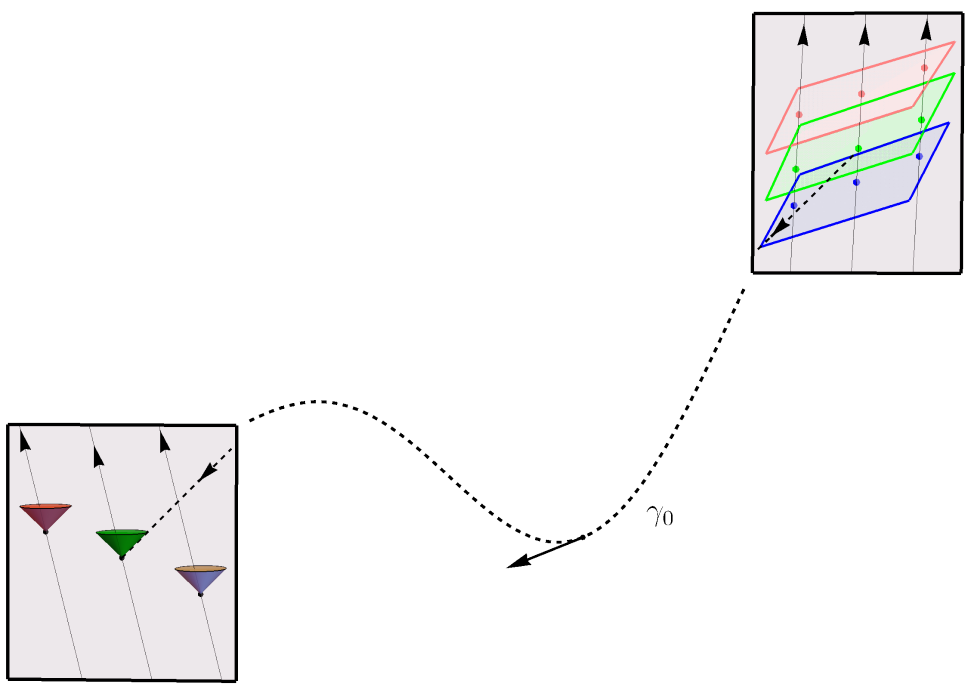
Weighing spacetime along the line of sight using times of arrival of electromagnetic signals



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Theory of Relativity Seminar
May 27th, 2022

Idea

Times of arrival of electromagnetic signals (TOA's) - one of most important observables in relativity

- signal method in special relativity
- Shapiro delays - provide a GR test [Shapiro 1964; Shapiro et al 1968]
- frequency shift due to gravity [Pound-Rebka 1959]
- measurement of the Hubble parameter using time delays between multiple images in strongly lensing systems [Refsdahl 1964; H0LiCOW 2016]
- pulsar timing
  - binary pulsars (and double pulsar) - proof of existence of GW [Hulse, Taylor 1974]
  - provide GR tests (quadrupole formula for GW, relativistic orbit deformation, Shapiro delay…)
  - pulsar timing arrays - low frequency GW [Sazhin 1978; Detweiler 1979; Hellings, Downs 1983…]
- clock-based gravitational compasses - curvature measurements using an ensemble of clocks and frequency comparisons [Neumann et al 2020; Puetzfeld et al 2016; Puetzfeld et al 2018]

Idea

TOA variations between slightly displaced points in two distant regions of spacetime

Possible to combine them into a measurement of curvature along the null geodesic connecting the region

Algebra rather complicated, but geometric interpretation is quite nice

Idea based on interesting geometric relations between the curvature and TOA's

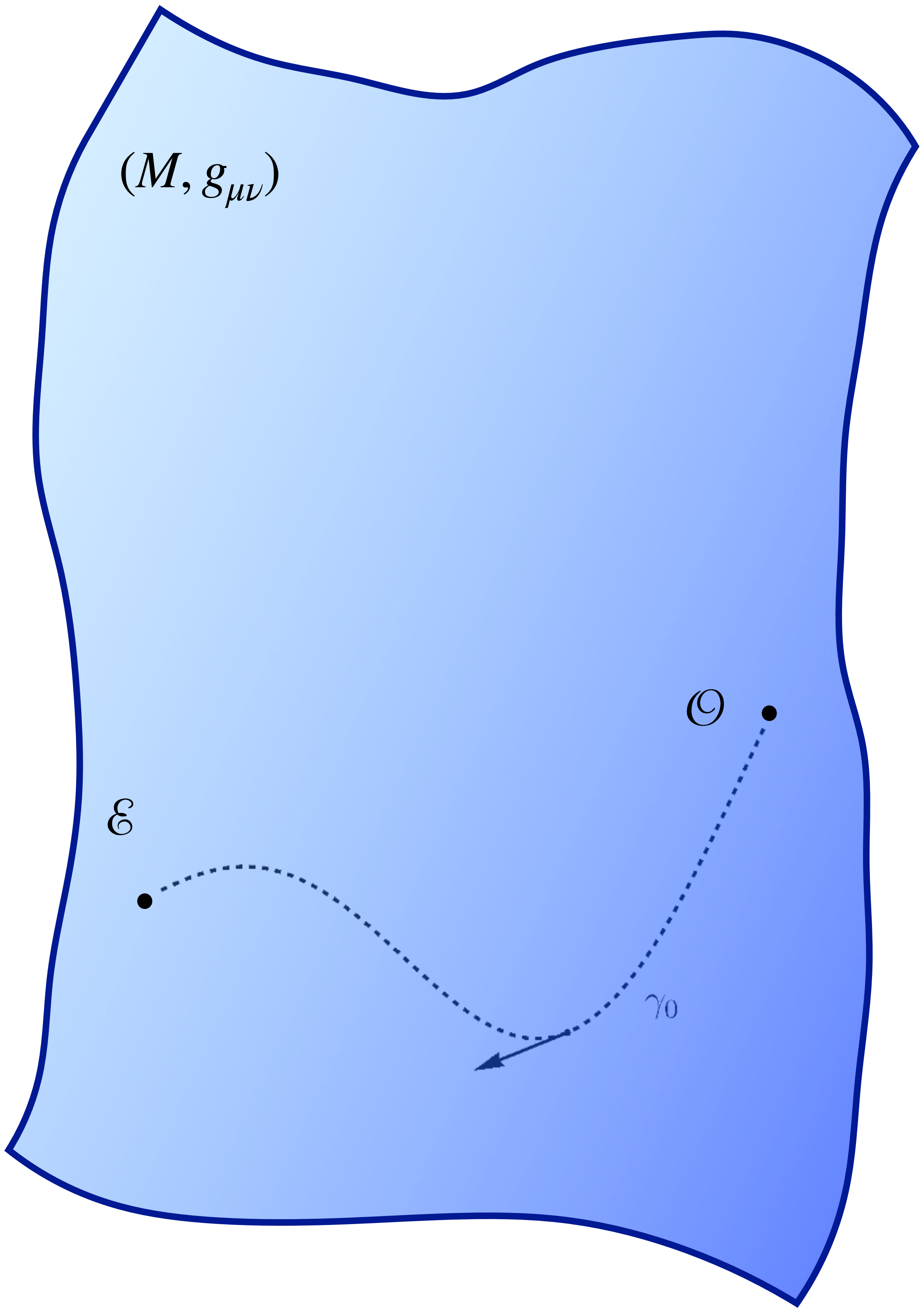
May have interesting implications for the theory of light propagation in curved spacetime

- based on papers:

MK, J. Miśkiewicz, J. Serbenta, „*Weighing the spacetime along the line of sight using times of arrival of electromagnetic signals*”, Phys. Rev. D **104**, 024026 (2021)

M. Grasso, MK, J. Serbenta, „*Geometric optics in general relativity using bilocal operators*”, Phys. Rev. D **99**, 064038 (2019)

Idea of the measurement



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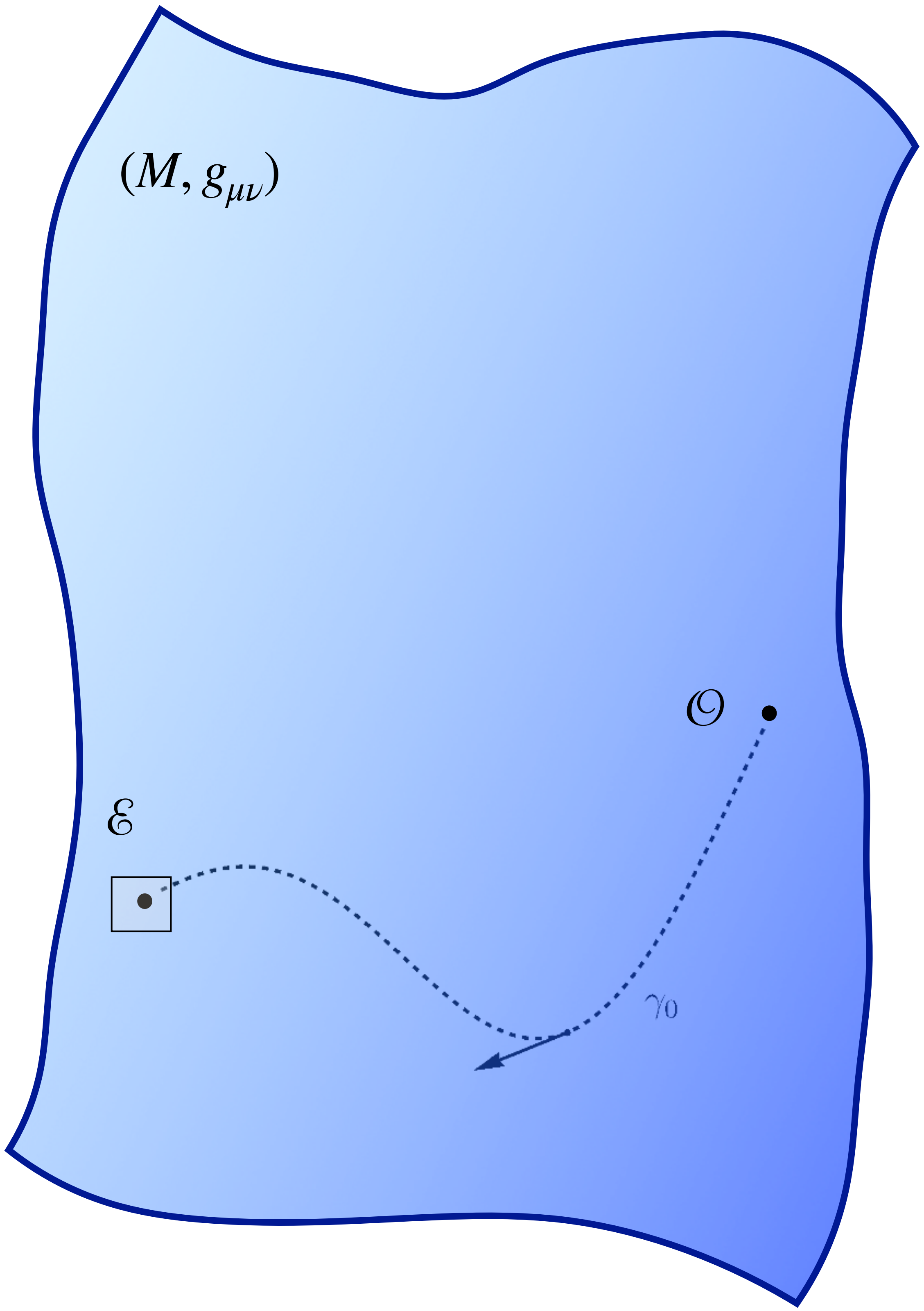

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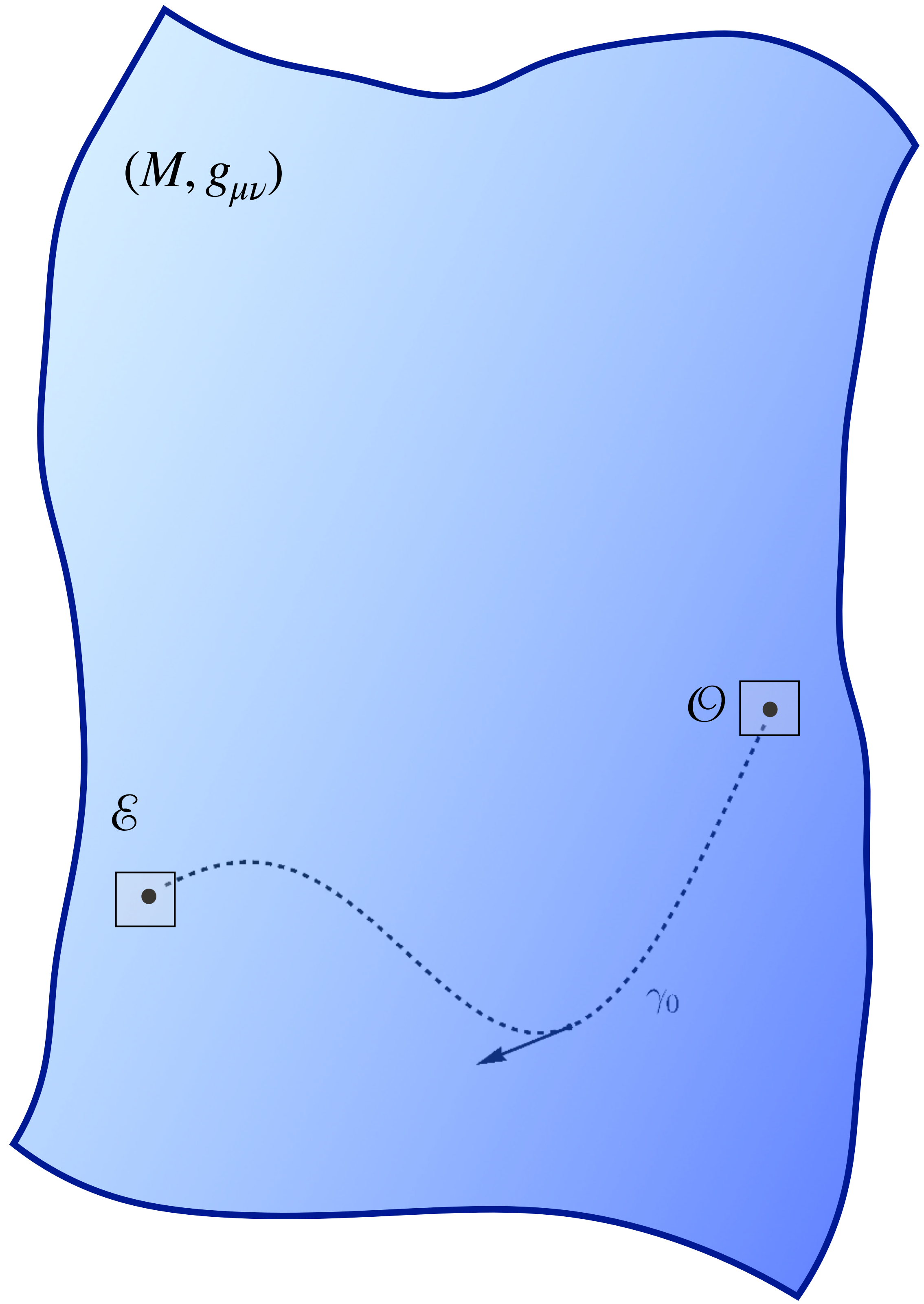

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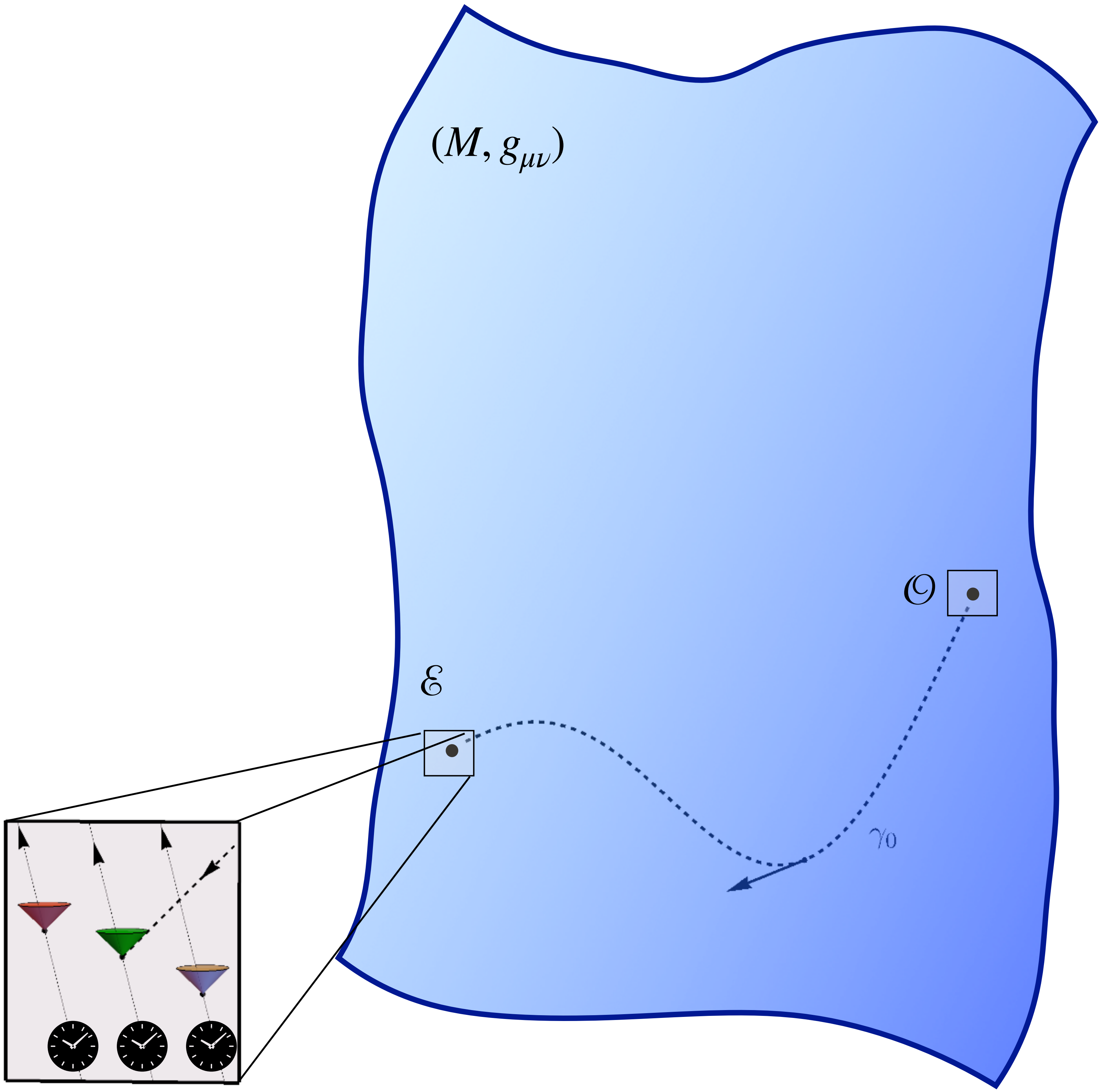

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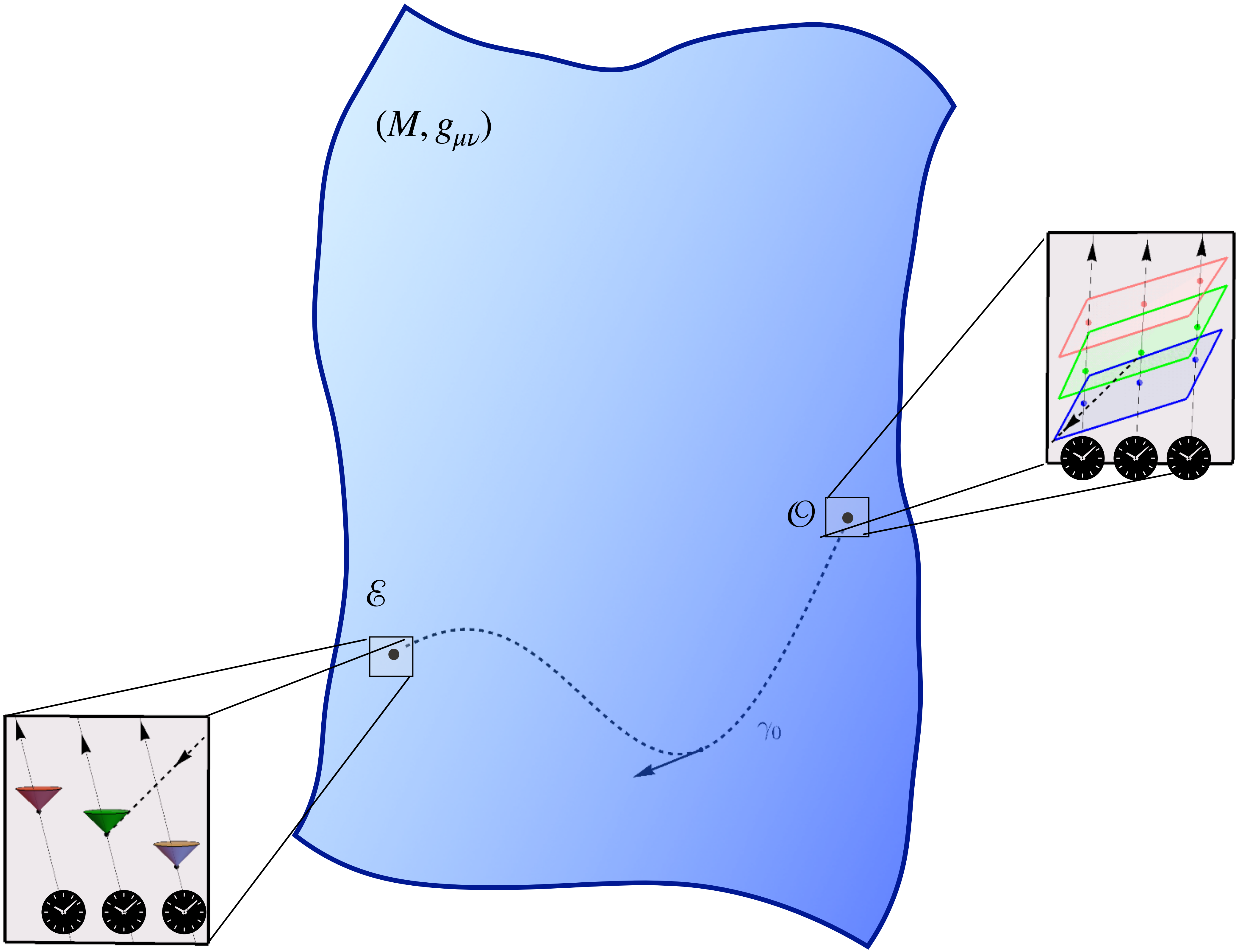
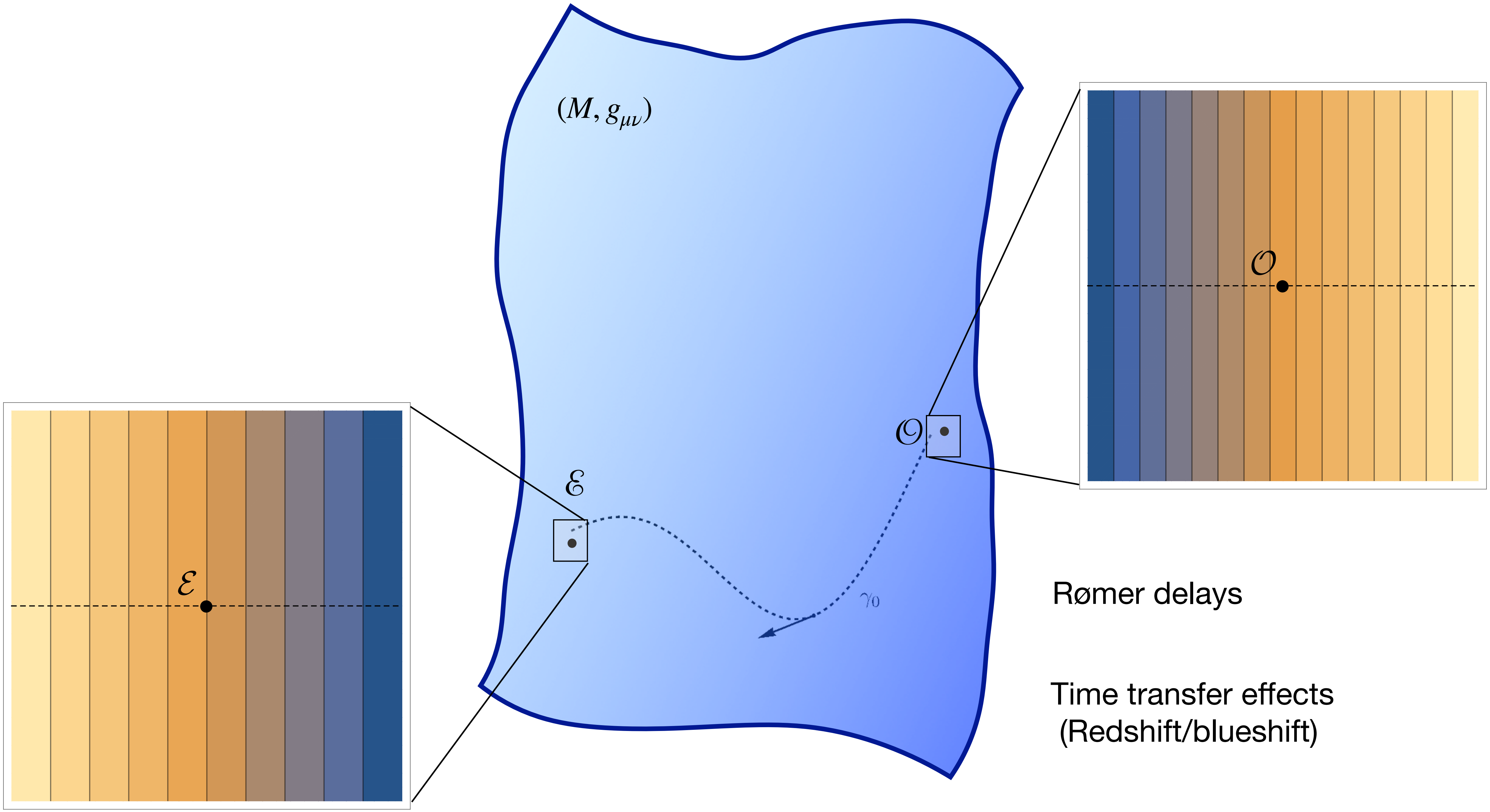


Idea of the measurement

Time of arrival variations, dependence on spatial positions

Linear order in position variations



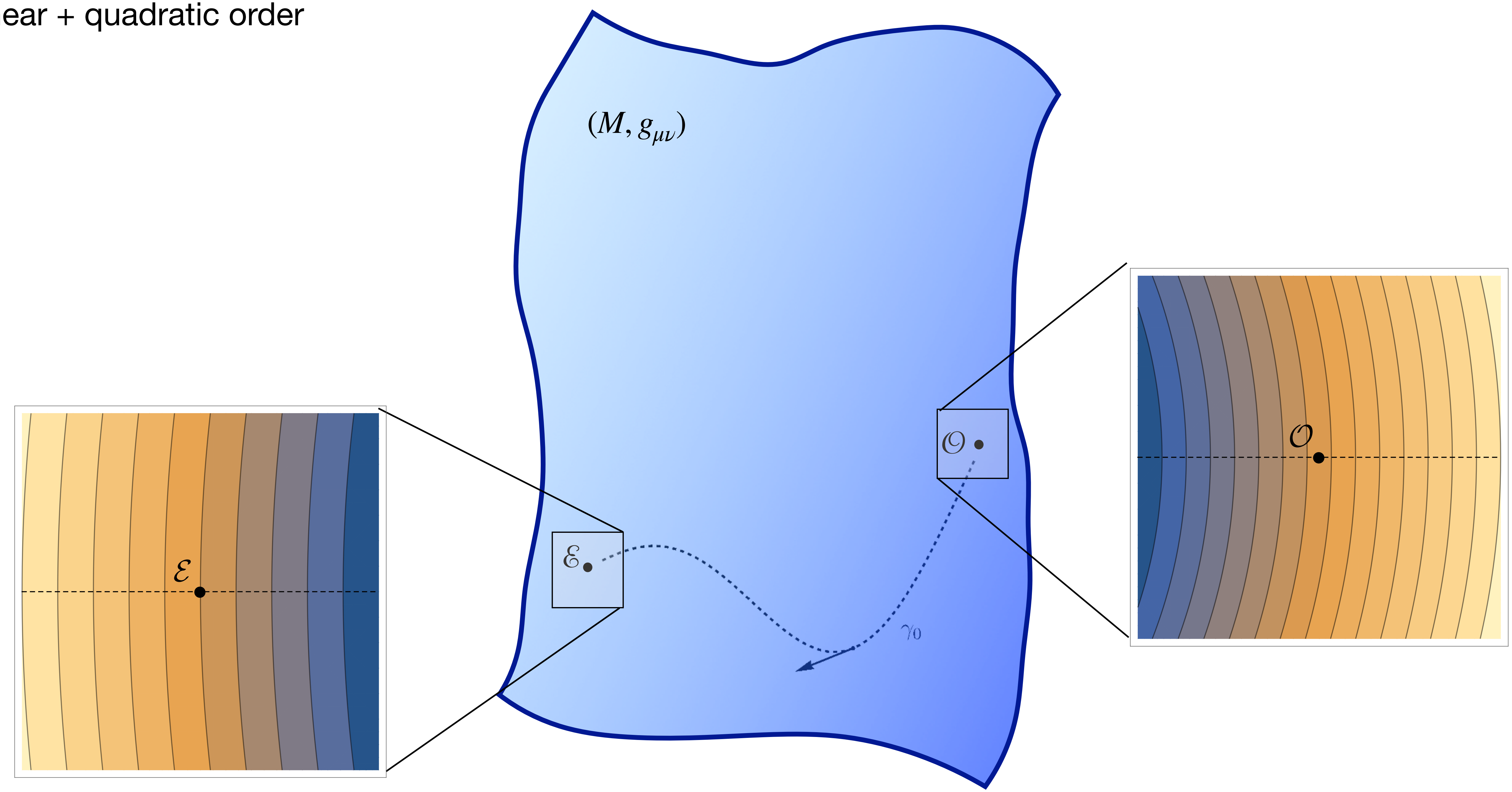
Rømer delays

Time transfer effects
(Redshift/blueshift)

Idea of the measurement

Time of arrival variations, dependence on spatial positions

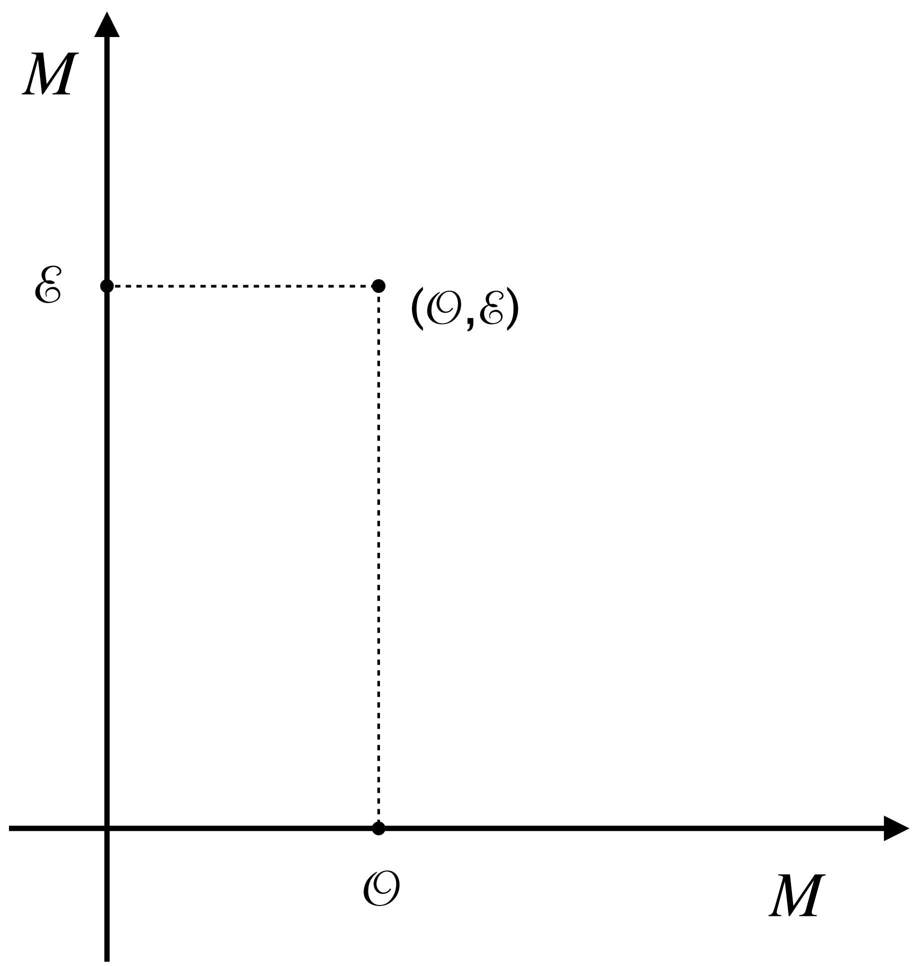
Linear + quadratic order

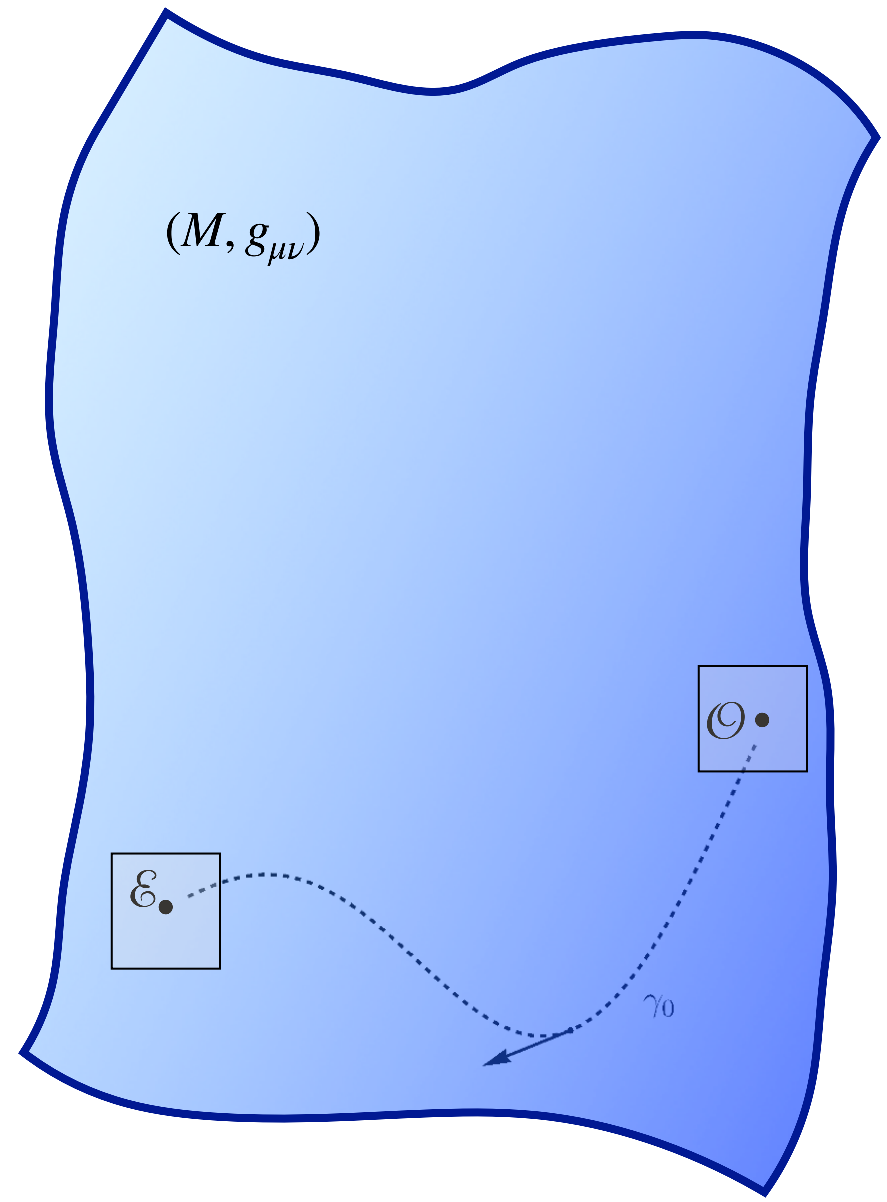


Quadratic term = finite distance effects + curvature imprint

Product manifold

The product manifold $M \times M$ $(\dim = 8)$





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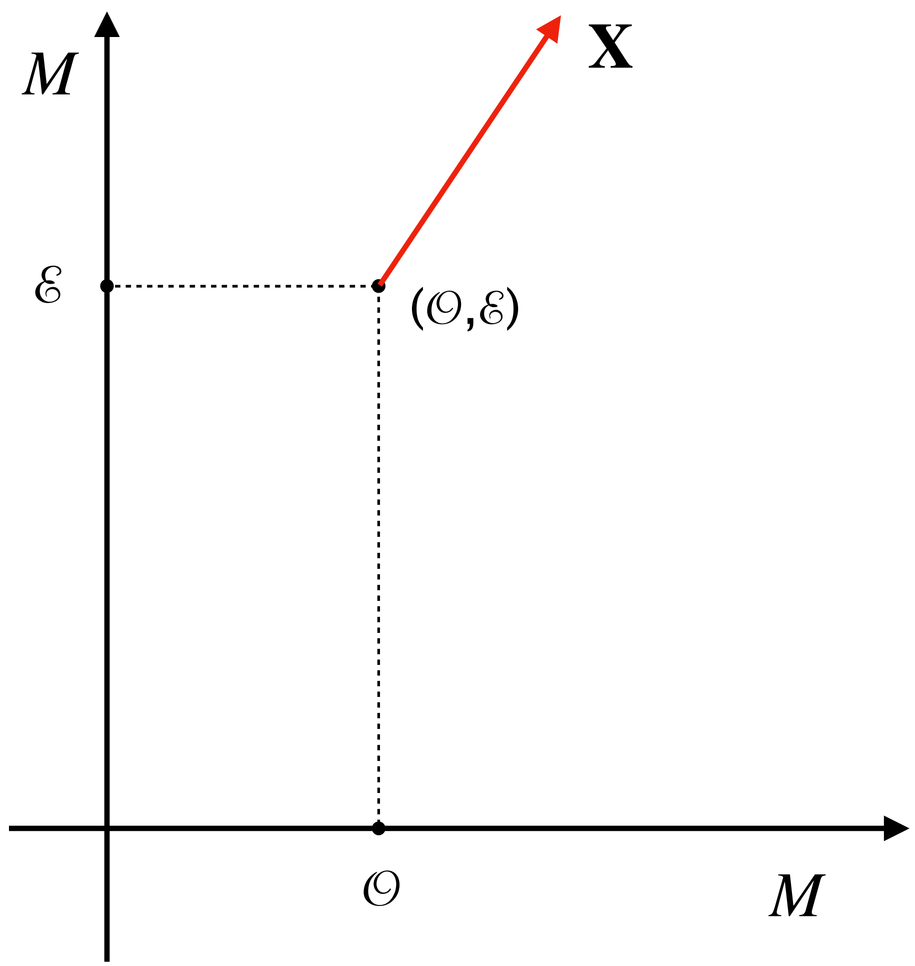
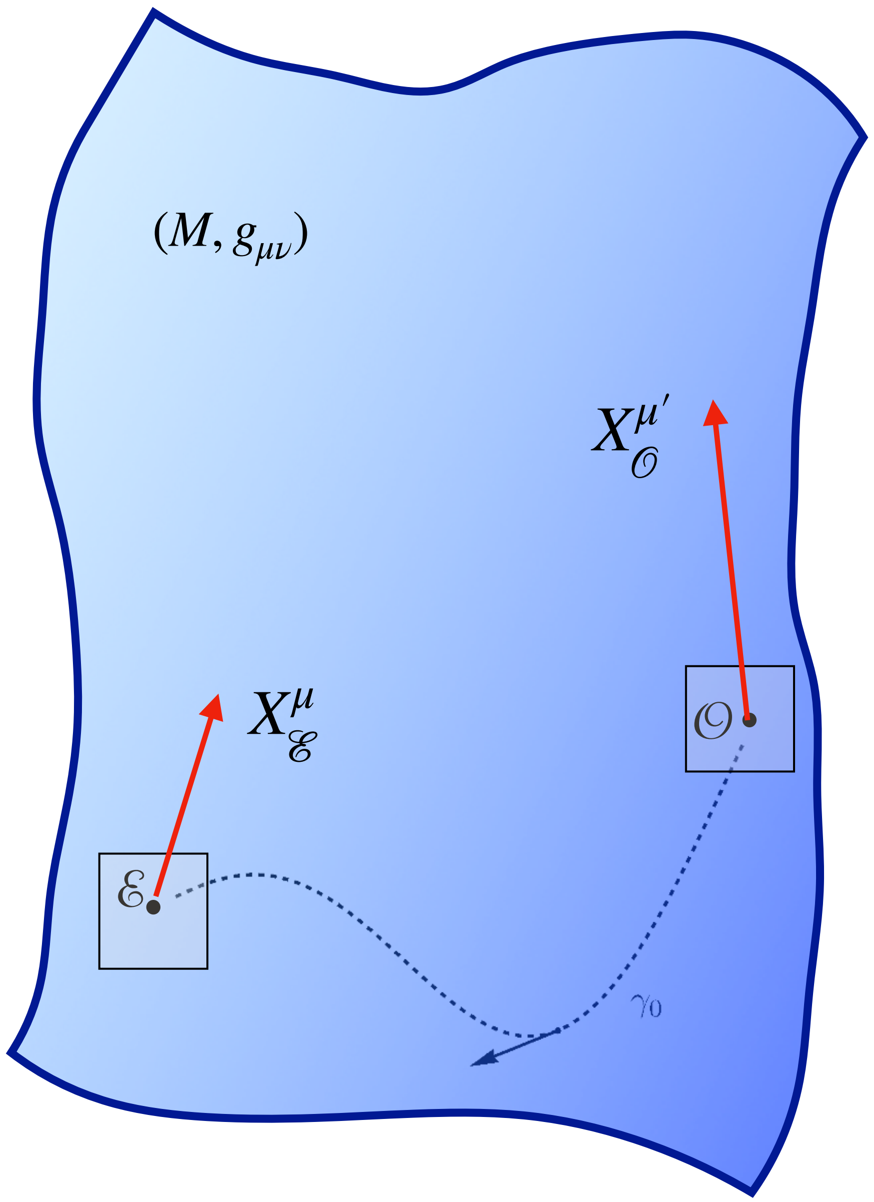
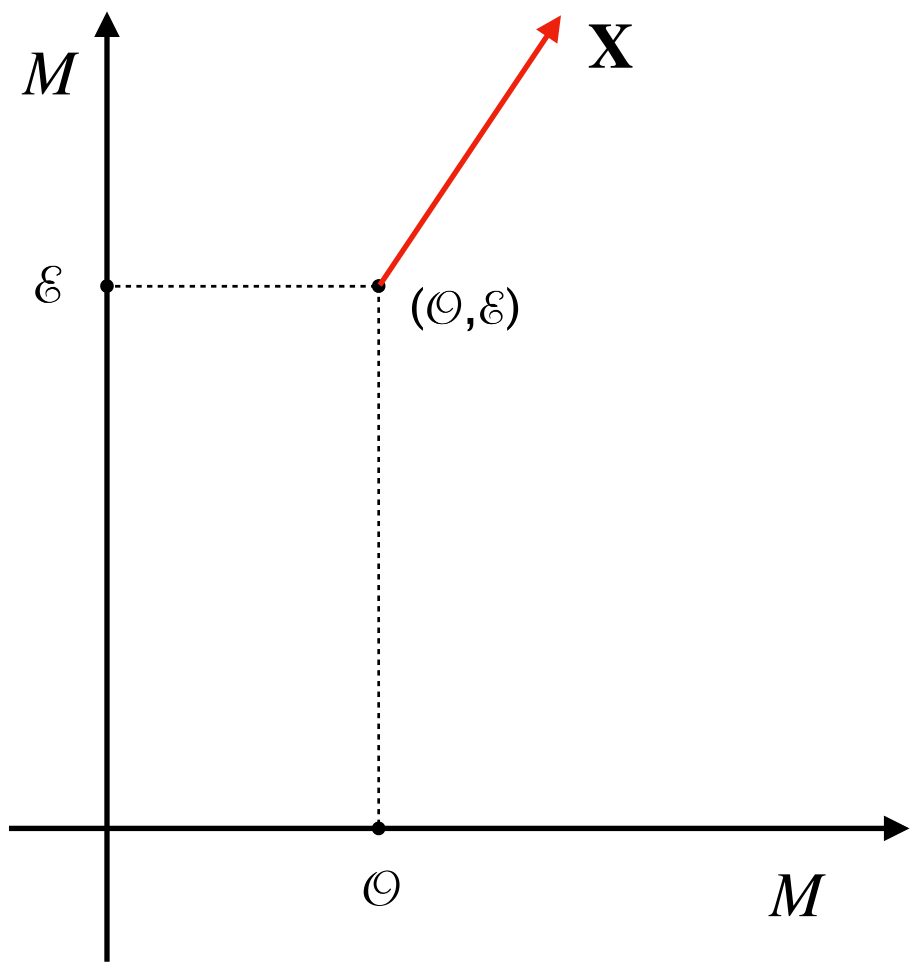


