Wald-Zoupas charges in asymptotically de Sitter spacetimes

Maciek Kolanowski

Faculty of Physics, University of Warsaw

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Diamentowy

WYDZIAŁ

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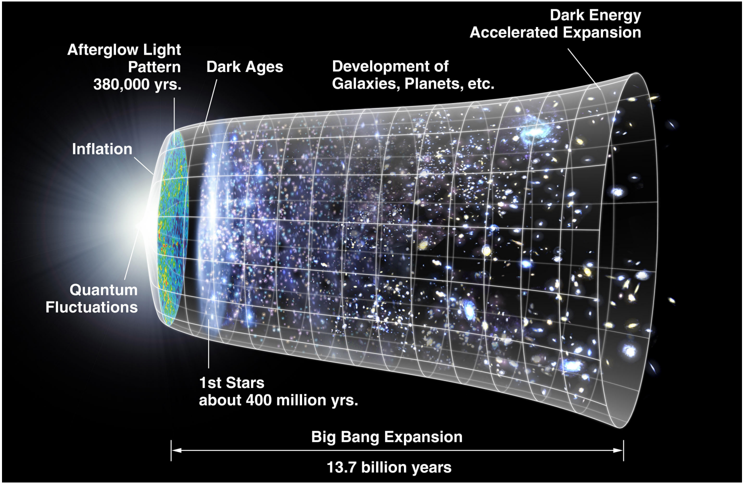

source: NASA

What's the problem?

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$\Lambda > 0$ is physical fact!

- $\mathcal{I}^+$ is spacelike
- No universal structure on $\mathcal{I}^+$
- Asymptotic symmetries: $Diff(\mathcal{I}^+)$ – no distinguished generators!

Can we measure differences using GW astronomy?

Outline

1. Asymptotically de Sitter spacetimes
2. Wald-Zoupas approach
   - Charges in the asymptotically dS
   - Uniqueness
3. de Sitter group
4. Examples
5. Conclusions

Definition

A physical spacetime is (M, g) and we change it to $(\tilde{M}, \tilde{g})$ where

$$\begin{aligned} \tilde{g} &= \Omega^2 g \\ \tilde{M} &= M \cup \mathcal{I} \\ \Omega|_{\mathcal{I}} &= 0 \\ d\Omega|_{\mathcal{I}} &\neq 0. \end{aligned} \tag{1}$$

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Moreover, we assume that g satisfies Einstein equations

$$R_{ab} - \frac{1}{R} g_{ab} + \Lambda g_{ab} = 8\pi T_{ab} \tag{2}$$

with such asymptotics that $\Omega^{-1} T_{ab}$ is smooth up to $\mathcal{I}$.

Consequences

This definition allows us to show that:

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- Weyl tensor vanishes on $\mathcal{I}$ (which does not imply any sort of conformal flatness!)

What is not specified:

- topology of $\mathcal{I}$
- boundary conditions at $\mathcal{I}$.

Example I - de Sitter

Let $\ell = \sqrt{\frac{3}{\Lambda}}$. The de Sitter metric reads

$$g = -d\tau^2 + \ell^2 \cosh^2\left(\frac{\tau}{\ell}\right) q_{S^3} \tag{3}$$

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Taking $\Omega = \cosh^{-1}\left(\frac{\tau}{\ell}\right)$, one can see that a metric induced is simply $\ell^2 q_{S^3}$ and $\mathcal{I} = S^3$.

Example II - $k = 0$ cosmology

Let us consider

$$g = a^2(\eta)\left(d\eta^2 + q_{\mathbb{R}^3}\right) \tag{4}$$

with a determined by Friedmann equations with Λ and a reasonable matter content (say, dust and radiation). Then, we can take $\Omega = a^{-1}$, metric induced on $\mathcal{I}$ is simply $q_{\mathbb{R}^3}$ and $\mathcal{I} = \mathbb{R}^3 = S^3 \setminus \{p\}$. If there is any matter, we cannot choose different Ω to enlarge $\mathcal{I}$ to S^3.

Example III - Kottler black hole

Let us consider

$$g = -\left(1 - \frac{\Lambda r^2}{3} - \frac{2M}{r}\right) du^2 - 2dudr + r^2 \mathring{\gamma}_{AB} dx^A dx^B. \tag{5}$$

This metric describes Schwarzschild-like BH, it satisfies Λ-Einstein equations. Taking $\Omega = r^{-1}$ we see that it is asymptotically de Sitter. In this case $\mathcal{I} = \mathbb{R} \times S^2 = S^3 \setminus \{p_1, p_2\}$ and the induced metric is

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Coincidence? Yes.

Example IV - Taub-NUT

(Kerr)-de Sitter-Taub-Nut describes topological deformation of a black hole. It was recently shown that such spacetimes are (after an appropriate gluing) smooth despite the fact their horizons are not [Lewandowski and Ossowski 2021].

