Status of Birkhoff 's theorem in polymerized semiclassical regime of Loop Quantum Gravity

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LC, Jerzy Lewandowski (2024) arXiv:2403.01910

Introduction

Classically: Spherically symmetric vacuum → Schwarzschild

Semiclassically: Spherically symmetric vacuum → ?

Collapse of a spherically symmetric cloud of dust

- Modified Einstein's equations
- Oppenheimer-Snyder model

EINSTEIN'S EQUATIONS

Classical theory

Spherically symmetric spacetime: $(G = c = 1)$

$$ds^2 = -N d\tau^2 + \frac{(E^\varphi)^2}{E^x}(dx + N^x\, d\tau)^2 + E^x\,(d\theta^2 + sin^2\theta\; d\varphi^2)$$

$N = N(\tau, x)$ → Lapse function

$N^x = N^x(\tau, x)$ → x-component of the Shift vector

$$E_i^a = \sqrt{q} e_i^a$$

$$A_a^i = \omega_a^i + \gamma K_a^i$$

$$E_1^x = E^x \sin\theta \qquad \gamma K_x^1 = K_x$$

$$E_2^\theta = E^\varphi \sin\theta \qquad \gamma K_\theta^2 = K_\varphi$$

$$E_3^\varphi = E^\varphi \qquad \gamma K_\varphi^3 = K_\varphi \sin\theta$$

Classical theory

Gravity + Dust:

$$S = \int d\tau \int dx \left[\frac{\dot{K}_x E^x + 2\dot{K}_\varphi E^\varphi}{2\gamma} + 4\pi \dot{\mathcal{T}} p_{\mathcal{T}} - N(\mathcal{H}^g + \mathcal{H}^d) - N^x(\mathcal{H}_x^g + \mathcal{H}_x^d) \right]$$

$$\mathcal{H}^g = -\frac{1}{2\gamma^2}\left[2K_x K_\varphi \sqrt{E^x} + \frac{E^\varphi}{\sqrt{E^x}}\left(K_\varphi^2 + \gamma^2\right) - \frac{\gamma^2}{4}\frac{(\partial_x E^x)^2}{E^\varphi \sqrt{E^x}} - \gamma^2 \sqrt{E^x}\, \partial_x\left(\frac{\partial_x E^x}{E^\varphi}\right) \right]$$

$$\mathcal{H}^d = 4\pi \sqrt{p_{\mathcal{T}}^2 + \frac{E^x}{(E^\varphi)^2} p_{\mathcal{T}}^2 (\partial_x \mathcal{T})^2}$$

$$\mathcal{H}_x^g = \frac{1}{2\gamma}\left(2E^\varphi \partial_x K_\varphi - K_x \partial_x E^x\right)$$

$$\mathcal{H}_x^d = -4\pi p_{\mathcal{T}} \partial_x \mathcal{T}$$

V. Husain, T. Pawlowski (2012)
J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Classical theory

Dust Gauge $(\mathcal{T} = \tau)$ → $N = 1$

$$\mathcal{H}^d = 4\pi\sqrt{p_{\mathcal{T}}^2 + \frac{E^x}{(E^\varphi)^2}p_{\mathcal{T}}^2(\partial_x\mathcal{T})^2} = 4\pi p_{\mathcal{T}}$$

$$\mathcal{H}_x^d = -4\pi p_{\mathcal{T}}\partial_x\mathcal{T} = 0$$

Areal Gauge $(E^x = x^2)$ → $N^x = -\frac{K_\varphi}{\gamma}$

$$\mathcal{H}_x^g = \frac{1}{2\gamma}\left(2E^\varphi\partial_x K_\varphi - K_x\partial_x E^x\right) \longrightarrow K_x = \frac{E^\varphi}{x}\partial_x K_\varphi$$

$\dot{K}_x E^x$ is a total derivative

One dynamical pair $\{K_\varphi(y_1), E^\varphi(y_2)\} = \gamma\delta(y_1 - y_2)$

V. Husain, T. Pawlowski (2012)
J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Classical theory

Gravity + Dust: $S = \int d\tau \int dx \left[\frac{\dot{K}_x E^x + 2\dot{K}_\varphi E^\varphi}{2\gamma} + 4\pi\, \dot{\mathcal{T}} p_{\mathcal{T}} - N(\mathcal{H}^g + \mathcal{H}^d) - N^x(\mathcal{H}_x^g + \mathcal{H}_x^d)\right]$