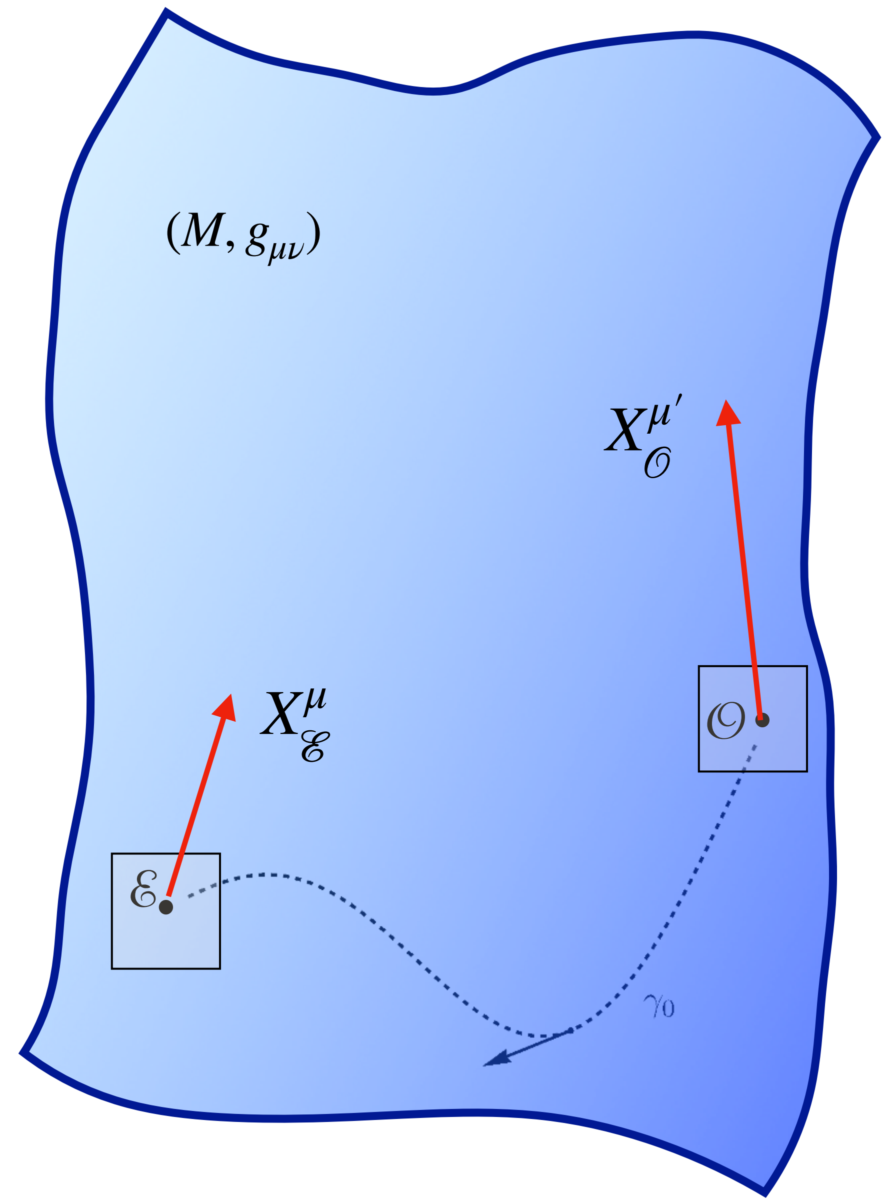


Tangent vectors $\mathbf{X} = \begin{pmatrix} X_{\mathcal{O}}^{\mu'} \\ X_{\mathcal{E}}^{\mu} \end{pmatrix}$

Product manifold

The product manifold $M \times M \quad (\dim = 8)$





Tangent vectors $\quad \mathbf{X} = \begin{pmatrix} X^{\mu'}_{\mathscr{O}} \\ X^{\mu}_{\mathscr{E}} \end{pmatrix}$

$$T_{(\mathscr{O},\mathscr{E})}(M \times M) \cong T_{\mathscr{O}}M \oplus T_{\mathscr{E}}M$$

Product manifold

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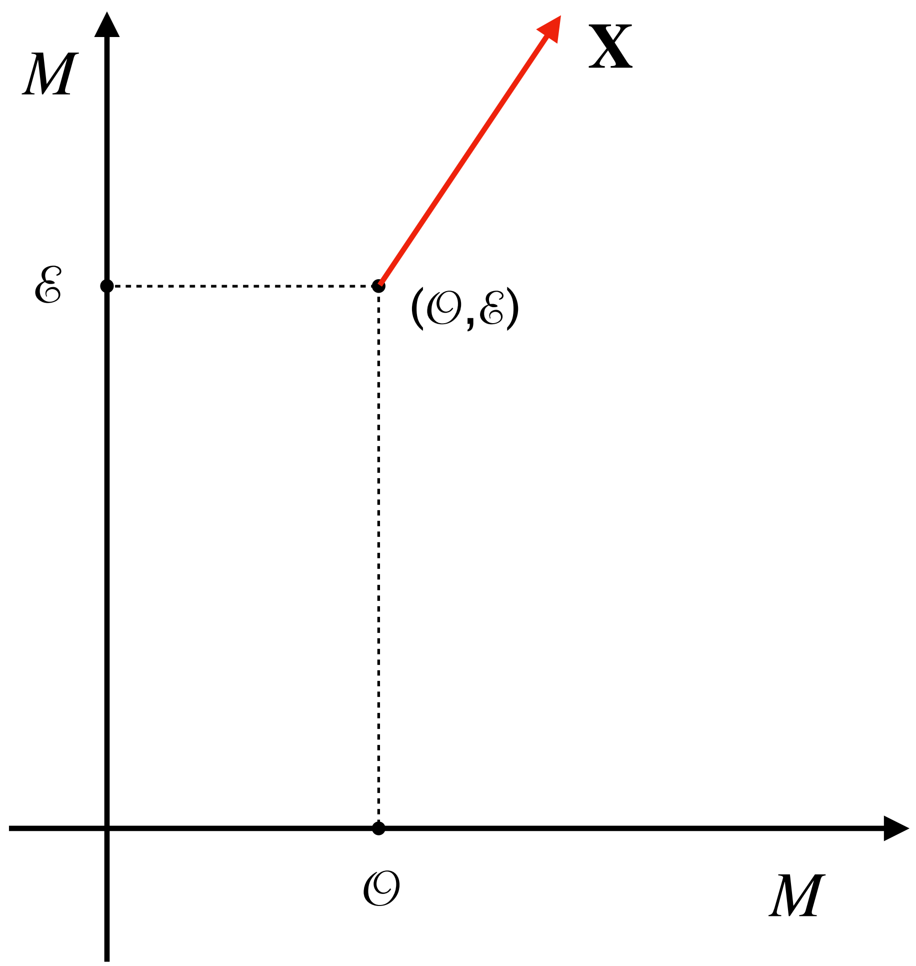

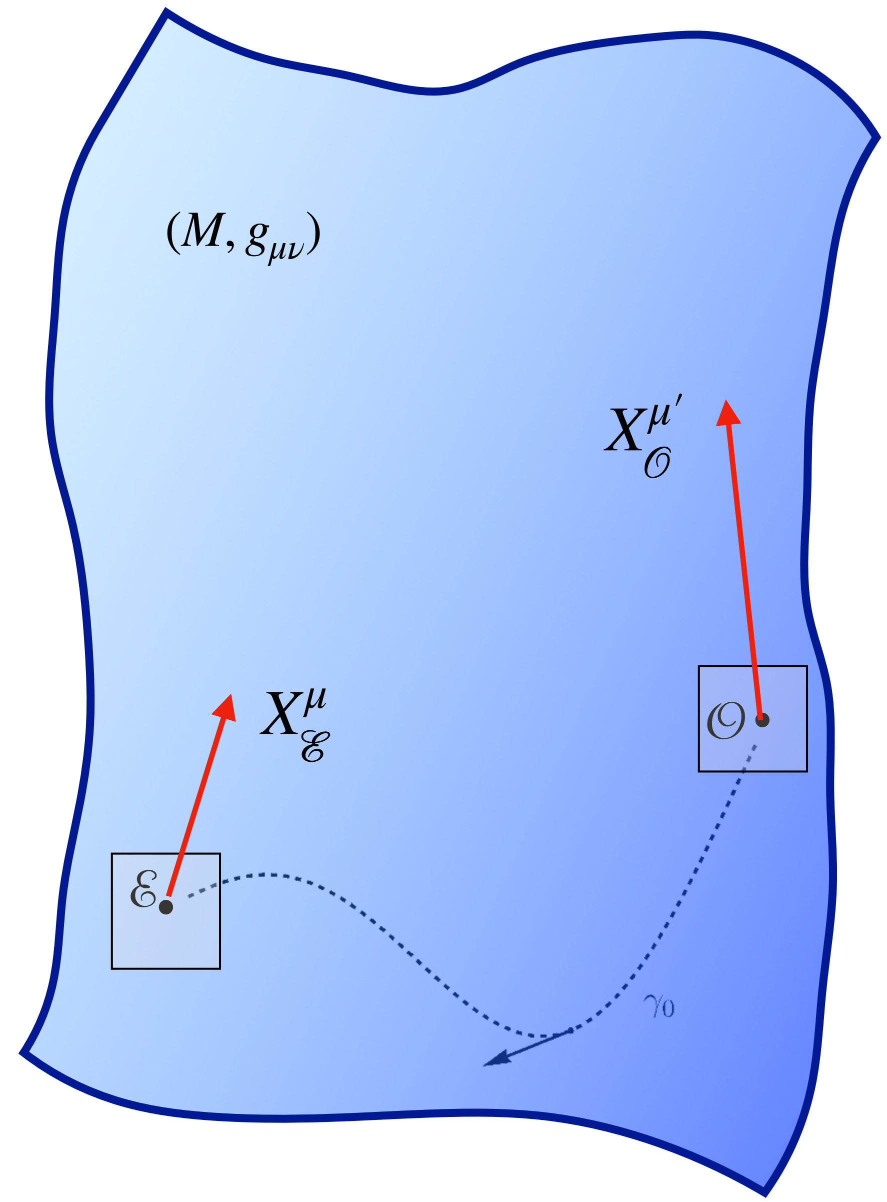

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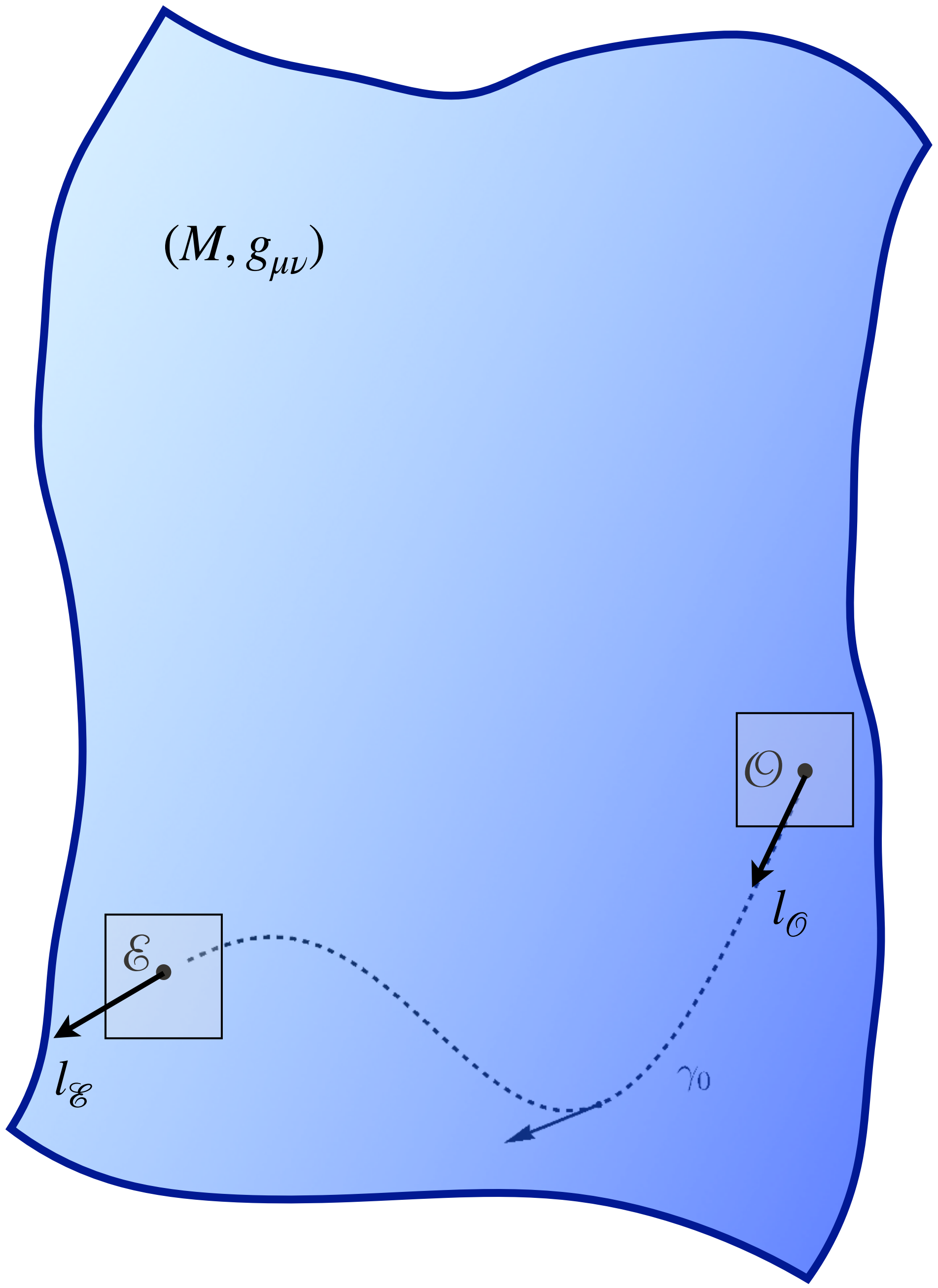
$$T_{(\mathscr{O},\mathscr{E})}(M \times M) \cong T_{\mathscr{O}}M \oplus T_{\mathscr{E}}M$$

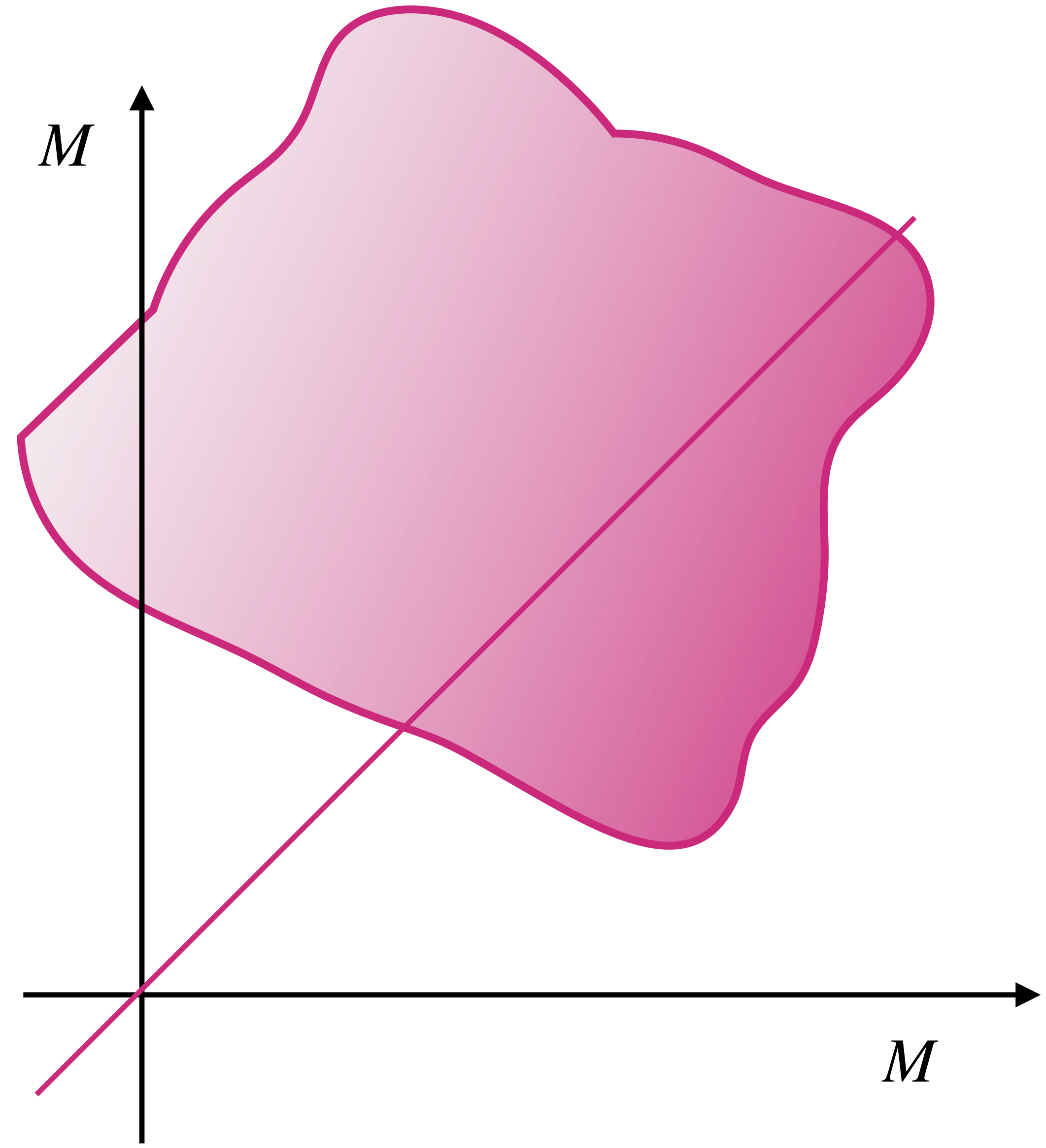
Metric tensor $\mathbf{h}(\mathbf{X}, \mathbf{Y}) \equiv g_{\mathscr{O}}(X_{\mathscr{O}}, Y_{\mathscr{O}}) - g_{\mathscr{E}}(X_{\mathscr{E}}, Y_{\mathscr{E}})$

$$\operatorname{sgn} \mathbf{h} = (4,4)$$

Local surface of communication



Geometric locus of pairs of points connected by a null geodesic



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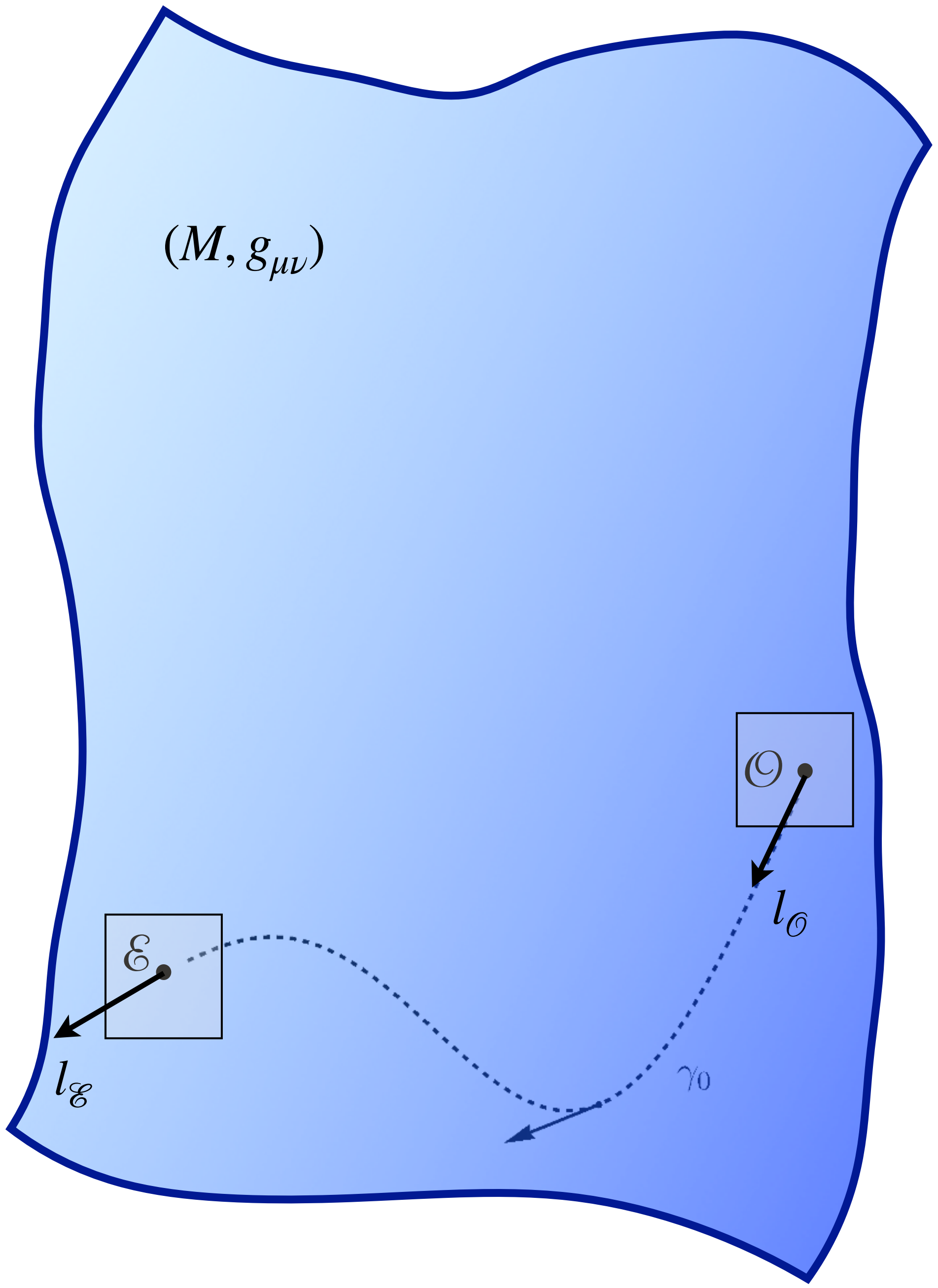

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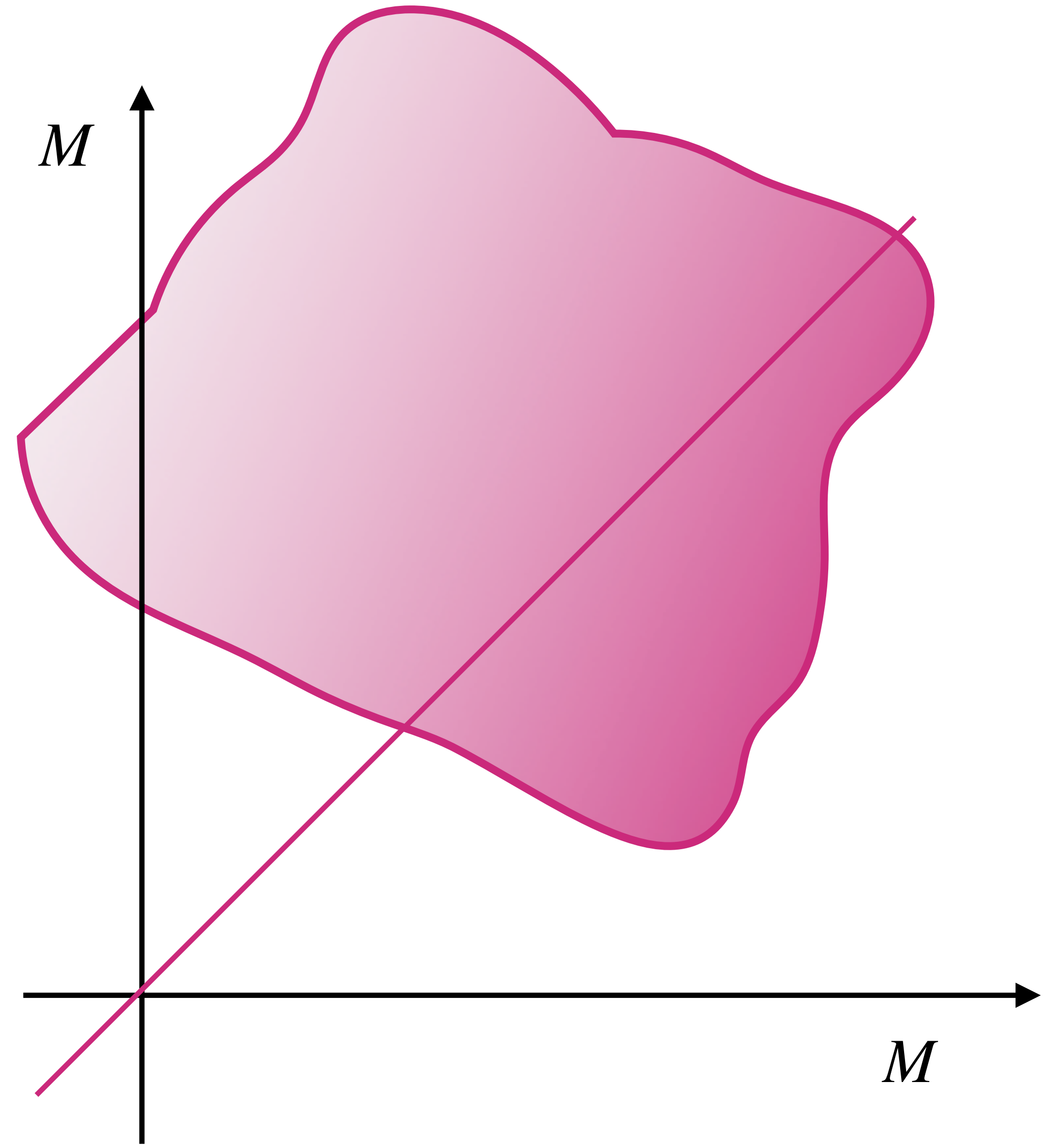
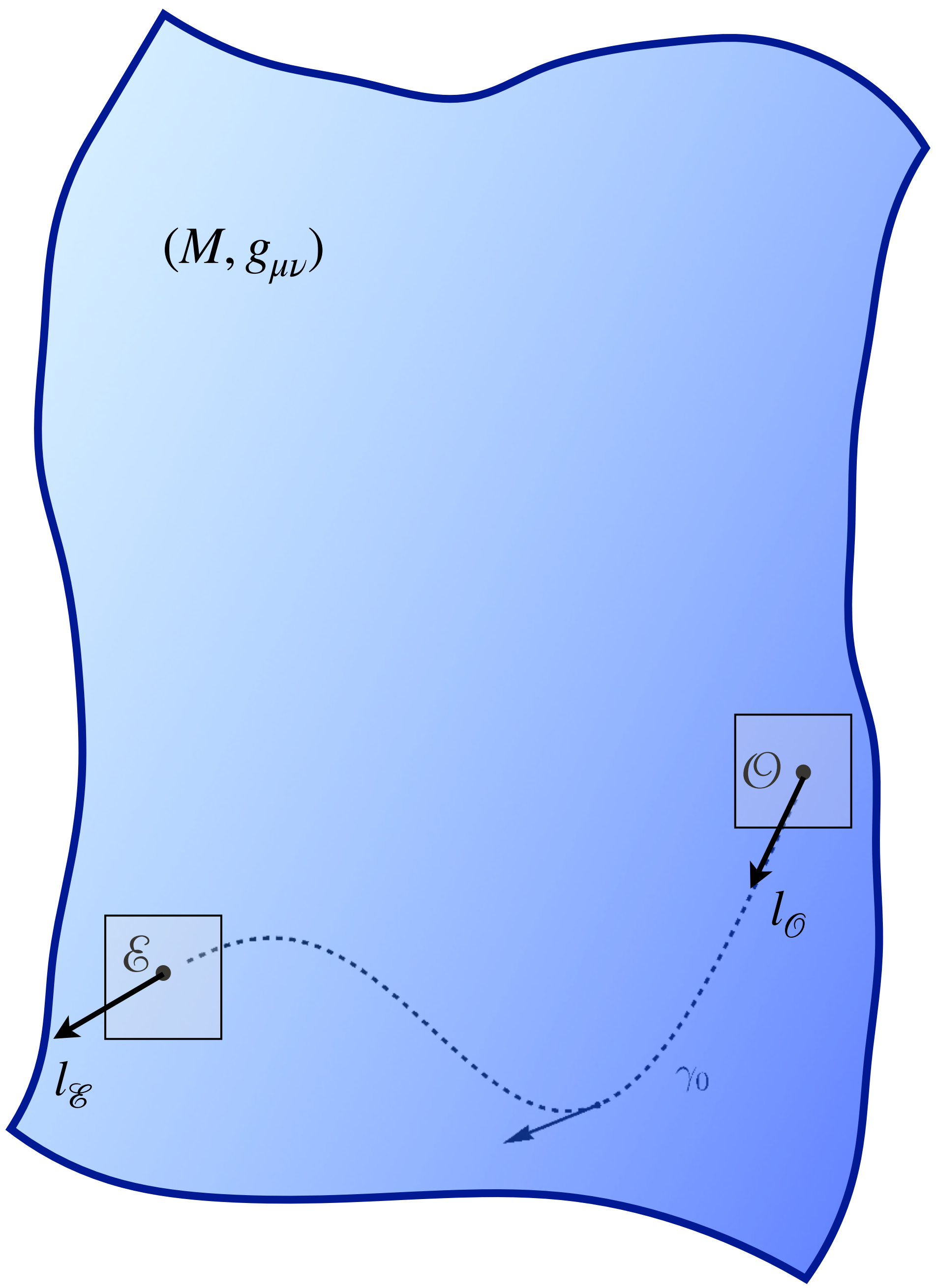


Defines the shapes of **all** light cones!

Local surface of communication



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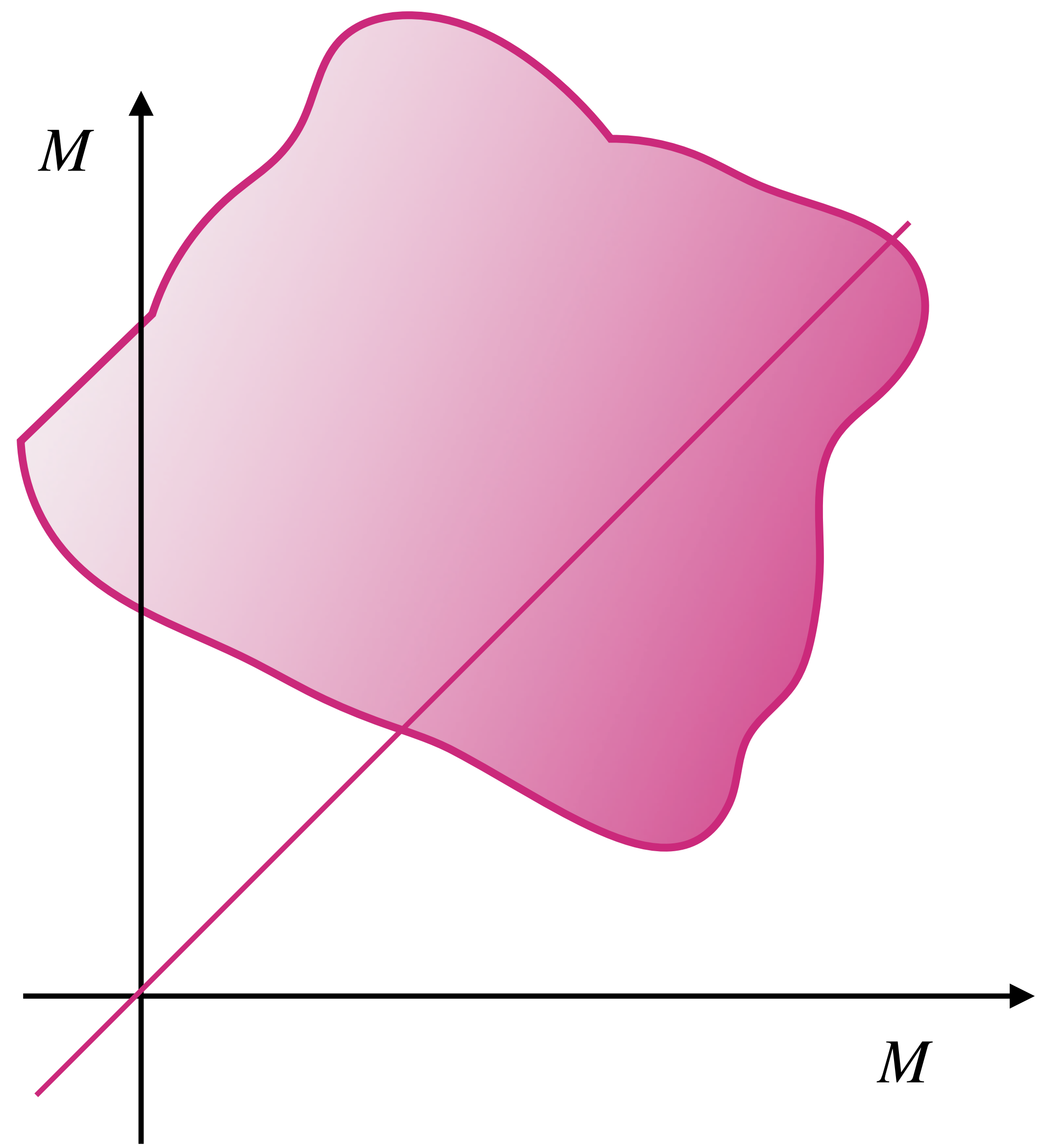
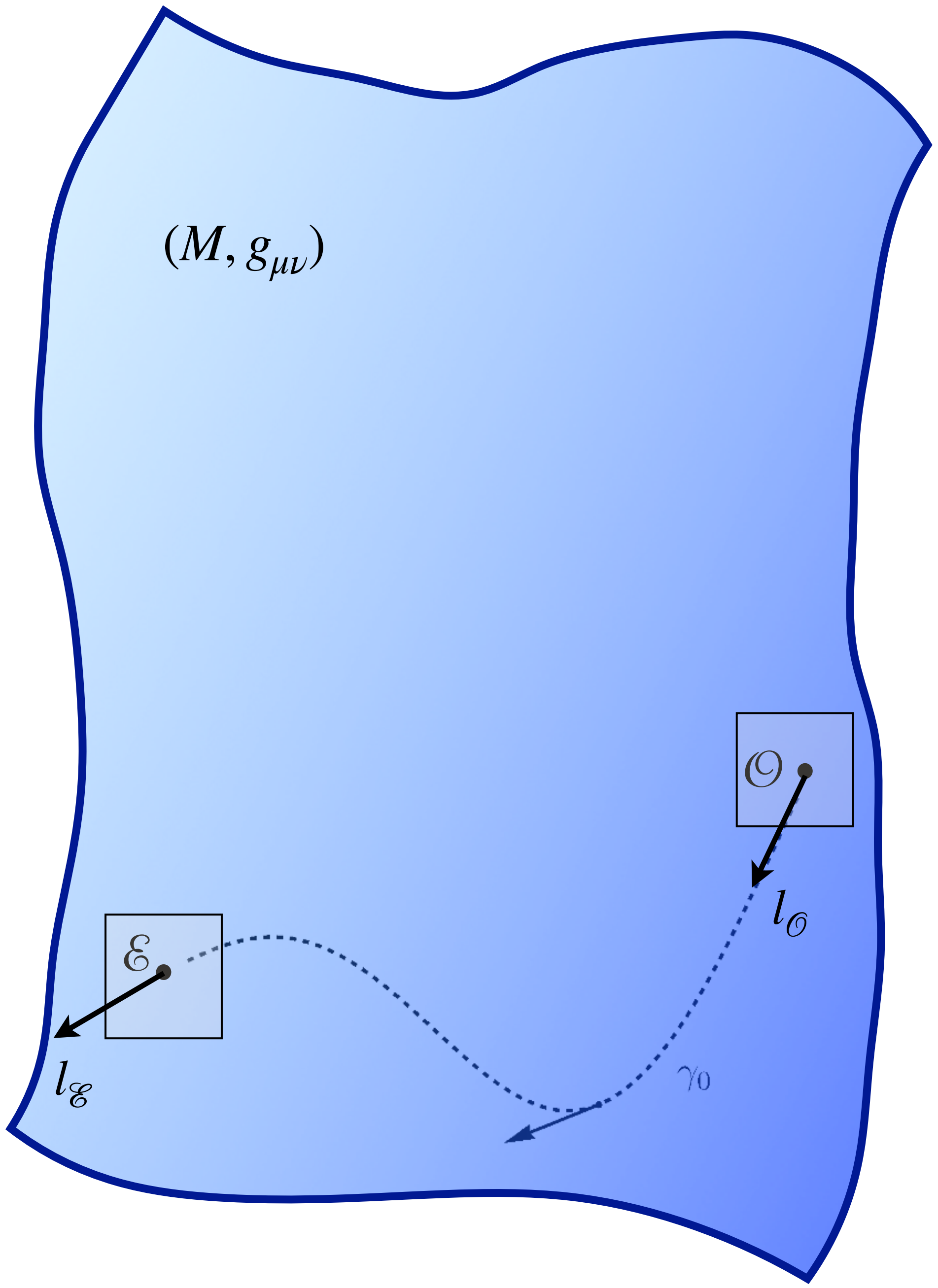


Defines the shapes of **all** light cones!

