



# IMPROVED EFFECTIVE DYNAMICS OF LOOP-QUANTUM- GRAVITY BLACK HOLE AND NARIAI LIMIT

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KTWiG

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In collaboration with Muxin Han

# LQG Black Hole and Nariai Limit

- We propose a new model of the spherical symmetric LQG black hole with **infinitely many DOFs** and study the effective dynamics of **both interior and exterior** of the black hole
- Classical limit is recovered in low curvature regime. The black hole singularity is **resolved** and replaced by the **non-singular bounce**
- After the bounce, the evolution stabilizes asymptotically to the charged Nariai limit  $dS_2 \times S^2$
- Black hole evaporation and the quantum tunneling proposes a scenario of BH--WH transition on  $dS_2 \times S^2$
- $dS_2 \times S^2$  have infinite zero mode quantum degeneracy: an example of Wheeler's bag of gold

# Reduced Phase Space Formulation

Couple gravity and standard matter fields to clock fields at classical level

Gaussian dust (GD)

[Kuchar and Torre 90  
[Giesel and Thiemann 15

$$S_{\text{GD}} [\rho, g_{\mu\nu}, T, S^j, W_j] = - \int_M d^4y \sqrt{|\det(g)|} \left[ \frac{\rho}{2} (g^{\mu\nu} \partial_\mu T \partial_\nu T + 1) + g^{\mu\nu} \partial_\mu T (W_j \partial_\nu S^j) \right]$$

heat-conducting dust fluid

$T, S^j$  clock fields

$\rho, W^j$  Lagrange multipliers

Dirac observables = parametrizing gravity variables with values of dust fields

$$T(x) \equiv \tau$$

$\tau$  physical time variable

$$S^j(x) \equiv \sigma^j$$

$\sigma^j$  physical space variable

[Rovelli, Dittrich, Thiemann...]

# Reduced Phase Space

Gravity Dirac observables

$$A(\tau, \sigma) = A(x)|_{T(x)=\tau, S^j(x)=\sigma^j}, \quad E(\tau, \sigma) = E(x)|_{T(x)=\tau, S^j(x)=\sigma^j}$$

SU(2) Ashtekar-Barbero connection  $A = \beta K + \Gamma$       Densitized triad

$$\{E_a^i(\sigma, t), A_j^b(\sigma', t)\} = -\frac{1}{2}\kappa\beta \delta_j^i \delta_a^b \delta^3(\sigma, \sigma') \quad \kappa = 16\pi G$$

Physical Hamiltonian:

Gaussian dust

$$\mathbf{H}_0 = \int_{\mathcal{S}} d^3\sigma \mathcal{C}(\sigma) \quad \mathcal{C}^{GR} = \frac{1}{\kappa} [F_{jk}^a - (\beta^2 + 1) \varepsilon_{ade} K_j^d K_k^e] \varepsilon^{abc} \frac{E_b^j E_c^k}{\sqrt{\det(q)}} + \frac{2\Lambda}{\kappa} \sqrt{\det(q)}$$

$$\mathcal{C}_a^{GR} = \frac{2}{\kappa\beta} F_{jk}^b \frac{E_b^k E_a^j}{\sqrt{\det(q)}}.$$

Classical EoMs:

$$\frac{df}{d\tau} = \{\mathbf{H}_0, f\}, \quad \mathbf{H}_0 = \int_{\delta} d^3\sigma h$$

# Black holes: symmetry reduction

Dust space with  $\mathcal{S} \simeq \mathbb{R} \times S^2$  with coordinate  $\sigma = (x, \theta, \phi)$

[Chiou, Ni and Tang 2012

Gauss constraint is solved with gauge fixing s.t. E is diagonal:

[Gambini, Olmedo, Pullin 2013

$$\begin{aligned} E_1^1(\sigma) &= E^x(x) \sin(\theta), & E_2^2(\sigma) &= E^\varphi(x) \sin(\theta), & E_3^3(\sigma) &= E^\varphi(x), \\ A_1^1(\sigma) &= 2\beta K_x(x), & A_2^2(\sigma) &= \beta K_\varphi(x), & A_3^3(\sigma) &= \beta K_\varphi(x) \sin(\theta), \\ A_3^1(\sigma) &= \cos(\theta), & A_3^2(\sigma) &= -\sin(\theta) \frac{E^{x'}(x)}{2E^\varphi(x)}, & A_2^3(\sigma) &= \frac{E^{x'}(x)}{2E^\varphi(x)}. \end{aligned}$$