Dust Gauge $(\mathcal{T} = \tau)$ → $N = 1$

Areal Gauge $(E^x = x^2)$ → $N^x = -\frac{K_\varphi}{\gamma}$

$$S = \int d\tau \int dx \left[\frac{\dot{K}_\varphi E^\varphi}{\gamma} - H\right] \quad \longrightarrow \quad \{K_\varphi(y_1), E^\varphi(y_2)\} = \gamma\delta(y_1 - y_2)$$

$$H = -4\pi p_{\mathcal{T}} = -\frac{1}{2\gamma}\left[\frac{E^\varphi}{\gamma x}\,\partial_x\left(x K_\varphi^2\right) + \frac{\gamma E^\varphi}{x} + \frac{\gamma x}{E^\varphi} - 2\gamma\partial_x\left(\frac{x^2}{E^\varphi}\right)\right]$$

V. Husain, T. Pawlowski (2012)
J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Classical theory – Dust density ρ

From the Dust Gauge: $\mathcal{H}^d = 4\pi p_{\mathcal{T}}$

By solving the Scalar Constraint: $\mathcal{H}^g = -4\pi p_{\mathcal{T}} = H$

The density ρ is defined by $\mathcal{H}^d = \int d\Omega \sqrt{q}\,\rho$

$$\rho = \frac{p_{\mathcal{T}}}{x E^{\varphi}} = -\frac{H}{4\pi x E^{\varphi}}$$

J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Classical theory – LTB model

$$ds^2 = -d\tau^2 + \frac{(E^\varphi)^2}{x^2}(dx + N^x\, d\tau)^2 + x^2 d\Omega^2$$

PG: $(\tau, x, \theta, \varphi)$

$$E^\varphi = \pm \frac{x}{\sqrt{1 + \varepsilon(\tau, x)}}$$

$x = \xi(T, R)$
$\tau = T$
$N^x = -\partial_T \xi$
$\varepsilon(\tau, x) = E(R)$

$$ds^2 = -dT^2 + \frac{(\partial_R \xi)^2}{1+E(R)} dR^2 + \xi^2 d\Omega^2$$

$$\xi = \xi(T, R)$$

LTB: (T, R, θ, φ)

Classical theory - recap

$$\{K_\varphi(y_1), E^\varphi(y_2)\} = \gamma\delta(y_1 - y_2)$$

$$H = \frac{1}{2\gamma}\left[\frac{E^\varphi}{\gamma x}\,\partial_x\left(xK_\varphi^2\right) + \frac{\gamma E^\varphi}{x} + \frac{\gamma x}{E^\varphi} - 2\gamma\partial_x\left(\frac{x^2}{E^\varphi}\right)\right]$$

$$\rho = -\frac{H}{4\pi x E^\varphi}$$

Three operators corresponding to $E^\varphi, K_\varphi, \frac{1}{E^\varphi}$

Quantum theory

$$H = -\frac{1}{2\gamma}\left[\frac{E^\varphi}{\gamma x}\,\partial_x\left(xK_\varphi^2\right) + \frac{\gamma E^\varphi}{x} + \frac{\gamma x}{E^\varphi} - 2\gamma\partial_x\left(\frac{x^2}{(E^\varphi)^2}\right)\right]$$

1) Discretization:

$$x \longrightarrow x_j \quad \Longrightarrow \quad \omega_j = x_{j+1} - x_j \approx \ell_p$$

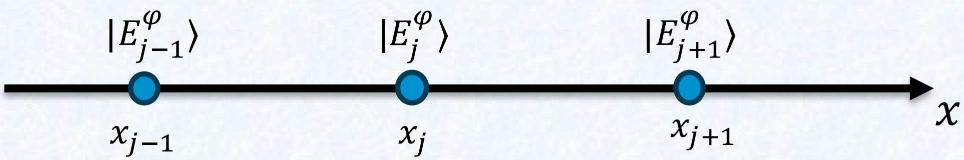
$$f(x) \longrightarrow f_j \coloneqq f(x_j)$$

$$\partial_x f(x) \longrightarrow \frac{f_{j+1} - f_j}{\omega_j}$$

Quantum theory

2) Quantization: $\left(E^{\varphi}, K_{\varphi}, \frac{1}{E^{\varphi}}\right) \longrightarrow \left(\hat{E}^{\varphi}_j, \hat{U}_j, \widehat{\frac{1}{E^{\varphi}_j}}\right)$

1-dimensional graph:



- Triad operator: $\hat{E}^{\varphi}_j\,|E^{\varphi}_j\rangle = E^{\varphi}_j|E^{\varphi}_j\rangle$

- Holonomy: $\hat{U}_j = e^{i\,\bar{\mu}_j K_{\varphi}(x_j)}$ $\xrightarrow{\text{Polymerization}}$ $\frac{\hat{U}_j - \hat{U}^{\dagger}_j}{2\,i\,\bar{\mu}_j} = \frac{\sin(\bar{\mu}_j K_{\varphi}(x_j))}{\bar{\mu}_j}$