Globally it is **not** an immersed sub-manifold of dim=7, but…

Local surface of communication



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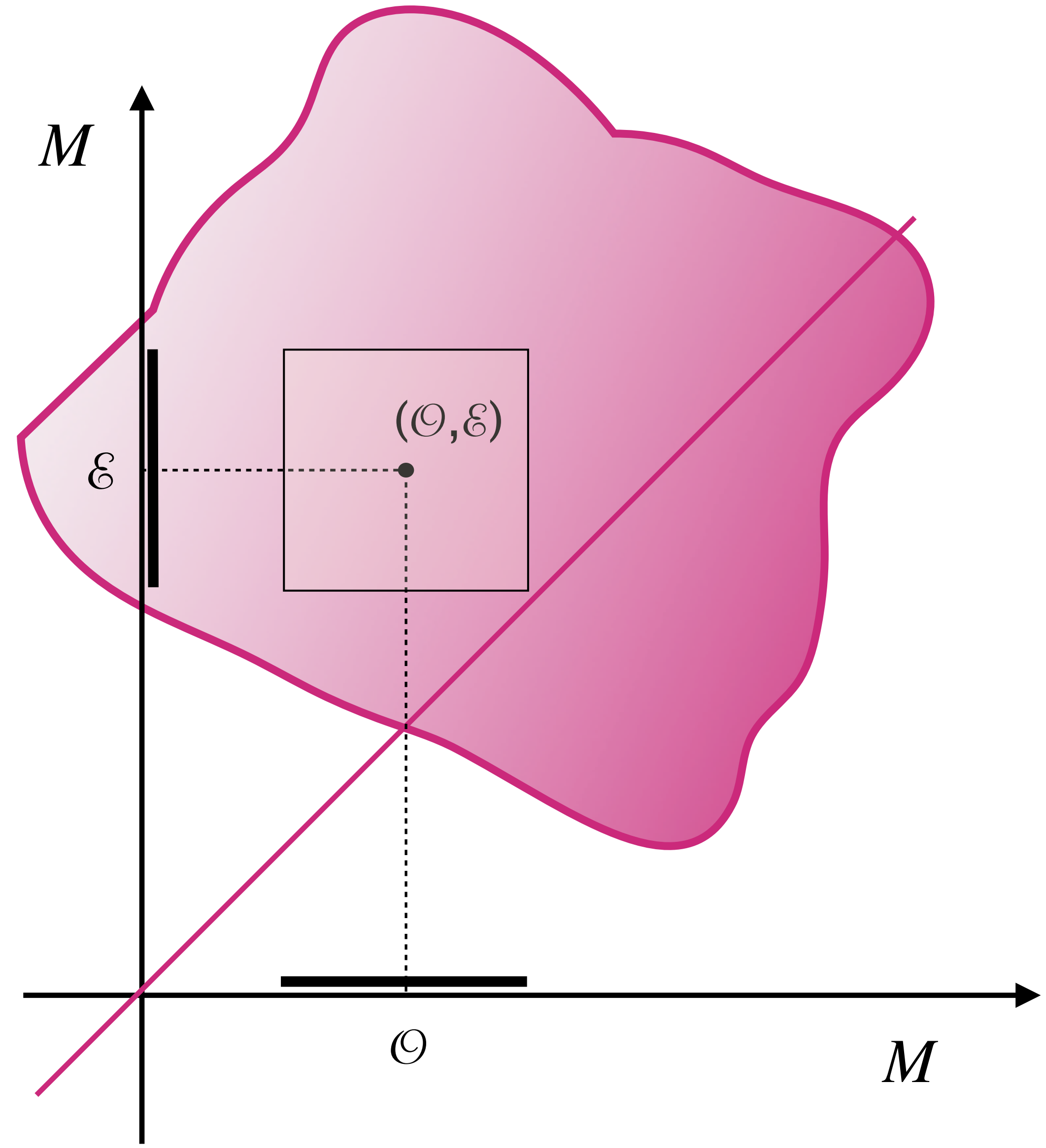
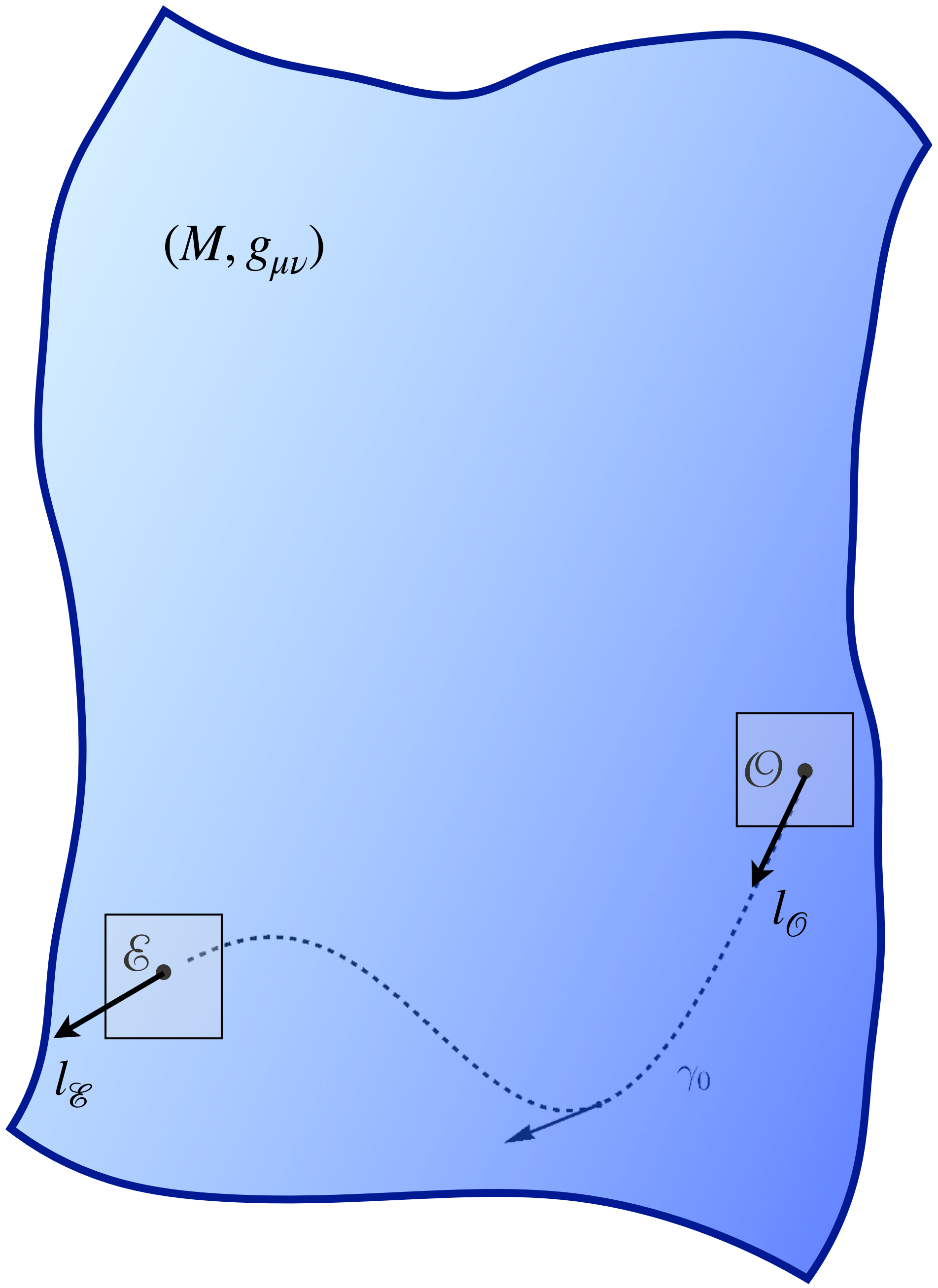


Defines the shapes of **all** light cones!

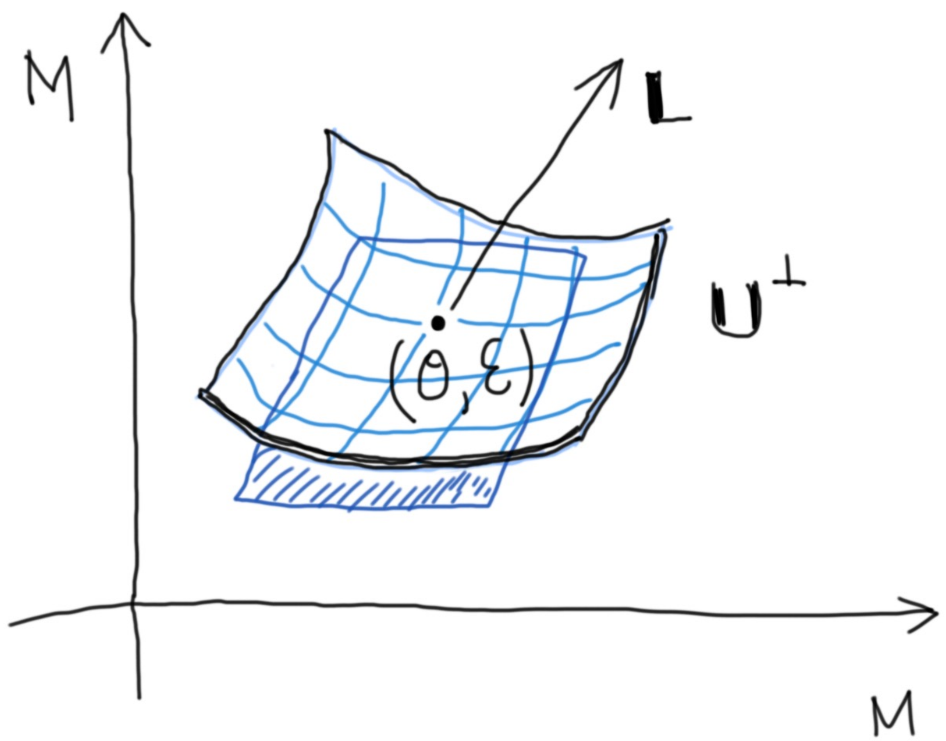
Globally it is **not** an immersed sub-manifold of dim=7, but…

…locally it is (far from caustics)

Local surface of communication



In a small neighbourhood of ($\mathscr{O}$,$\mathscr{E}$)



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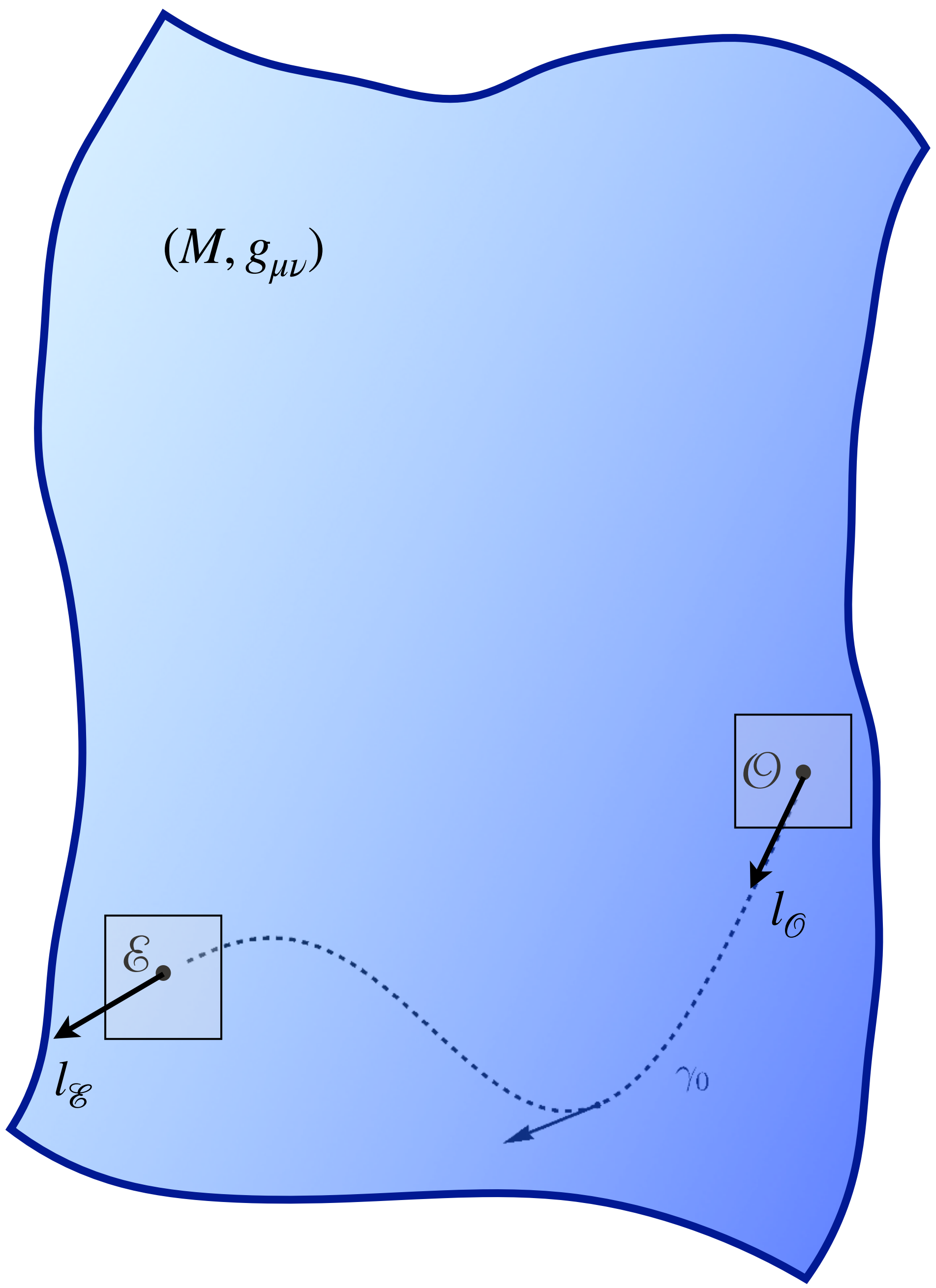
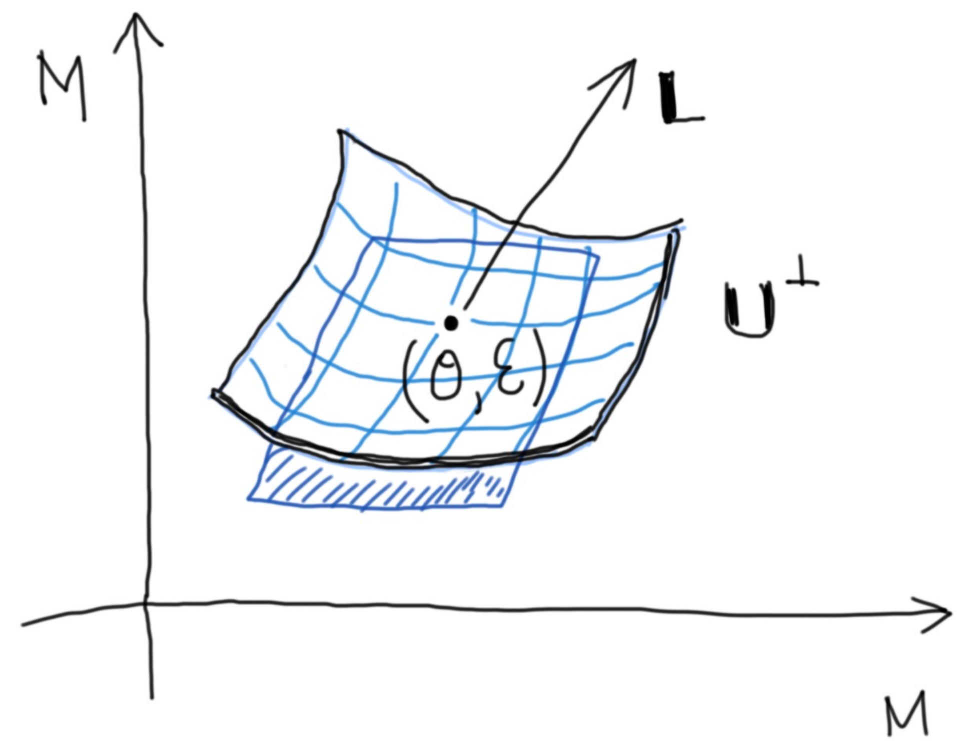


In a small neighbourhood of ($\mathcal{O}$,$\mathcal{E}$)



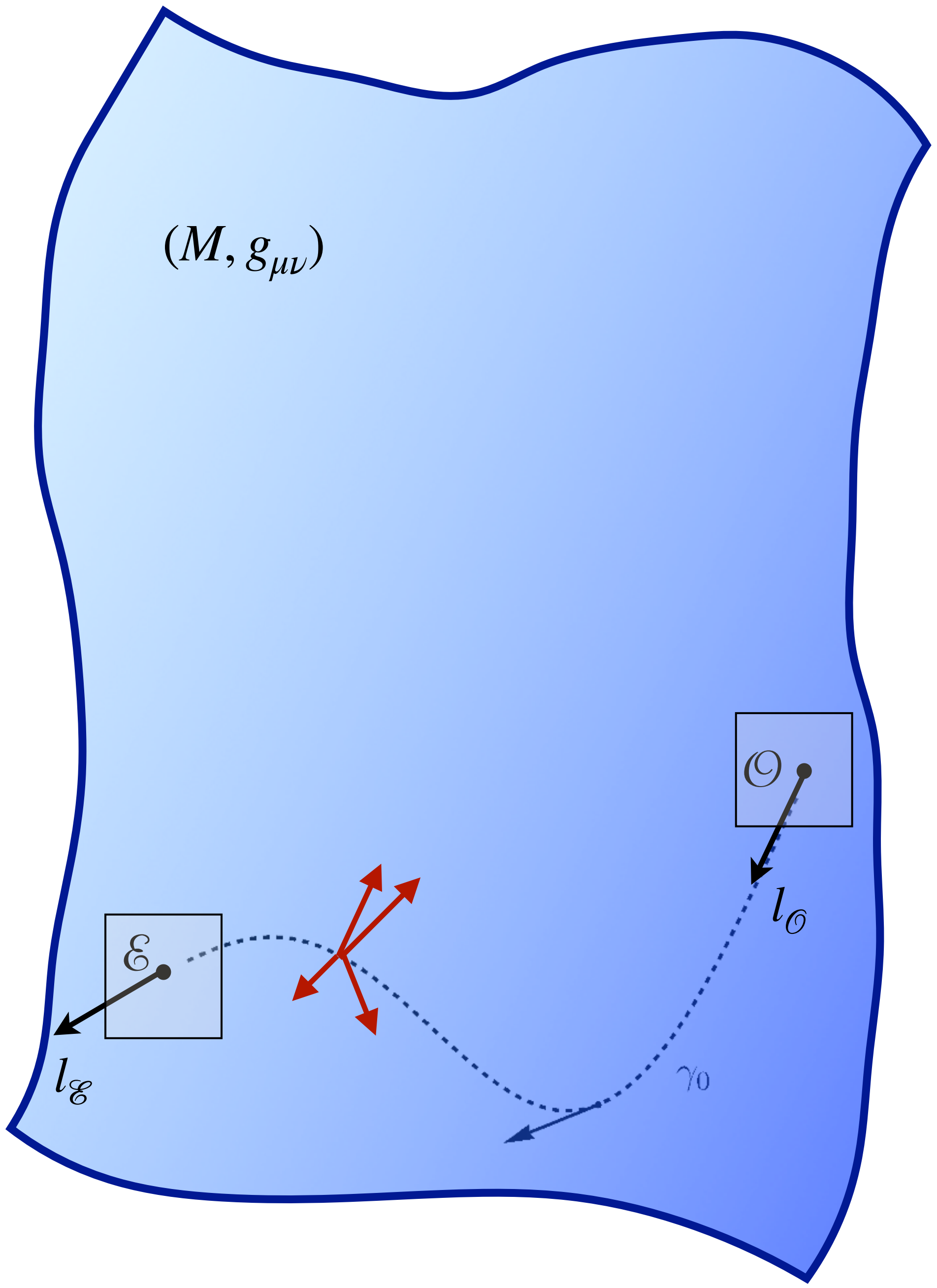
Normal 1-form $\mathbf{L} = \left(l_{\mathcal{O}\,\mu'} \quad - l_{\mathcal{E}\,\mu} \right)$

Extrinsic curvature $\mathbf{U}^{\perp}$ given by the distance and the spacetime curvature along γ_0

$$\mathbf{L} \to c \cdot \mathbf{L}, \quad \mathbf{U}^{\perp} \to c \cdot \mathbf{U}^{\perp}$$

Proof: via Synge's world function formalism [Synge 1968], [Teyssandier, Le Poncin-Lafitte, Linet 2008]…

Local surface of communication



Resolvent of the first order geodesic deviation equation

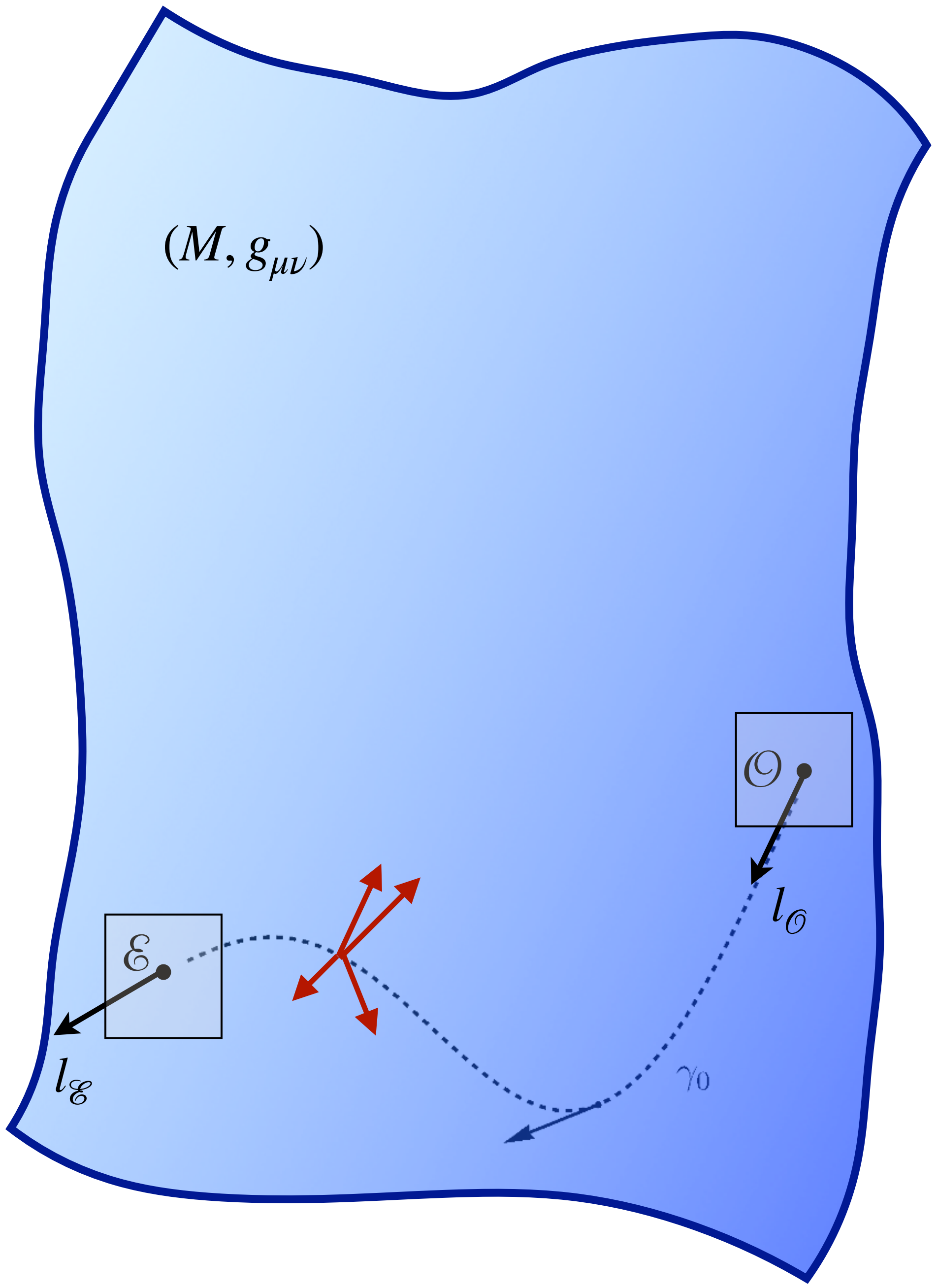
$$\mathbf{W} = \begin{pmatrix} {W_{XX}}^{\mu}{}_{\nu} & {W_{XL}}^{\mu}{}_{\sigma} \\ {W_{LX}}^{\rho}{}_{\nu} & {W_{LL}}^{\rho}{}_{\sigma} \end{pmatrix}$$

[Grasso, MK, Serbenta 2019]
[Uzun 2020], [Fleury 2014]…

$$\frac{d}{d\lambda}\mathbf{W} = \begin{pmatrix} 0 & \delta^{\alpha}{}_{\beta} \\ R^{\gamma}{}_{\mu\nu\epsilon}(\lambda)\, l^{\mu}\, l^{\nu} & 0 \end{pmatrix} \mathbf{W}(\lambda)$$

$$\mathbf{W}(\mathscr{O}) = \mathrm{I}_8$$

Local surface of communication



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[Grasso, MK, Serbenta 2019]
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$$\begin{pmatrix} 0 & \delta^{\alpha}{}_{\beta} \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ R^{\gamma}{}_{\mu\nu\epsilon}(\lambda)\, l^{\mu}\, l^{\nu} & 0 \end{pmatrix}$$

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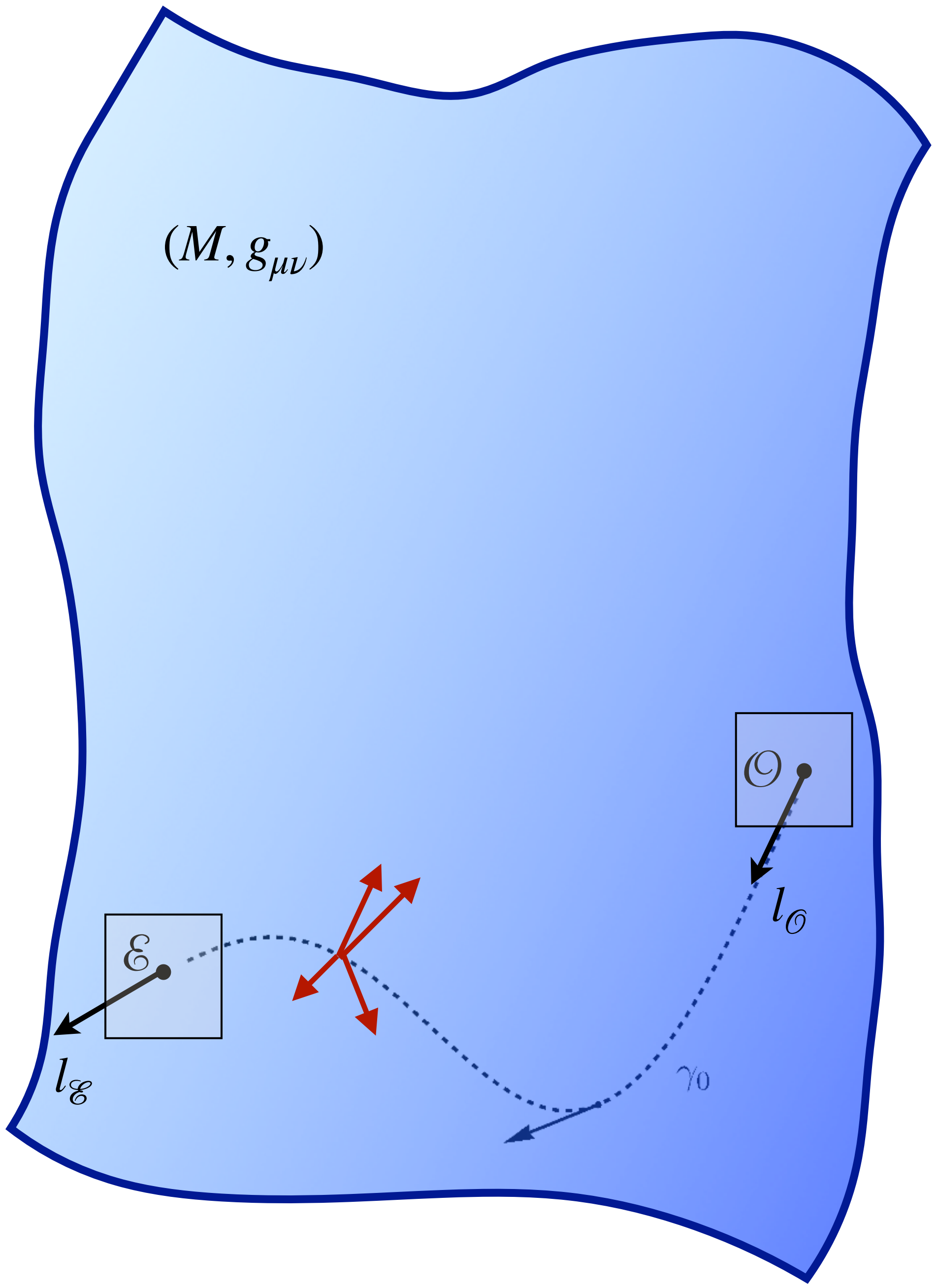

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$$\mathbf{W}(\mathscr{O}) = \mathrm{I}_8$$

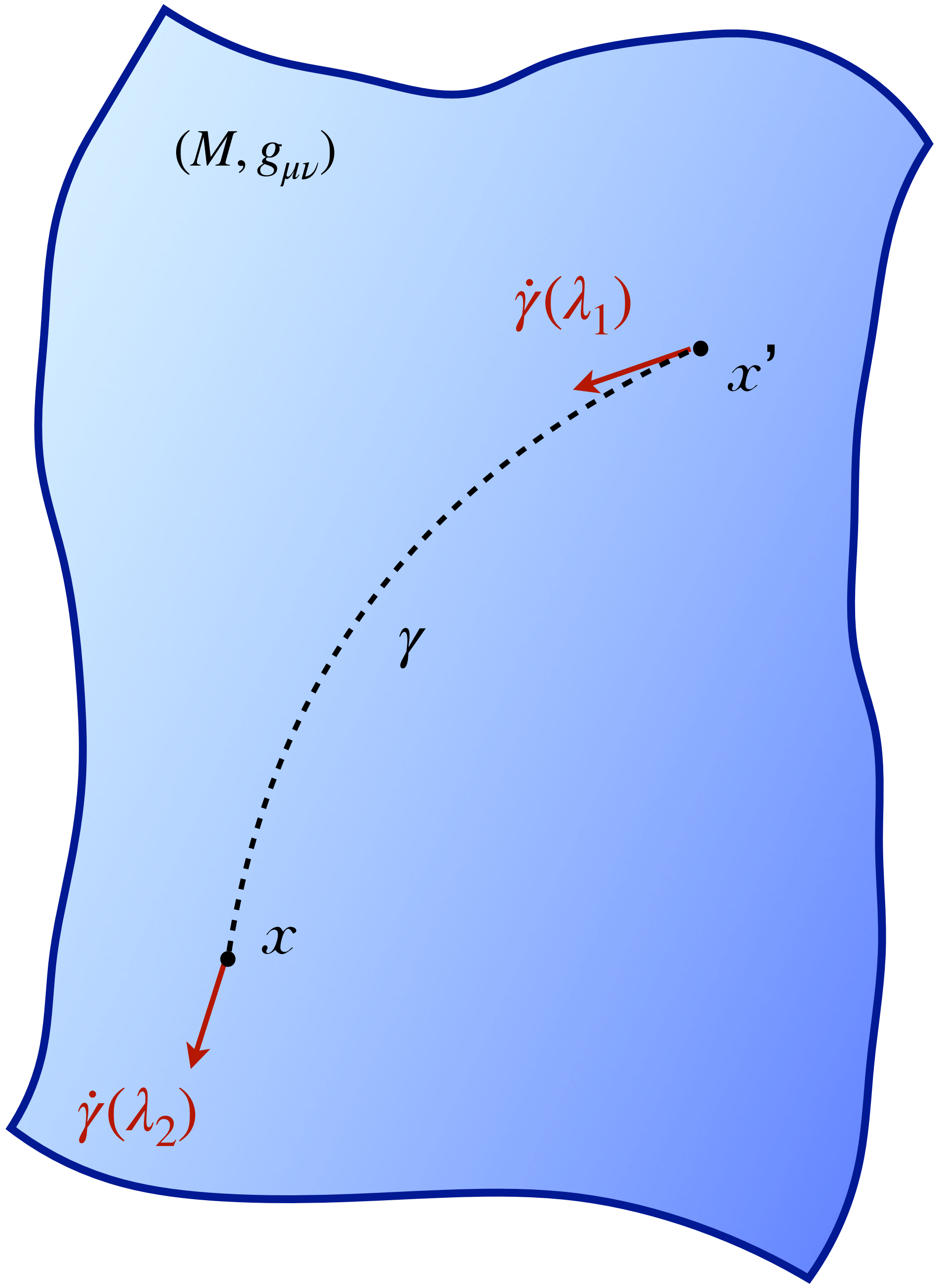
Symmetric 2-tensor on $T_{(\mathscr{O},\mathscr{E})}(M \times M)$

$$\mathbf{U} = \begin{pmatrix} U_{\mathscr{O}\mathscr{O}} & U_{\mathscr{O}\mathscr{E}} \\ U_{\mathscr{E}\mathscr{O}} & U_{\mathscr{E}\mathscr{E}} \end{pmatrix} = \begin{pmatrix} -g_{\mathscr{O}}\, W_{XL}^{-1}\, W_{XX} & g_{\mathscr{O}}\, W_{XL}^{-1} \\ g_{\mathscr{E}}\left(W_{LL}\, W_{XL}^{-1}\, W_{XX} - W_{LX}\right) & -g_{\mathscr{E}}\, W_{LL}\, W_{XL}^{-1} \end{pmatrix}$$

$$\mathbf{U}(\mathbf{X},\mathbf{Y}) = \mathbf{U}(\mathbf{Y},\mathbf{X})$$

Extrinsic curvature of LSC: $\mathbf{U}^{\perp} = \mathbf{U}\Big|_{\mathbf{L}^{\perp}}$ (restriction to the tangent space to LSC)

Synge's world function



Introduced by Synge in 1960

bi-scalar $\sigma : M \times M \supset \mathcal{U} \to \mathbf{R}$

$$\sigma(x, x') = \frac{\lambda_2 - \lambda_1}{2} \int_{\lambda_1}^{\lambda_2} g_{\mu\nu}\, \dot\gamma^{\mu}(\lambda)\, \dot\gamma^{\nu}(\lambda)\, d\lambda$$

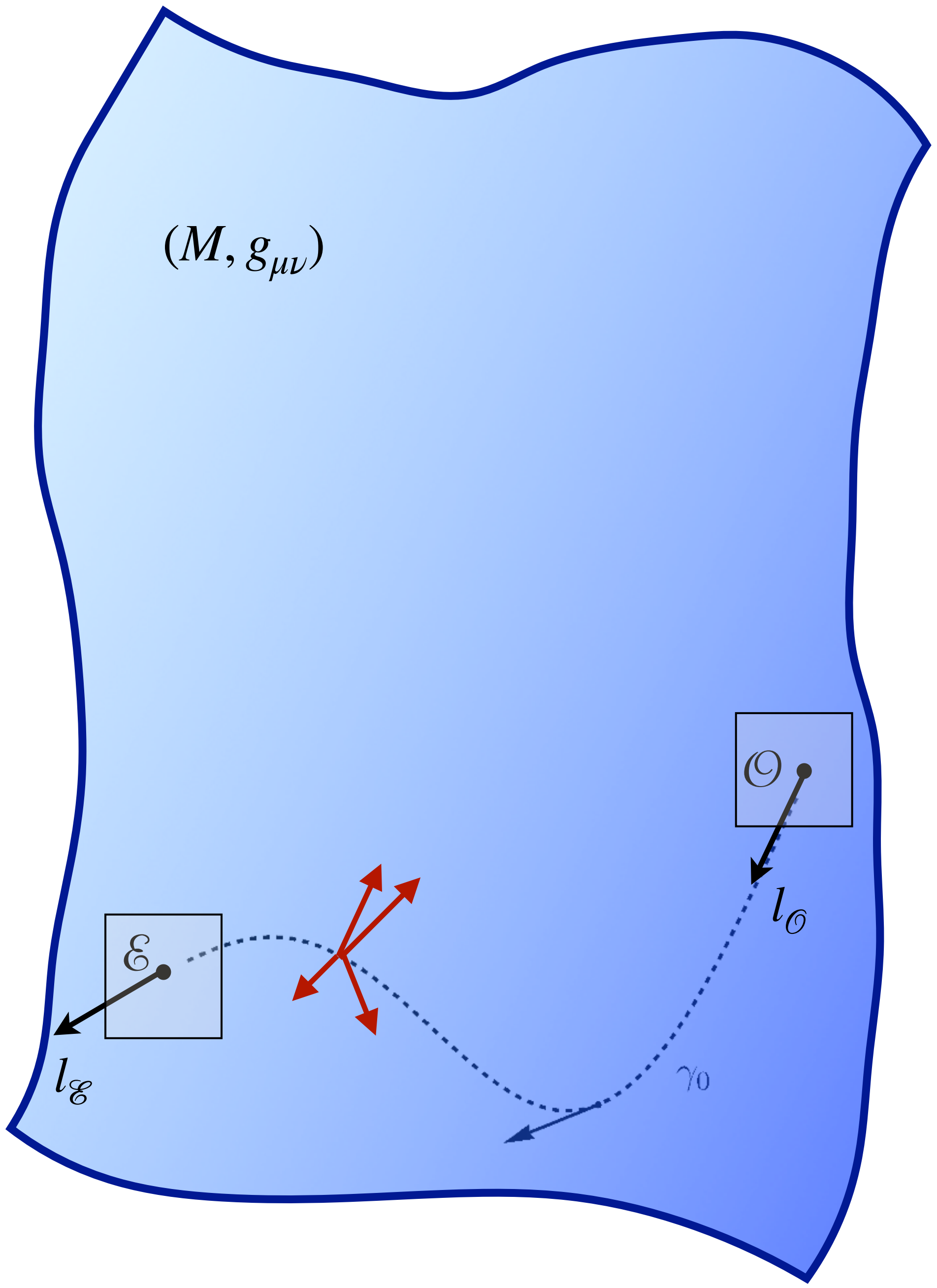
Properties:

$$\sigma_{,\nu} \equiv \frac{\partial \sigma}{\partial x^{\nu}} = (\lambda_2 - \lambda_1)\, \dot\gamma^{\mu}(\lambda_2)\, g_{\mu\nu}$$

$$\sigma_{,\nu'} \equiv \frac{\partial \sigma}{\partial x^{\nu'}} = -\,(\lambda_2 - \lambda_1)\, \dot\gamma^{\mu'}(\lambda_1)\, g_{\mu'\nu'}$$