Symplectic structure

$$\Omega = -\frac{2}{\kappa\beta} \int d^3\sigma [\delta A_j^a(\sigma) \wedge \delta E_a^j(\sigma)] = -\frac{16\pi}{\kappa} \int dx [\delta K_x(x) \wedge \delta E^x(x) + \delta K_\varphi(x) \wedge \delta E^\varphi(x)],$$

Canonical structure:

$$\{K_x(x), E^x(x')\} = G\delta(x - x'), \quad \{K_\varphi(x), E^\varphi(x')\} = G\delta(x - x').$$

Metric is given by:

$$ds^2 = -dt^2 + \Lambda(t, x)^2 dx^2 + R(t, x)^2 [d\theta^2 + \sin^2(\theta) d\varphi^2], \quad \Lambda = \frac{E^\varphi}{\sqrt{|E^x|}}, \quad R = \sqrt{|E^x|}.$$

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Physical Hamiltonian:

$$\mathbf{H}_0 = \int dx \mathcal{C}(x),$$

$$\mathcal{C}(x) = \frac{4\pi \operatorname{sgn}(E^\varphi)}{\kappa \sqrt{|E^x|}} \left( -\frac{2E^x E^{x'} E^{\varphi'}}{E^{\varphi^2}} + \frac{4E^x E^{x''} + E^{x'^2}}{2E^\varphi} - 8E^x K_x K_\varphi - 2E^\varphi [K_\varphi^2 + 1] \right).$$

$$E^{x'} \equiv \partial_x E^x$$

The time evolution by  $\mathbf{H}_0$  has infinitely many conserved charges from spatial diffeomorphisms,

$$\mathcal{V}(N) = \int dx N(x) \mathcal{C}_x(x) \quad \{\mathbf{H}_0, \mathcal{V}(N)\} = 0$$

$$\mathcal{C}_x(x) = E^\varphi(x) K'_\varphi(x) - K_x(x) E^{x'}(x)$$

$$\mathcal{C}(x) \text{ conserved charge when } \mathcal{C}_x(x) = 0$$

1+1 dim field theory which contain infinitely many DOFs.

different from homogeneous Kantowski-Sachs models

# Black holes: symmetry reduction

$$ds^2 = -dt^2 + \Lambda(t, x)^2 dx^2 + R(t, x)^2 [d\theta^2 + \sin^2(\theta) d\varphi^2], \quad \Lambda = \frac{E^\varphi}{\sqrt{|E^x|}}, \quad R = \sqrt{|E^x|}.$$

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General solutions of EoMs  $\frac{df}{dt} = \{f, \mathbf{H}_0\}$  Lemaître-Tolman-Bondi (LTB) spacetime. [Geisel, Tambornino, Thiemann 10']

$$\Lambda(x)^2 = \frac{[\partial_x R(t, x)]^2}{1 + \mathcal{E}(x)}, \quad \partial_t R(t, x) = \pm \sqrt{\mathcal{E}(x) + \frac{\mathcal{F}(x)}{R(t, x)}}$$

$\mathcal{F}$  gravitation mass  
 $\mathcal{E}$  unit mass energy of dust  
 Lemaître-type coordinates,  $t, x$

Schwarzschild solution is given when arbitrary function (can be taken as  $x$ )

$$\mathcal{E}(x) = 0 : \quad R(t, x) = \left[ \frac{3}{2} \sqrt{\mathcal{F}(x)} (f(x) - t) \right]^{2/3}, \quad \mathcal{F} = R_s = 2GM$$

$f(x) = t$  classical singularity

# Improved Hamiltonian

A  $\bar{\mu}$ -scheme regularization from fixed-edge-length holonomies  $h_\Delta(A_1^1), h_\Delta(A_2^2), h_\Delta(A_3^3)$

$$K_\varphi(x) \rightarrow \frac{\sqrt{|E^x|}}{\beta\sqrt{\Delta}} \sin \left[ \frac{\sqrt{\Delta}}{\sqrt{|E^x|}} \beta K_\varphi(x) \right], \quad K_x(x) \rightarrow \frac{E^\varphi}{2\beta\sqrt{\Delta}\sqrt{|E^x|}} \sin \left[ \frac{\sqrt{\Delta}\sqrt{|E^x|}}{E^\varphi} 2\beta K_x(x) \right].$$

s.t. they give the fixed area  $\Delta$  to every plaquette

The effective Physical Hamiltonian:

[Chiou, et al 2012,  
[Bojowald and Swiderski 2005,  
[Gambini, Olemmedo, Pullin,  
...