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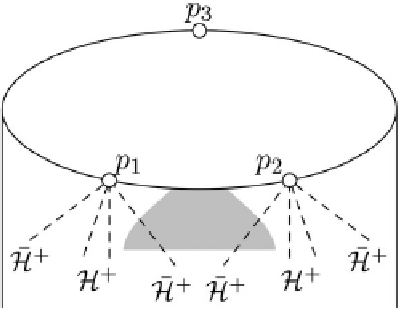
Q: Allowed parameters? Do E and B commute? Global hyperbolicity?

Example V - many BHs



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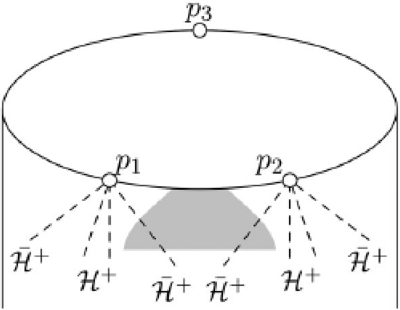

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Q: Scattering? Topology changes?

Behaviour near $\mathcal{I}$

We use Fefferman-Graham gauge

$$ds^2 = -\frac{3}{\Lambda}\frac{d\rho^2}{\rho^2} + \gamma_{ab}(\rho, x^c)dx^a dx^b \tag{7}$$

where γ_{ab} has an expansion

$$\gamma_{ab} = \rho^{-2} g^{(0)}_{ab} + \rho^{-1} g^{(1)}_{ab} + g^{(2)}_{ab} + \rho g^{(3)}_{ab} + O(\rho^2). \tag{8}$$

$g^{(1)}_{ab}$ and $g^{(2)}_{ab}$ are determined by $g^{(0)}_{ab}$ and Einstein equations, $g^{(3)}_{ab}$ is freely prescribed up to the constraints

$$g^{(0)ab} g^{(3)}_{ab} = 0 \tag{9}$$

$$D^{(0)a} g^{(3)}_{ab} = 0. \tag{10}$$

For convenience let us write

$$T_{ab} = \frac{\sqrt{3\Lambda}}{16\pi G} g^{(3)}_{ab}. \tag{11}$$

Initial data

Such $g^{(0)}$ and T_{ab} are defined up to conformal transformations:

$$(g^{(0)}, T) \sim (\Omega^2 g^{(0)}, \Omega^{-1} T). \quad (12)$$

From the work of Friedrich it follows that $[(g^{(0)}, T)]$ uniquely defines solution (at least in some neighborhood of de Sitter). In the Fefferman-Graham gauge, all $g^{(n)}$ with $n > 3$ satisfy recurrence equations which express them unambiguously in terms of $(g^{(0)}, T)$.

General idea

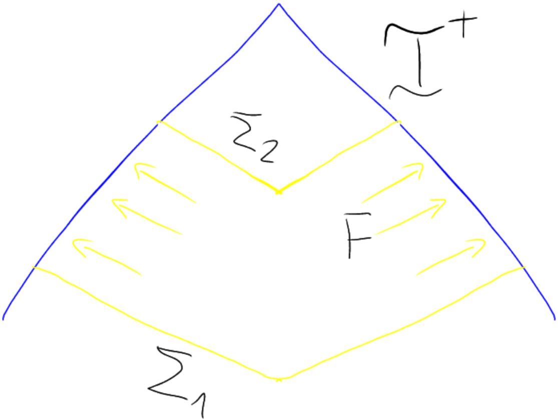

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Figure for $\Lambda = 0$

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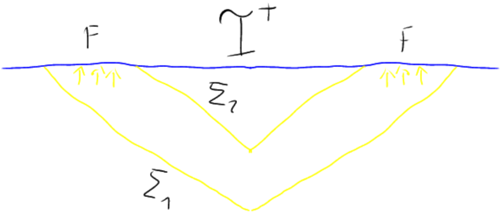

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Figure for $\Lambda > 0$

Symplectic way

From the definition we have

$$\not{\delta} H_\xi = \int_\Sigma \omega(\phi; \delta\phi, \mathcal{L}_\xi \phi) = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) \tag{13}$$

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We can take 'the second' variation

$$\delta_1 \not{\delta}_2 H_\xi - \delta_2 \not{\delta}_1 H_\xi = - \int_{\partial\Sigma} \xi \cdot \omega(\phi, \delta_1\phi, \delta_2\phi)$$

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$$\delta_1 \not{\delta}_2 H_\xi - \delta_2 \not{\delta}_1 H_\xi = - \int_{\partial\Sigma} \xi \cdot \omega(\phi, \delta_1\phi, \delta_2\phi) \neq 0 \tag{14}$$

The idea is to change $\not{\delta} H_\xi$ into a true variation by adding something.

Physics at the boundaries

In practice $\delta\Sigma$ is a cross-section of $\mathcal{I}$ so let us denote

$$\lim_{\to\mathcal{I}^+} \omega = \overline{\omega} \tag{15}$$

and let Θ be such that

$$\overline{\omega}(\phi; \delta_1\phi, \delta_2\phi) = 2\delta_{[1}\Theta(\phi; \delta_{2]}\phi). \tag{16}$$

Then, a 'correct' Hamiltonian is given by

$$\delta H_\xi = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) + \int_{\partial\Sigma} \xi \cdot \Theta \tag{17}$$

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and $F_\xi = \Theta(\phi, \mathcal{L}_\xi \phi)$.
Of course, we have an ambiguity $\Theta \mapsto \Theta + \delta W(\phi)$.

