

Wald-Zoupas charges in asymptotically de Sitter spacetimes

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Joint work with J. Lewandowski & A. Ashtekar

based on JHEP 05 (2021) 063 & yet unpublished work

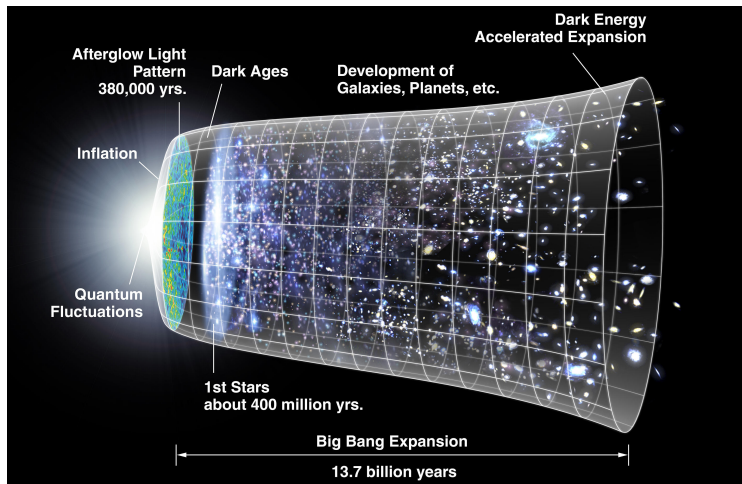


Why bother?

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source: NASA

What's the problem?

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- I^+ is spacelike
- No universal structure on I^+
- Asymptotic symmetries: $D_i(I^+)$ – no distinguished generators!

Can we measure differences using GW astronomy?

Outline

- 1 Asymptotically de Sitter spacetimes
- 2 Wald-Zoupas approach
 - Charges in the asymptotically dS
 - Uniqueness
- 3 de Sitter group
- 4 Examples
- 5 Conclusions

Definition

A physical spacetime is (M, g) and we change it to (\tilde{M}, \tilde{g}) where

$$\begin{aligned}
 \tilde{g} &= \Omega^2 g \\
 \tilde{M} &= M \quad / \\
 \Omega|_I &= 0 \\
 d\Omega|_I &= 0.
 \end{aligned}
 \tag{1}$$

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A physical spacetime is (M, g) and we change it to (\tilde{M}, \check{g}) where

$$\begin{aligned}\check{g} &= \Omega^2 g \\ \tilde{M} &= M \setminus I \\ \Omega|_I &= 0 \\ d\Omega|_I &= 0.\end{aligned}\tag{1}$$

Moreover, we assume that g satisfies Einstein equations

$$R_{ab} - \frac{1}{R}g_{ab} + \Lambda g_{ab} = 8\pi T_{ab}\tag{2}$$

with such asymptotics that $\Omega^{-1}T_{ab}$ is smooth up to I .

Consequences

This definition allows us to show that:

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- Weyl tensor vanishes on Σ (which does not imply any sort of conformal flatness!)

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What is not specified:

- topology of Σ
- boundary conditions at Σ .

Example I - de Sitter

Let $\ell = \sqrt{\frac{3}{\Lambda}}$. The de Sitter metric reads

$$g = -d\tau^2 + \ell^2 \cosh^2\left(\frac{\tau}{\ell}\right) q_{S^3} \quad (3)$$

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Taking $\Omega = \cosh^{-1}\left(\frac{\tau}{\ell}\right)$, one can see that a metric induced is simply $\ell^2 q_{S^3}$ and $I = S^3$.

Example II - $k = 0$ cosmology

Let us consider

$$g = a^2(\eta) (d\eta^2 + q_{\mathbb{R}^3}) \quad (4)$$

with a determined by Friedmann equations with Λ and a reasonable matter content (say, dust and radiation). Then, we can take $\Omega = a^{-1}$, metric induced on I is simply $q_{\mathbb{R}^3}$ and $I = \mathbb{R}^3 = S^3 \setminus \{p\}$. If there is any matter, we cannot choose different Ω to enlarge I to S^3 .

Example III - Kottler black hole

Let us consider

$$g = - \left(1 - \frac{\Lambda r^2}{3} - \frac{2M}{r} \right) du^2 - 2dudr + r^2 \gamma_{AB} dx^A dx^B. \quad (5)$$

This metric describes Schwarzschild-like BH, it satisfies Λ -Einstein equations. Taking $\Omega = r^{-1}$ we see that it is asymptotically de Sitter. In this case $I = \mathbb{R} \times S^2 = S^3 \setminus \{p_1, p_2\}$ and the induced metric is

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Example IV - Taub-NUT

(Kerr)-de Sitter-Taub-Nut describes topological deformation of a black hole. It was recently shown that such spacetimes are (after an appropriate gluing) smooth despite the fact their horizons are not [Lewandowski and Ossowski 2021].