  $\bar{\mu}_j = \frac{\sqrt{\Delta}}{x_j}$ $\qquad \hat{U}_j|E^{\varphi}_j\rangle = |E^{\varphi}_j + \bar{\mu}_j\rangle$

- Inverse triad:

$$\widehat{\frac{1}{E^{\varphi}_j}}|E^{\varphi}_j\rangle = \begin{cases} 0 & \text{If } \hat{E}^{\varphi}_j\,|E^{\varphi}_j\rangle = 0 \\ 1/E^{\varphi}_j\,|E^{\varphi}_j\rangle & \text{If } \hat{E}^{\varphi}_j\,|E^{\varphi}_j\rangle = E^{\varphi}_j|E^{\varphi}_j\rangle \end{cases}$$

R. Gambini, J. Olmedo, J. Pullin (2023)
V. Husain, J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Quantum theory - semiclassical limit

Classical: $$H = -\frac{1}{2\gamma}\left[\frac{E^{\varphi}}{\gamma x}\,\partial_x\left(xK_{\varphi}^2\right) + \frac{\gamma E^{\varphi}}{x} + \frac{\gamma x}{E^{\varphi}} - 2\gamma\partial_x\left(\frac{x^2}{E^{\varphi}}\right)\right]$$

Quantum: $$\hat{H}_j = -\frac{1}{2\gamma}\left\{\frac{1}{\gamma x_j}\widehat{\sqrt{E_j^{\varphi}}}\,\frac{1}{\omega_j}\left[\frac{x_{j+1}^3}{\Delta}sin^2\left(\frac{\sqrt{\Delta}\hat{K}_{\varphi,j+1}}{x_{j+1}}\right) - \frac{x_j^3}{\Delta}sin^2\left(\frac{\sqrt{\Delta}\hat{K}_{\varphi,j}}{x_j}\right)\right]\widehat{\sqrt{E_j^{\varphi}}} + \frac{\gamma\hat{E}_j^{\varphi}}{x_j} + \frac{\gamma x_j}{\hat{E}_j^{\varphi}} + \frac{1}{\omega_j}\left(\frac{x_{j+1}^2}{\hat{E}_{j+1}^{\varphi}} - \frac{x_j^2}{\hat{E}_j^{\varphi}}\right)\right\}$$

The Hamiltonian operator is then written in terms of classical variables and finally the continuum limit is recovered by $x_j \to x$.

Semiclassical: $$H = -\frac{1}{2\gamma}\left\{\frac{E^{\varphi}}{\gamma\Delta x}\,\partial_x\left[x^3 sin^2\left(\frac{\sqrt{\Delta}K_{\varphi}}{x}\right)\right] + \frac{\gamma E^{\varphi}}{x} + \frac{\gamma x}{E^{\varphi}} - 2\gamma\partial_x\left(\frac{x^2}{E^{\varphi}}\right)\right\}$$

EOM: $\dot{A} = \left\{A, \int dx\, H\right\}$

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Semiclassical theory

$$ds^2 = -d\tau^2 + \frac{(E^\varphi)^2}{x^2}(dx + N^x\, d\tau)^2 + x^2 d\Omega^2$$

$$\beta := \frac{\sqrt{\Delta}}{x} K_\varphi$$

$$\dot{E}^\varphi = -\frac{x^2}{\gamma\sqrt{\Delta}}\partial_x\left(\frac{E^\varphi}{x}\right)\sin\beta\cos\beta$$

$$\dot{K}^\varphi = \frac{\gamma x}{2(E^\varphi)^2} - \frac{\gamma}{2x} - \frac{\partial_x(x^3 sin^2\beta)}{2\gamma\Delta x}$$

Polymerized Einstein Field Equations (PEFE)

$$\rho = -\frac{H}{4\pi x E^\varphi} \longrightarrow \rho = \frac{1}{8\pi x E^\varphi}\left[\frac{E^\varphi}{\gamma^2\Delta x}\partial_x(x^3 sin^2\beta) + \frac{x}{E^\varphi} + \frac{E^\varphi}{x} - 2\partial_x\left(\frac{x^2}{E^\varphi}\right)\right]$$

$$N^x = -\frac{K_\varphi}{\gamma} \longrightarrow N^x = -\frac{x}{\gamma\sqrt{\Delta}}\sin\beta\cos\beta$$

J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2020)
V. Husain, J. G. Kelly, R. Santacruz, E. Wilson-Ewing. (2022)