$$\mathbf{H}_\Delta = \int_{-\infty}^{\infty} dx \mathcal{C}_\Delta(x),$$

$$\mathcal{C}_\Delta(x) = \frac{4\pi \operatorname{sgn}(E^\varphi)}{\kappa \sqrt{|E^x|}} \left( -\frac{2E^x E^{x'} E^{\varphi'}}{E^{\varphi^2}} + \frac{4E^x E^{x''} + E^{x'^2}}{2E^\varphi} - \frac{4E^x E^\varphi}{\beta^2 \Delta} \sin \left[ \frac{\sqrt{\Delta}\sqrt{|E^x|}}{E^\varphi} 2\beta K_x(x) \right] \sin \left[ \frac{\sqrt{\Delta}}{\sqrt{|E^x|}} \beta K_\varphi(x) \right] - \frac{2E^\varphi |E^x|}{\beta^2 \Delta} \sin^2 \left[ \frac{\sqrt{\Delta}}{\sqrt{|E^x|}} \beta K_\varphi(x) \right] - 2E^\varphi \right).$$

Boundary term: 
$$\mathbf{H}_{bdy} = -\frac{8\pi}{\kappa} \left( \frac{\sqrt{E^x} E^{x'}}{E^\varphi} - 2\sqrt{E^x} \right) \Big|_{x=L>0}$$

0 at  $x \rightarrow +\infty$  if we impose asymptotically flatness (Schwarzschild bdy condition)

0 at  $x \rightarrow -\infty$  we impose the Neumann boundary condition  $E^{x'} \sim 0$  as  $x \rightarrow -\infty$

$\mathcal{V}(N^x)$  are still conserved  $\mathcal{C}_\Delta(x)$  does not

$$\mathcal{V}(N^x) = \int dx N^x(x) \mathcal{C}_x(x) \quad \{\mathbf{H}_\Delta, \mathcal{V}(N)\} = 0 \quad \text{with } \mathcal{C}_x(x) \text{ not modified}$$

# Effective dynamics

EoMs from physical Hamiltonian:

Nonlinear PDEs for  $K_x(t, x)$ ,  $K_\varphi(t, x)$ ,  $E_x(t, x)$ ,  $E_\varphi(t, x)$

EoMs for  $E_x > 0$  and  $E_{\tilde{x}} < 0$  are related by spacetime inversion  $\tilde{x} \rightarrow -x, \tilde{t} \rightarrow -t$  and

$$K_{\tilde{x}}(\tilde{t}, \tilde{x}) = K_x(t, x), \quad K_{\tilde{\varphi}}(\tilde{t}, \tilde{x}) = -K_\varphi(t, x), \quad E^{\tilde{x}}(\tilde{t}, \tilde{x}) = -E^x(t, x), \quad E^{\tilde{\varphi}}(\tilde{t}, \tilde{x}) = E^\varphi(t, x)$$

Introducing  $z = x - t$ ,  Stationary solutions

$z$  parametrizes the spatial slice when fixing  $t$ ,  
 $t$  parametrizes the time evolution when fixing  $x$ .

Nonlinear PDEs   $\frac{d}{dz} \begin{bmatrix} E^x \\ E^\varphi \\ K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} f^x(E^x, E^\varphi, K_1, K_2) \\ f^\varphi(E^x, E^\varphi, K_1, K_2) \\ f_1(E^x, E^\varphi, K_1, K_2) \\ f_2(E^x, E^\varphi, K_1, K_2) \end{bmatrix}$ . 1<sup>st</sup> order ODEs

Initial values at  $z = z_0 \gg 0$ : Schwarzschild spacetime

$$E^x(z_0) = \left(\frac{3}{2}\sqrt{R_s}z_0\right)^{4/3}, \quad E^\varphi(z_0) = \sqrt{R_s} \left(\frac{3}{2}\sqrt{R_s}z_0\right)^{1/3}, \quad z=0 \text{ singularity}$$

$$K_x(z_0) = \frac{R_s}{3 \times 2^{2/3} 3^{1/3} (\sqrt{R_s}z_0)^{4/3}}, \quad K_\varphi(z_0) = -\frac{\left(\frac{2}{3}\right)^{1/3} \sqrt{R_s}}{(\sqrt{R_s}z_0)^{1/3}}$$