Uniqueness

- Θ is built locally out of ϕ and universal background structure
- $\Theta(\phi; \delta\phi) = 0$ whenever ϕ is stationary
- Θ depends analytically upon ϕ
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Limit of the symplectic current reads [Compere, Fiorucci, Ruzziconi 2019, see also Jezierski 2008]

$$\overline{\omega} = \frac{1}{2\ell^2}\delta\left(\sqrt{g^{(0)}}T^{ab}\right) \wedge \delta g^{(0)}_{ab} \tag{18}$$

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One immediately sees that

$$\Theta = \frac{1}{2\ell^2}\sqrt{g^{(0)}}\,T^{ab}\delta g^{(0)}_{ab} \tag{19}$$

First guess

According to the general prescription we have

$$\delta H_\xi = \int_{\partial\Sigma} (\delta Q - \xi \cdot \theta) + \int_{\partial\Sigma} \xi \cdot \Theta. \tag{20}$$

With our choice of Θ, H_ξ truly exists: [Anninos, Seng Ng, Strominger 2010]

$$H_\xi = \int_{\partial\Sigma} d^2x \sqrt{\sigma} n^i \xi^j T_{ij} \tag{21}$$

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$$W = \int_{\mathcal{I}} d^3x \sqrt{g^{(0)}} w(g^{(0)}, T) \tag{22}$$

where w's conformal weight is -3. We can expand it in a series power (in $g, R_{ab}, \epsilon, T \ldots$) and demand that each term has weight -3 under constant rescalings. The only such term is $g^{ab}T_{ab}$ which happens to be zero.

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$$w = \sqrt{C^{abc}C_{abc}}, \tag{23}$$

where C_{abc} is a Cotton tensor of $g^{(0)}$.

Problem

It follows that all diffeomorphism of $\mathcal{I}$ are asymptotic symmetries (in contrast to e.g. AF spacetimes). Our prescription thus generates way too many charges with non-zero fluxes even on 'stationary' solutions! However, constraint

$$D^a T_{ab} = 0 \tag{24}$$

shows that at least some of those are gauge transformations. It can be shown [Ashtekar 2016] that the quotient is actually de Sitter! (At least when $\mathcal{I} \sim \mathbb{R} \times S^2$.)

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Q: Can we actually distinguish de Sitter algebra within $\Gamma(T\mathcal{I})$?

Minimal surfaces

The idea (geometrical idea, physical content is not yet clear) is to introduce a frame in which the induced metric reads [Compère, Fiorucci, Ruzziconi 2019]

$$g^{(0)} = du^2 + q_{AB} dx^A dx^B \tag{25}$$

and $\det q = q(x^A)$ is fixed. In AF spacetime context, equivalent condition is well-known [Kijowski 1984]. It follows from [Chruściel 1985] that if $[g^{(0)}]$ becomes conformally flat quickly enough, such a frame exists and is in fact unique. Then, we can propagate de Sitter algebra from $u \to -\infty$ to the whole $\mathcal{I}$ using a vector field ∂_u.

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Q: What is the physical meaning of this? What kind of initial and final conditions are allowed?

de Sitter and Schwarzschild-de Sitter

We can use Killing vectors of the de Sitter to calculate charges and fluxes of perturbations of dS. In particular, energy flux reads:

$$\mathbf{F}_{\partial_u} = \frac{1}{16\pi H} \mathcal{E}_{cd} \mathcal{L}_{\partial_u} g^{(0)}_{ab} \mathring{g}^{(0)ac} \mathring{g}^{(0)bd} \sqrt{\mathring{g}^{(0)}} d^3x, \tag{26}$$

which coincides with the results from the linearized theory [Chruściel, Hoque, Smołka 2020; MK, Lewandowski 2020] and have the correct limit as $\Lambda \to 0$.
We can similarly treat perturbations around the Schwarzschild-de Sitter because we have distinguished rotations generators and 'time'-translation generator (up to a scale) among Killing vectors.
Funnily enough, waves with only magnetic part have no zero energy density (which integrates to zero, though).

Kerr-de Sitter

We have 2dim space of Killing vectors: $\mathrm{span}\{\partial_t, \partial_\phi\}$. Angular momentum is defined uniquely by the requirement that generator's orbits are closed with a period 2π – it is ∂_ϕ. Having that, energy generator T is picked up as being perpendicular to ∂_ϕ: $T = \partial_u + a(a^2 + l^2)^{-1}\partial_\phi$.
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How to extend this choice far away from the Kerr-de Sitter?

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- They are unique under highly natural conditions
- Their expansion around symmetric spacetimes agrees with the linearized theory and recovers classical results in the limit $\Lambda \to 0$.
- The question of how to define energy in the full theory is still open

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- Observational perspectives [Bonga's PhD thesis 2017]
- (Loop?) quantization

THANK YOU
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