Interior $(\rho \neq 0, \quad \partial_x \rho = 0)$

$$E^\varphi = \pm \frac{x}{\sqrt{1+\varepsilon(\tau,x)}}$$

PEFE
$$\begin{cases} \dot{\varepsilon} = \varepsilon' \sqrt{\frac{8\pi}{3}\rho x^2 + \varepsilon} \sqrt{1 - \frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{\varepsilon}{x^2}} \\ sin^2 \beta = \gamma^2 \Delta \left(\frac{8\pi}{3}\rho + \frac{\varepsilon}{x^2}\right) \end{cases}$$

$$\rho_c \coloneqq \frac{3}{8\pi\gamma^2\Delta}$$

$x = \xi(T,R)$
$\tau = T$
$N^x = -\partial_T \xi$
$\varepsilon = E(R)$

$$\longrightarrow \quad ds^2 = -dT^2 + \frac{(\partial_R \xi)^2}{1+E(R)} dR^2 + \xi^2 d\Omega^2$$

$$\left(\frac{\partial_T \xi}{\xi}\right)^2 = \left(\frac{8\pi}{3}\rho + \frac{E}{\xi^2}\right)\left(1 - \frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{E}{\xi^2}\right)$$

M. Bojowald, J. D. Reyes, R. Tibrewala (2009)
K. Giesel, H. Liu, E. Rullit, P. Singh, S. A. Weigl (2023)

Interior – Friedmann equation

From $ds^2 = -dT^2 + \frac{(\partial_R \xi)^2}{1+E(R)} dR^2 + \xi^2 d\Omega^2$

$$x = \xi = a(T)\chi_k(R)$$
$$\varepsilon = -k\left(\frac{x}{a}\right)^2 = -k\chi_k^2 = E(R)$$

The Friedmann dust ball: $ds^2 = -dT^2 + a^2 dR^2 + a^2\chi_k^2 d\Omega^2$ with $\chi_k(R) = \frac{1}{\sqrt{k}}\sin\left(\sqrt{k}R\right)$

$$\left(\frac{\partial_T \xi}{\xi}\right)^2 = \left(\frac{8\pi}{3}\rho + \frac{E}{\xi^2}\right)\left(1 - \frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{E}{\xi^2}\right) \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho - \frac{k}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{k}{a^2}\right)$$

$$\rho = \frac{C}{a^3}$$

Static Exterior $(\rho = 0)$

$$\dot{E}^{\varphi} = -\frac{x^2}{\gamma\sqrt{\Delta}}\partial_x\left(\frac{E^{\varphi}}{x}\right)\sin\beta\cos\beta = 0$$

$$E^{\varphi} = Ax = \pm\frac{x}{\sqrt{1+B}} \longrightarrow \dot{K}^{\varphi} = 0$$

$$sin^2\beta = \gamma^2\Delta\left(\frac{B}{x^2}+\frac{2M}{x^3}\right) \longrightarrow (N^x)^2 = \frac{2M}{x} - \frac{\alpha}{x^2}\left(\frac{M}{x}+\frac{B}{2}\right)^2 + B \qquad \alpha \coloneqq 4\gamma^2\Delta$$

$$ds^2 = -d\tau^2 + A^2(dx + N^x\, d\tau)^2 + x^2 d\Omega^2$$

In Schwarzschild coordinates: $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r}+\frac{B}{2}\right)^2$$

Static Exterior - $\sin\beta$

$$0 \leq sin^2\beta = \gamma^2\Delta\left(\frac{B}{x^2} + \frac{2M}{x^3}\right) \leq 1$$

$$\gamma^2\Delta\left(\frac{B}{x^2} + \frac{2M}{x^3}\right) = 1 \quad \longrightarrow \quad x_b = (2\gamma^2\Delta\, M)^{1/3} + \frac{B}{6M}(2\gamma^2\Delta\, M)^{2/3} + O(\Delta^{4/3})$$

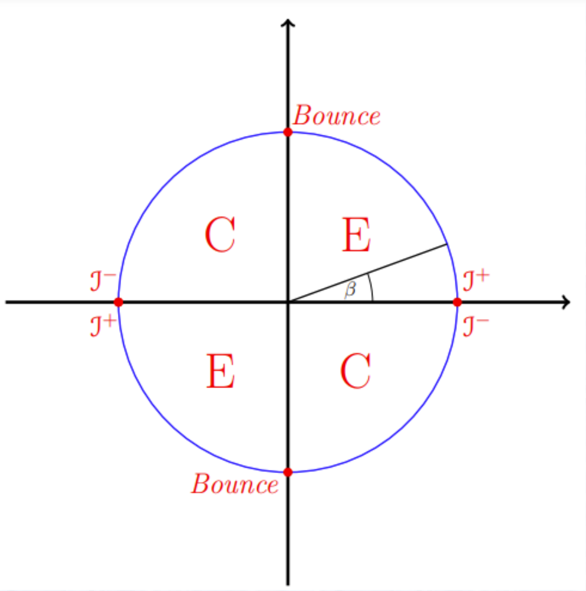
$$\gamma^2\Delta\left(\frac{B}{x^2} + \frac{2M}{x^3}\right) = 0 \quad \longrightarrow \quad \begin{cases} x = \infty, & B \geq 0 \\ x = \dfrac{2M}{|B|}, & B < 0 \end{cases}$$