Recall Schwarzschild solution in Lemaitre coordinate with  $f(x) = x$

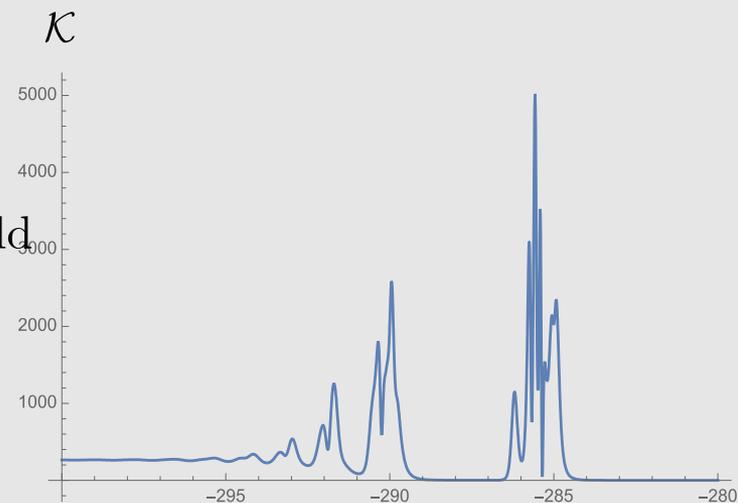
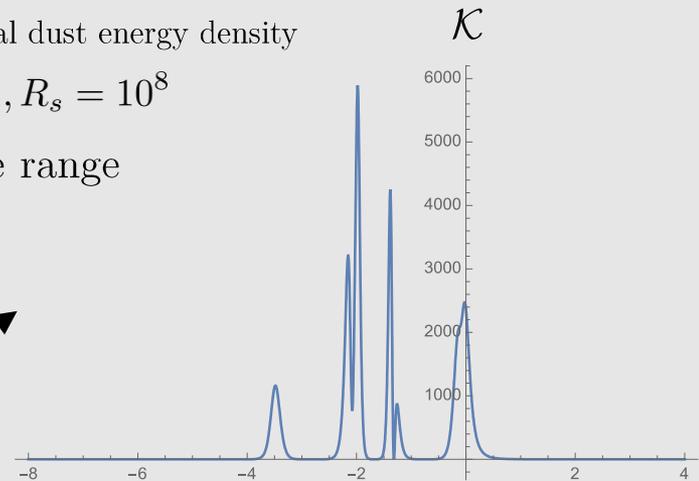
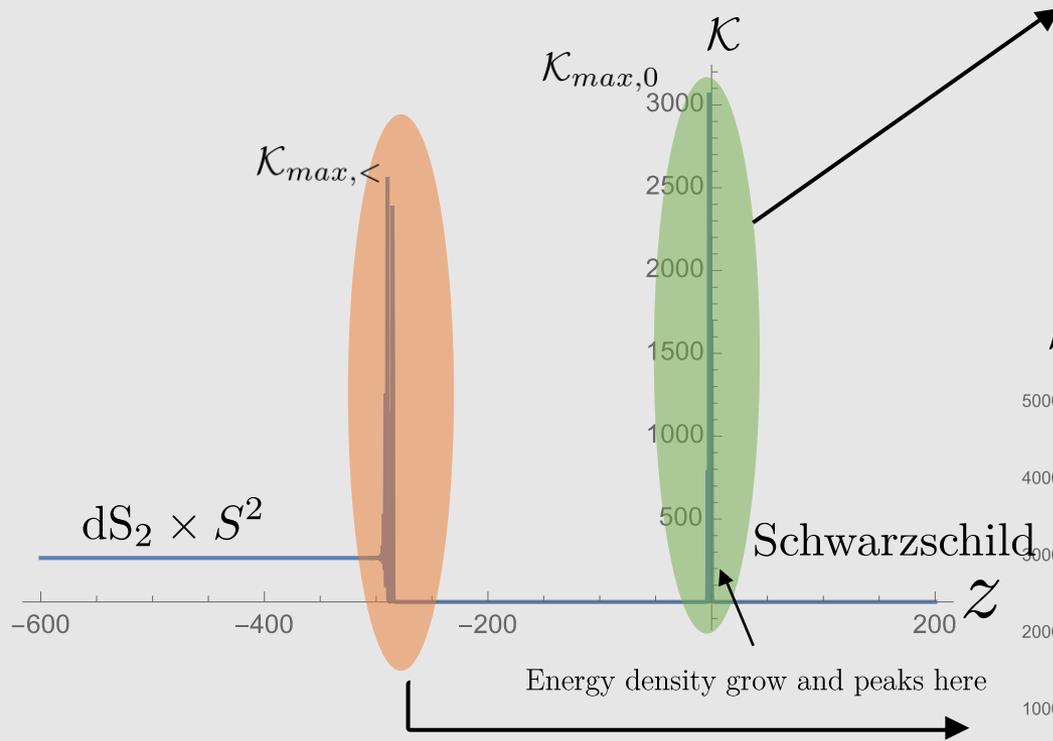
$$\frac{(E^\phi)^2}{|E^x|} = \Lambda^2 = (\partial_x R)^2 = R_s \left[\frac{3}{2}\sqrt{R_s}(x-t)\right]^{-2/3}, \quad \sqrt{|E^x|} = R = \left[\frac{3}{2}\sqrt{R_s}(x-t)\right]^{2/3}$$