$\sigma > 0$		γ spacelike
$\sigma = 0$	iff	γ null
$\sigma < 0$		γ timelike

Synge's world function



Locally LSC can be identified with the zero level set of σ [Teyssandier, Leponcin-Lafitte]

$$LSC = \left\{(x, x') \in M \times M \,|\, \sigma(x, x') = 0\right\}$$

Taylor expansion in coordinates locally flat at $\mathcal{O}$ and $\mathcal{E}$

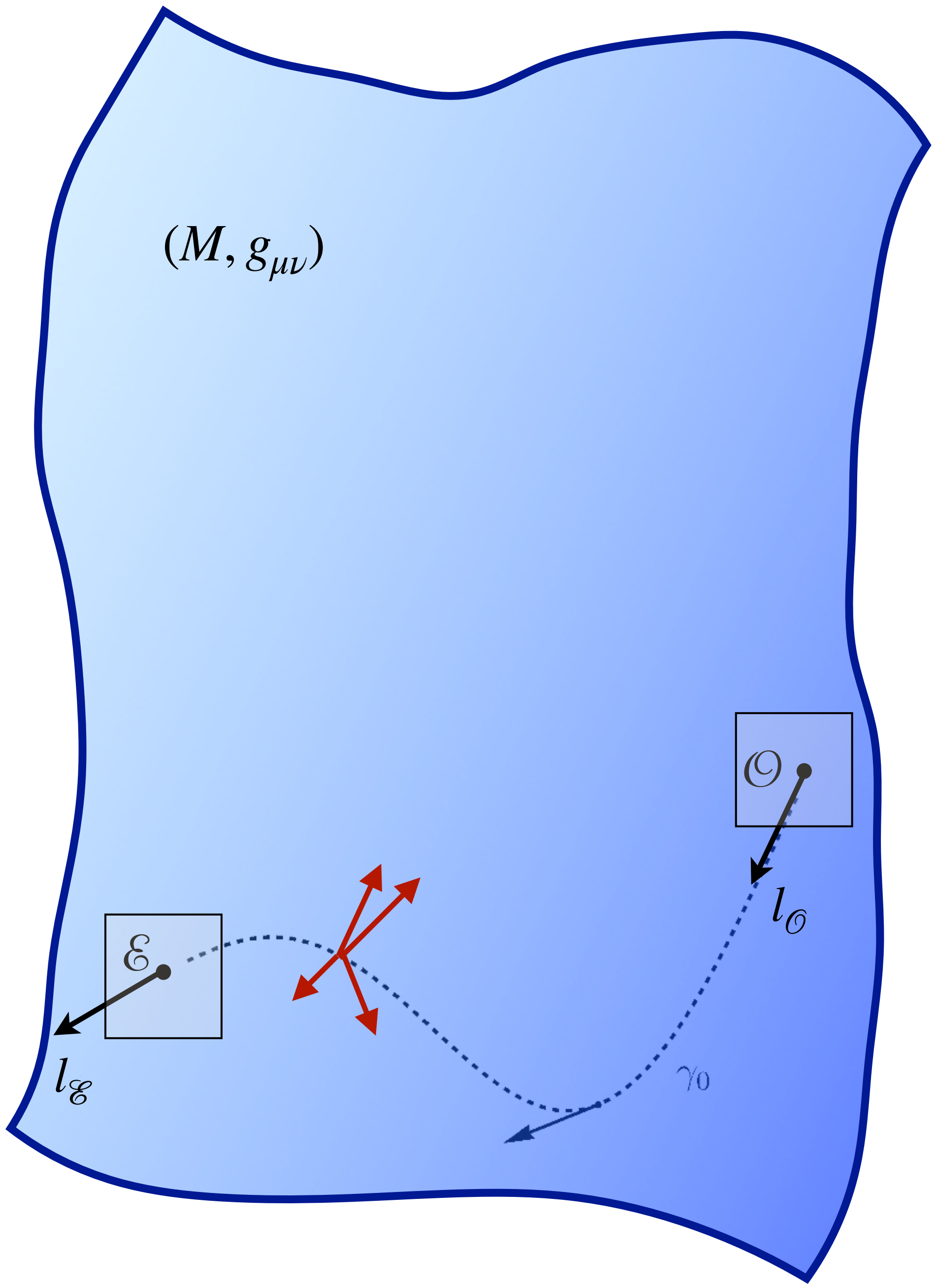
$$\sigma(x_{\mathcal{E}} + \delta x_{\mathcal{E}}, x_{\mathcal{O}} + \delta x_{\mathcal{O}}) = \sigma_{,\mu'}\,\delta x_{\mathcal{O}}^{\mu'} + \sigma_{,\mu}\,\delta x_{\mathcal{E}}^{\mu}$$
$$+\frac{1}{2}\sigma_{;\mu\nu}\,\delta x_{\mathcal{E}}^{\mu}\,\delta x_{\mathcal{E}}^{\nu} + \frac{1}{2}\sigma_{;\mu'\nu'}\,\delta x_{\mathcal{O}}^{\mu'}\,\delta x_{\mathcal{O}}^{\nu'} + \sigma_{;\mu\nu'}\,\delta x_{\mathcal{E}}^{\mu}\,\delta x_{\mathcal{O}}^{\nu'}$$
$$+O(\delta x^3)$$

Also a locally flat coordinate system on $M \times M$ near $(\mathcal{E}, \mathcal{O})$

LSC condition can be rewritten $\quad \mathbf{L}(\mathbf{X}) + \frac{1}{2}\mathbf{U}(\mathbf{X}, \mathbf{X}) = 0$

$$\mathbf{X} = \begin{pmatrix} \delta x_{\mathcal{O}}^{\mu'} \\ \delta x_{\mathcal{E}}^{\mu} \end{pmatrix} \qquad \mathbf{L} = \begin{pmatrix} l_{\mathcal{O}\,\mu'} & -l_{\mathcal{E}\,\mu} \end{pmatrix} \qquad \mathbf{U} = -\frac{1}{\lambda_{\mathcal{E}} - \lambda_{\mathcal{O}}}\begin{pmatrix} \sigma_{;\mu'\nu'} & \sigma_{;\mu\nu'} \\ \sigma_{;\mu'\nu} & \sigma_{;\mu\nu} \end{pmatrix}$$

Synge's world function



Relation between $\mathbf{U}$ and the spacetime curvature:

$\mathbf{U}$ gives relation between endpoints variations and tangent vector variations

$$\begin{pmatrix} \Delta l_{\mathcal{O}\,\mu'} \\ -\Delta l_{\mathcal{E}\,\nu} \end{pmatrix} = \mathbf{U} \begin{pmatrix} \delta x_{\mathcal{O}}^{\mu'} \\ \delta x_{\mathcal{E}}^{\nu} \end{pmatrix}$$

on the other hand, the geodesic deviation equation relates initial data variations at $\mathcal{O}$ to variations at $\mathcal{E}$

$$\begin{pmatrix} \delta x_{\mathcal{E}}^{\mu} \\ \Delta l_{\mathcal{E}}^{\nu} \end{pmatrix} = \mathbf{W} \begin{pmatrix} \delta x_{\mathcal{O}}^{\mu'} \\ \Delta l_{\mathcal{O}}^{\nu'} \end{pmatrix}$$

$$\frac{d}{d\lambda}\mathbf{W} = \begin{pmatrix} 0 & \delta^{\alpha}{}_{\beta} \\ R^{\gamma}{}_{\mu\nu\epsilon}(\lambda)\, l^{\mu}\, l^{\nu} & 0 \end{pmatrix} \mathbf{W}(\lambda)$$

$$\mathbf{W}(\mathcal{O}) = \mathrm{I}_8$$

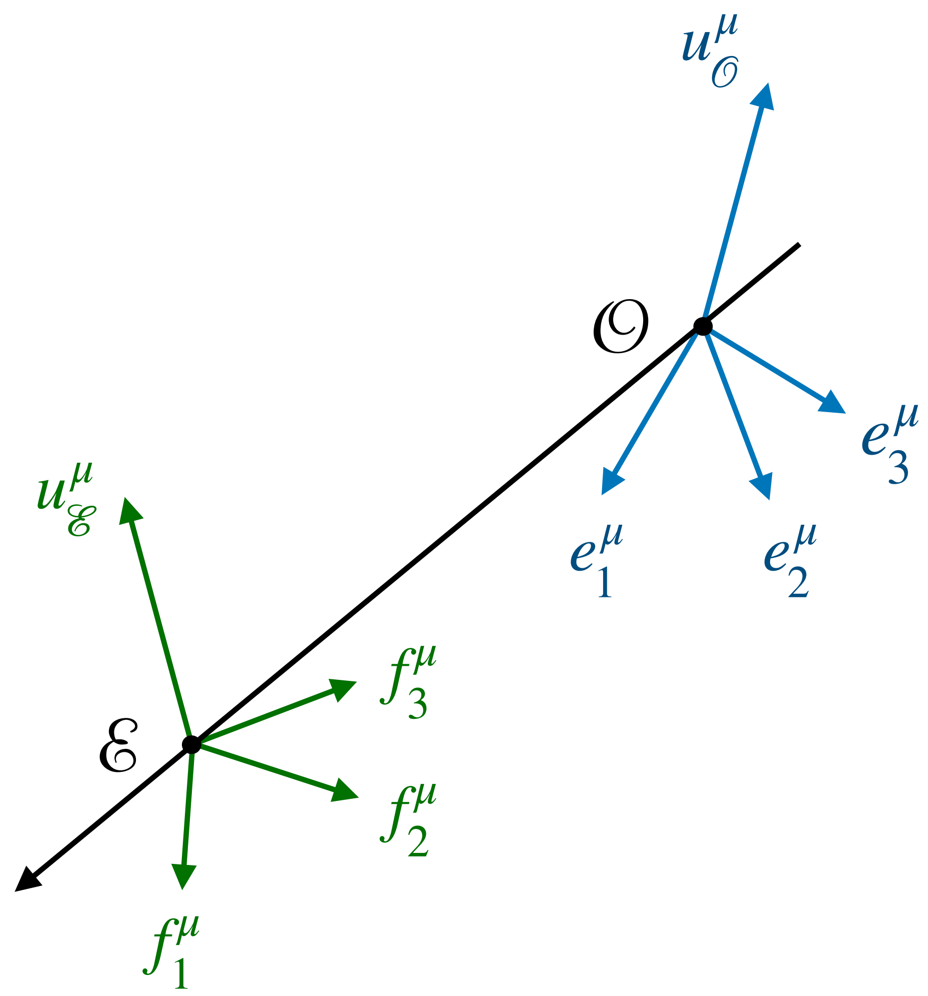
after a bit of algebra:

$$\mathbf{U} = \begin{pmatrix} -g_{\mathcal{O}}\, W_{XL}^{-1}\, W_{XX} & g_{\mathcal{O}}\, W_{XL}^{-1} \\ g_{\mathcal{E}}\left(W_{LL}\, W_{XL}^{-1}\, W_{XX} - W_{LX}\right) & -g_{\mathcal{E}}\, W_{LL}\, W_{XL}^{-1} \end{pmatrix}$$

Relation partially known [Dixon 1970, Vines 2015…]

Time of arrival of a signal

Introduce orthonormal tetrads at $\mathcal{O}$ and $\mathcal{E}$



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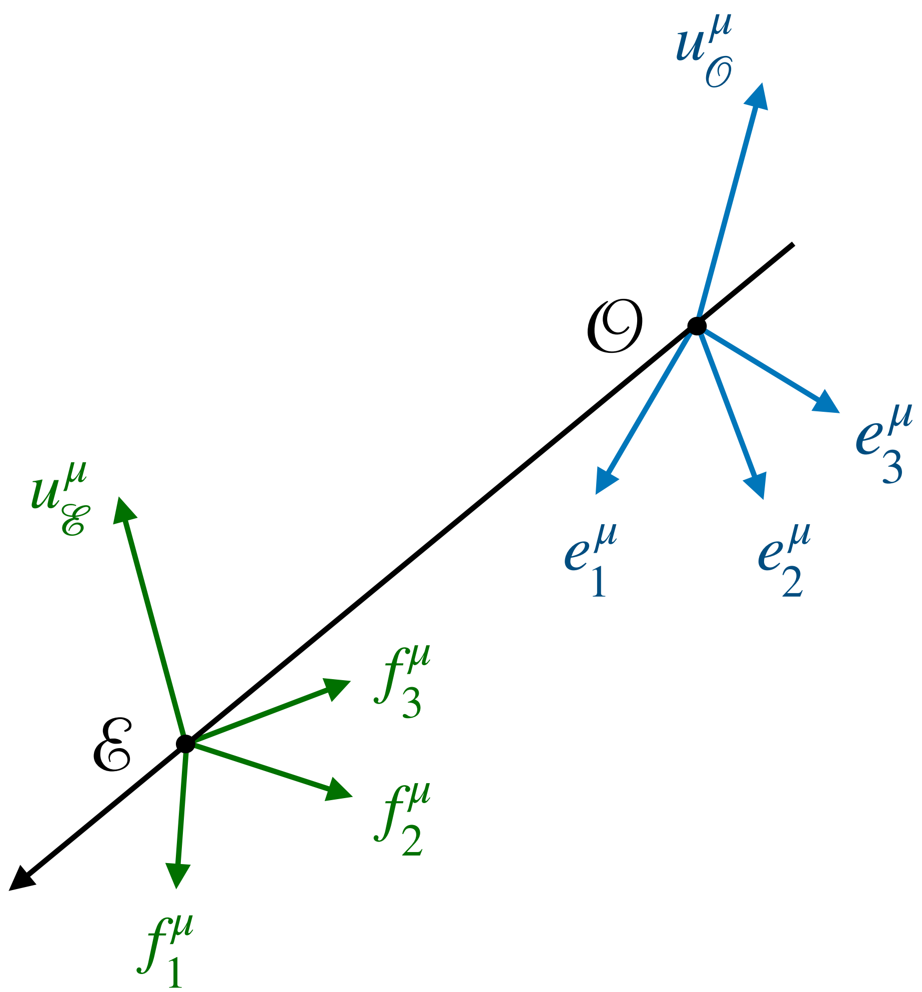

Locally flat coordinate systems near $\mathcal{O}$ and $\mathcal{E}$

points in *M* near $\mathcal{O}$ and $\mathcal{E}$ $\longleftrightarrow$ tangent vectors at $\mathrm{T}_\mathcal{O}M$, $\mathrm{T}_\mathcal{E}M$

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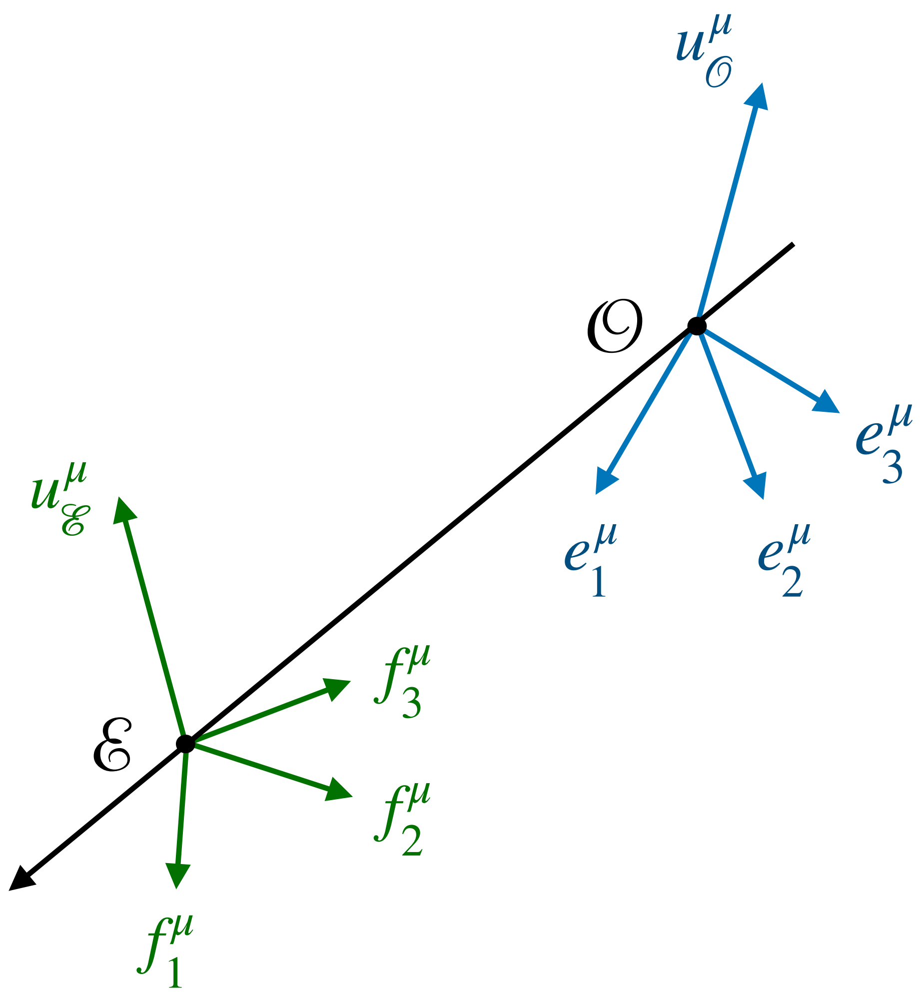
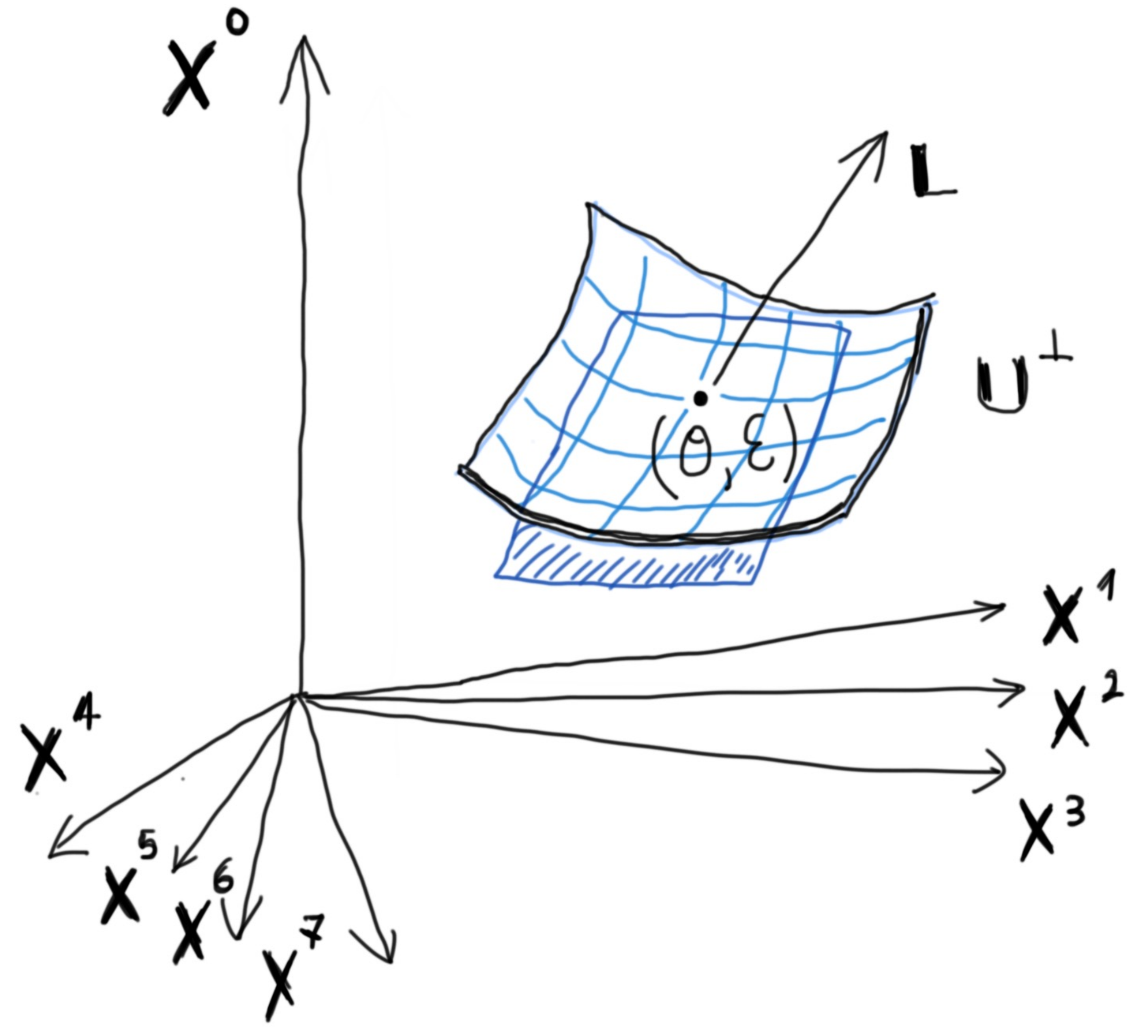




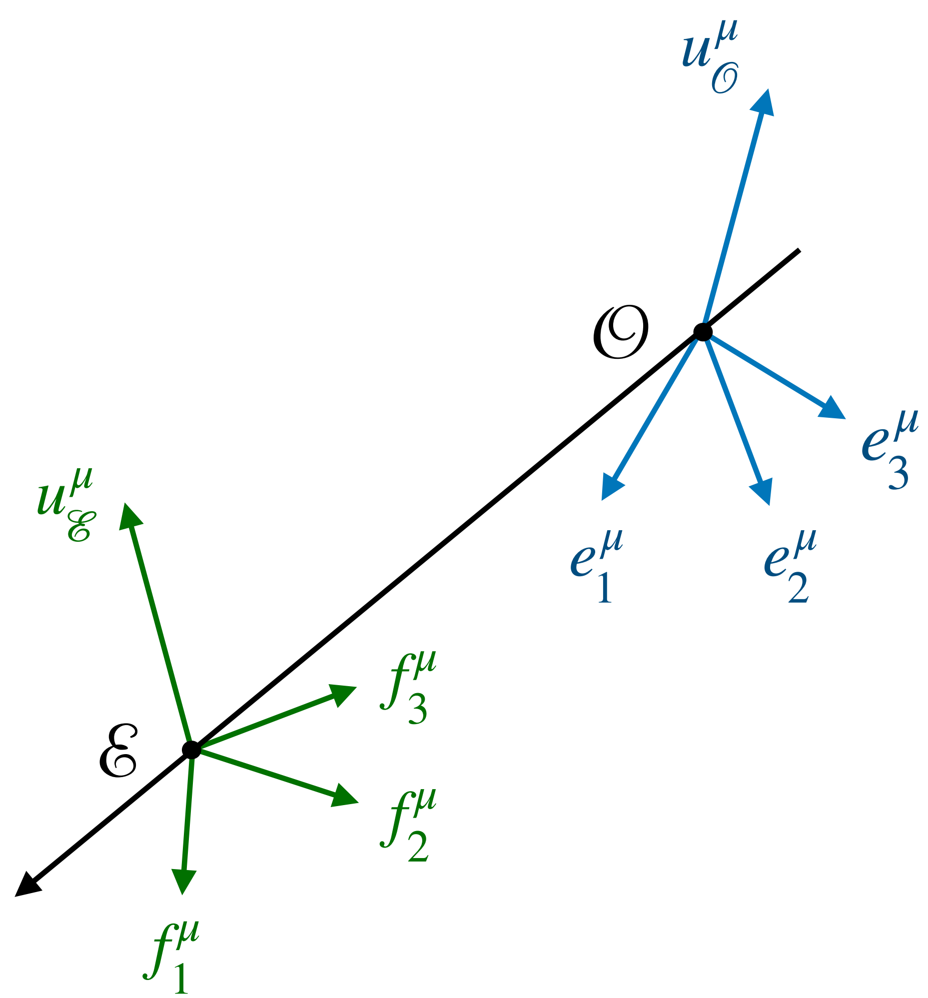
Locally flat coordinate systems near $\mathcal{O}$ and $\mathcal{E}$

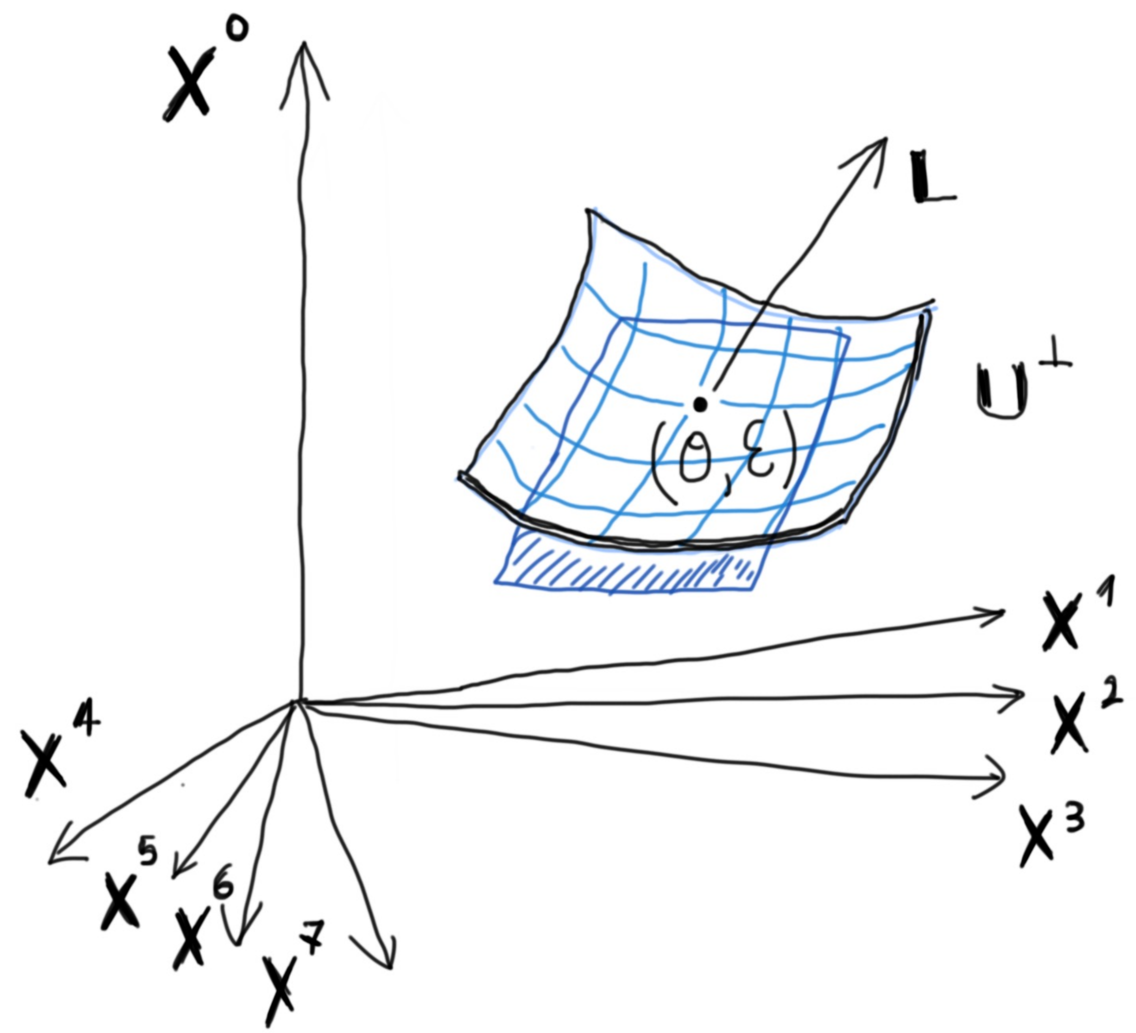
points in M near $\mathcal{O}$ and $\mathcal{E} \longleftrightarrow$ tangent vectors at $\mathrm{T}_{\mathcal{O}}M$, $\mathrm{T}_{\mathcal{E}}M$

Locally flat coordinate system ($\mathbf{X}^0,\ldots,\mathbf{X}^7$) in $M{\times}M$ near ($\mathcal{O}$, $\mathcal{E}$)

Time of arrival of a signal

Introduce orthonormal tetrads at $\mathcal{O}$ and $\mathcal{E}$





Locally flat coordinate systems near $\mathcal{O}$ and $\mathcal{E}$

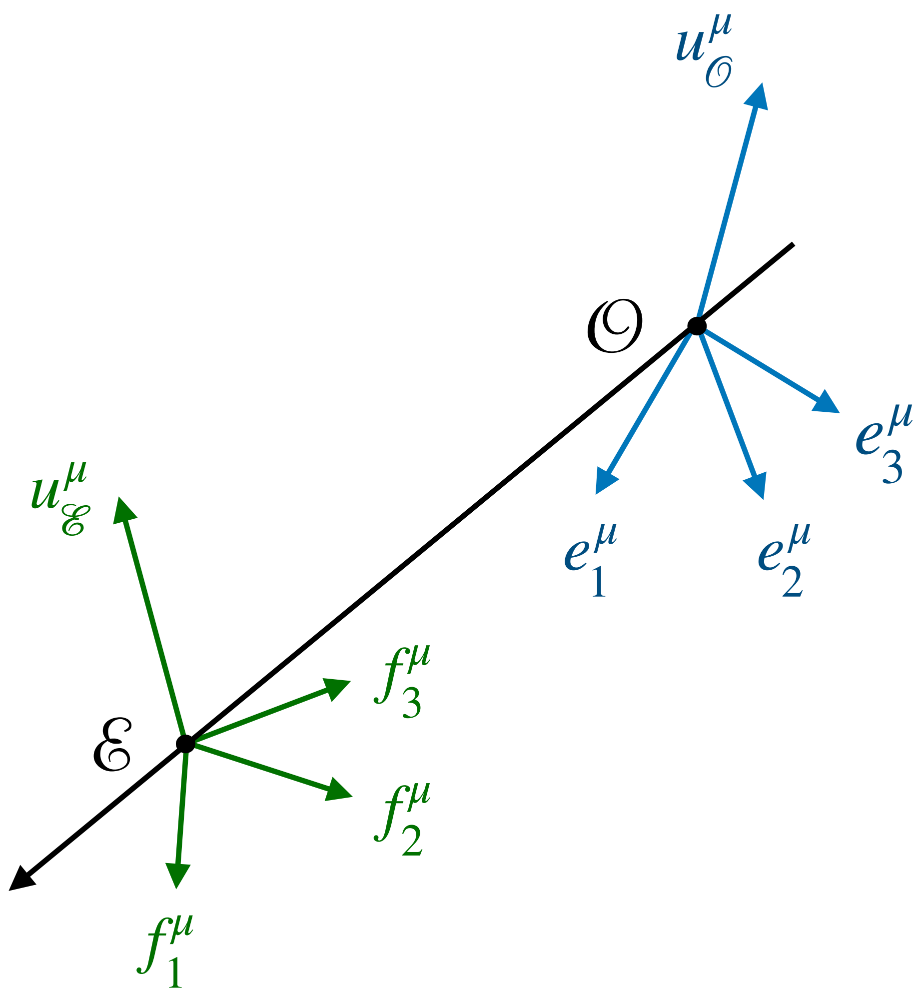
points in M near $\mathcal{O}$ and $\mathcal{E}$ $\longleftrightarrow$ tangent vectors at $\mathrm{T}_{\mathcal{O}}M$, $\mathrm{T}_{\mathcal{E}}M$

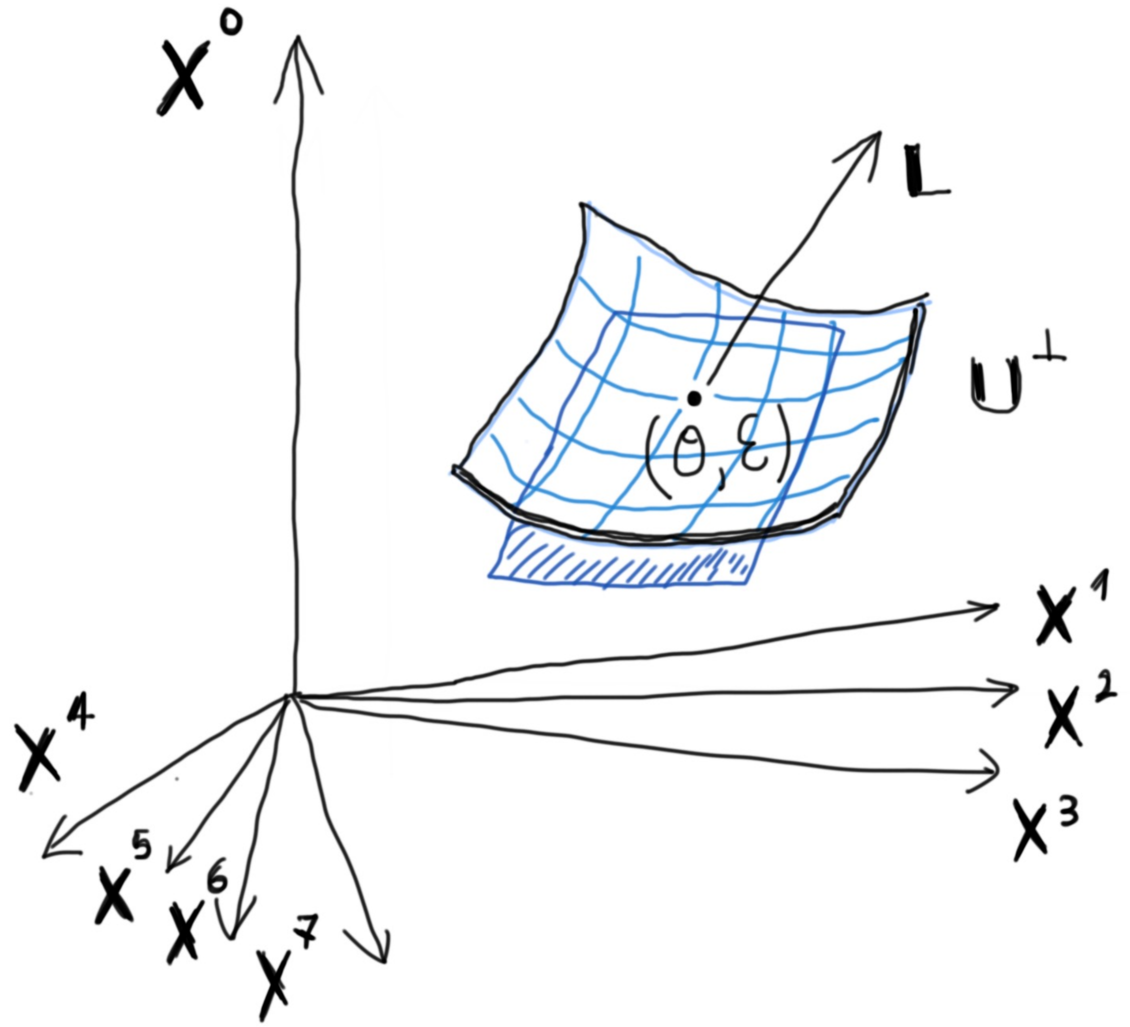
Locally flat coordinate system ($\mathbf{X}^0,\ldots,\mathbf{X}^7$) in $M \times M$ near ($\mathcal{O}$, $\mathcal{E}$)

Gauge choice: $\mathbf{L_0} = 1$

Time of arrival of a signal

Introduce orthonormal tetrads at $\mathcal{O}$ and $\mathcal{E}$





Locally flat coordinate systems near $\mathcal{O}$ and $\mathcal{E}$

points in M near $\mathcal{O}$ and $\mathcal{E}$ $\longleftrightarrow$ tangent vectors at $\mathrm{T}_{\mathcal{O}}M$, $\mathrm{T}_{\mathcal{E}}M$

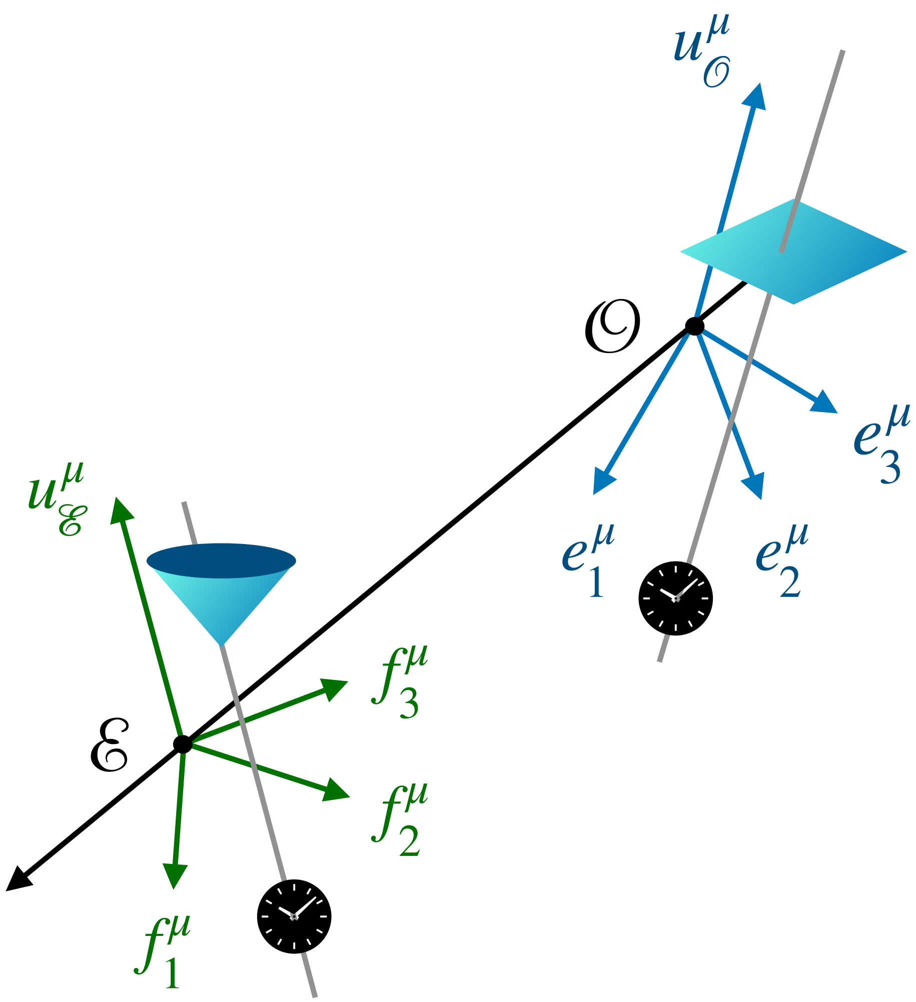
$$\mathbf{X^0} \equiv \mathbf{X^0}\left(\mathbf{X^1}, \ldots, \mathbf{X^7}\right)$$

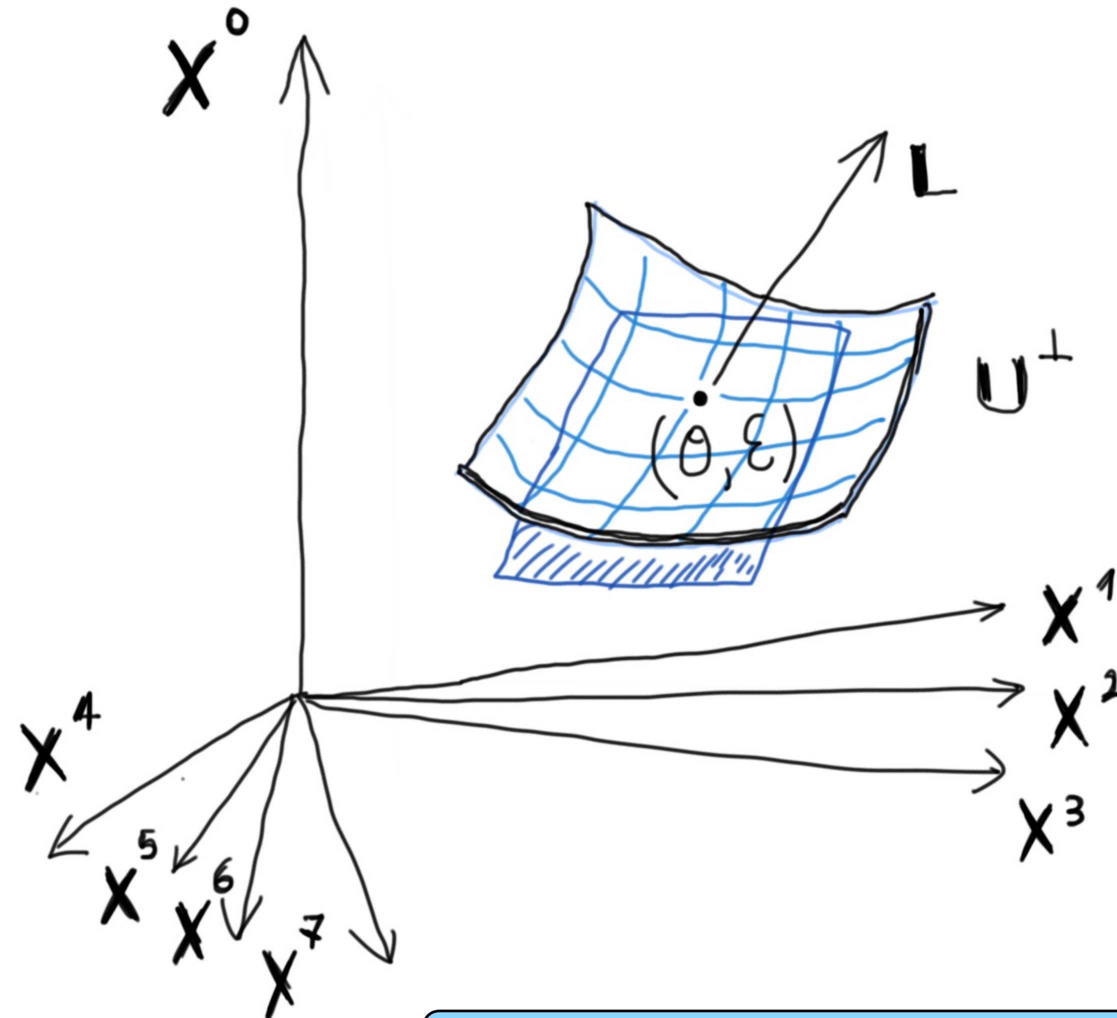
Locally flat coordinate system ($\mathbf{X}^0,\ldots,\mathbf{X}^7$) in $M\times M$ near ($\mathcal{O}$, $\mathcal{E}$)

Gauge choice: $\mathbf{L_0} = 1$

Time of arrival of a signal

Introduce orthonormal tetrads at $\mathcal{O}$ and $\mathcal{E}$





Time of arrival

Position of the receiver + position and time of emission

$$\mathbf{X^0} \equiv \mathbf{X^0}\left(\mathbf{X^1}, \ldots, \mathbf{X^7}\right)$$