Static Exterior - $\sin\beta$

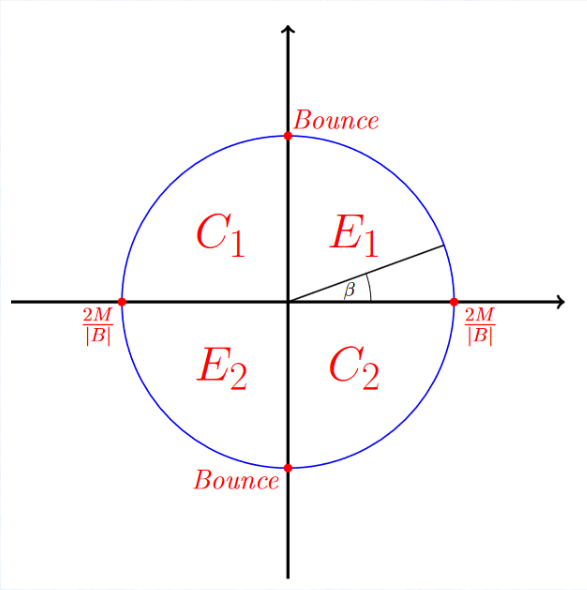
What is the sign of $\sin\beta$?

Collapse: $\sin\beta\cos\beta < 0$
Expansion: $\sin\beta\cos\beta > 0$

$B \geq 0 \rightarrow x \in [x_b, \infty)$



$B < 0 \rightarrow x \in \left[x_b, \frac{2M}{|B|}\right]$



Time dependent Exterior $(\rho = 0)$

$$\dot{E}^{\varphi} = -\frac{x^2}{\gamma\sqrt{\Delta}}\partial_x\left(\frac{E^{\varphi}}{x}\right)\sin\beta\cos\beta \neq 0 \quad \longrightarrow \quad E^{\varphi} = \pm\frac{x}{\sqrt{1+\varepsilon(\tau,x)}}$$

PEFE
$$\begin{cases} \dot{\varepsilon} = \varepsilon'\sqrt{\varepsilon + \frac{2M}{x}}\sqrt{1-\gamma^2\Delta\left(\frac{\varepsilon}{x^2}+\frac{2M}{x^3}\right)} \\ sin^2\beta = \gamma^2\Delta\left(\frac{\varepsilon}{x^2}+\frac{2M}{x^3}\right) \end{cases}$$

$\varepsilon = const$ or $\varepsilon = \varepsilon(\tau, x)$

If $\varepsilon \neq \varepsilon(\tau,x)$ then the only line element is given by $f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r}+\frac{B}{2}\right)^2$

BIRKHOFF’S THEOREM !!

Oppenheimer-Snyder model

Oppenheimer-Snyder model

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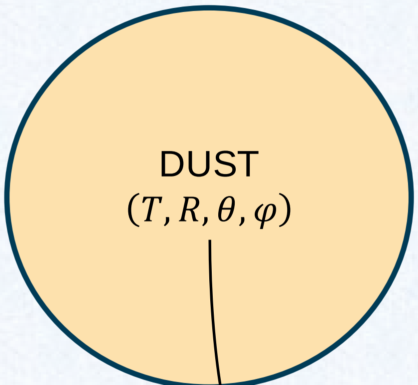
$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r} - \frac{k\chi_{k,0}^2}{2}\right)^2$$



$$\rho = \frac{C}{a^3}$$

$$ds^2 = -dT^2 + a^2 dR^2 + a^2 \chi_k^2 d\Omega^2 \quad \text{with} \quad \chi_k(R) = \frac{1}{\sqrt{k}}\sin\left(\sqrt{k}R\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho - \frac{k}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{k}{a^2}\right)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{k}{a^2}\right) + \left(\frac{8\pi}{3}\rho - \frac{k}{a^2}\right)\left(\frac{3}{2}\frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{k}{a^2}\right)$$