# Numerical evaluation

Evaluation with very tiny initial dust energy density

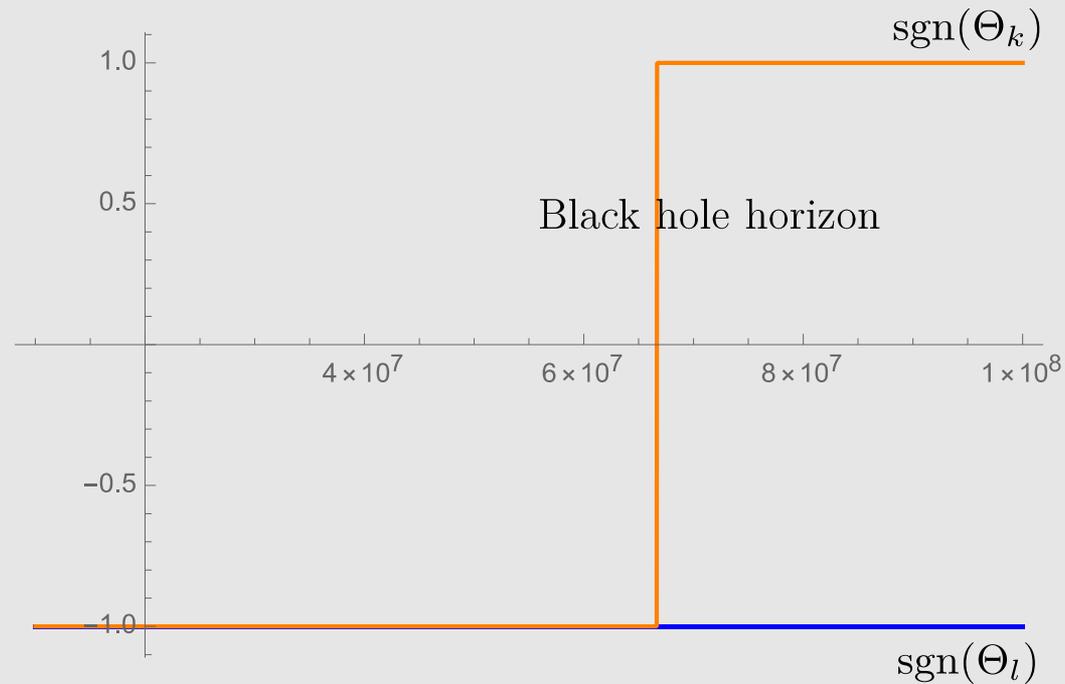
Numerical evaluation with  $z_0 = 3 \times 10^8, \Delta = 0.1, \beta = 1, R_s = 10^8$

- The spacetime curvature is finite on the entire range
- Two local maxima for  $\mathcal{K}$
- Asymptotically Schwarzschild and dS2