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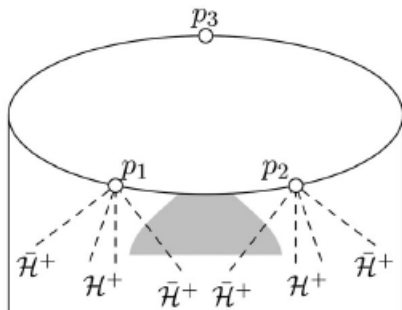
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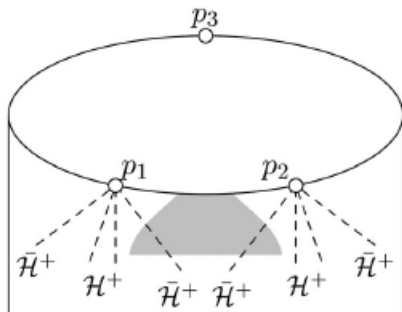
Q: Allowed parameters? Do E and B commute? Global hyperbolicity?

Example V - many BHs



source: Hintz 2021

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Q: Scattering? Topology changes?

Behaviour near /

We use Fefferman-Graham gauge

$$ds^2 = -\frac{3}{\Lambda} \frac{d\rho^2}{\rho^2} + \gamma_{ab}(\rho, x^c) dx^a dx^b \quad (7)$$

where γ_{ab} has an expansion

$$\gamma_{ab} = \rho^{-2} g_{ab}^{(0)} + \rho^{-1} g_{ab}^{(1)} + g_{ab}^{(2)} + \rho g_{ab}^{(3)} + O(\rho^2). \quad (8)$$

$g_{ab}^{(1)}$ and $g_{ab}^{(2)}$ are determined by $g_{ab}^{(0)}$ and Einstein equations, $g_{ab}^{(3)}$ is freely prescribed up to the constraints

$$g^{(0)ab} g_{ab}^{(3)} = 0 \quad (9)$$

$$D^{(0)a} g_{ab}^{(3)} = 0. \quad (10)$$

For convenience let us write

$$T_{ab} = \frac{\overline{3\Lambda}}{16\pi G} g_{ab}^{(3)}. \quad (11)$$

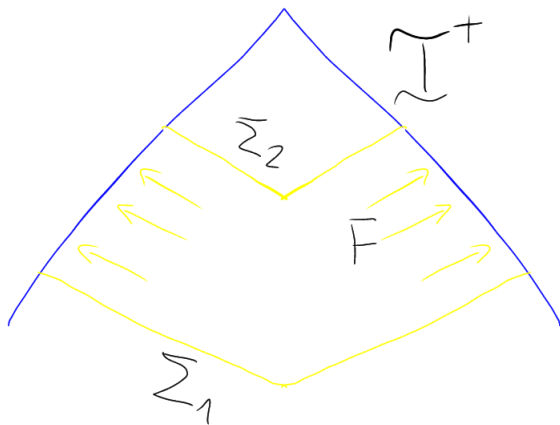
Initial data

Such $g^{(0)}$ and T_{ab} are defined up to conformal transformations:

$$(g^{(0)}, T) \sim (\Omega^2 g^{(0)}, \Omega^{-1} T). \quad (12)$$

From the work of Friedrich it follows that $[(g^{(0)}, T)]$ uniquely defines solution (at least in some neighborhood of de Sitter). In the Fefferman-Graham gauge, all $g^{(n)}$ with $n > 3$ satisfy recurrence equations which express them unambiguously in terms of $(g^{(0)}, T)$.

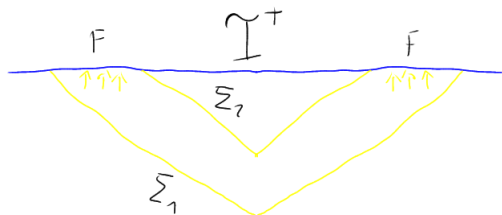
General idea



$$H[\Sigma_1] = H[\Sigma_2] + F$$

Figure for $\Lambda = 0$

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Figure for $\Lambda > 0$

Symplectic way

From the definition we have

$$\delta H_\xi = \int_\Sigma \omega(\phi; \delta\phi, L_\xi\phi) = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) \quad (13)$$

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$$\delta_1\delta_2 H_\xi - \delta_2\delta_1 H_\xi = - \int_{\partial\Sigma} \xi \cdot \omega(\phi, \delta_1\phi, \delta_2\phi) = 0 \quad (14)$$

The idea is to change δH_ξ into a true variation by adding something.

Physics at the boundaries

In practice $\delta\Sigma$ is a cross-section of I so let us denote

$$\lim_{I^+} \omega = \bar{\omega} \quad (15)$$

and let Θ be such that

$$\bar{\omega}(\phi; \delta_1\phi, \delta_2\phi) = 2\delta_{[1}\Theta(\phi; \delta_2]\phi). \quad (16)$$

Then, a 'correct' Hamiltonian is given by

$$\delta H_\xi = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) + \int_{\partial\Sigma} \xi \cdot \Theta \quad (17)$$

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Of course, we have an ambiguity $\Theta \rightarrow \Theta + \delta W(\phi)$.