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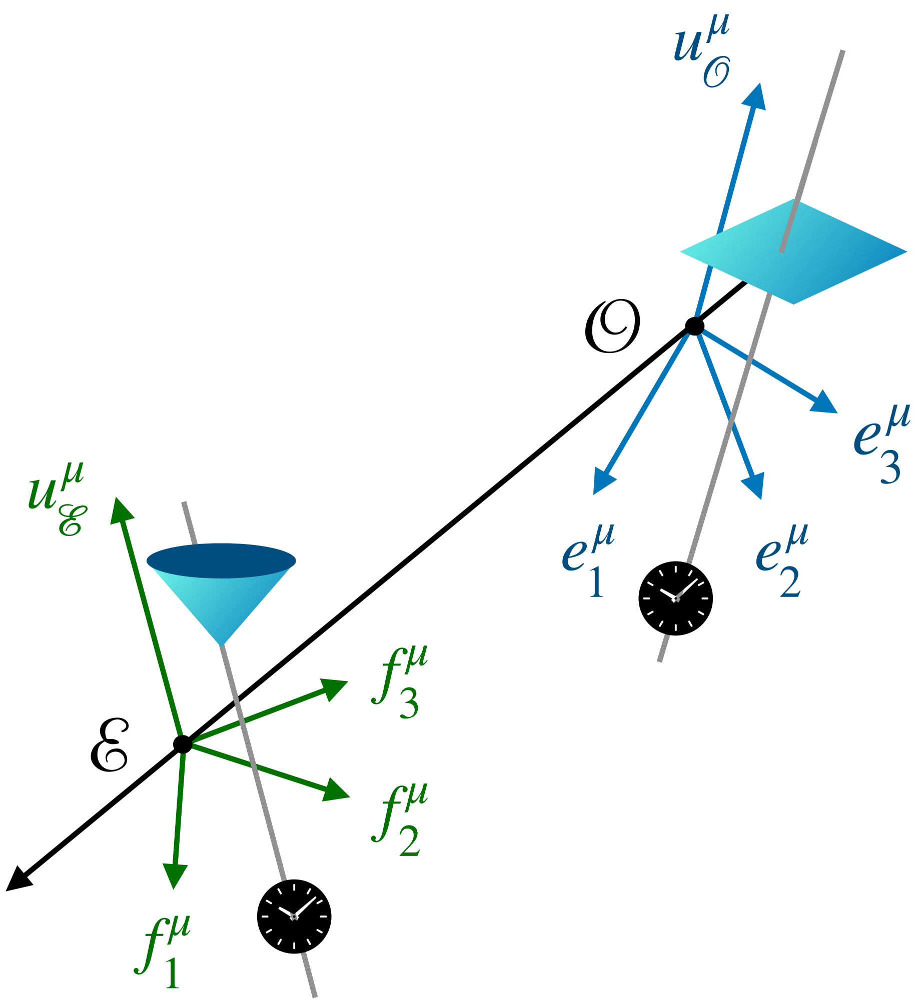

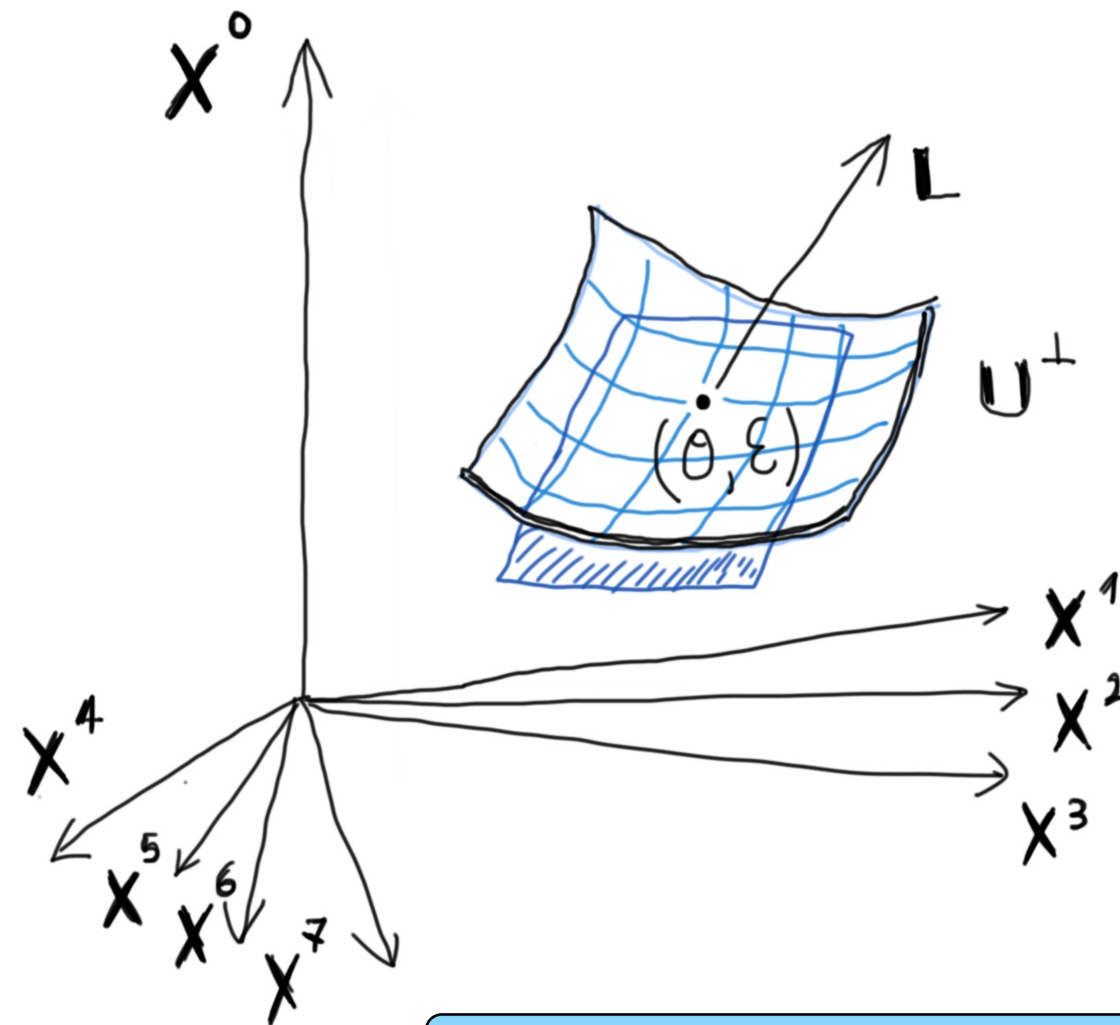

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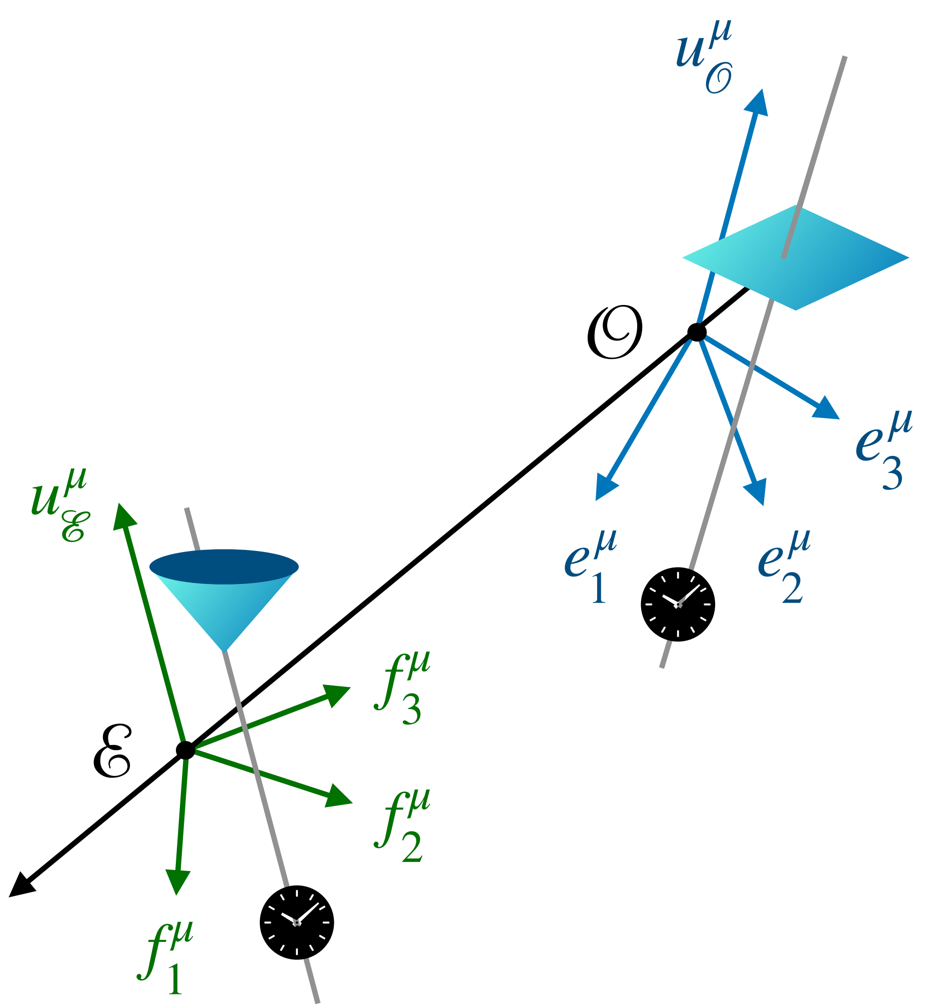
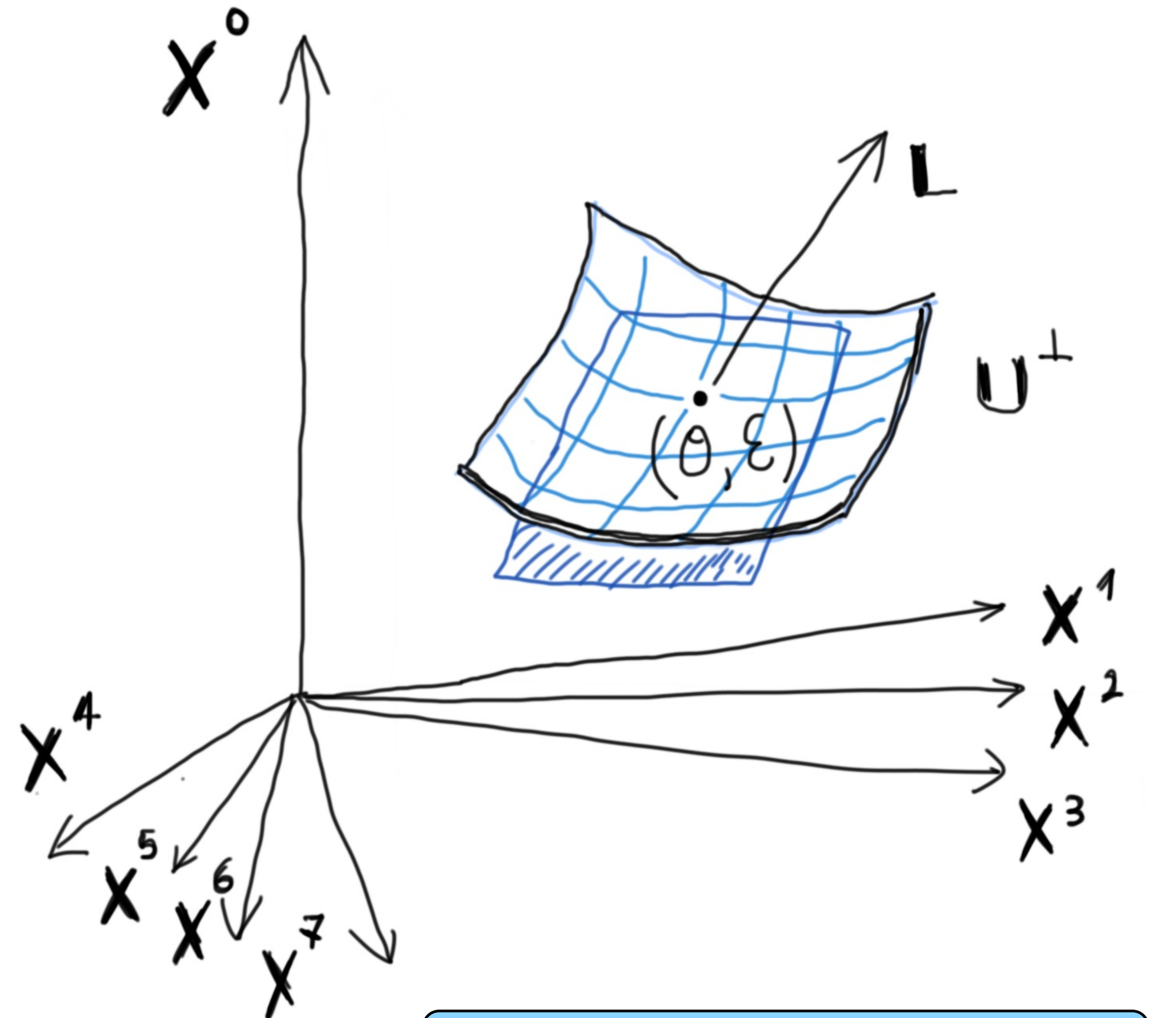
$$\mathbf{X^0} = -\mathbf{L_i}\,\mathbf{X^i} - \frac{1}{2}\mathbf{Q_{ij}}\,\mathbf{X^i}\,\mathbf{X^j} + O\left((\mathbf{X^i})^3\right)$$

Time of arrival of a signal

Introduce orthonormal tetrads at $\mathscr{O}$ and $\mathscr{E}$





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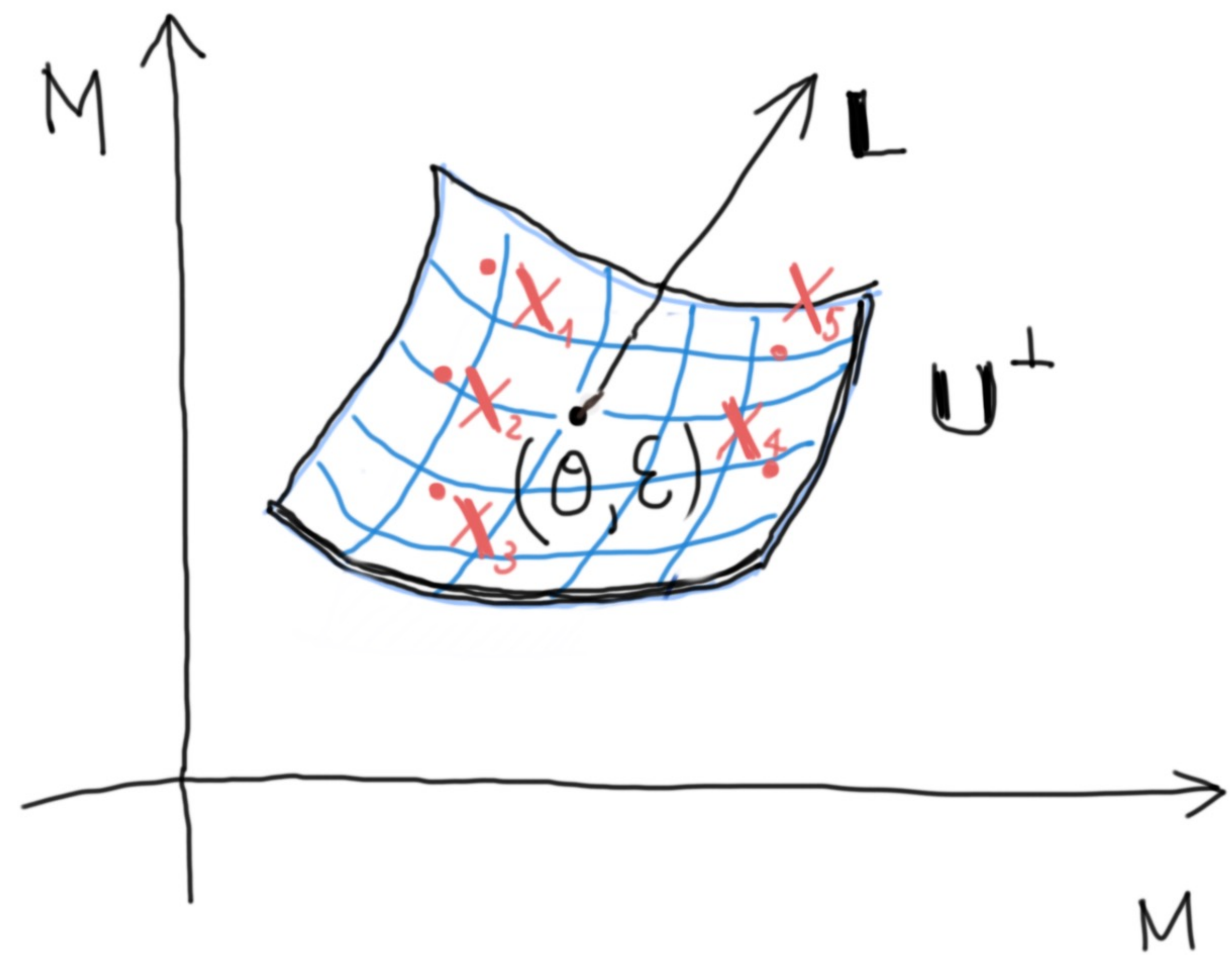
extrinsic curvature $\mathbf{U}^\perp$

Inverse problem

Assume we have a sample of TOA's (= pairs of points)

Can we reconstruct $\mathbf{L}$ and $\mathbf{U}^{\perp}$?



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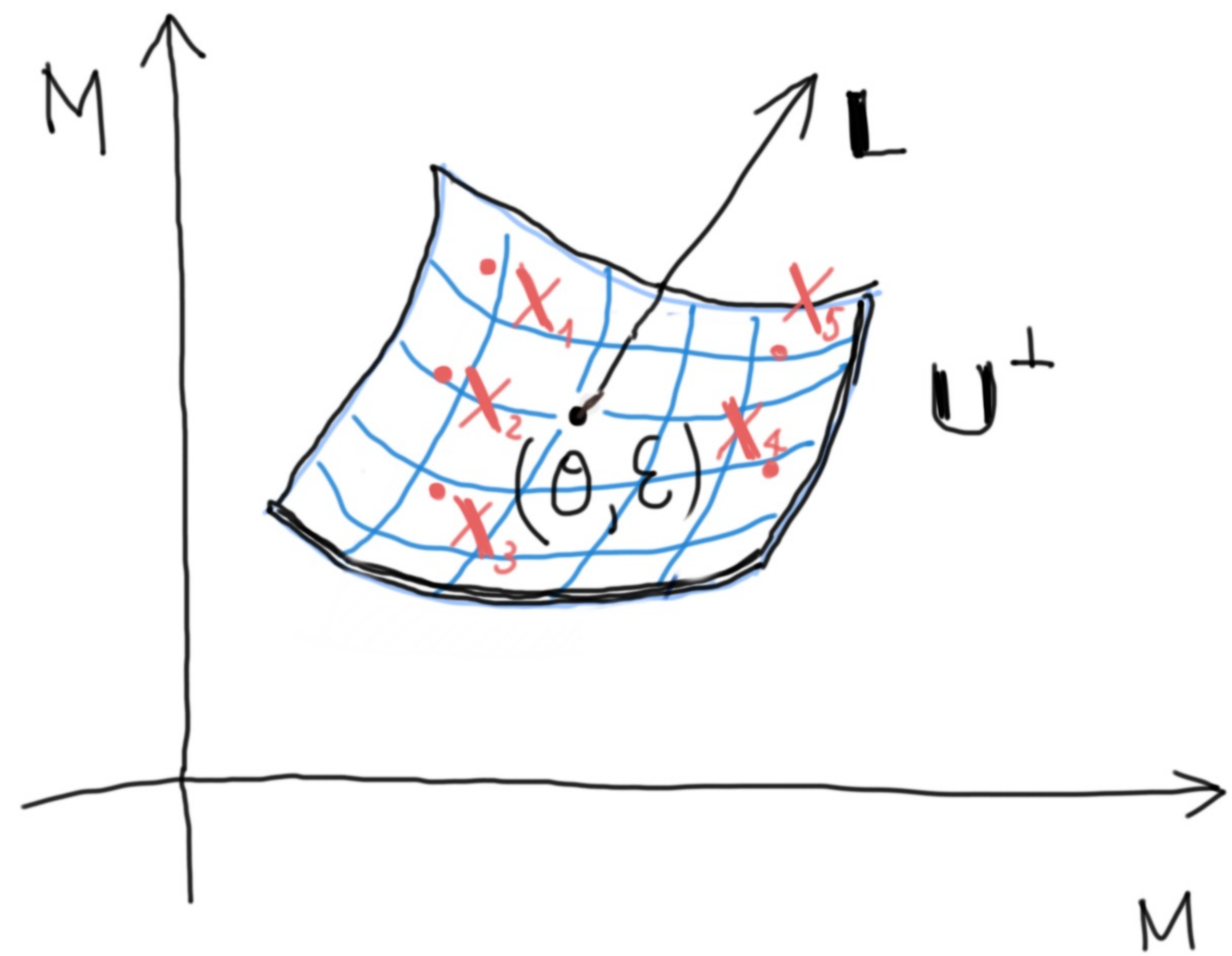
known



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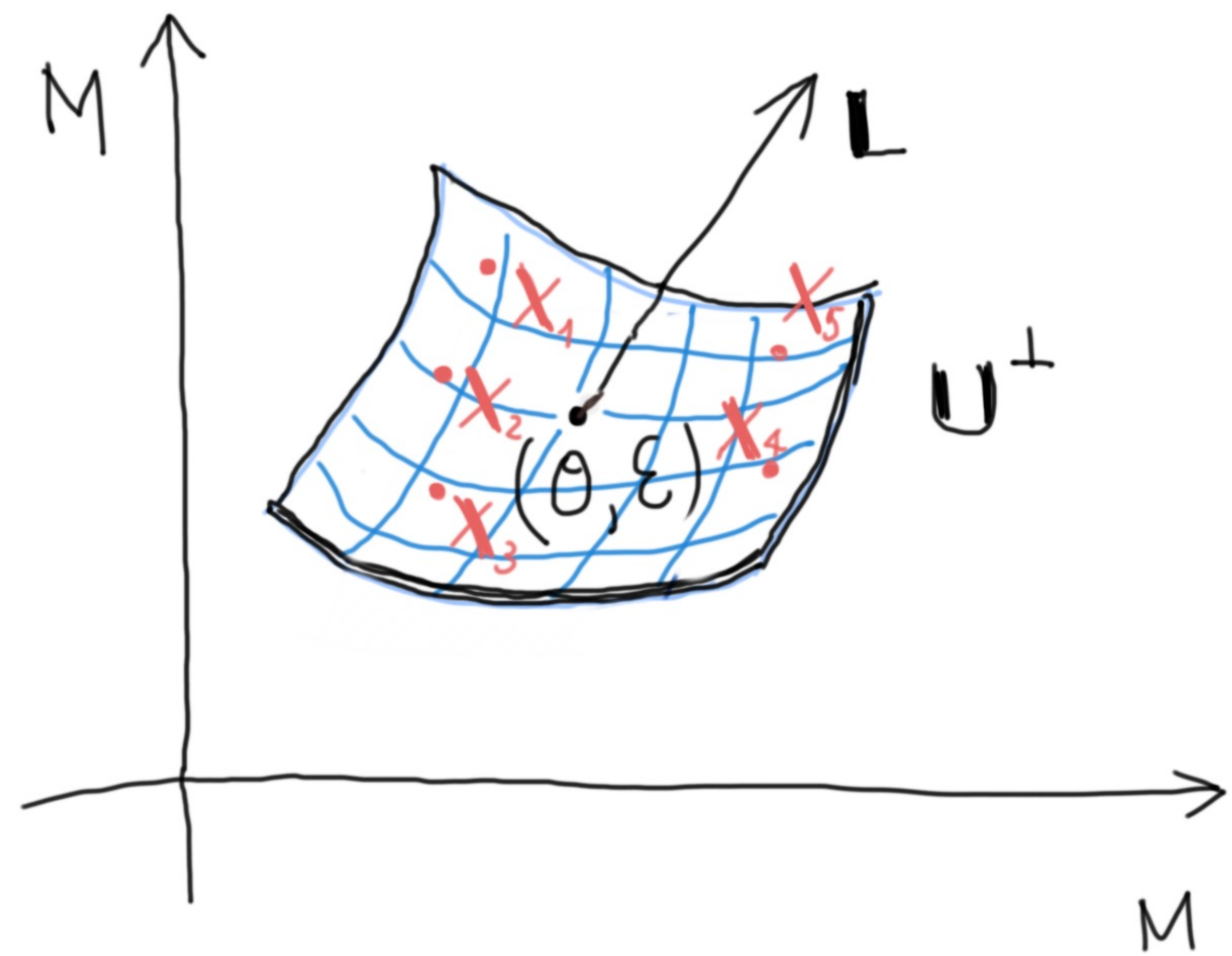
$$\mathbf{X}^{\mathbf{0}}_2 = -\mathbf{L}_{\mathbf{i}}\,\mathbf{X}^{\mathbf{i}}_2 - \frac{1}{2}\mathbf{Q}_{\mathbf{ij}}\,\mathbf{X}^{\mathbf{i}}{}_2\,\mathbf{X}^{\mathbf{j}}{}_2$$

$$\vdots$$



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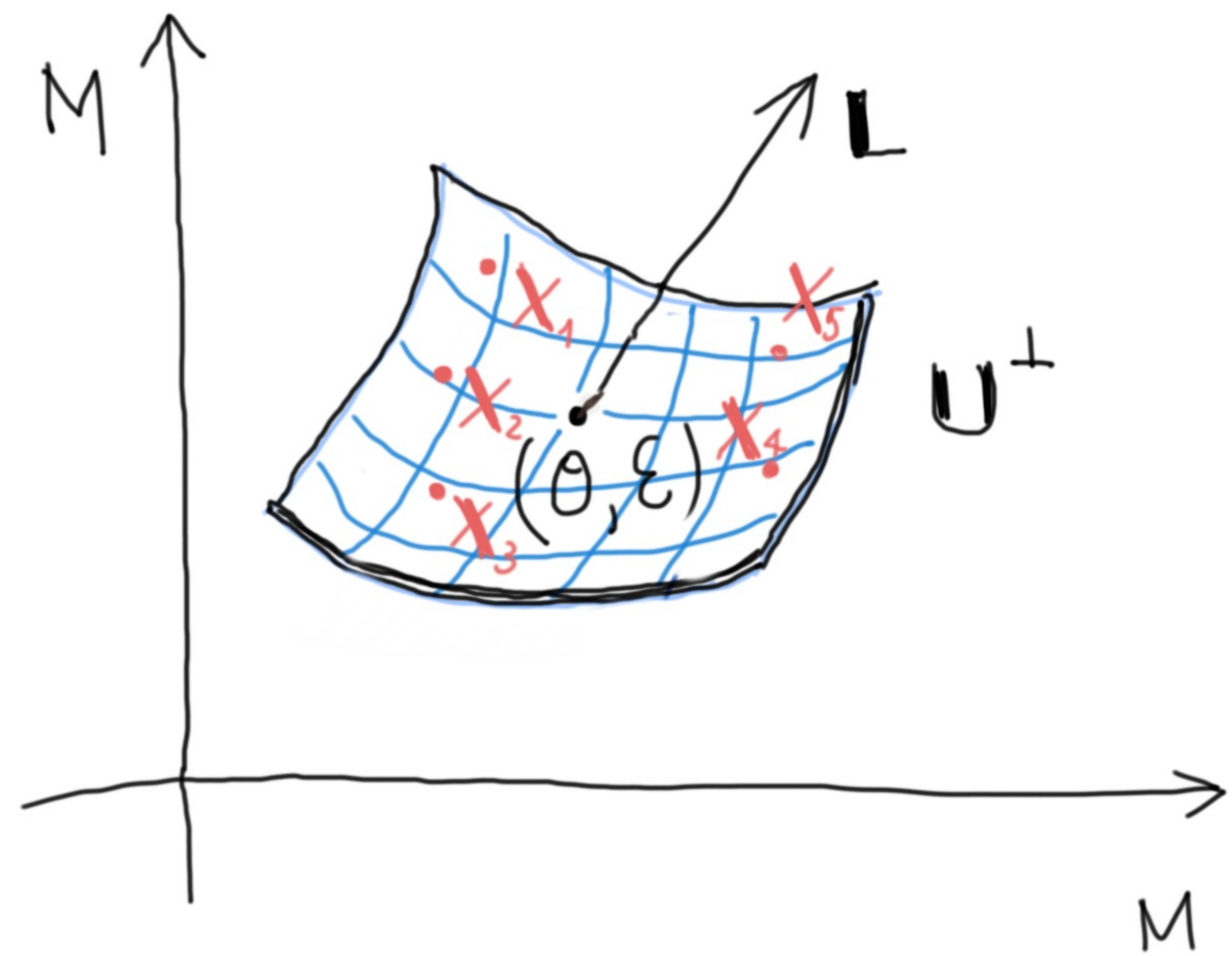
$$\vdots$$

$\mathbf{L_i}, \mathbf{Q_{ij}}$

solved for



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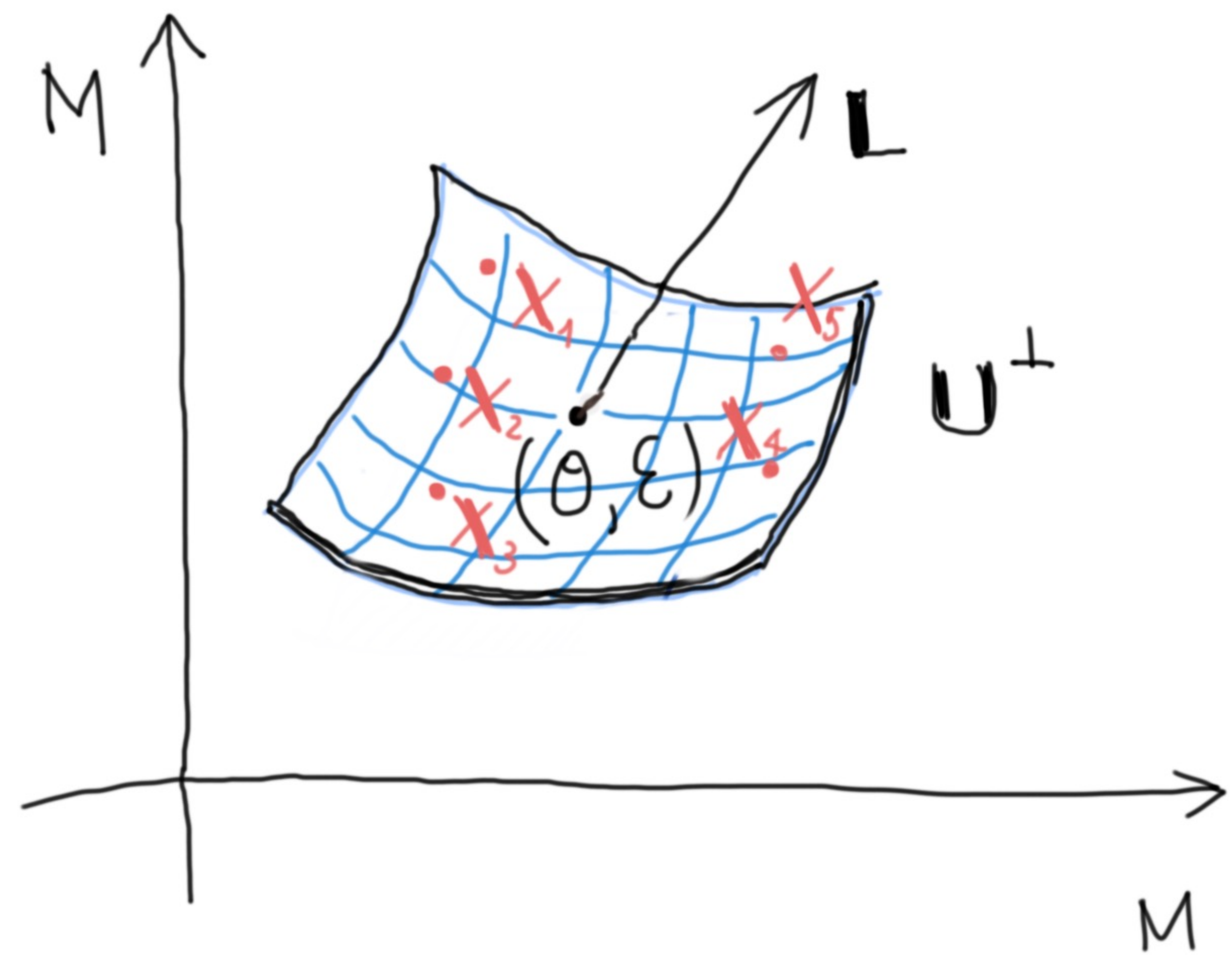

Finding a unique quadric through a set of points

Inverse problem

Assume we have a sample of TOA's (= pairs of points)

Can we reconstruct $\mathbf{L}$ and $\mathbf{U}^{\perp}$?

$$\mathbf{X}_k = \begin{pmatrix} x^{\mu}_{obs,k} \\ x^{\mu}_{em,k} \end{pmatrix}$$

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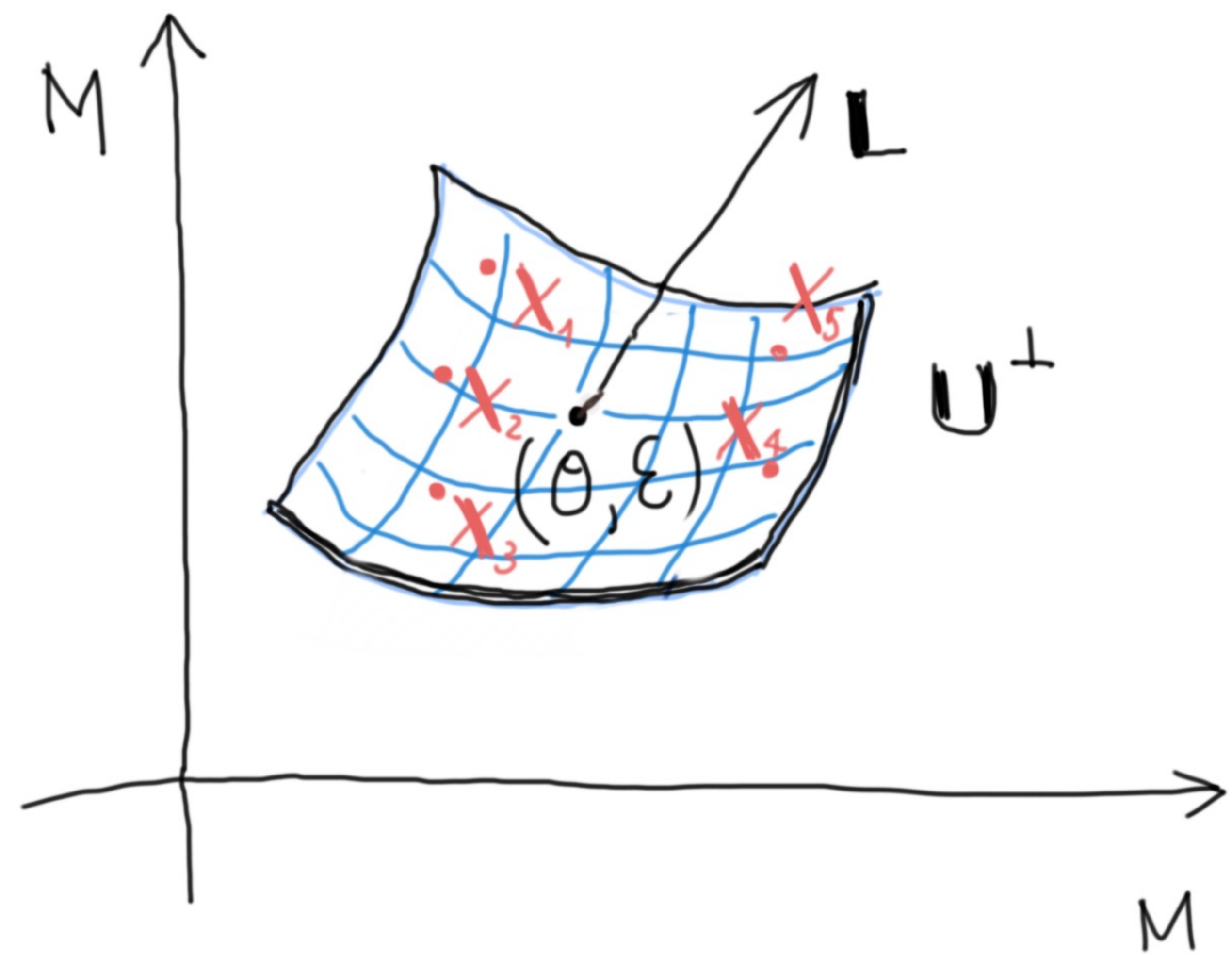
$$\vdots$$

$\mathbf{L}_i, \mathbf{Q}_{ij}$ solved for



Finding a unique quadric through a set of points

Possible if the sampling done right

Inverse problem

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Can we reconstruct $\mathbf{L}$ and $\mathbf{U}^{\perp}$?

$$\mathbf{X}_k = \begin{pmatrix} x^{\mu}_{obs,k} \\ x^{\mu}_{em,k} \end{pmatrix}$$ known

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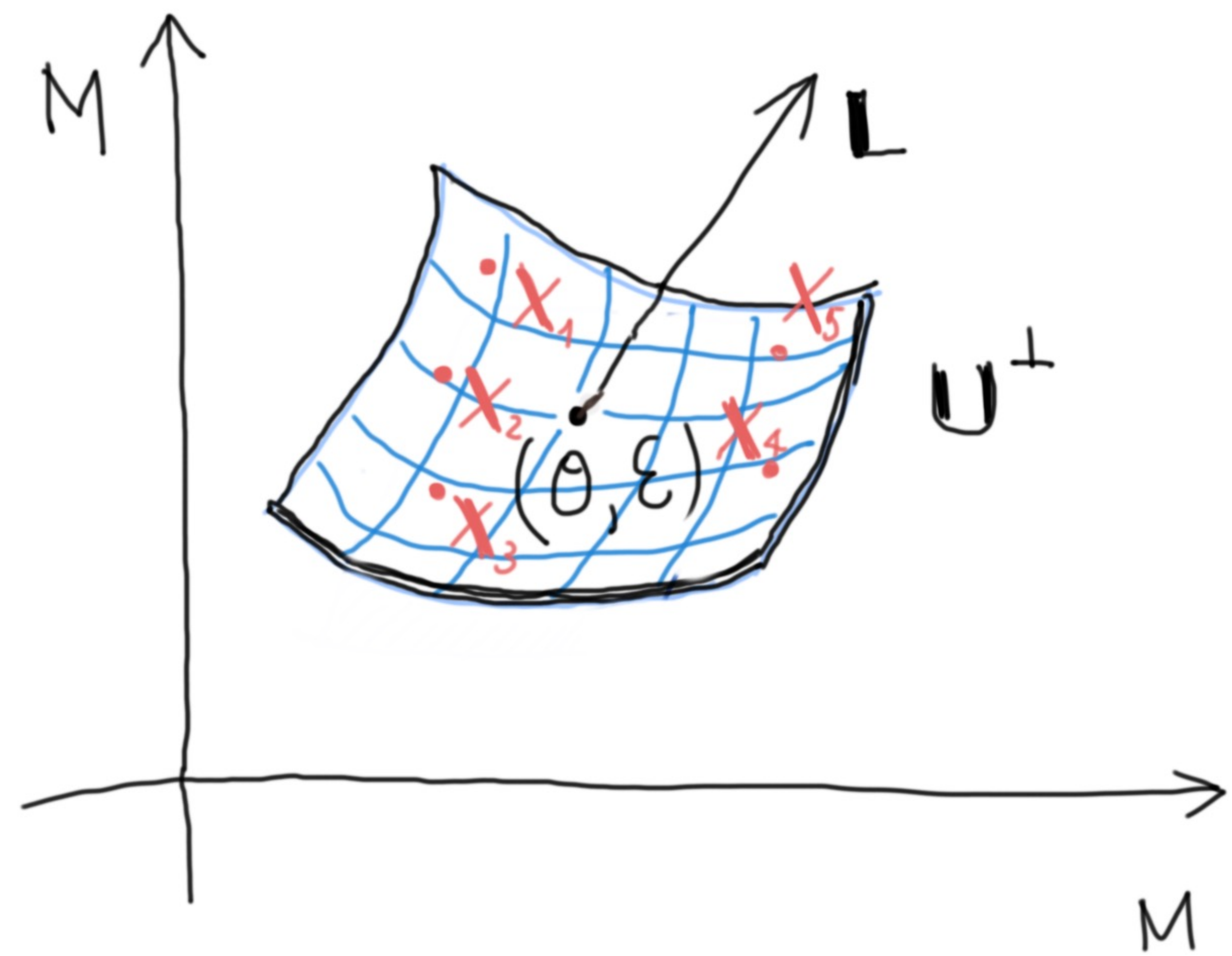
$$\vdots$$