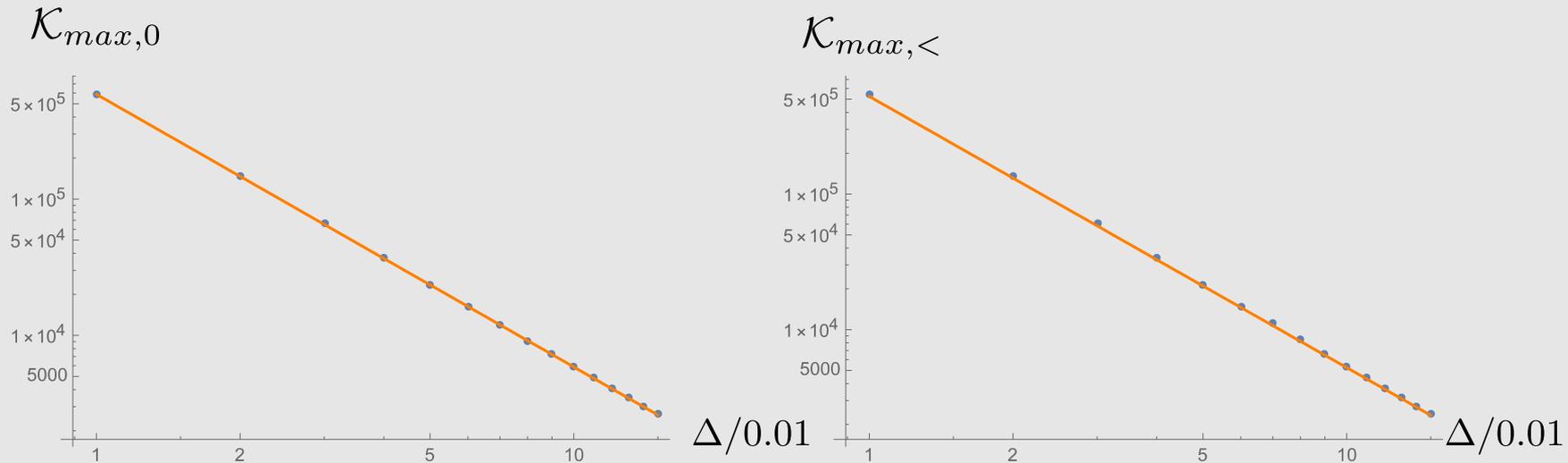
# Numerical evaluation

Null expansions



Quantum correction at the black hole horizon are negligible

# Two local curvature maxima



$$\mathcal{K}_{max,0}|_{\beta=1} \simeq \frac{1}{\Delta^2} k_0(R_s), \quad \mathcal{K}_{max,<}|_{\beta=1} \simeq \frac{k_{<}(R_s)}{\Delta^2} \left[ 1 + \tilde{k}_{<}(R_s) \log(\Delta) \right]$$

# $dS_2 \times S^2$ region ( $z \rightarrow -\infty$ )

The solution approaches  $z \rightarrow -\infty$

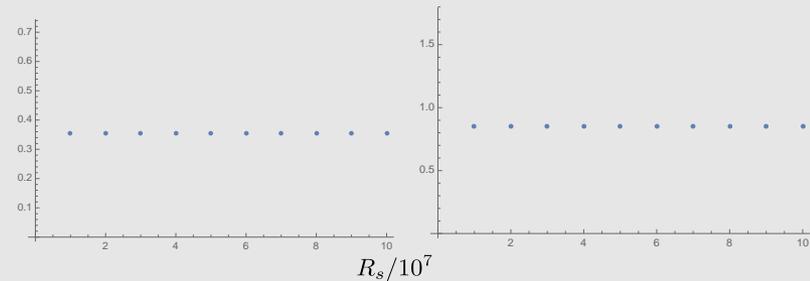
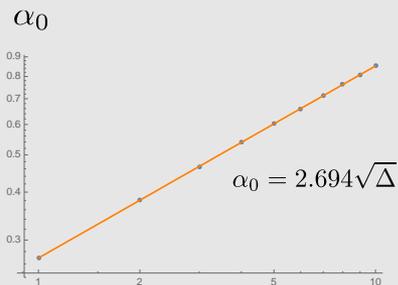
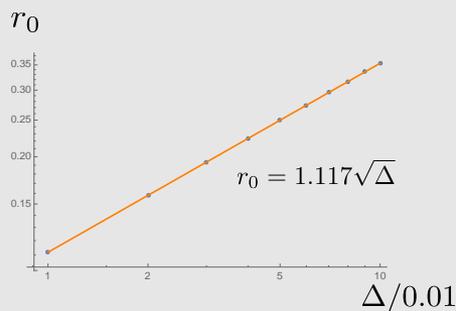
$$E^x(z) \sim r_0^2, \quad \Lambda(z) = \frac{E^\varphi(z)}{\sqrt{E^x(z)}} \sim e^{-\alpha_1 - \alpha_0^{-1}z}, \quad z \rightarrow -\infty$$

A  $dS_2 \times S^2$  metric:  $ds^2 \sim -dt^2 + e^{-2\alpha_1 + 2\alpha_0^{-1}(t-x)} dx^2 + r_0^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$

$\uparrow$   $S^2$  Radius  
 $\uparrow$   $dS_2$  Radius  
 Quantum charged Nariai geometry

$r_0, \alpha_0$  does not depend on  $R_S$ , and they are **Planckian**  $\rightarrow$  stable under perturbation

$$r_0 \simeq 1.11724\Delta^{1/2}, \quad \alpha_0 \simeq 2.69371\Delta^{1/2}, \quad (\text{at } \beta = 1) \quad \mathcal{K} \sim 4 \left( \frac{1}{\alpha_0^4} + \frac{1}{r_0^4} \right) \simeq \frac{2.64325}{\Delta^2}$$