J. Lewandowski, Y. Ma, J. Yang, C. Zhang (2023)

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Critical mass and horizons

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r} - \frac{k\chi_{k,0}^2}{2}\right)^2$$

- If $M_- \leq M \leq M_+ \quad \Rightarrow \quad \nexists$ real solutions of $f(r) = 0$ (no horizons)

$$M_\pm^2 = \frac{\alpha}{216}\left[64 - 96\,k\chi_{k,0}^2 + 30\,k^2\chi_{k,0}^4 + k^3\chi_{k,0}^6 \pm \left(16 - 16\,k\chi_{k,0}^2 + k^2\chi_{k,0}^4\right)^{3/2}\right]$$

- If $M \geq M_+ \quad \cup \quad M \leq M_- \quad \Rightarrow \quad \exists$ 2 real solutions to $f(r) = 0$

$$r_- = \left(\frac{\alpha M}{2}\right)^{1/3} + \frac{1 - 2\,k\chi_{k,0}^2}{6M}\left(\frac{\alpha M}{2}\right)^{2/3} + \frac{\left(1 - k\chi_{k,0}^2\right)^2}{24\,M}\alpha + O\left(\alpha^{4/3}\right)$$

$$r_+ = 2M - \frac{\left(1 - k\chi_{k,0}^2\right)^2}{8\,M}\alpha + O\left(\alpha^{4/3}\right)$$

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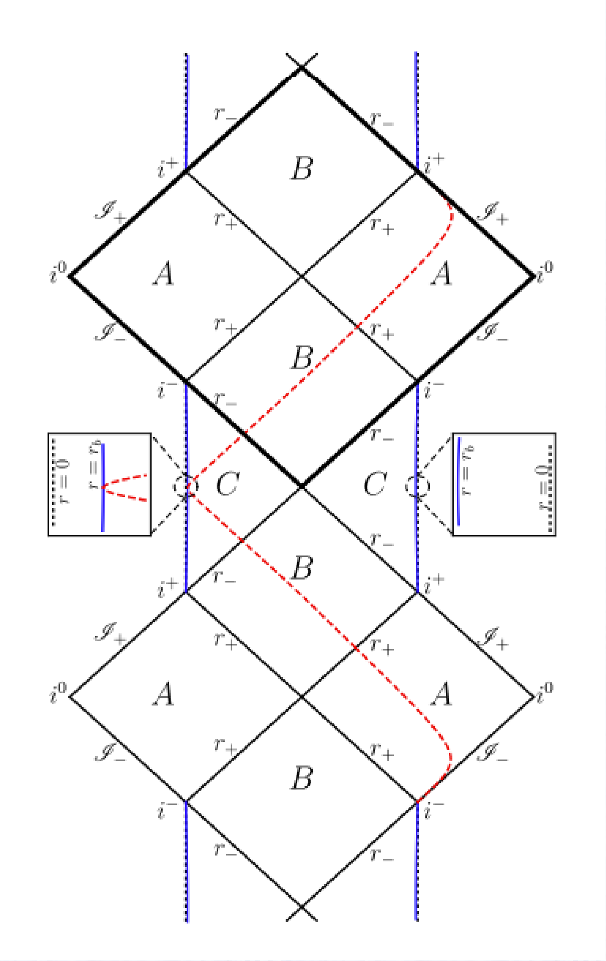
k=0

- $f(r) = 1 - \frac{2M}{r} + \alpha \frac{M^2}{r^4}$

- $M_- = 0, \quad M_+ = \frac{4}{3\sqrt{3}}\sqrt{\alpha}$

Exact solution to the PEFE with $B = 0$



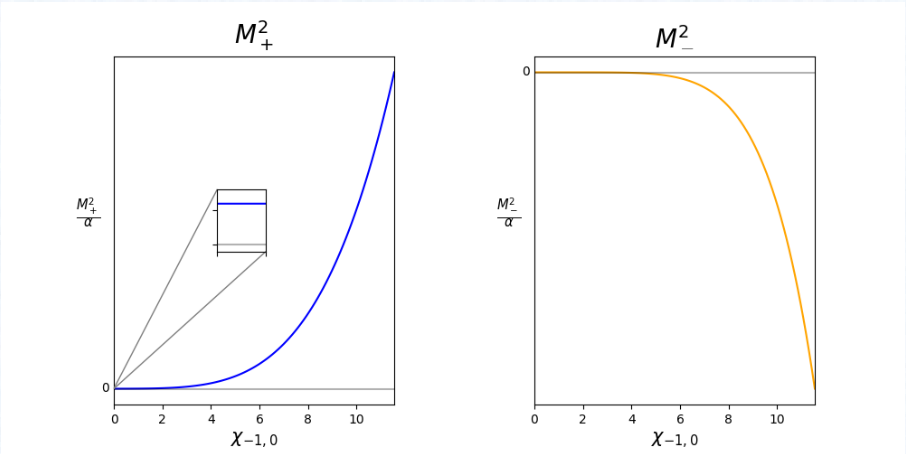
$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho\right)\left(1 - \frac{\rho}{\rho_c}\right)$$