Uniqueness

- Θ is built locally out of ϕ and universal background structure
- $\Theta(\phi; \delta\phi) = 0$ whenever ϕ is stationary
- Θ depends analytically upon ϕ
- Θ does not depend upon any arbitrary choices like a conformal factor

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Limit of the symplectic current reads [Compere, Fiorucci, Ruzziconi 2019, see also Jezierski 2008]

$$\bar{\omega} = \frac{1}{2\ell^2} \delta \left(\sqrt{g^{(0)}} T^{ab} \right) \delta g_{ab}^{(0)} \quad (18)$$

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One immediately sees that

$$\Theta = \frac{1}{2\ell^2} \sqrt{g^{(0)}} T^{ab} \delta g_{ab}^{(0)} \quad (19)$$

First guess

According to the general prescription we have

$$\delta H_\xi = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) + \int_{\partial\Sigma} \xi \cdot \Theta. \quad (20)$$

With our choice of Θ , H_ξ truly exists: [Anninos, Seng Ng, Strominger 2010]

$$H_\xi = \int_{\partial\Sigma} d^2x \bar{\sigma} n^i \xi^j T_{ij} \quad (21)$$

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$$W = \int_I d^3x \sqrt{g^{(0)}} w(g^{(0)}, T) \quad (22)$$

where w 's conformal weight is -3 . We can expand it in a series power (in $g, R_{ab}, \epsilon, T \dots$) and demand that each term has weight -3 under constant rescalings. The only such term is $g^{ab} T_{ab}$ which happens to be zero.

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Notice that analytical properties are of importance here. Otherwise, we could take for example

$$w = \sqrt{C^{abc} C_{abc}}, \quad (23)$$

where C_{abc} is a Cotton tensor of $g^{(0)}$.

Problem

It follows that all diffeomorphism of M are asymptotic symmetries (in contrast to e.g. AF spacetimes). Our prescription thus generates way too many charges with non-zero fluxes even on 'stationary' solutions!

However, constraint

$$D^a T_{ab} = 0 \quad (24)$$

shows that at least some of those are gauge transformations. It can be shown [Ashtekar 2016] that the quotient is actually de Sitter! (At least when $M = \mathbb{R} \times \mathcal{S}^2$.)

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Q: Can we actually distinguish de Sitter algebra within $\Gamma(T/M)$?

Minimal surfaces

The idea (geometrical idea, physical content is not yet clear) is to introduce a frame in which the induced metric reads [Compère, Fiorucci, Ruzziconi 2019]

$$g^{(0)} = du^2 + q_{AB} dx^A dx^B \quad (25)$$

and $\det q = q(x^A)$ is fixed. In AF spacetime context, equivalent condition is well-known [Kijowski 1984]. It follows from [Chruściel 1985] that if $[g^{(0)}]$ becomes conformally flat quickly enough, such a frame exists and is in fact unique. Then, we can propagate de Sitter algebra from $u = -\infty$ to the whole \mathcal{I}^+ using a vector field ∂_u .

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Q: What is the physical meaning of this? What kind of initial and final conditions are allowed?

de Sitter and Schwarzschild-de Sitter

We can use Killing vectors of the de Sitter to calculate charges and fluxes of perturbations of dS. In particular, energy flux reads:

$$\mathbf{F}_{\partial_u} = \frac{1}{16\pi H} E_{cd} L_{\partial_u} g_{ab}^{(0)} \dot{g}^{(0)ac} \dot{g}^{(0)bd} \sqrt{\dot{g}^{(0)}} d^3x, \quad (26)$$

which coincides with the results from the linearized theory [Chruściel, Hoque, Smořka 2020; MK, Lewandowski 2020] and have the correct limit as $\Lambda \rightarrow 0$.

We can similarly treat perturbations around the Schwarzschild-de Sitter because we have distinguished rotations generators and 'time'-translation generator (up to a scale) among Killing vectors.

Funnily enough, waves with only magnetic part have no zero energy density (which integrates to zero, though).

Kerr-de Sitter

We have 2dim space of Killing vectors: $\text{span}\{\partial_t, \partial_\phi\}$. Angular momentum is defined uniquely by the requirement that generator's orbits are closed with a period 2π – it is ∂_ϕ . Having that, energy generator T is picked up as being perpendicular to ∂_ϕ : $T = \partial_u + a(a^2 + l^2)^{-1}\partial_\phi$.

Question: is there any physical reason to choose this T ?

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How to extend this choice far away from the Kerr-de Sitter?

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- They are unique under highly natural conditions
- Their expansion around symmetric spacetimes agrees with the linearized theory and recovers classical results in the limit $\Lambda \rightarrow 0$.
- The question of how to define energy in the full theory is still open

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- Observational perspectives [Bonga's PhD thesis 2017]
- (Loop?) quantization

THANK YOU
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