$\mathbf{L_i}, \mathbf{Q_{ij}}$ solved for



Finding a unique quadric through a set of points

Possible if the sampling done right

Determining the shape of the LSC from variations of TOA's

Curvature measurement

Flat spacetime

$$\mathbf{U}^{\perp} = D_{\mathcal{O}}^{-1}\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{pmatrix}$$

diagonalising basis

Curved spacetime

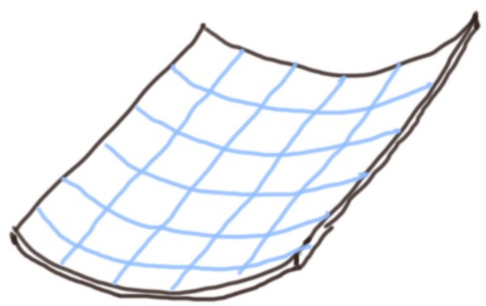
$$\mathbf{U}^{\perp} = \begin{pmatrix}*&*&*&*&*&0&0\\*&*&*&*&*&0&0\\*&*&*&*&*&0&0\\*&*&*&*&*&0&0\\*&*&*&*&*&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{pmatrix}$$

Curvature measurement

Flat spacetime

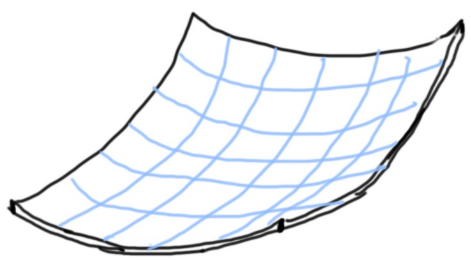
$$\mathbf{U}^{\perp} = D_{\mathcal{O}}^{-1}\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{pmatrix}$$

diagonalising basis



Curved spacetime

$$\mathbf{U}^{\perp} = \begin{pmatrix}*&*&*&*&*&0&0\\ *&*&*&*&*&0&0\\ *&*&*&*&*&0&0\\ *&*&*&*&*&0&0\\ *&*&*&*&*&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{pmatrix}$$

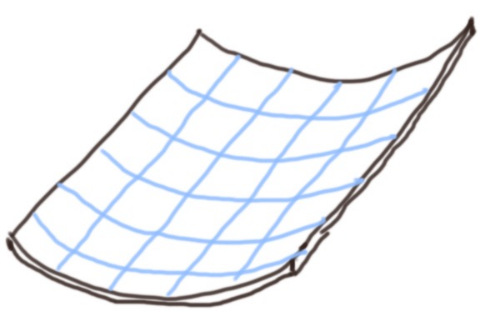


Curvature measurement

Flat spacetime

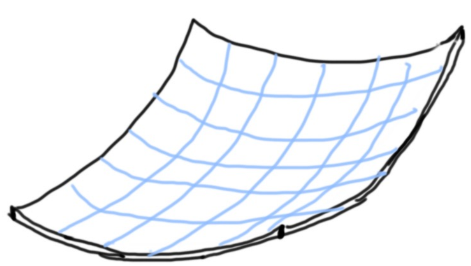
$$\mathbf{U}^{\perp} = D_{\mathcal{O}}^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

diagonalising basis



Curved spacetime

$$\mathbf{U}^{\perp} = \begin{pmatrix} * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



Curvature causes a change of shape of LSC

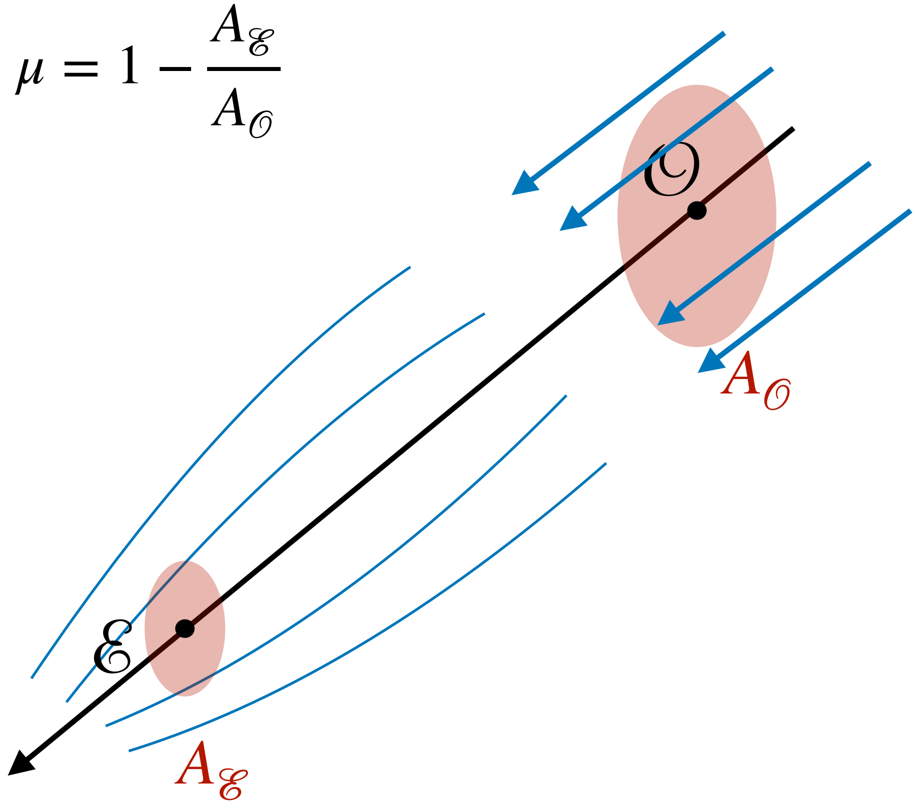
Need to detect the imprint of curvature in the extrinsic curvature

Problem: we have no control over the orthonormal tetrads systems at $\mathcal{O}$ and $\mathcal{E}$

Look for 2-sided Lorentz invariants

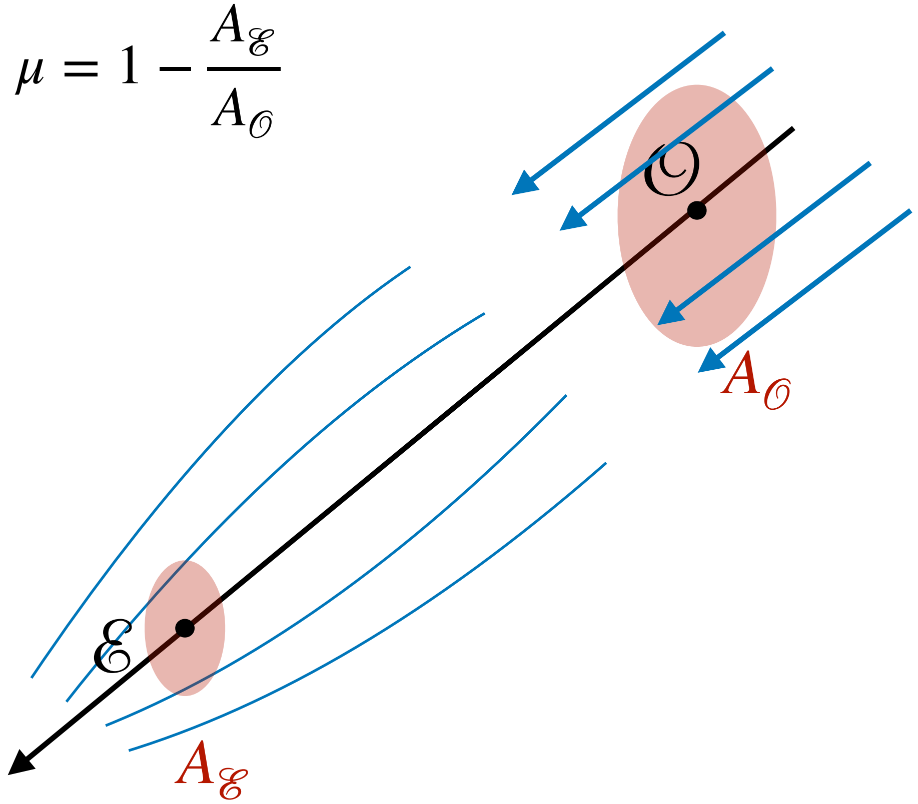
Distance slip

$$\mu = 1 - \frac{A_{\mathscr{E}}}{A_{\mathscr{O}}}$$



Distance slip

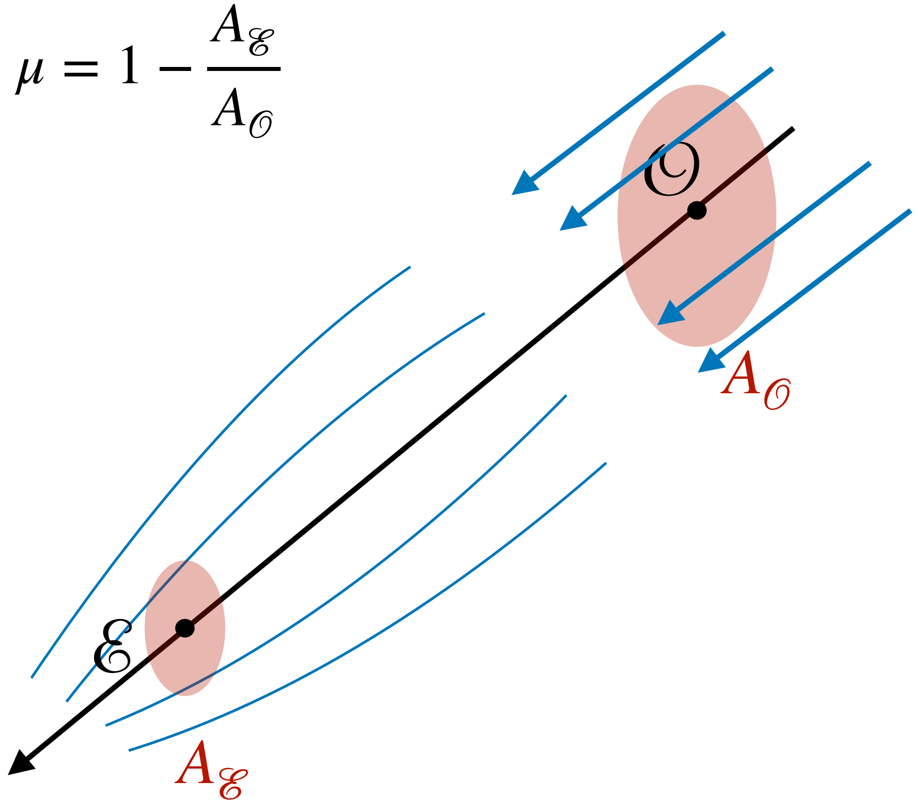
$$\mu = 1 - \frac{A_{\mathscr{E}}}{A_{\mathscr{O}}}$$



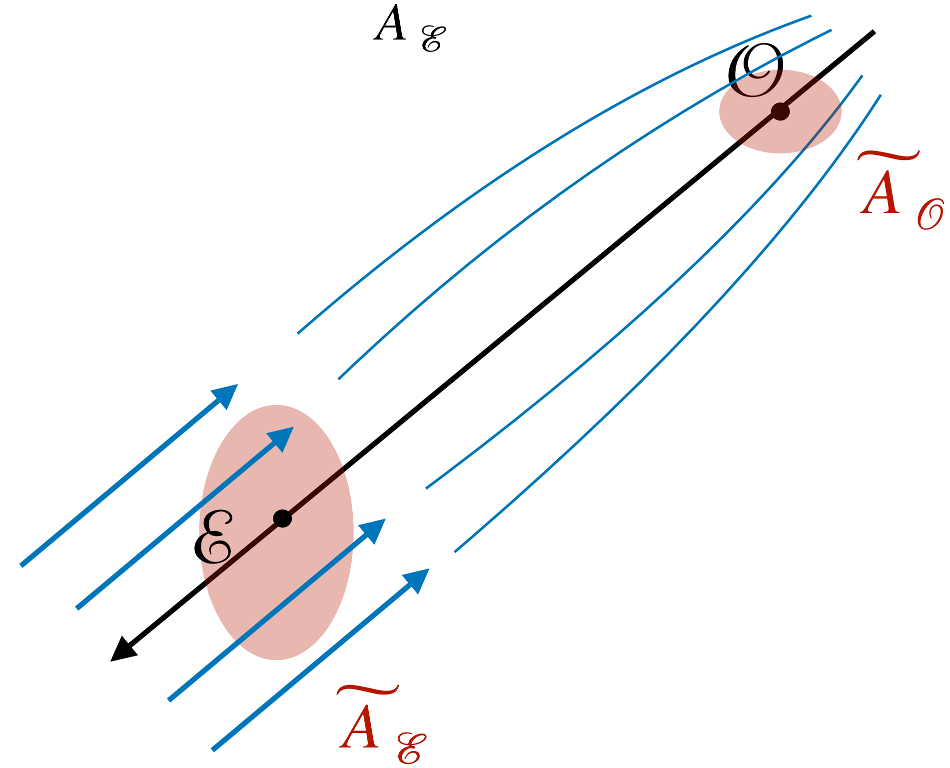
- Independent of the states of motion
- Vanish in a flat spacetime
- For short distances given by an integral of the stress-energy tensor

Distance slip

$$\mu = 1 - \frac{A_{\mathscr{E}}}{A_{\mathscr{O}}}$$



$$\nu = 1 - \frac{\widetilde{A}_{\mathscr{O}}}{\widetilde{A}_{\mathscr{E}}}$$



- Independent of the states of motion
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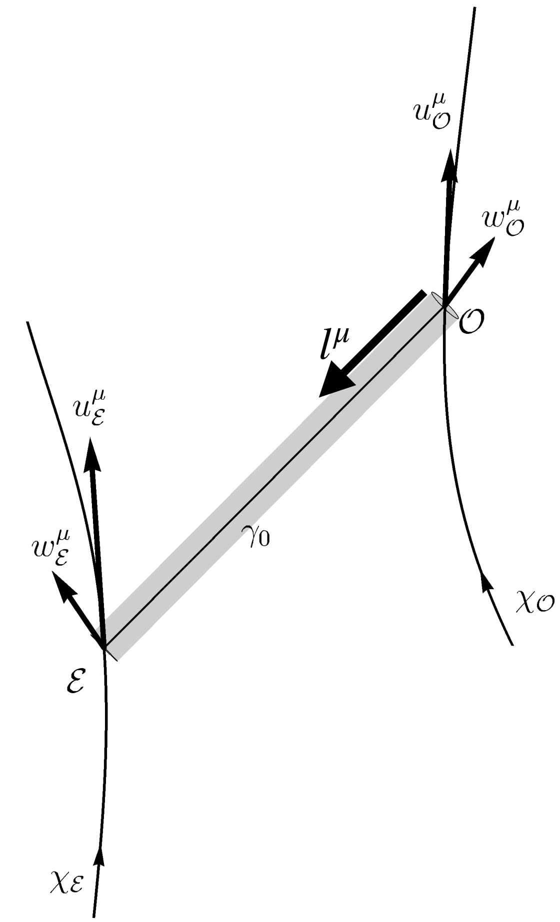
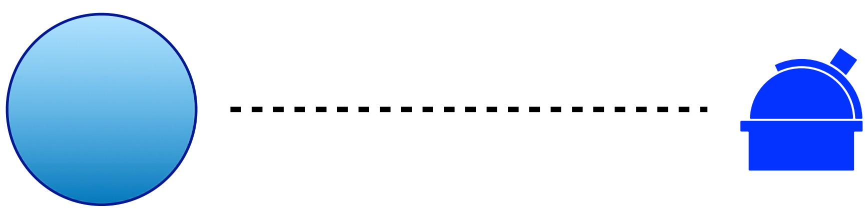
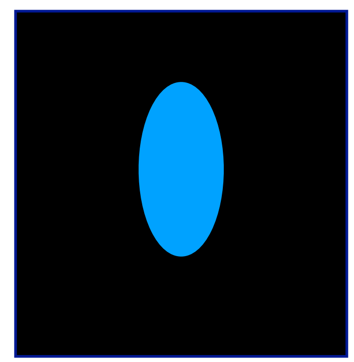
Distance slip

Distance measures in astrometry Grasso, Korzyński, Serbenta 2019



- Angular diameter distance (area distance) [Perlick 2004]





$$D_{ang} = \left(\frac{A_\mathscr{E}}{\Omega_\mathscr{O}}\right)^{1/2}$$

$$D_{ang} \equiv D_{ang}(u^\mu_\mathscr{O}, R^\mu{}_{\alpha\beta\nu})$$

- Luminosity distance

$$D_{lum} = \left(\frac{I}{4\pi F}\right)^{1/2}$$

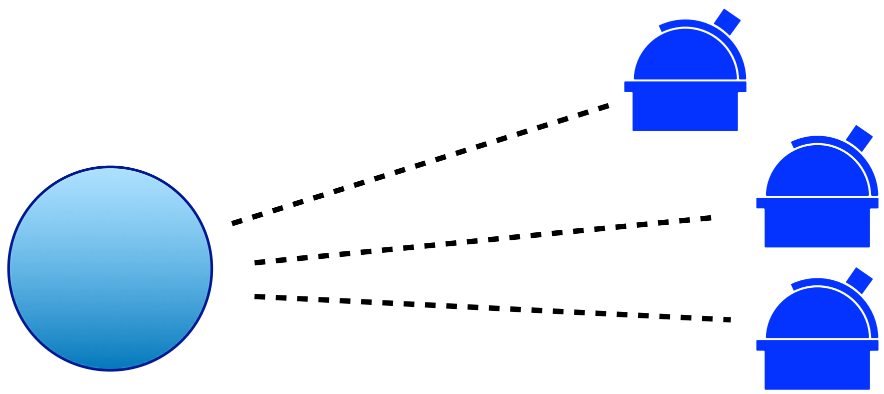
$$D_{lum} \equiv D_{lum}(u^\mu_\mathscr{O}, u^\mu_\mathscr{E}, R^\mu{}_{\alpha\beta\nu})$$

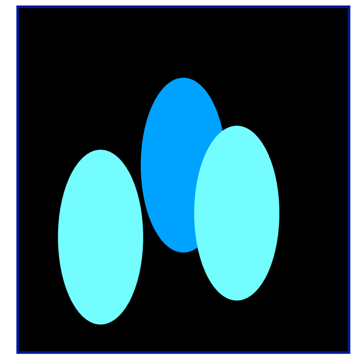
Etherington's reciprocity relation [Etherington 1933]

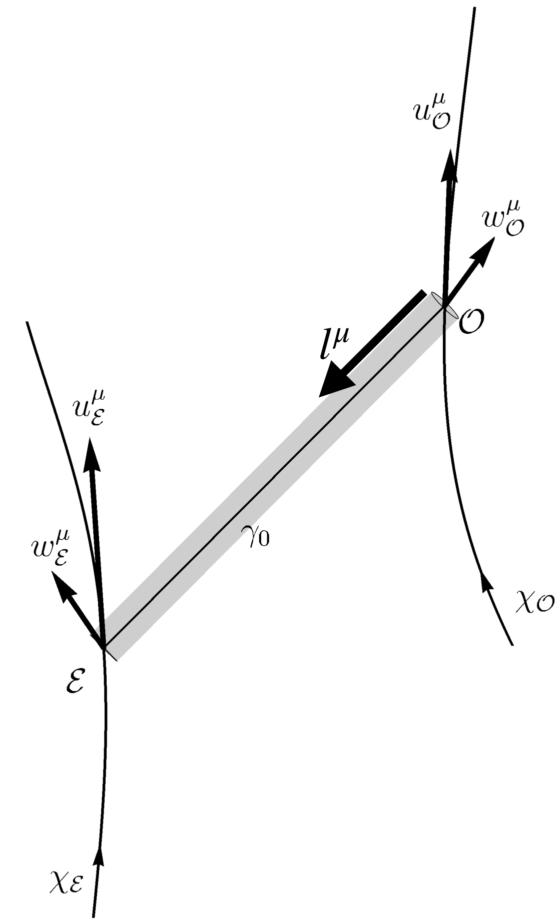
$$D_{lum} = (1+z)^2\, D_{ang}$$

Distance slip

Parallax distance







flat spacetime $\delta\theta^A = -\, D_{\mathcal{O}}^{-1}\, \delta x^A_{\mathcal{O}}$

general spacetime $\delta\theta^A = -\, \Pi^A{}_B\, \delta x^B_{\mathcal{O}}$

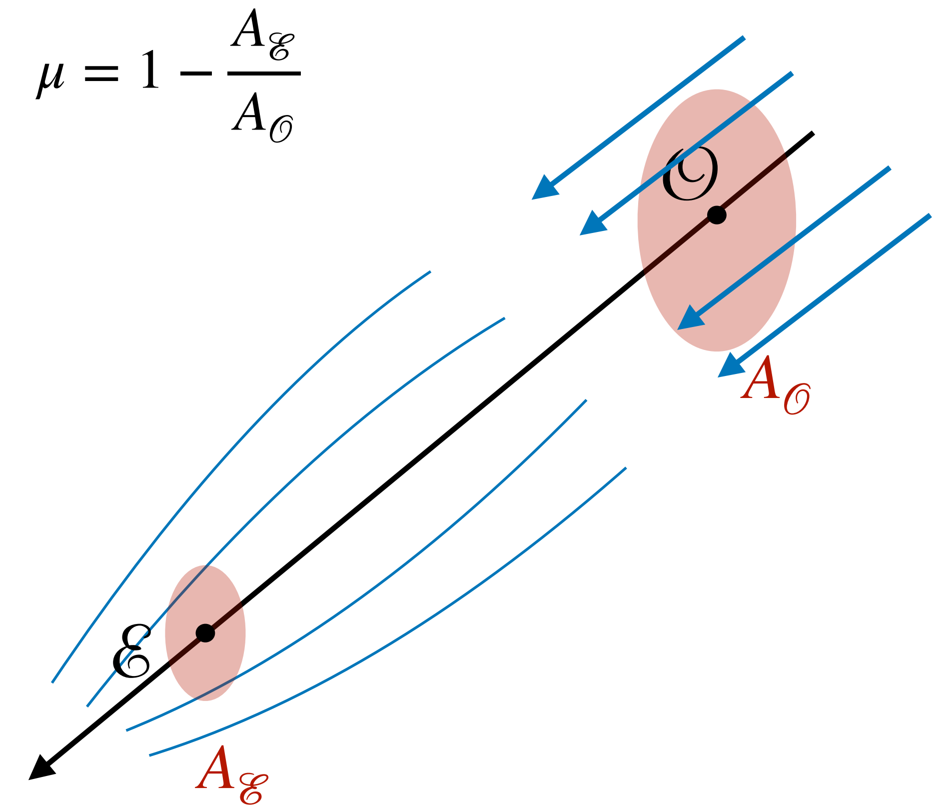
baseline-averaged parallax distance:

$$D_{par} = \left|\det \Pi^A{}_B\right|^{-1/2} = \left(\frac{A_{\mathcal{O}}}{\Omega}\right)^{1/2}$$

$$D_{par} \equiv D_{par}(u^\mu_{\mathcal{O}}, R^\alpha{}_{\mu\nu\beta})$$

Distance slip

$$\mu = 1 - \sigma \frac{D^2_{ang}}{D^2_{par}} = 1 - \sigma \frac{D^2_{lum}}{D^2_{par}}(1+z)^{-4}$$



$\sigma = \pm 1$ (almost always +1)

$\mu \equiv \mu(R^{\mu}{}_{\alpha\beta\nu})$

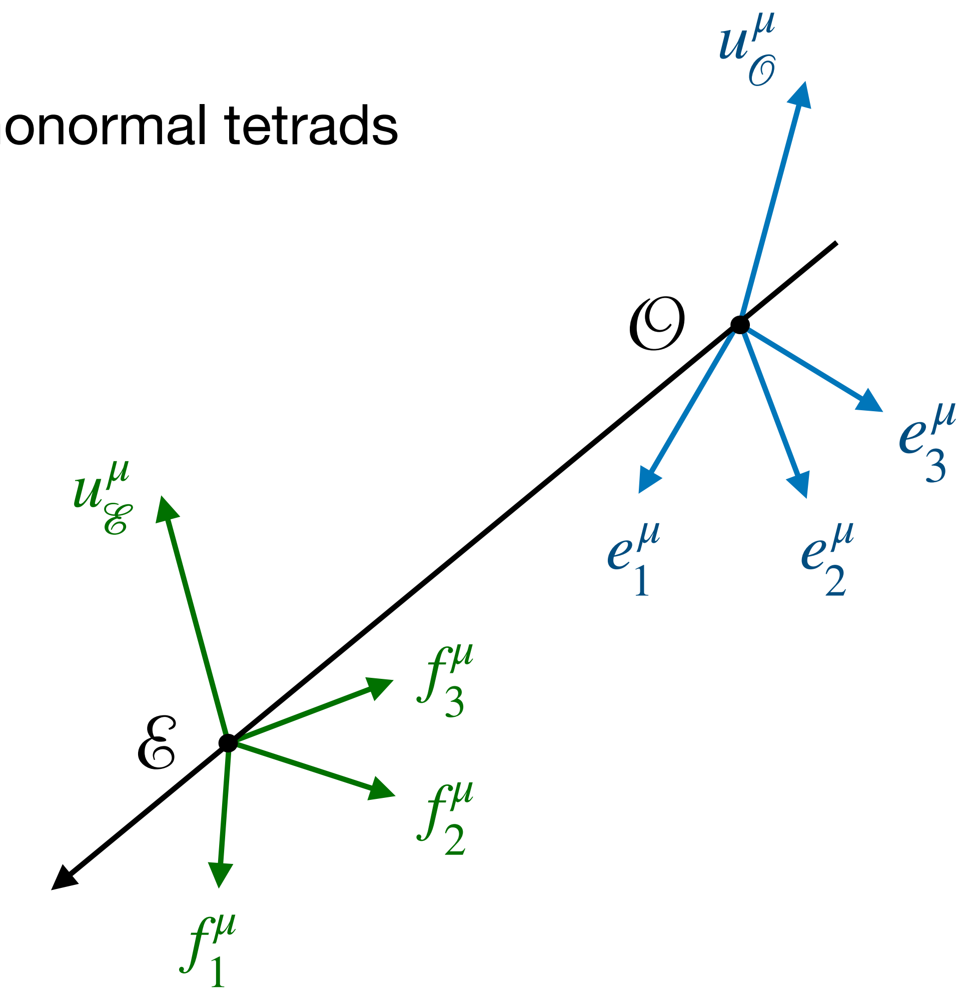
- Measurable as the relative difference between distance measurements to a single object (very small effect though)
- Independent of the states of motion
- Vanishes in a flat spacetime
- For short distances given by an integral of the stress-energy tensor

Probing curvature

- Lorentz-invariant expressions, work for any pair of oriented, orthonormal tetrads

$$\mu = 1 - \frac{Q_{\mathcal{O}\mathcal{O}\,i'j'}\,Q_{\mathcal{O}\mathcal{O}\,k'l'}\,\epsilon_{\mathcal{O}}^{i'k'}\,\epsilon_{\mathcal{O}}^{j'l'}}{Q_{\mathcal{O}\mathcal{E}\,i'j}\,Q_{\mathcal{O}\mathcal{E}\,k'l}\,\epsilon_{\mathcal{O}}^{i'k'}\,\epsilon_{\mathcal{E}}^{jl}} \qquad \epsilon_{\mathcal{O}\,i'j'} = \frac{l_{\mathcal{O}}^{k'}}{l_{\mathcal{O}}^{0'}}\,\varepsilon_{i'j'k'}$$

$$\nu = 1 - \frac{Q_{\mathcal{E}\mathcal{E}\,ij}\,Q_{\mathcal{E}\mathcal{E}\,kl}\,\epsilon_{\mathcal{E}}^{ik}\,\epsilon_{\mathcal{E}}^{jl}}{Q_{\mathcal{O}\mathcal{E}\,i'j}\,Q_{\mathcal{O}\mathcal{E}\,k'l}\,\epsilon_{\mathcal{O}}^{i'k}\,\epsilon_{\mathcal{E}}^{jl}} \qquad \epsilon_{\mathcal{E}\,ij} = \frac{l_{\mathcal{E}}^{k}}{l_{\mathcal{E}}^{0}}\,\varepsilon_{ijk}$$



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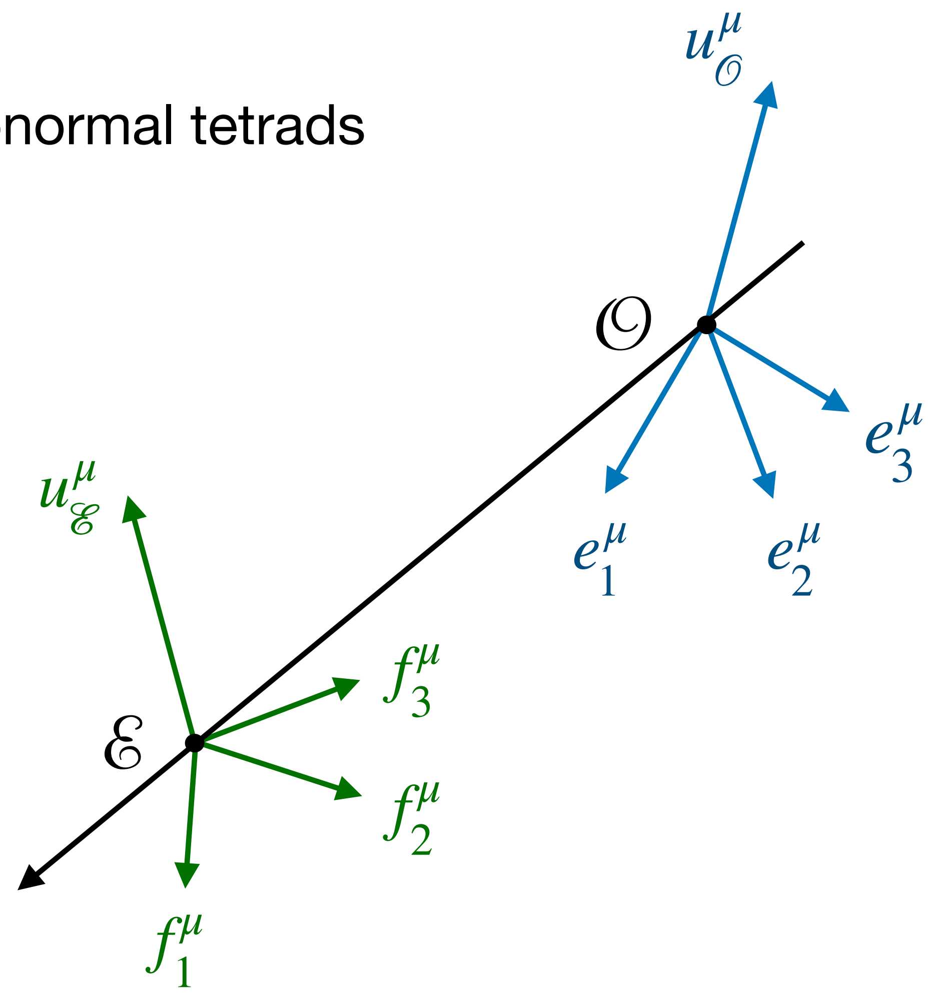


$$\mu \equiv \mu\left(\mathbf{U}^{\perp}, \mathbf{L}, (u_{\mathcal{O}}, e_i), (u_{\mathcal{E}}, f_i)\right)$$

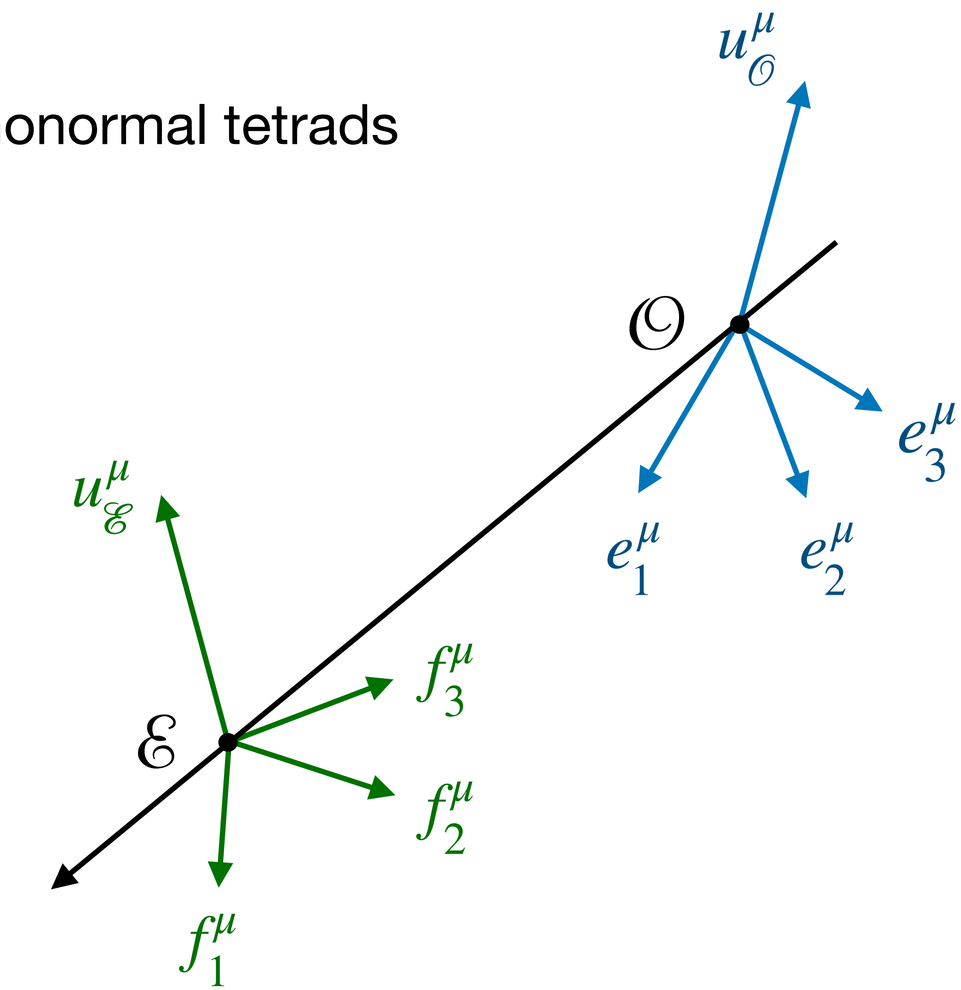
$$\nu \equiv \nu\left(\mathbf{U}^{\perp}, \mathbf{L}, (u_{\mathcal{O}}, e_i), (u_{\mathcal{E}}, f_i)\right)$$

Probing curvature

- Lorentz-invariant expressions, work for any pair of oriented, orthonormal tetrads

$$\mu = 1 - \frac{Q_{\mathcal{O}\mathcal{O}\,i'j'}\,Q_{\mathcal{O}\mathcal{O}\,k'l'}\,\epsilon_{\mathcal{O}}^{i'k'}\,\epsilon_{\mathcal{O}}^{j'l'}}{Q_{\mathcal{O}\mathcal{E}\,i'j}\,Q_{\mathcal{O}\mathcal{E}\,k'l}\,\epsilon_{\mathcal{O}}^{i'k'}\,\epsilon_{\mathcal{E}}^{jl}} \qquad \epsilon_{\mathcal{O}\,i'j'} = \frac{l_{\mathcal{O}}^{k'}}{l_{\mathcal{O}}^{0'}}\,\varepsilon_{i'j'k'}$$

$$\nu = 1 - \frac{Q_{\mathcal{E}\mathcal{E}\,ij}\,Q_{\mathcal{E}\mathcal{E}\,kl}\,\epsilon_{\mathcal{E}}^{ik}\,\epsilon_{\mathcal{E}}^{jl}}{Q_{\mathcal{O}\mathcal{E}\,i'j}\,Q_{\mathcal{O}\mathcal{E}\,k'l}\,\epsilon_{\mathcal{O}}^{i'k}\,\epsilon_{\mathcal{E}}^{jl}} \qquad \epsilon_{\mathcal{E}\,ij} = \frac{l_{\mathcal{E}}^{k}}{l_{\mathcal{E}}^{0}}\,\varepsilon_{ijk}$$



$$\mu \equiv \mu\left(\mathbf{U}^{\perp}, \mathbf{L}, (u_{\mathcal{O}}, e_i), (u_{\mathcal{E}}, f_i)\right)$$

$$\nu \equiv \nu\left(\mathbf{U}^{\perp}, \mathbf{L}, (u_{\mathcal{O}}, e_i), (u_{\mathcal{E}}, f_i)\right)$$

Small curvature limit

Expansion in powers of curvature

$$\mu = \frac{8\pi G}{c^4} \int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} T_{\mu\nu}\, l^\mu\, l^\mu \left(\lambda_{\mathcal{E}} - \lambda\right)\, d\lambda + O(\textbf{Riemann}^2)$$

$$\nu = \frac{8\pi G}{c^4} \int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} T_{\mu\nu}\, l^\mu\, l^\mu \left(\lambda - \lambda_{\mathcal{O}}\right)\, d\lambda + O(\textbf{Riemann}^2)$$

No $C^\mu{}_{\nu\alpha\beta}, \Lambda$

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Pressureless dust, weak gravity

$$\mu = \frac{8\pi G}{c^2} \int_0^D \rho(r)(D - r)\, dr$$

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$$\nu = \frac{8\pi G}{c^2} \int_0^D \rho(r)\, r\, dr$$

Average mass density and COM position of mass distribution along the LOS

$$\langle \rho \rangle \equiv D^{-1} \int_0^D \rho(r)\, dr = \frac{c^2(\mu + \nu)}{8\pi G\, D^2}$$

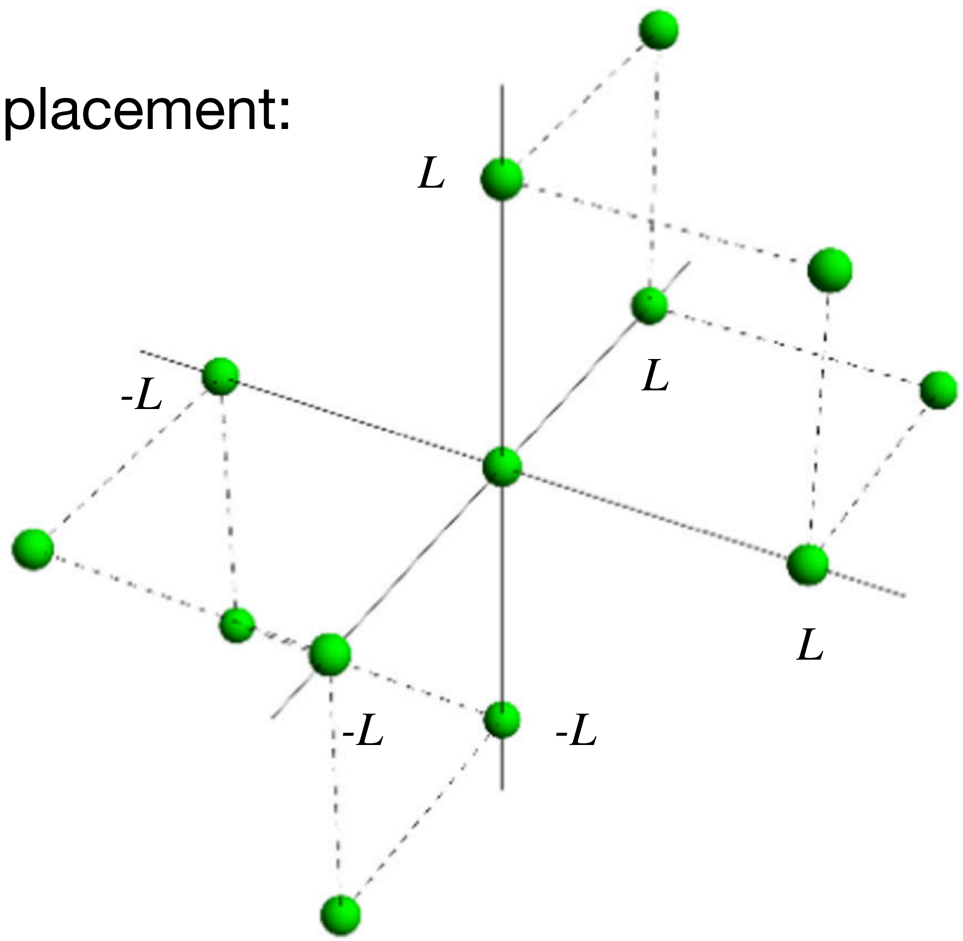
$$r_{CM} \equiv \frac{\int_0^D \rho(r)\, r\, dr}{\int_0^D \rho(r)\, dr} = \frac{D\,\nu}{\mu + \nu}$$

Measurement protocol

Proof of concept

13 clocks in both ensembles

placement:



(with respect to any pair of locally flat,
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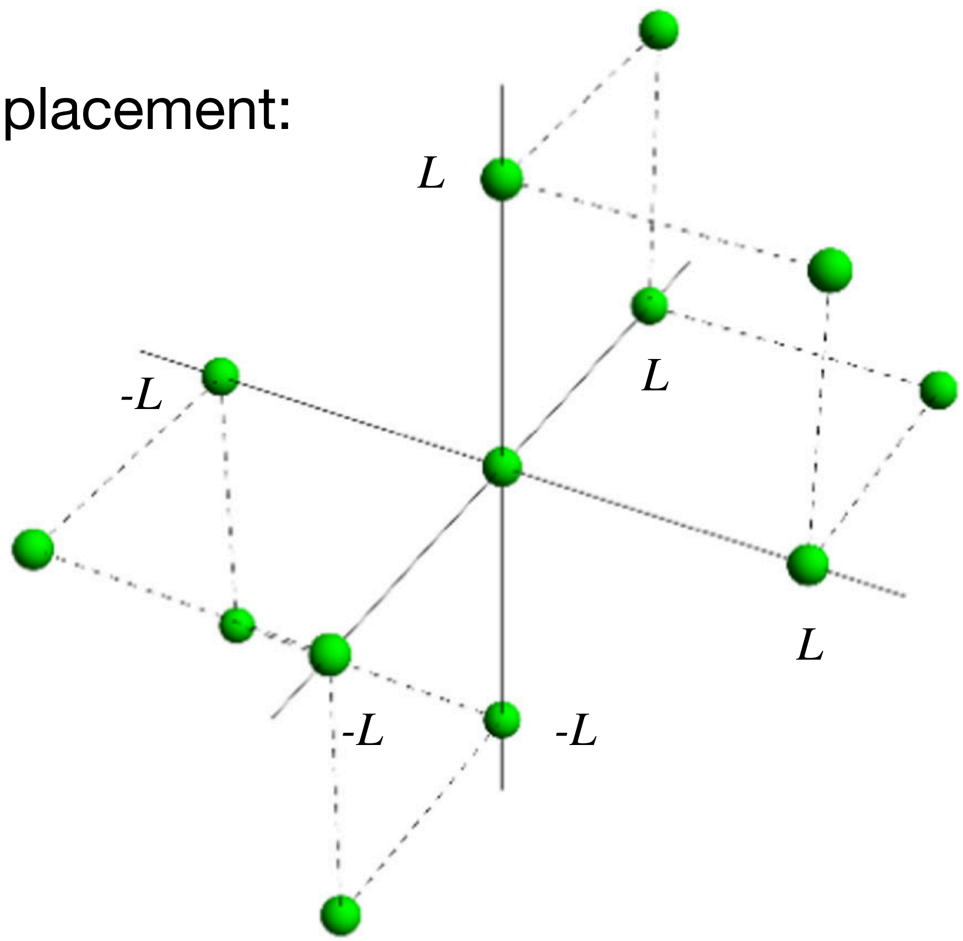


(with respect to any pair of locally flat, orthonormal coordinates near $\mathcal{O}$ and $\mathcal{E}$)

all emitters send 3 signals at

$$t_{\mathcal{E}} = -\frac{L}{c}, 0, \frac{L}{c}$$

We can derive exact expressions

$$\mathbf{L_i} \equiv \mathbf{L_i}(\mathbf{X}^{\mathbf{0}}_{(k)})$$

$$\mathbf{Q_{kl}} \equiv \mathbf{Q_{kl}}(\mathbf{X}^{\mathbf{0}}_{(k)})$$

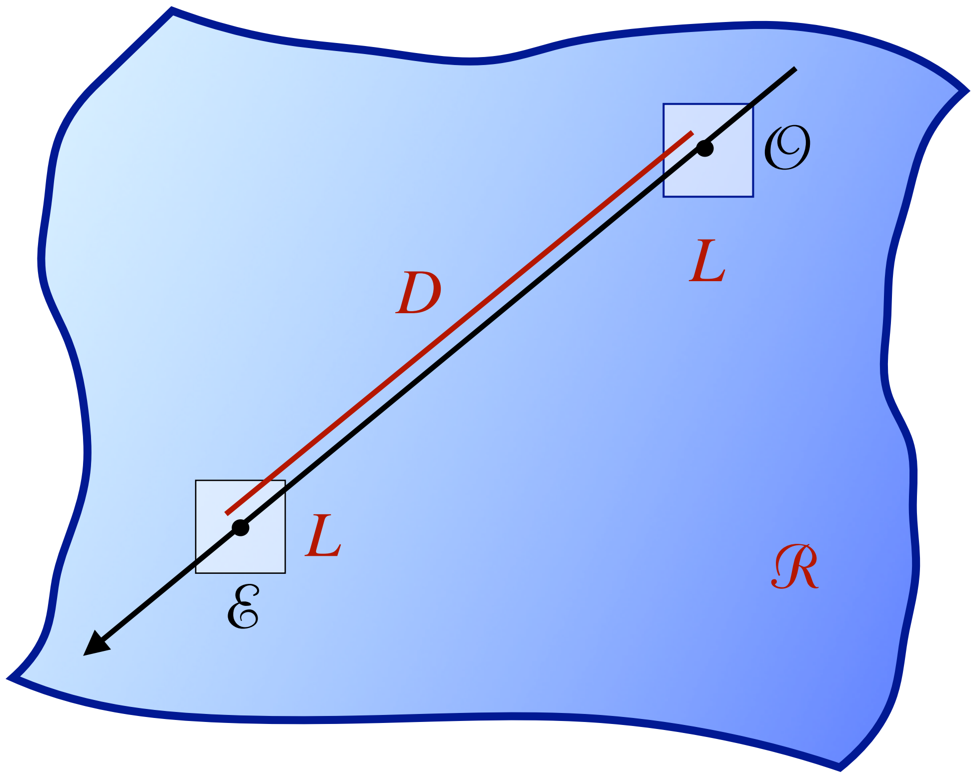
$$\mu \equiv \mu\left(\mathbf{Q_{ij}}, \mathbf{L_j}\right)$$

$$\nu \equiv \nu\left(\mathbf{Q_{ij}}, \mathbf{L_j}\right)$$

Estimating the signal

Assumptions:

$L \ll D \ll \mathscr{R}$



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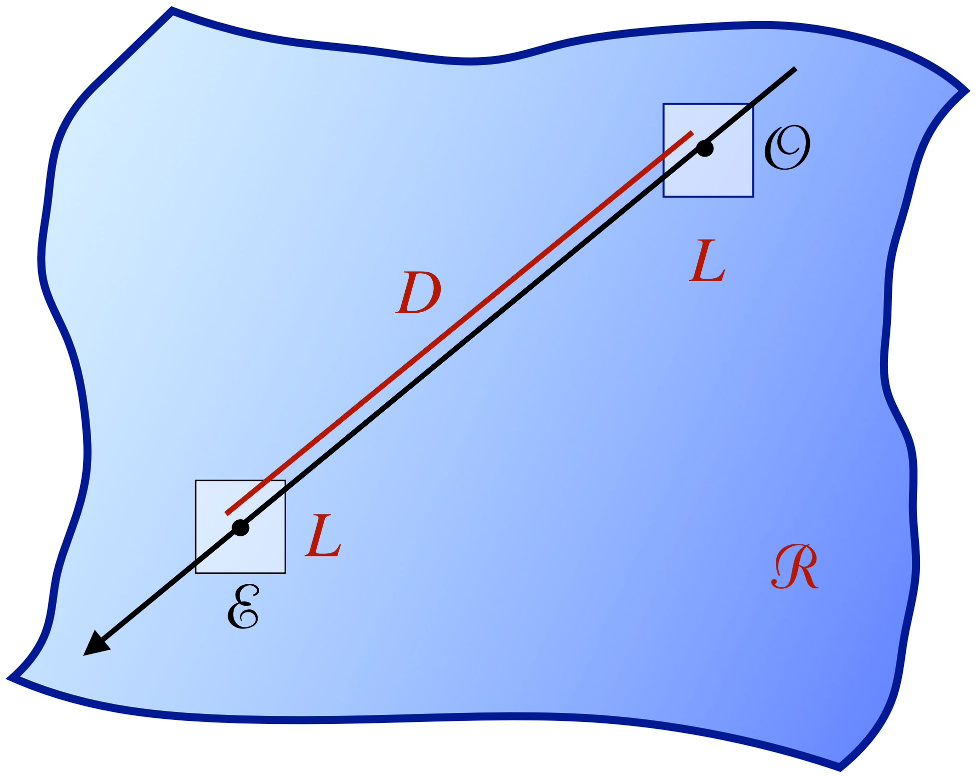

Dimensionless variables:

$$\mathbf{X} = L \cdot \widetilde{\mathbf{X}}$$

$$\lambda = D \cdot \widetilde{\lambda}$$

$$\mathbf{L} = \widetilde{\mathbf{L}}$$

$$R^{\mu}{}_{\nu\rho\sigma} = \mathscr{R}^{-2} \cdot \widetilde{R}^{\mu}{}_{\nu\rho\sigma}$$

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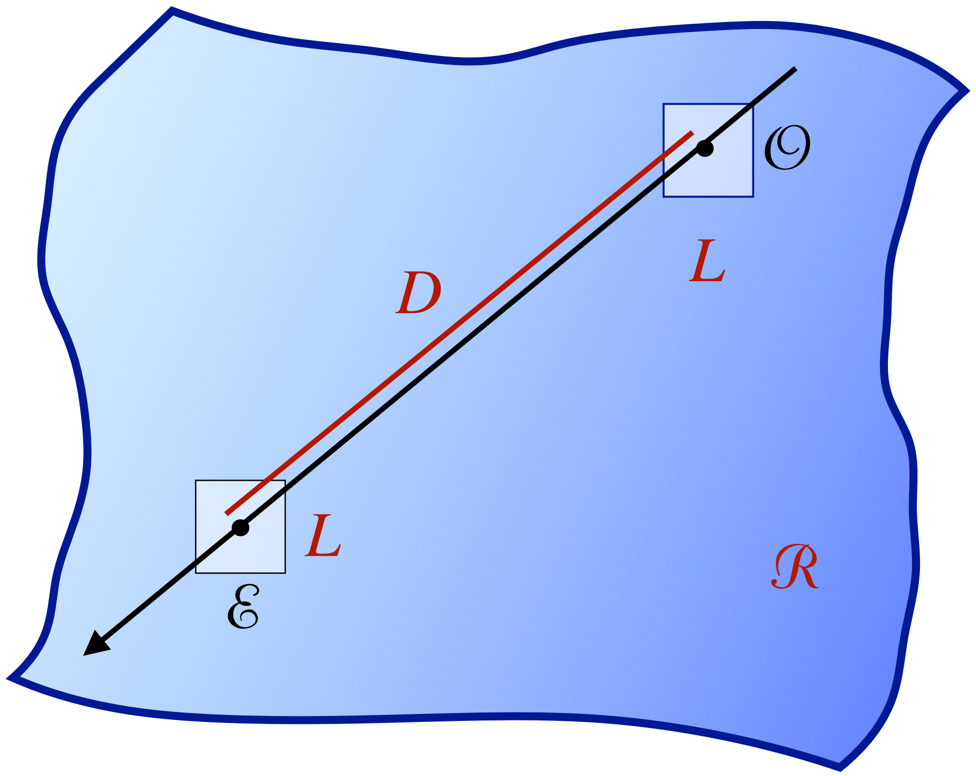

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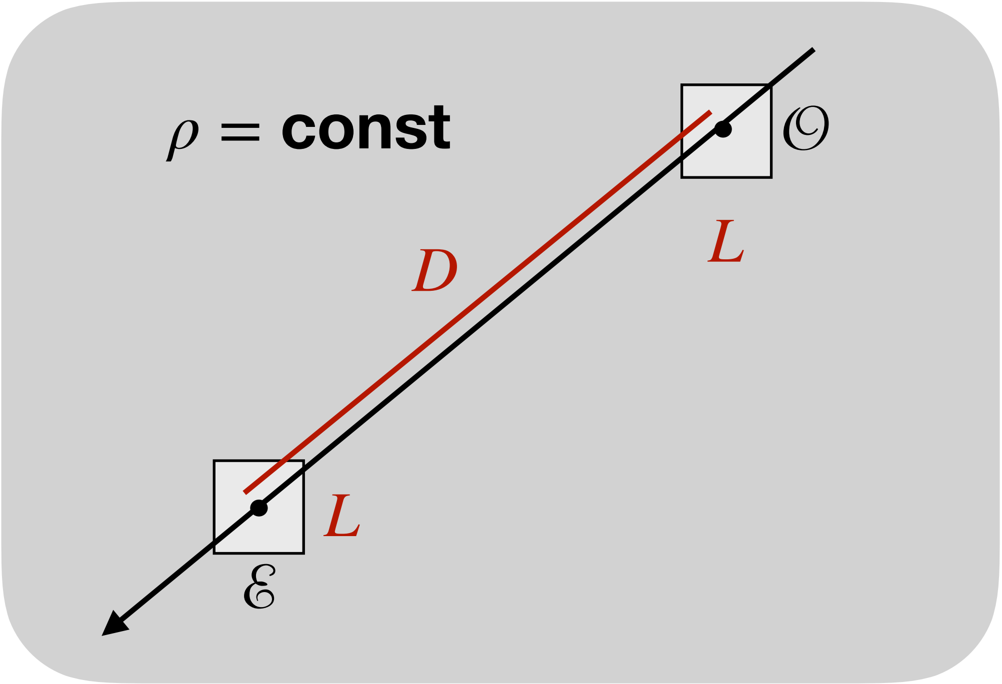
$$0 = \widetilde{\mathbf{L}}(\widetilde{\mathbf{X}}) + \frac{1}{2}\left(\frac{L}{D}\right)\widetilde{\mathbf{U}}_{flat}(\widetilde{\mathbf{X}}, \widetilde{\mathbf{X}}) + \frac{1}{2}\left(\frac{L}{D}\right)\left(\frac{D}{\mathscr{R}}\right)^{2}\widetilde{\mathbf{U}}_{curv}(\widetilde{\mathbf{X}}, \widetilde{\mathbf{X}})$$

Estimating the signal

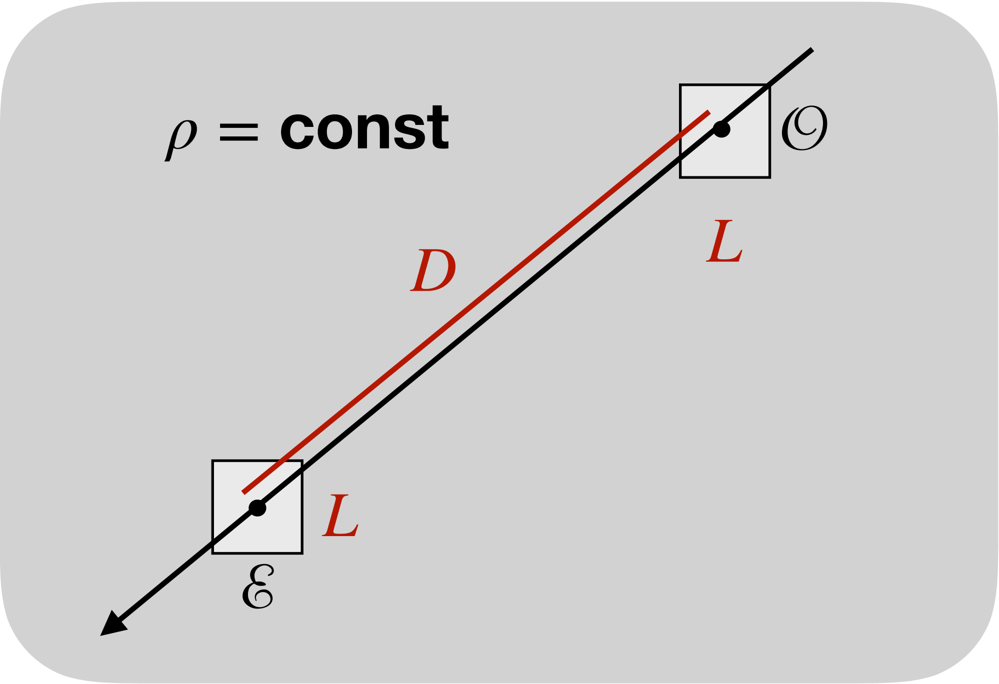
Constant mass density ρ



Estimating the signal

Constant mass density ρ

$$\mathscr{R}^{-2} = \frac{8\pi G\,\rho}{c^2}$$



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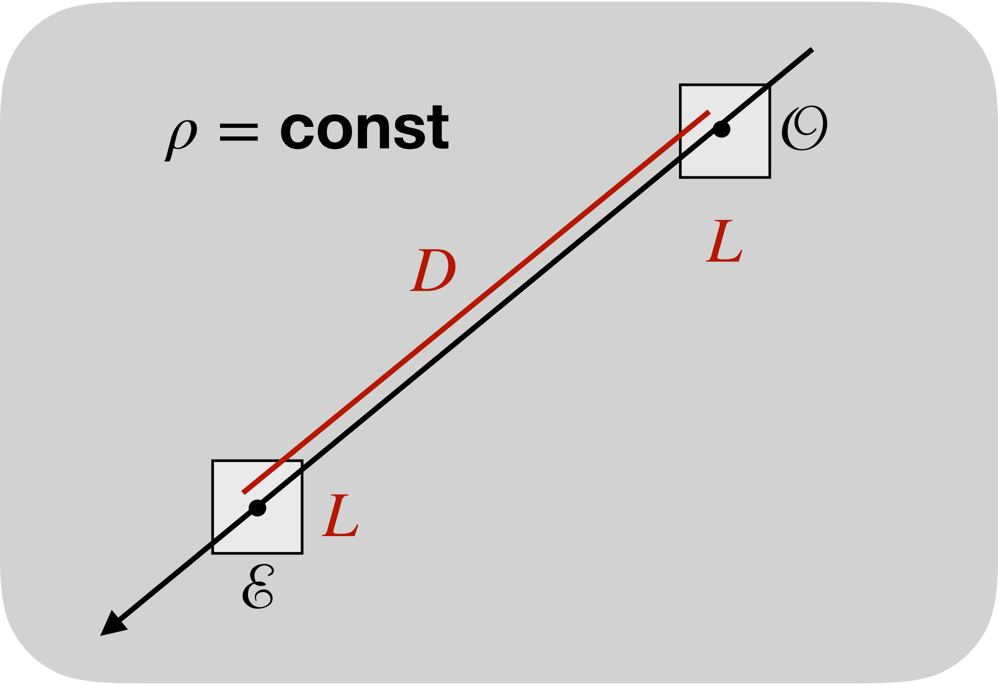

$$M_{tot} = \pi L^2\, D\,\rho$$

mass enclosed in the connecting cylinder

Estimating the signal

Constant mass density ρ

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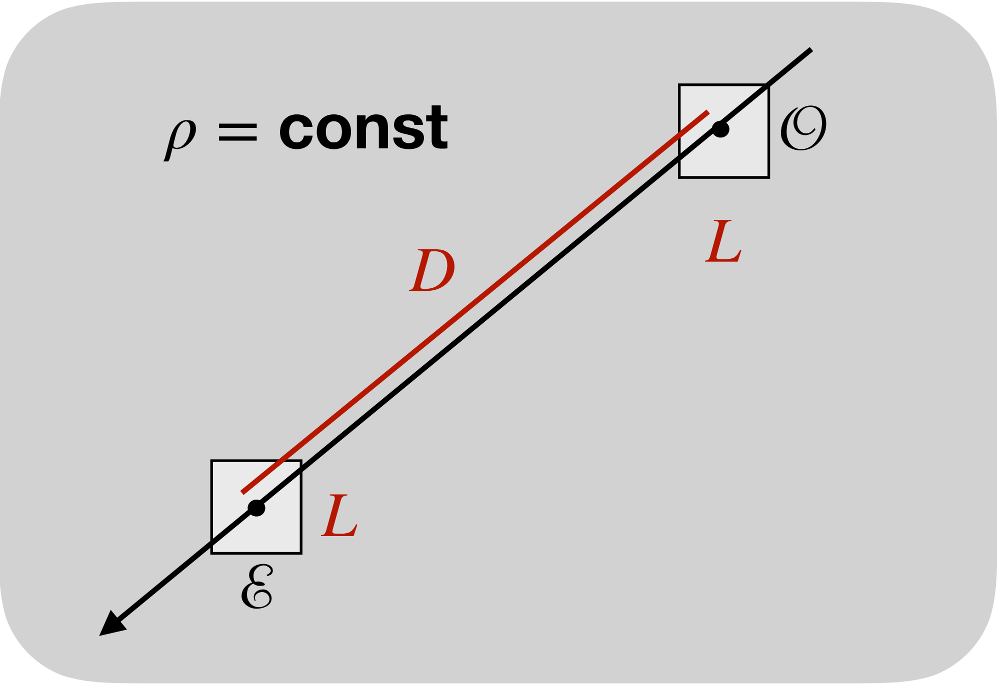

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$$\delta \mathbf{X}^{\mathbf{0}}_{curv} \approx \frac{1}{c} \cdot \frac{G\,M_{tot}}{c^2}$$

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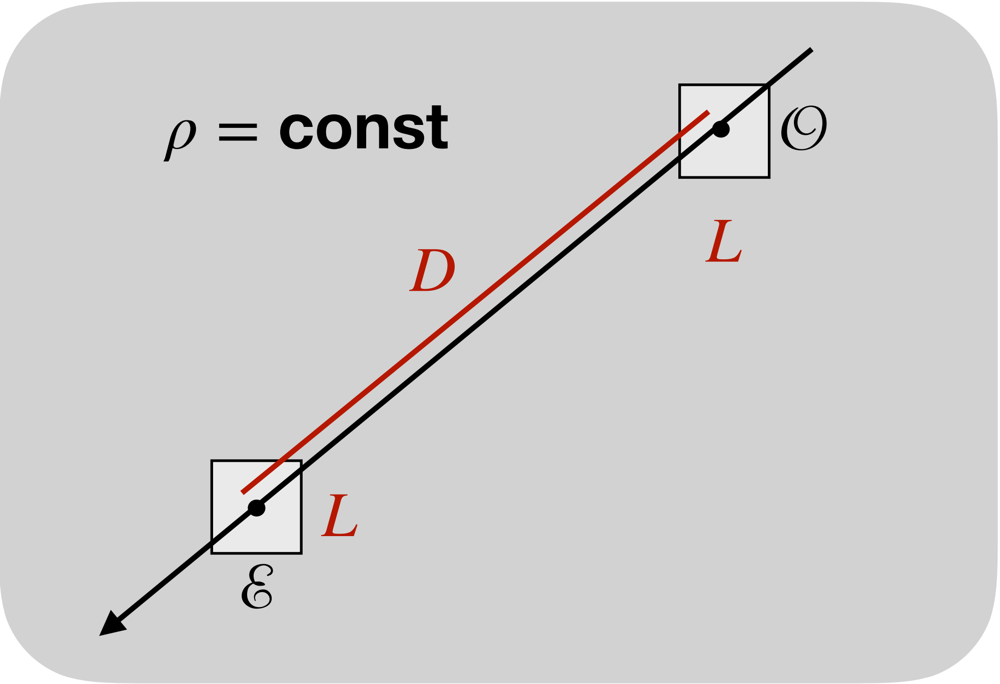

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Schwarzschild radius for the enclosed mass

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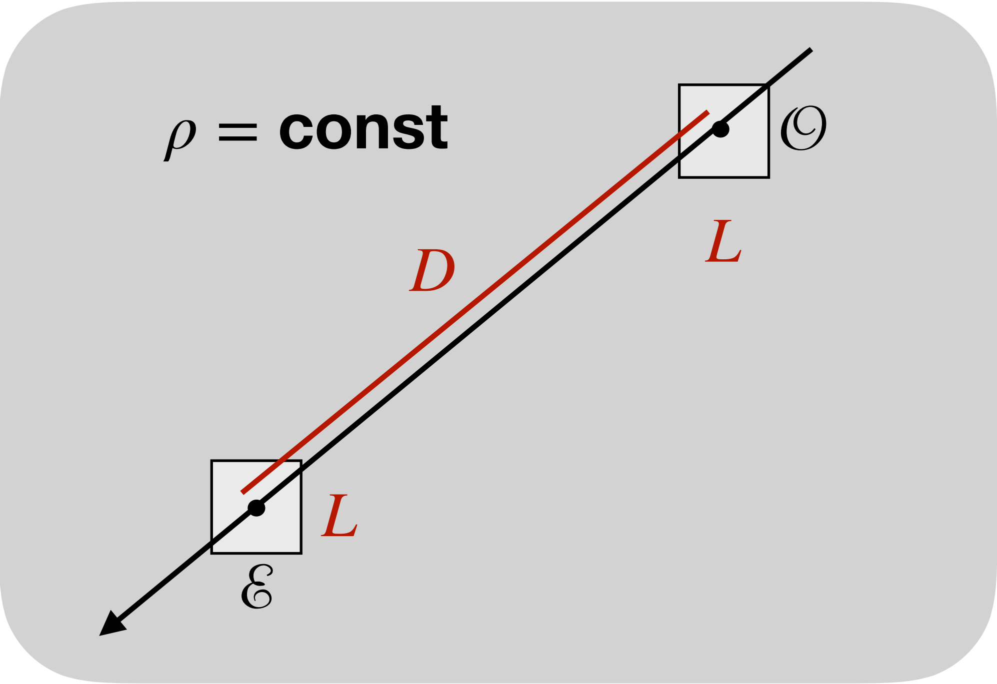

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$$\delta \mathbf{X}^{\mathbf{0}}_{curv} \approx \frac{1}{c} \cdot \frac{G\,M_{tot}}{c^2}$$

Schwarzschild radius for the enclosed mass

Small effect usually:

Jupiter mass M_J corresponds to $10\ \mathrm{ns}$, Solar mass $M_\odot$ to $10\ \mu\mathrm{s}$

Summary

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Variations of TOA's of the second order between two distant regions contain curvature corrections on top of finite distance effects

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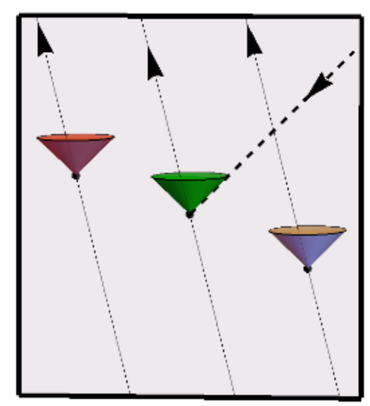
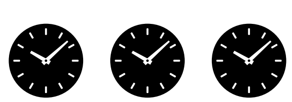
Small effects usually

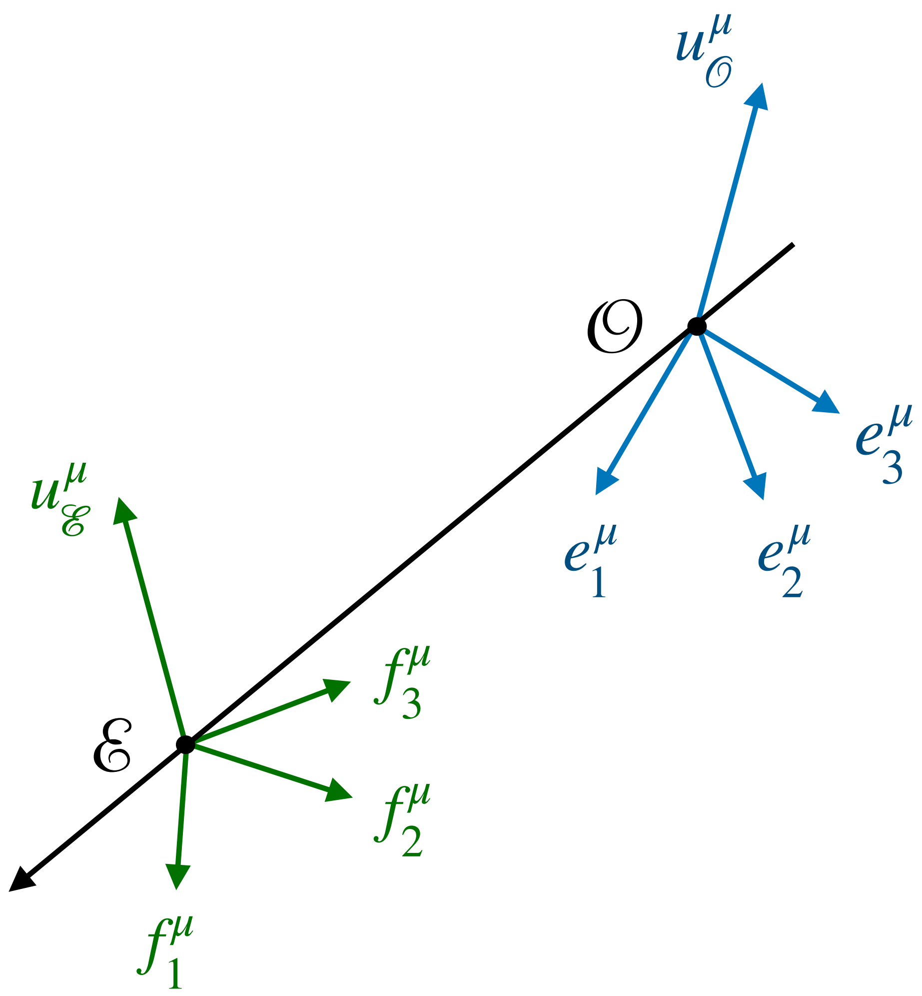
Applications: binary pulsar timing, direct curvature measurements etc.





Time of arrival of a signal

Introduce orthonormal tetrads at $\mathcal{O}$ and $\mathcal{E}$



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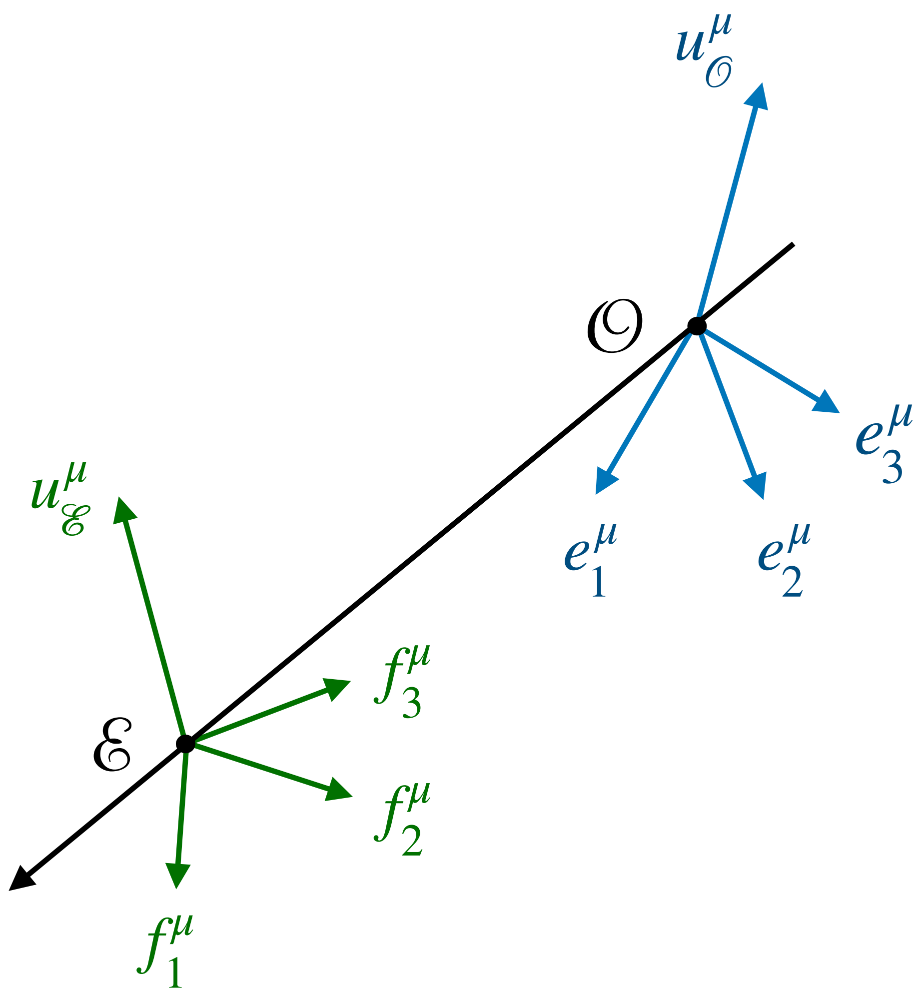

Locally flat coordinate systems near $\mathscr{O}$ and $\mathscr{E}$

points in M near $\mathscr{O}$ and $\mathscr{E}$ $\longleftrightarrow$ tangent vectors at $\mathrm{T}_{\mathscr{O}}M$, $\mathrm{T}_{\mathscr{E}}M$

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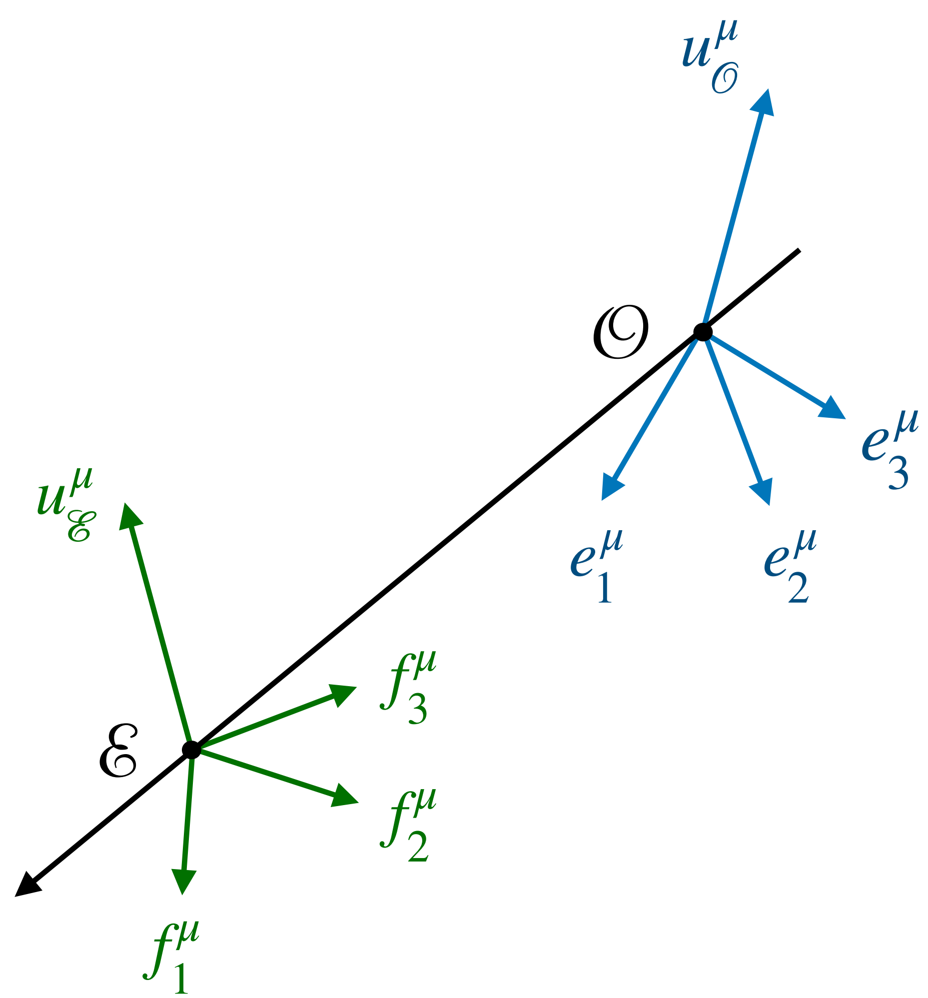

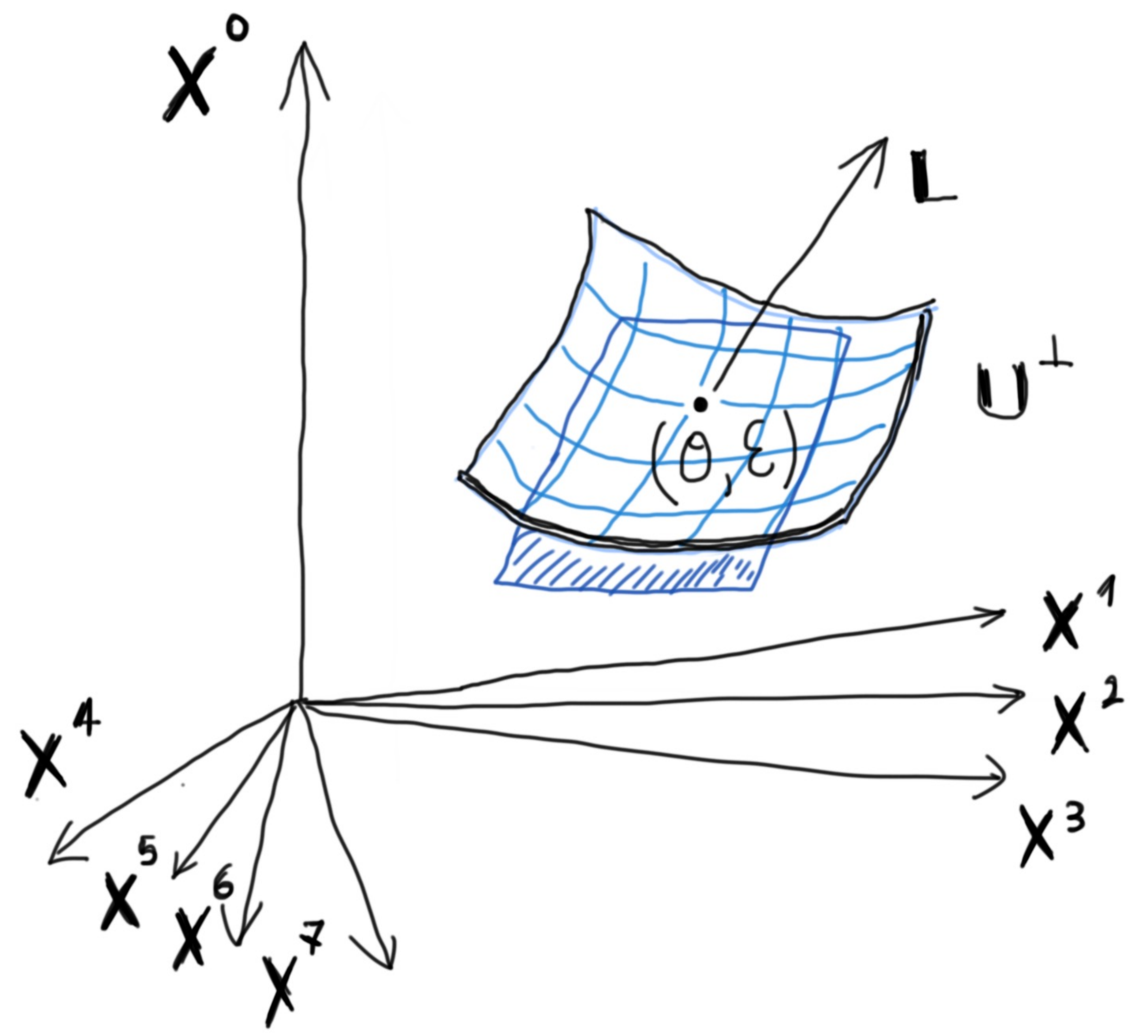

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Time of arrival of a signal

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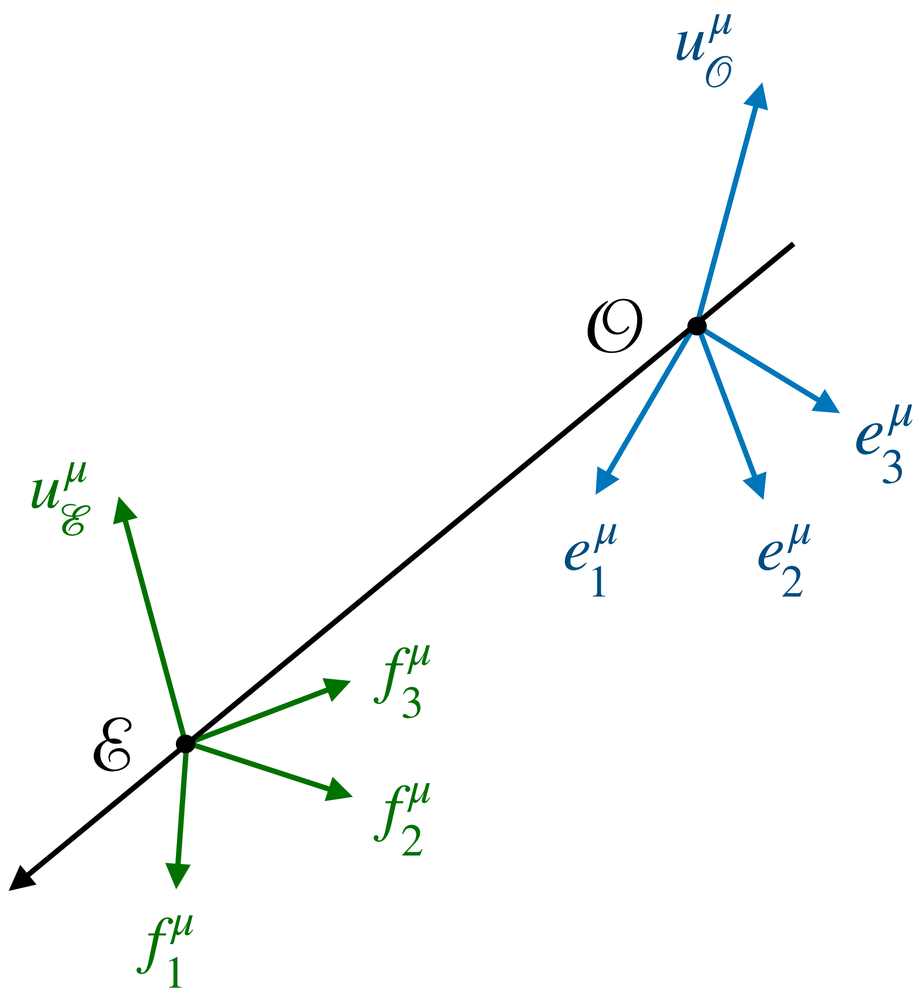