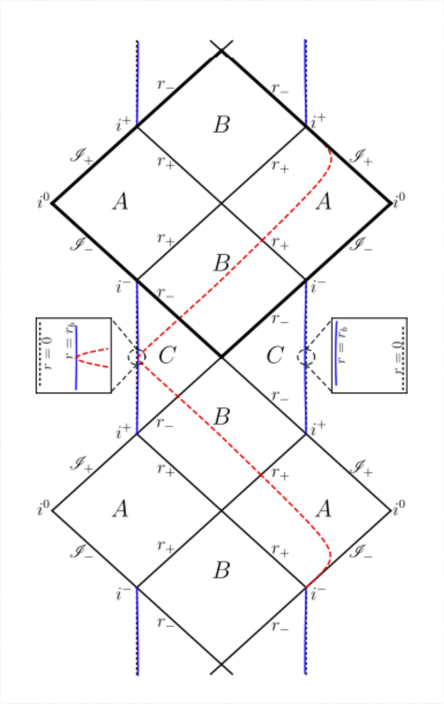
$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) + 4\pi\frac{\rho^2}{\rho_c}$$

H. Ziaie, Y. Tavakoli (2020)
A. Parvizi, T. Pawlowski, Y. Tavakoli, J. Lewandowski (2022)
J. Lewandowski, Y. Ma, J. Yang, C. Zhang (2023)

k= -1

- $f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r} + \frac{\chi^2_{-1,0}}{2}\right)^2$ Exact solution to the PEFE with $B = \chi^2_{-1,0}$





$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho + \frac{1}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right) + \left(\frac{8\pi}{3}\rho + \frac{1}{a^2}\right)\left(\frac{3}{2}\frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right)$$

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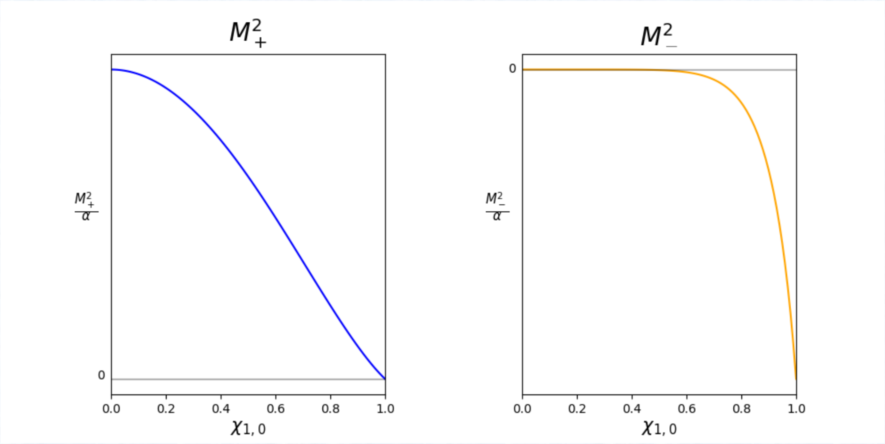
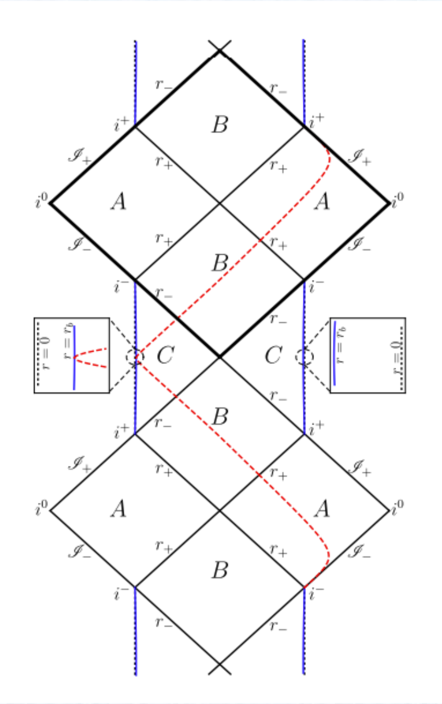
k=1

- $f(r) = 1 - \frac{2M}{r} + \frac{\alpha}{r^2}\left(\frac{M}{r} - \frac{\chi_{1,0}^2}{2}\right)^2$ Exact solution to the PEFE with $B = -\chi_{1,0}^2 < 0$!!!





