quantum theory

Hilbert space

$$\mathcal{H} = \overline{H_{\mathrm{AP}}(\mathsf{C})}^{\,\cdot} \otimes \Lambda[\psi_{A'}]$$

- reality condition in quantum theory can be solved exactly![Wilson-Ewing '15, KE+HS '20]

Quantum right SUSY constraint

$$\widehat{S}^R_{A'} = \frac{3g^{\frac{1}{2}}}{2\lambda_m}|v|^{\frac{1}{4}} \left((N_- - N_+) - \frac{\kappa\lambda_m}{6g|v|}\widehat{\Theta} \right) |v|^{\frac{1}{4}} \widehat{\phi}_{A'}$$

- λ_m: quantum area gap (full theory)

Loop quantum cosmology

- Quantum algebra between left and right SUSY constraint closes and reproduces Poisson algebra exactly!

Quantum algebra

$$[\widehat{S}^{LA'}, \widehat{S}^{R}_{A'}] = 2i\hbar\kappa\widehat{H} - \frac{\hbar\kappa}{6g^{\frac{1}{2}}|v|^{\frac{1}{2}}}\widehat{\pi}_{\psi}^{A'}\widehat{S}^{R}_{A'}$$

- $\rightarrow$ fixes some of the quantization ambiguities
- semiclassical limit: $\lambda_m \rightarrow 0$ i.e. corrections from quantum geometry negligible
- $\rightarrow$ **Hartle-Hawking state** as solution of constraints $\Leftrightarrow$[D'Eath '98]

Hartle-Hawking state

$$\Psi(a) = \exp\left(\frac{3a^2}{\hbar}\right)$$