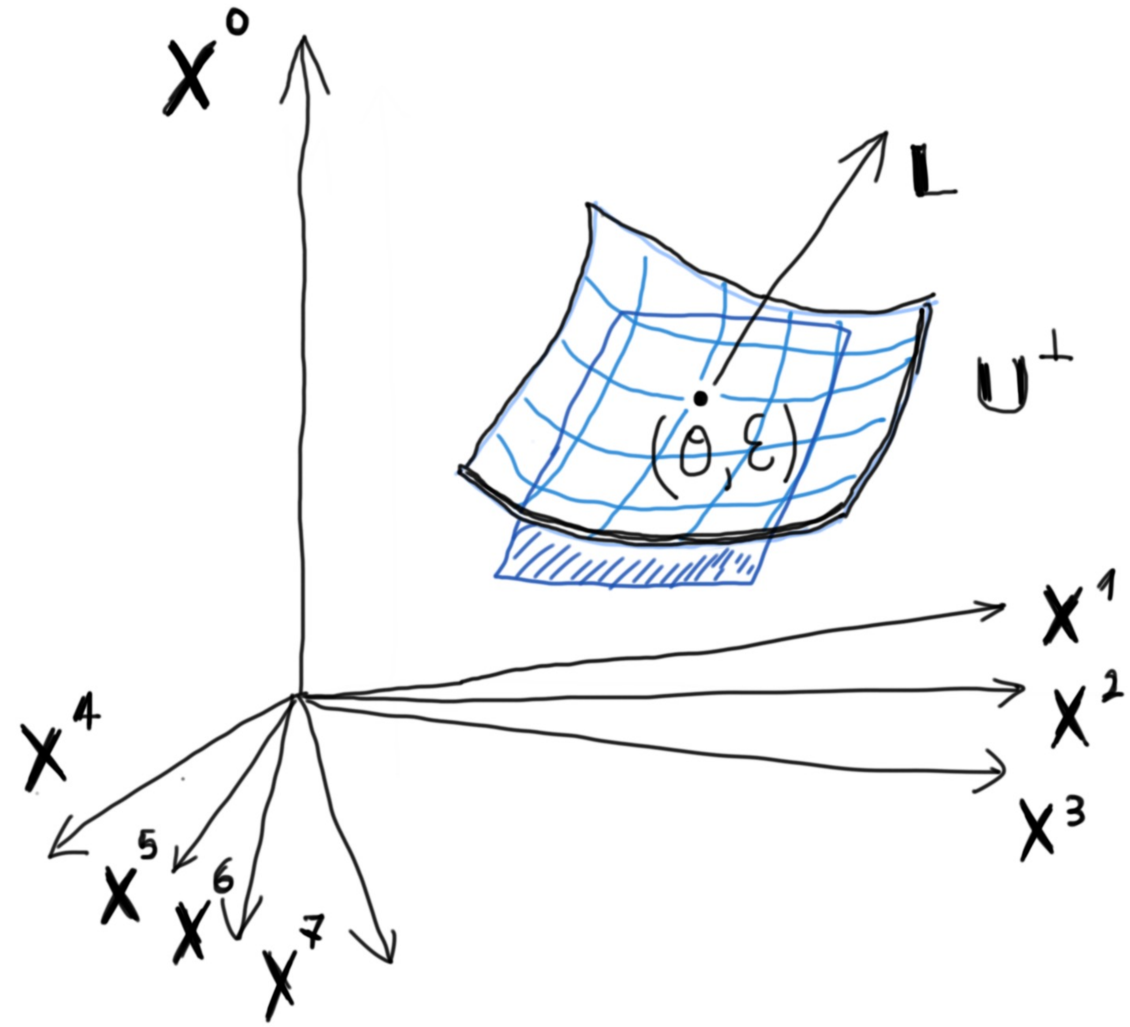

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Gauge choice: $\mathbf{L_0} = 1$

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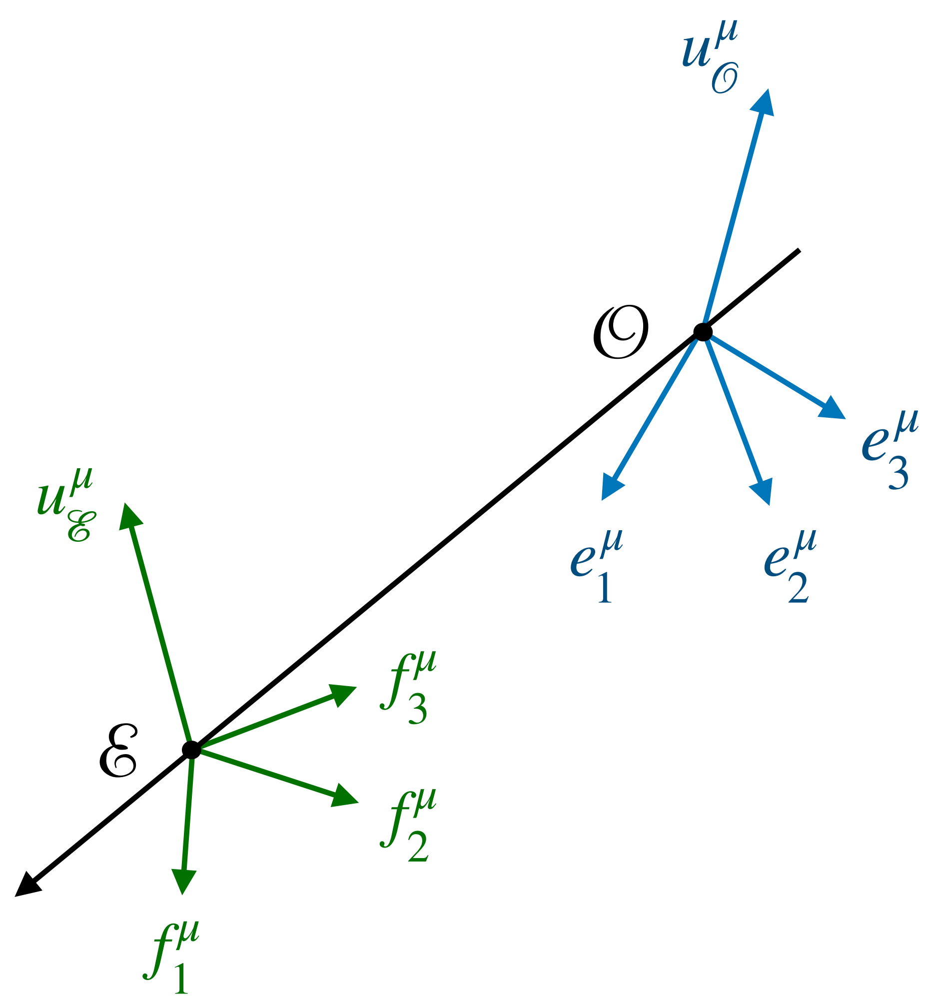

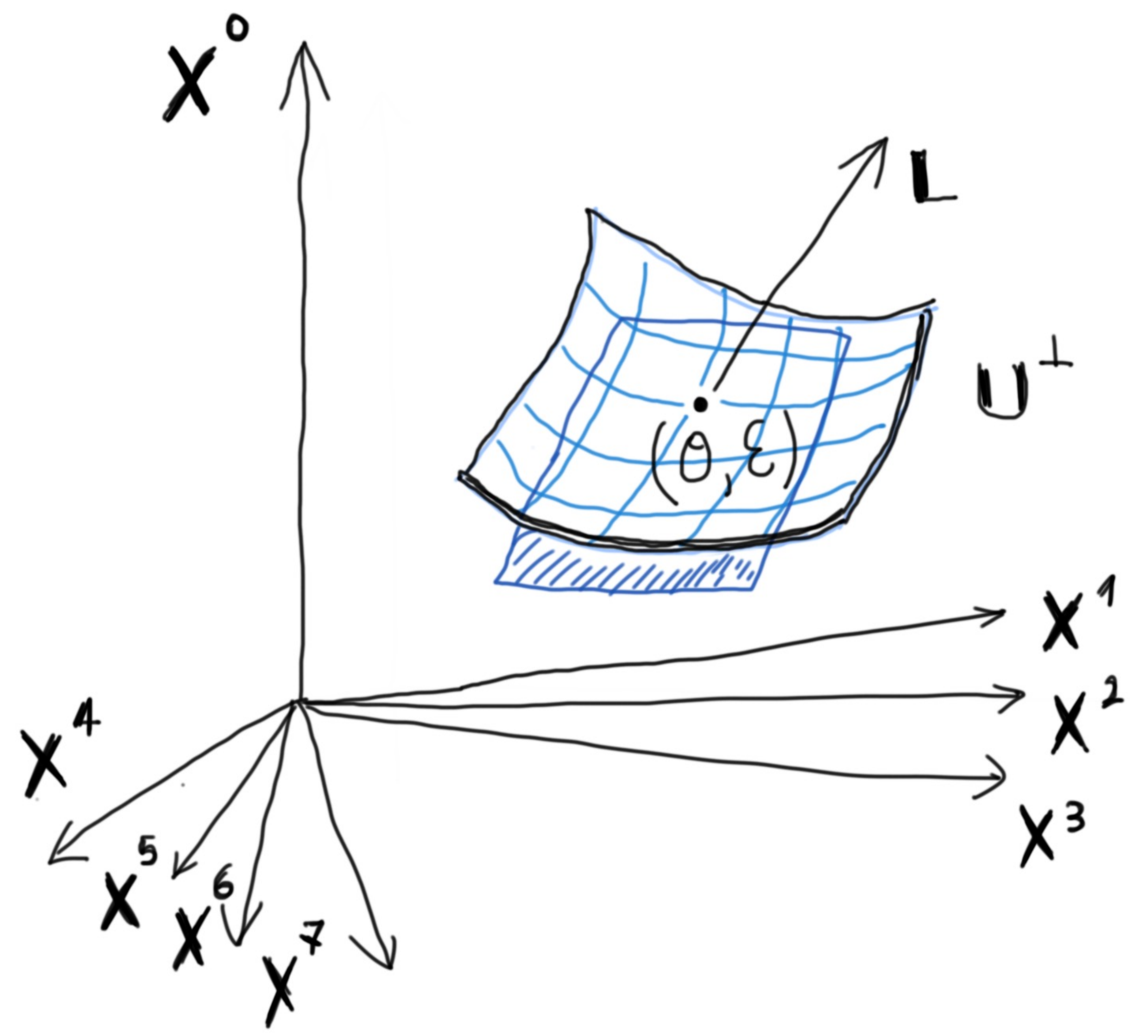

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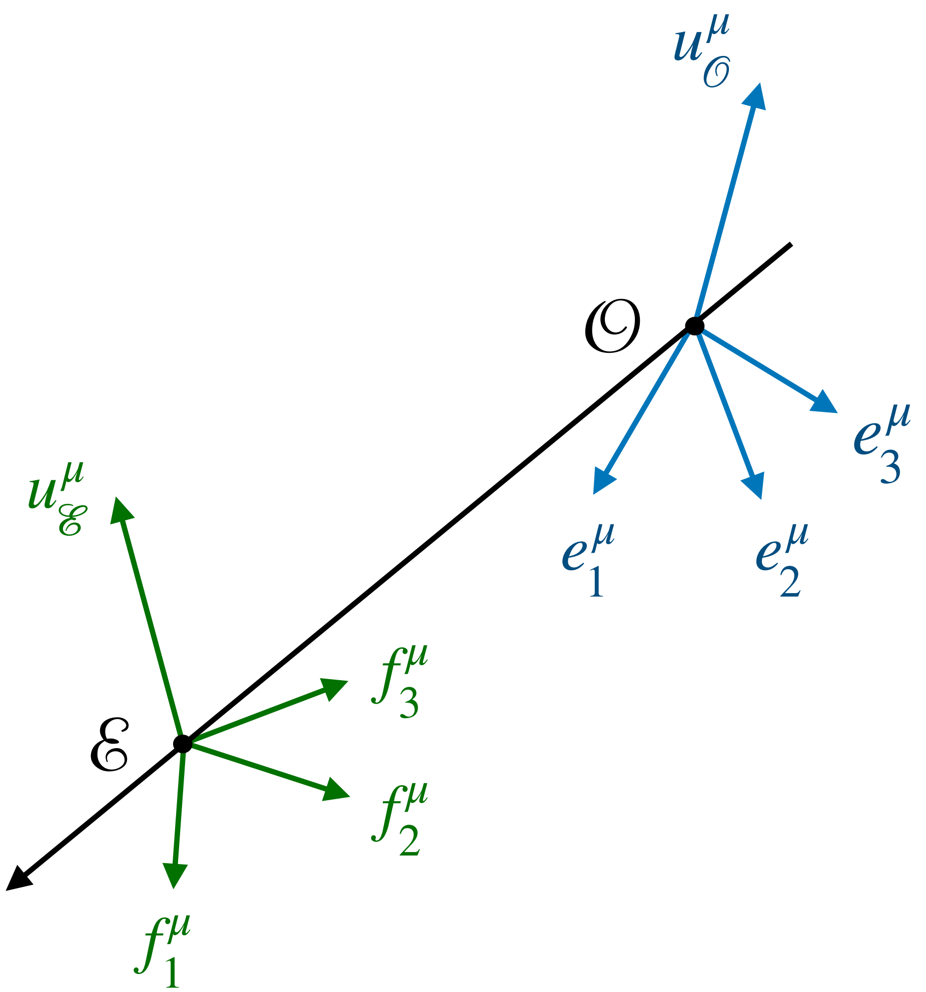

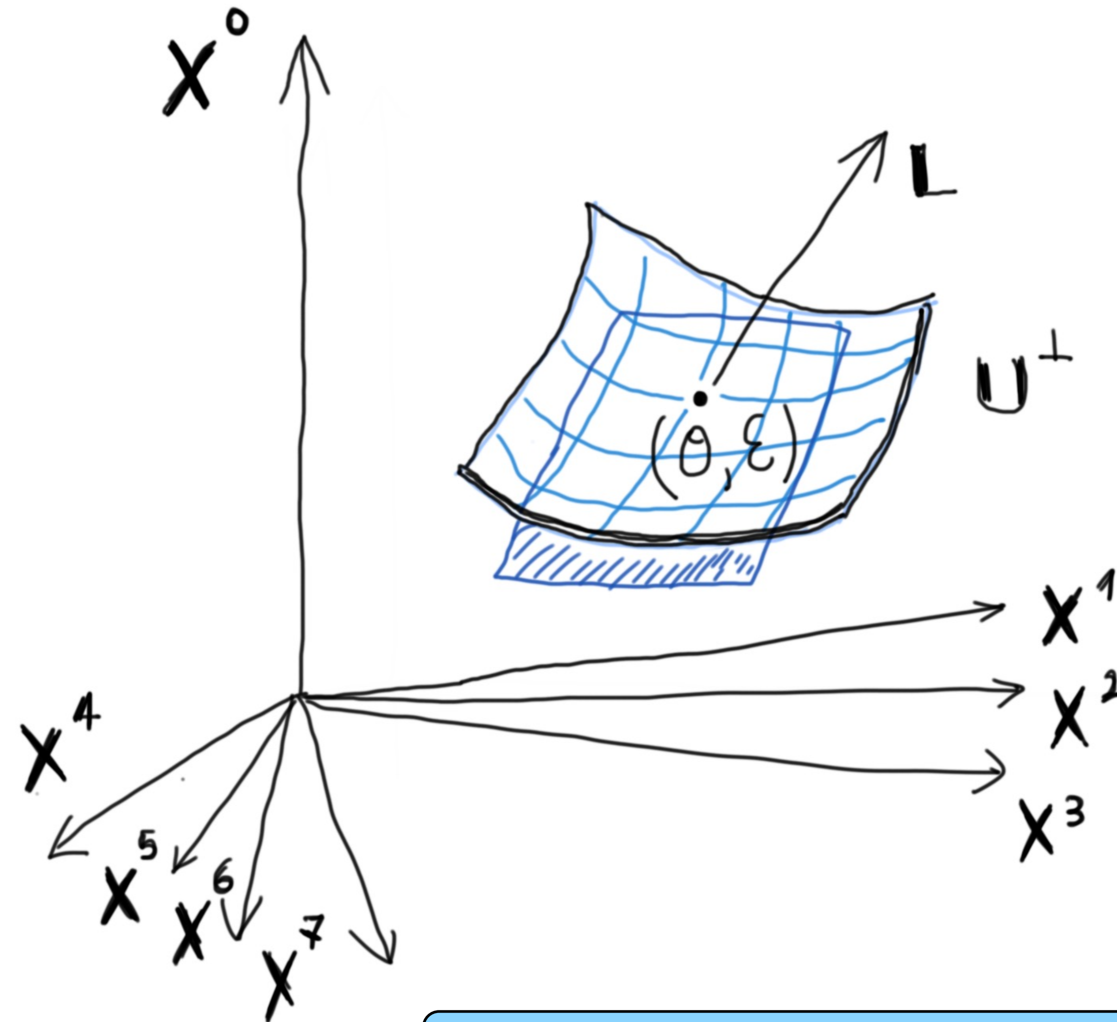

Time of arrival

Position of the receiver + position and time of emission

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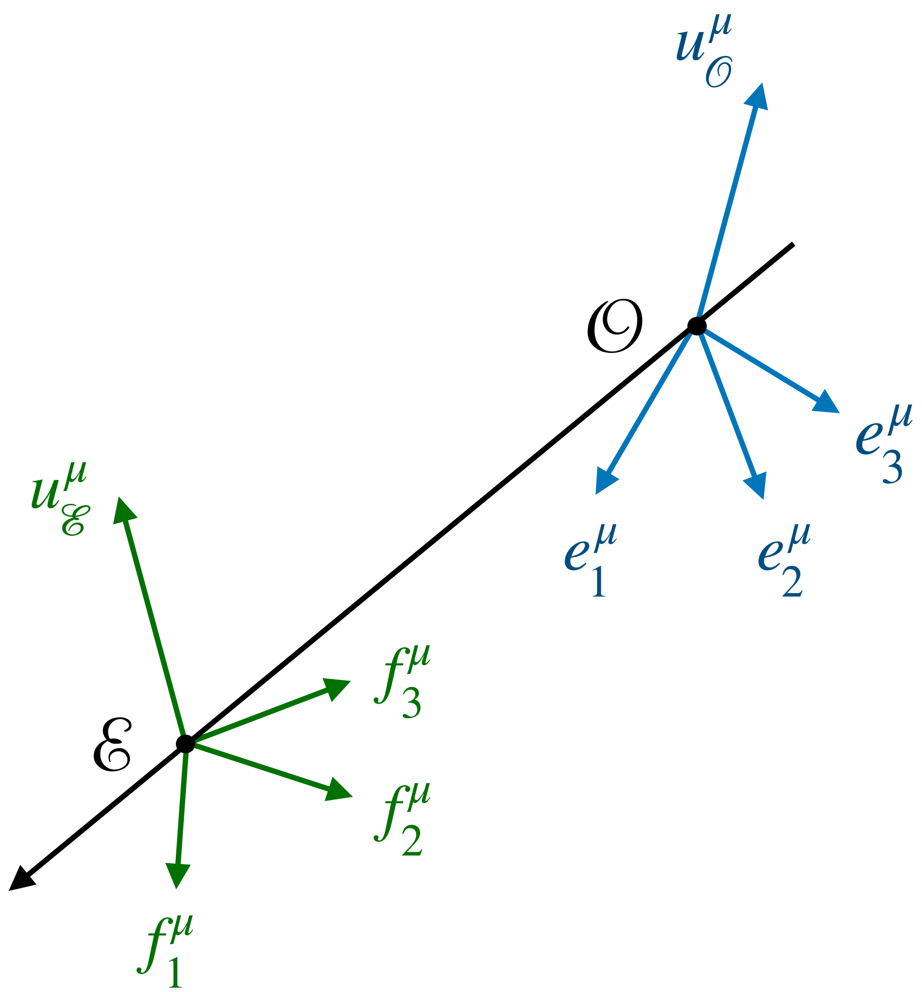

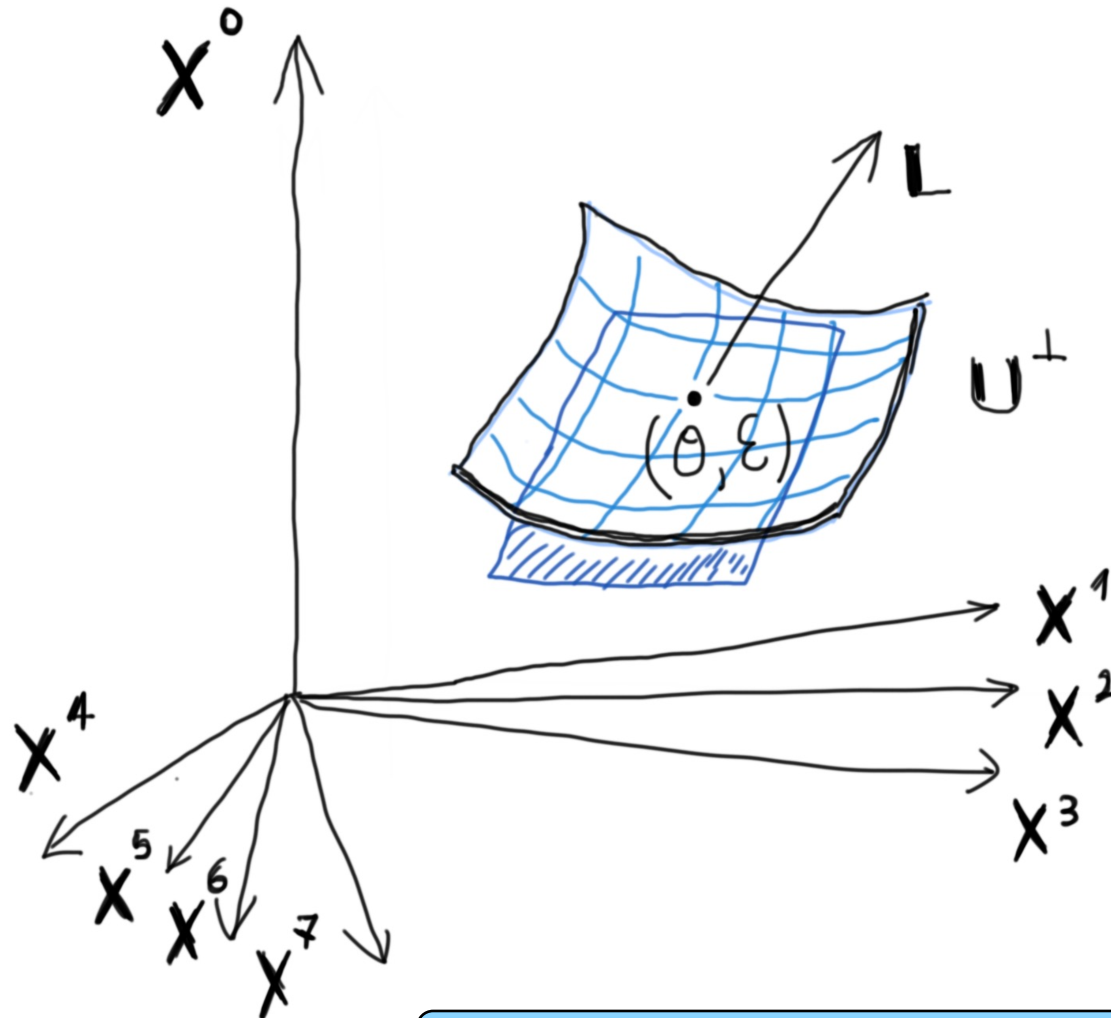

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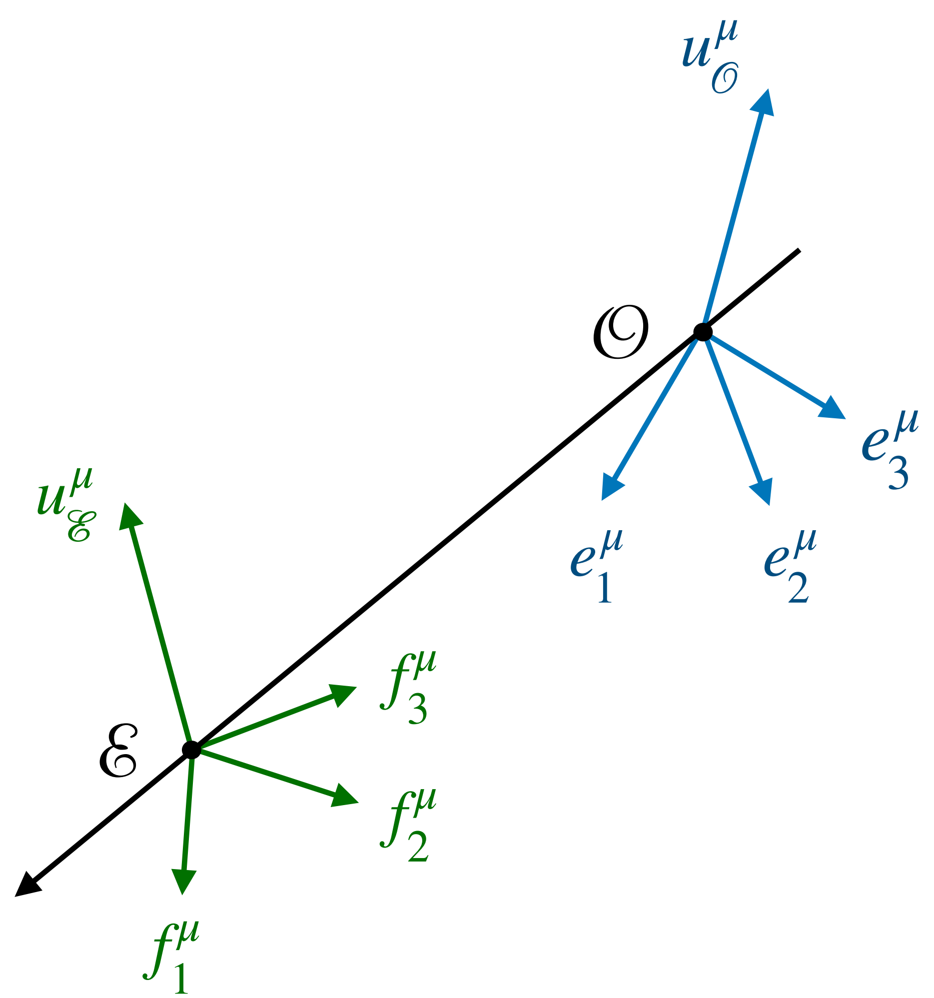

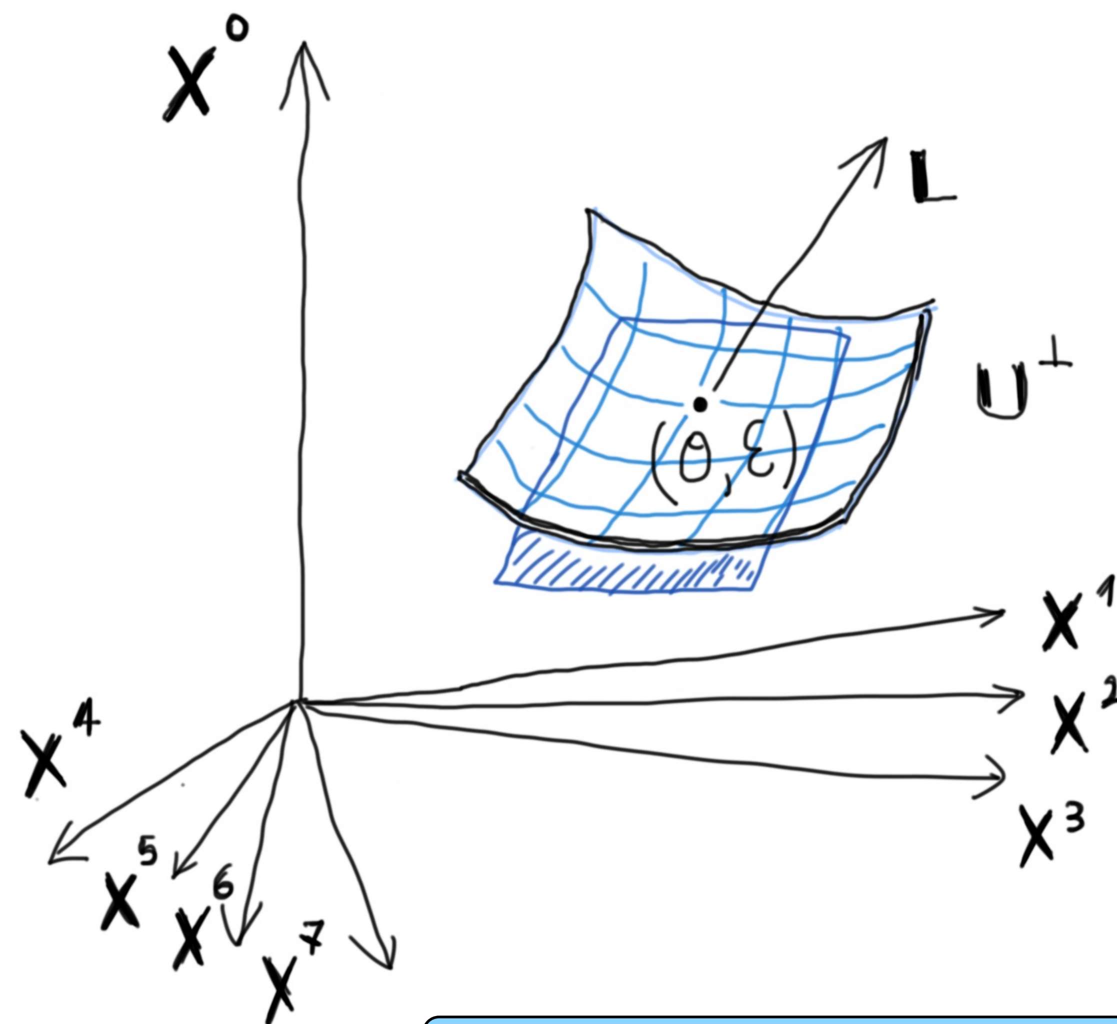

Locally flat coordinate systems near 𝒪 and ℰ

points in M near 𝒪 and ℰ ⟷ tangent vectors at $T_{\mathcal{O}}M$, $T_{\mathcal{E}}M$

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extrinsic curvature $\mathbf{U}^\perp$

Probing curvature

$\mu \equiv \mu\left(\mathbf{U}^{\perp}, \mathbf{L}\right) \qquad \nu \equiv \nu\left(\mathbf{U}^{\perp}, \mathbf{L}\right)$

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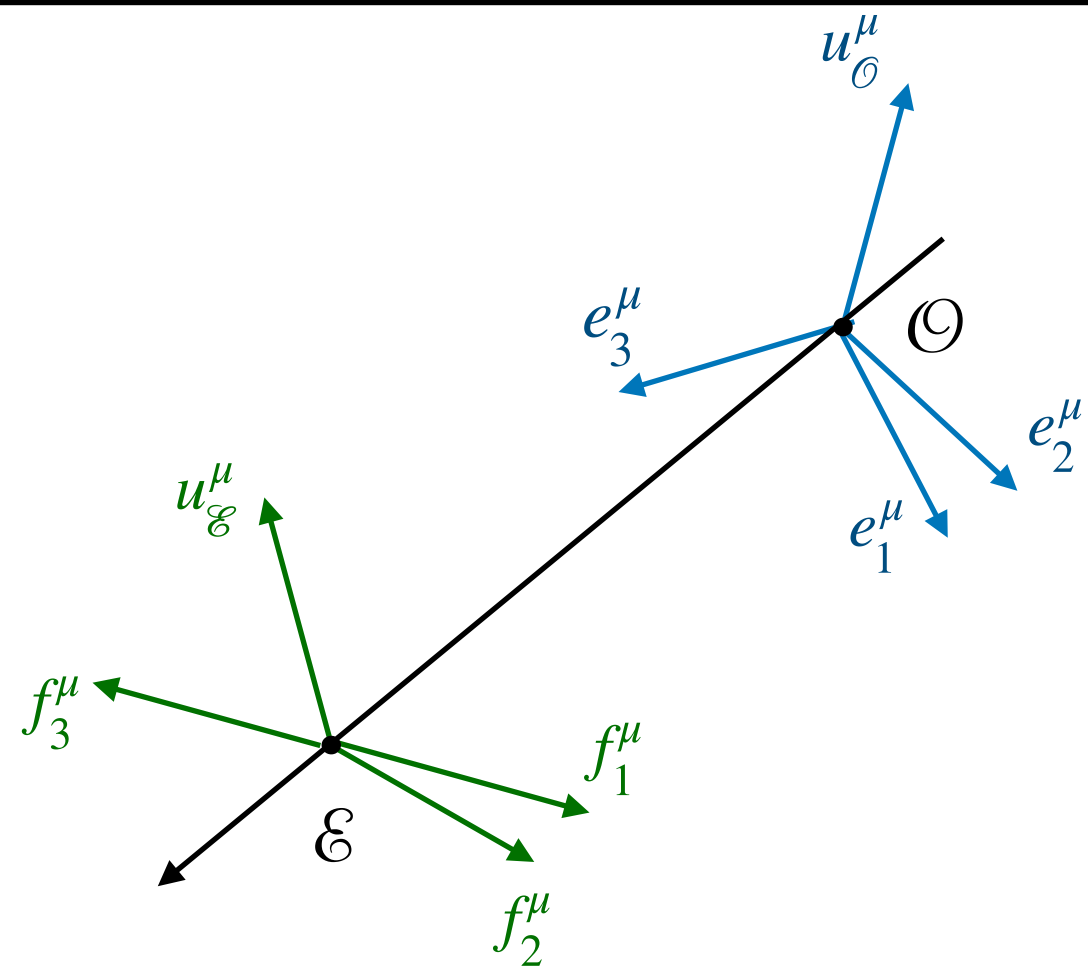
In an aligned orthonormal tetrad



$$l^{\mu'}_{\mathcal{O}} = a(-u^{\mu'}_{\mathcal{O}} + e^{\mu'}_3)$$

$$l^{\mu}_{\mathcal{E}} = b(-u^{\mu}_{\mathcal{E}} + f^{\mu}_3)$$

Probing curvature

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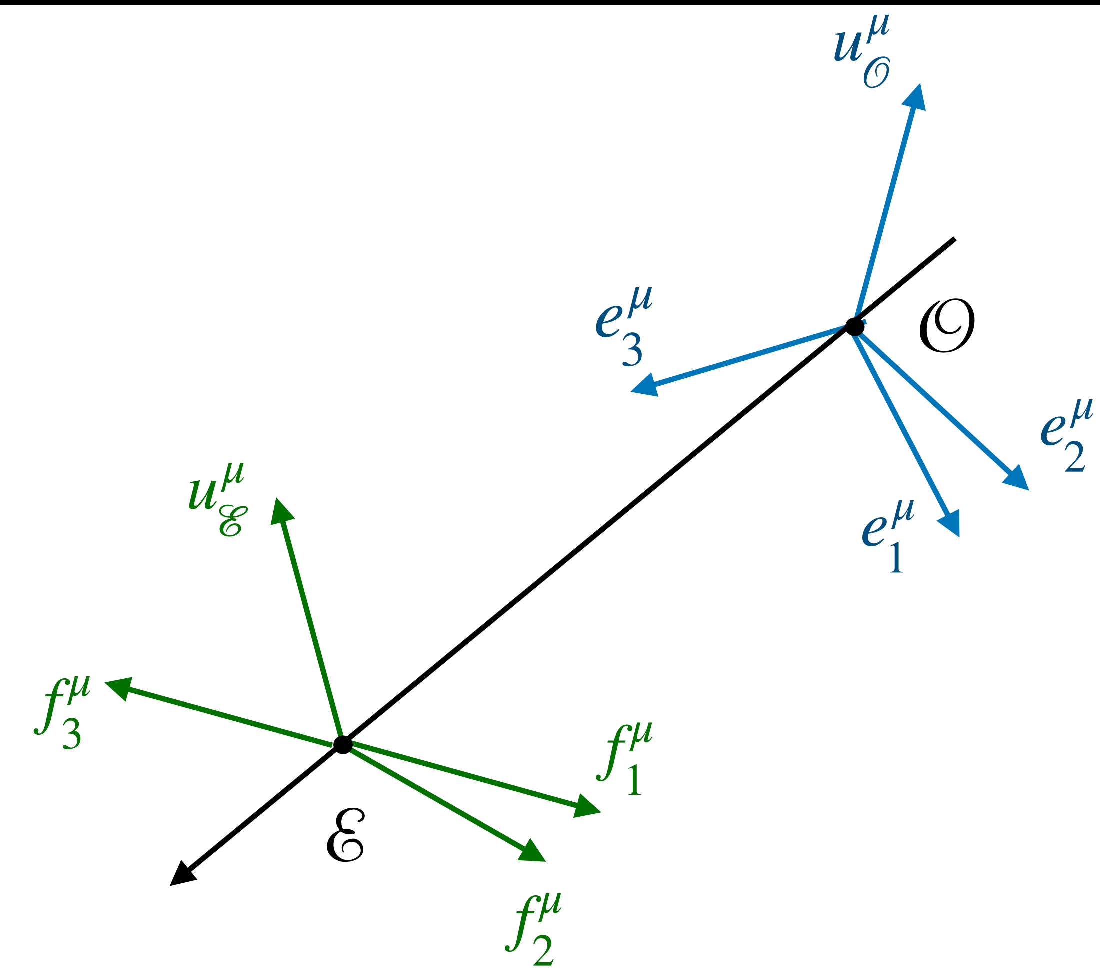

In an aligned orthonormal tetrad

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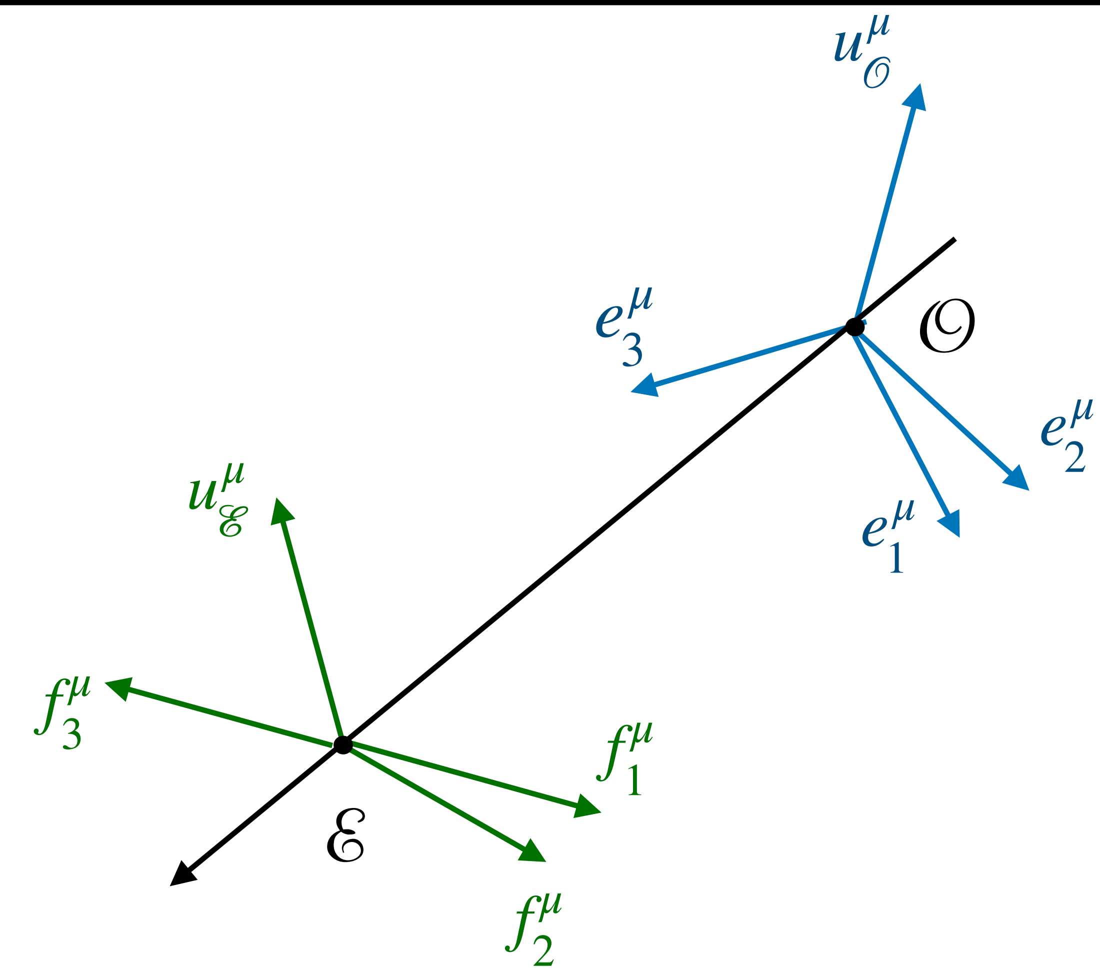

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$$l^{\mu}_{\mathcal{E}} = b(-u^{\mu}_{\mathcal{E}} + f^{\mu}_{3})$$

Independent of the choice of $u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}$

and rotations of the transverse vectors