$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho - \frac{1}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right) + \left(\frac{8\pi}{3}\rho - \frac{1}{a^2}\right)\left(\frac{3}{2}\frac{\rho}{\rho_c} - \frac{3}{8\pi\rho_c}\frac{1}{a^2}\right)$$

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k=1

Motionless ball at $T = T_0$:

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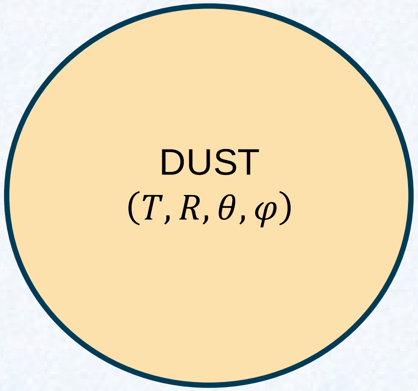
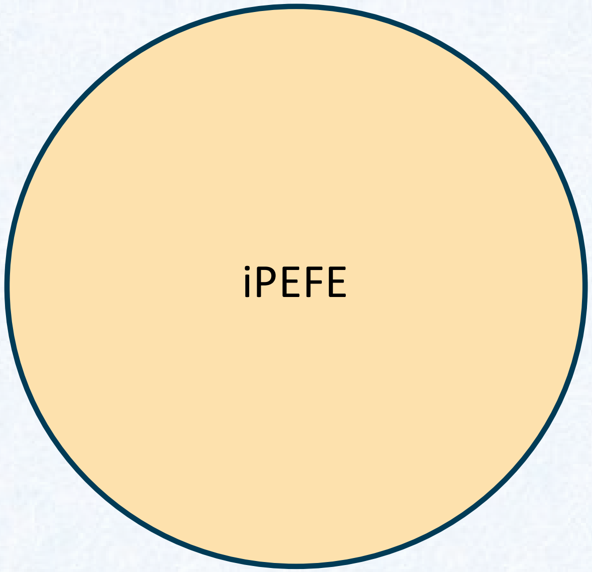


- $a_0 = \frac{2M}{\chi_{1,0}^3}, \ \dot{a} = 0, \quad \ddot{a} < 0$
- $r_0 = a_0 \chi_{1,0} = \frac{2M}{\chi_{1,0}^2}$
- $\rho_0 = \frac{3}{32\pi} \frac{\chi_{1,0}^6}{M^2}$

$B = -\chi_{1,0}^2 < 0$ → $sin^2 \beta = \gamma^2 \Delta \left(\frac{B}{x^2} + \frac{2M}{x^3} \right) > 0$ for $x = r < r_{MAX} = \frac{2M}{\chi_{1,0}^2}$

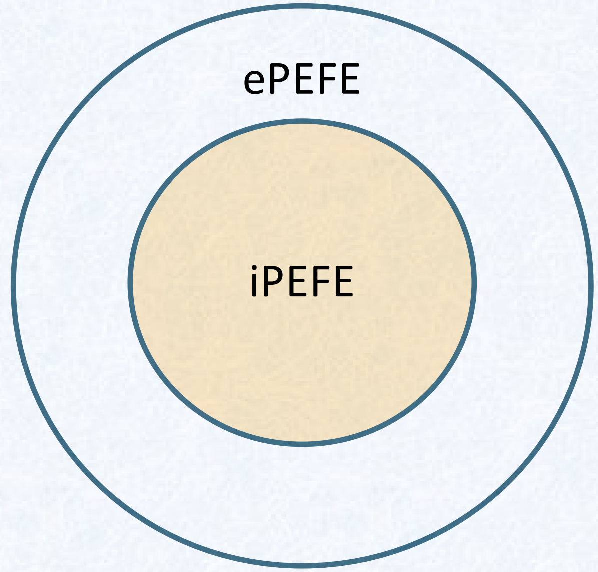
k=1

T_0



For $r < r_0$ the iPEFE are satisfied

$T_1 > T_0$



For $r < r_1$ the iPEFE are satisfied
For $r_1 < r < r_0$ the ePEFE are satisfied

For $r > r_0$ this metric is not a solution to the PEFE

Conclusion

- Static exterior solutions to the Einsten's equation are Schwarzschild-like but depend on two parameters (M and B).
- There may exist other non-static solutions.
  If this possibility is ruled out ⇒ Birkhoff's theorem.
- The OS model is everywhere a solution for $k = 0$ and $k = -1$.
  It is only a local solution ($r \leq r_0$) when $k = 1$. Bad choice of coordinates?