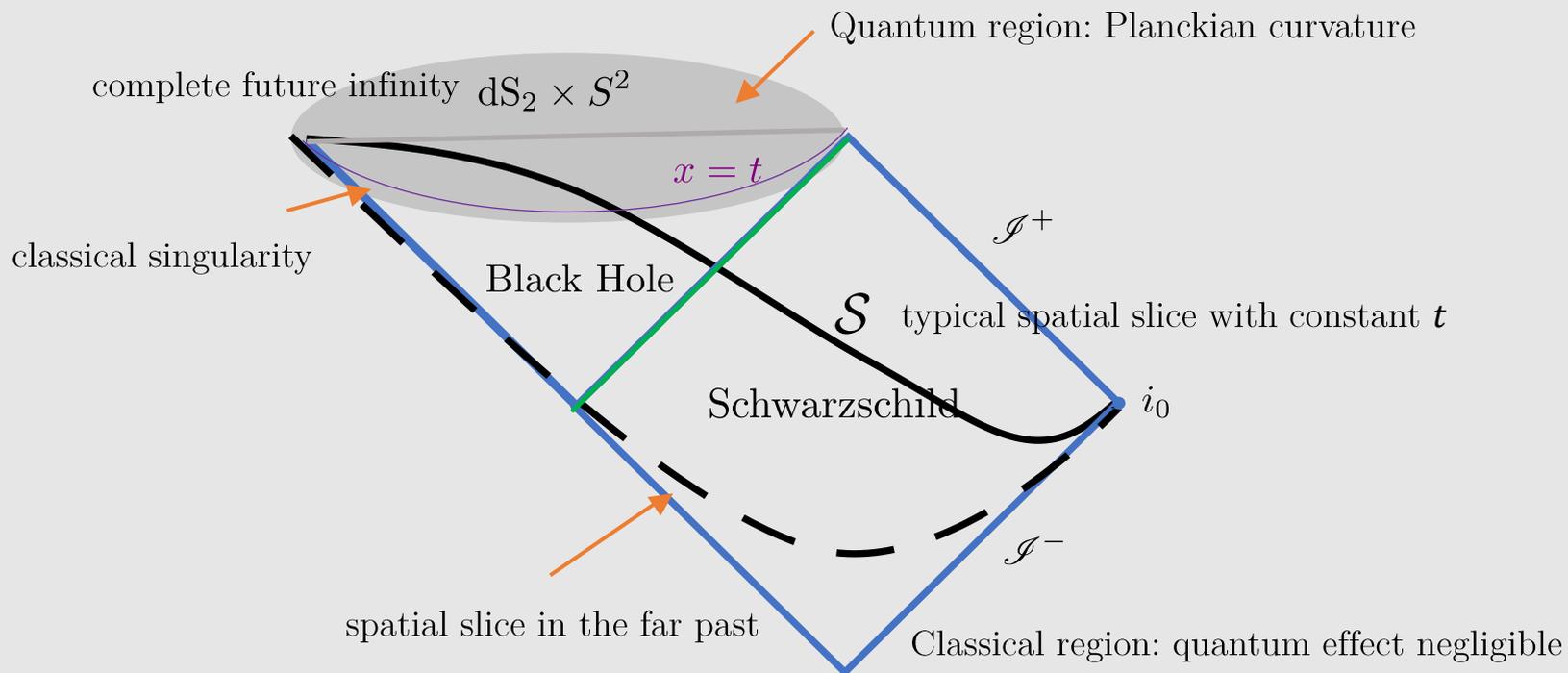


$\alpha_1$  nonlinearly depend on both  $R_S$  and  $\Delta$

Quantum states  $|r_0, \alpha_0; \alpha_1\rangle$  also depend also on  $\alpha_1$   $\rightarrow$  infinite degeneracy



# Effective spacetime diagram



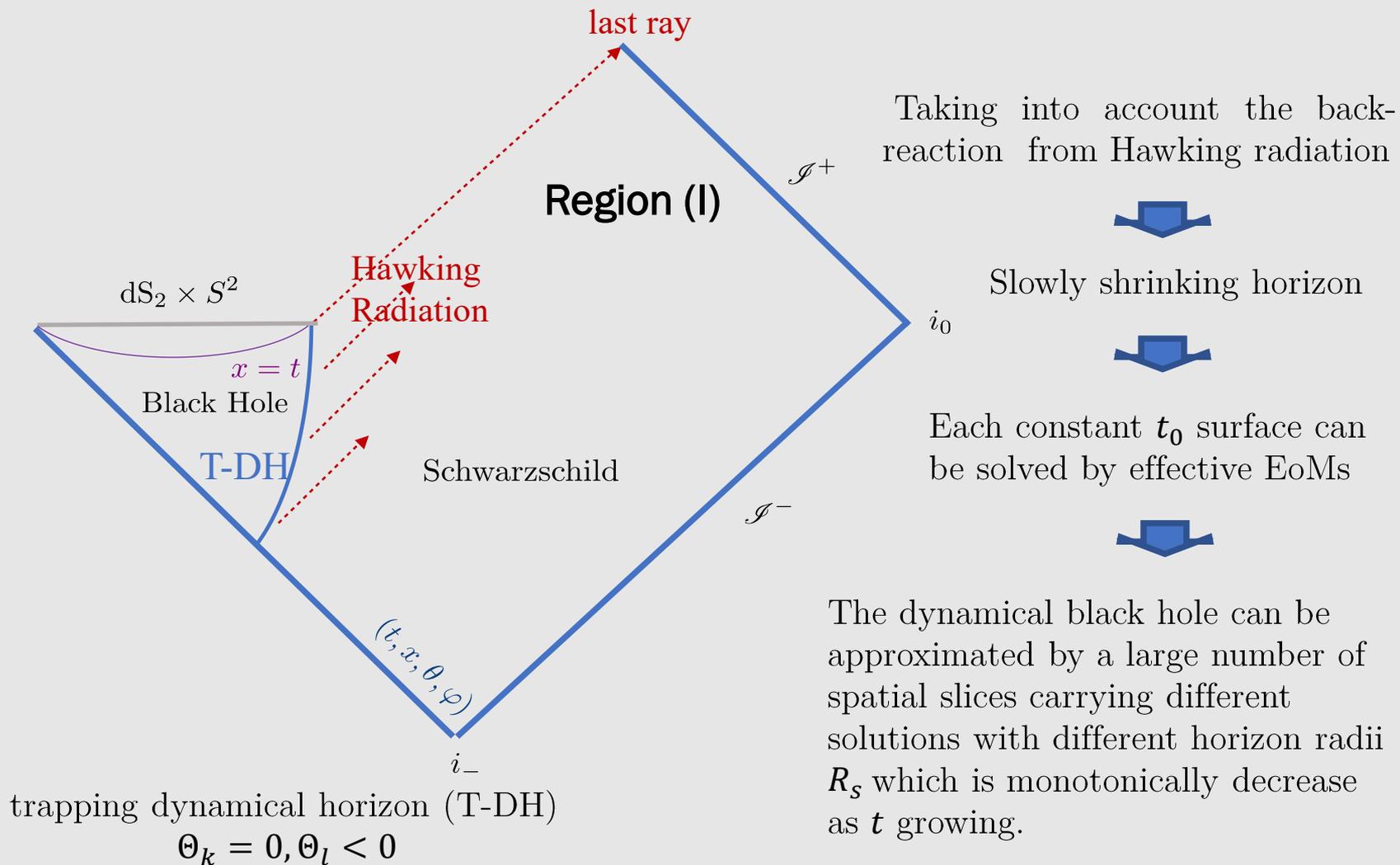
Linear perturbations are bounded from above:

$dS_2 \times S^2$  is stable classically but may be unstable quantum mechanically

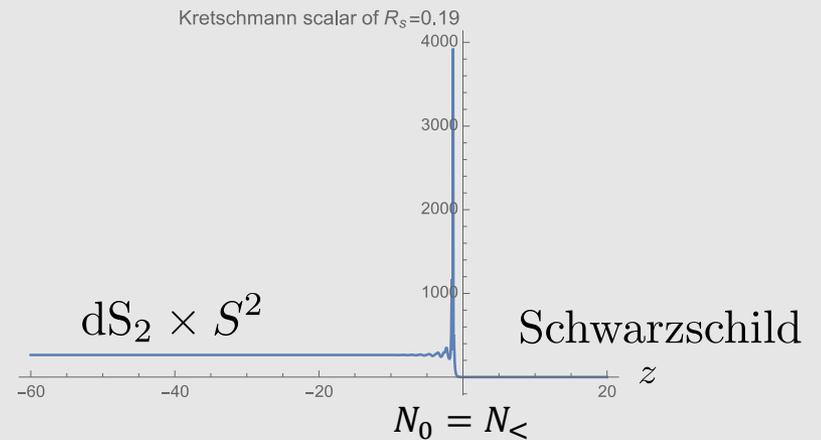
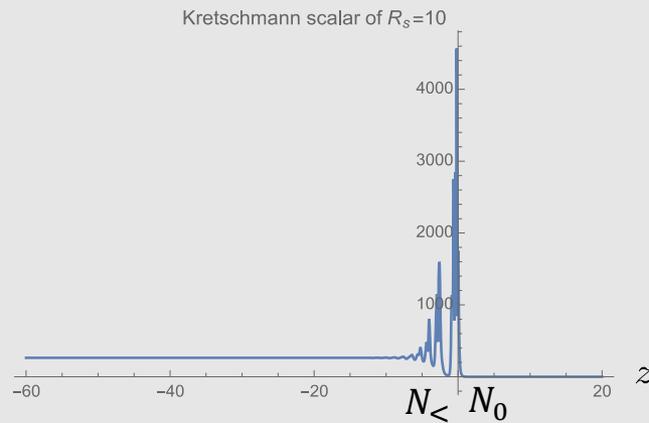
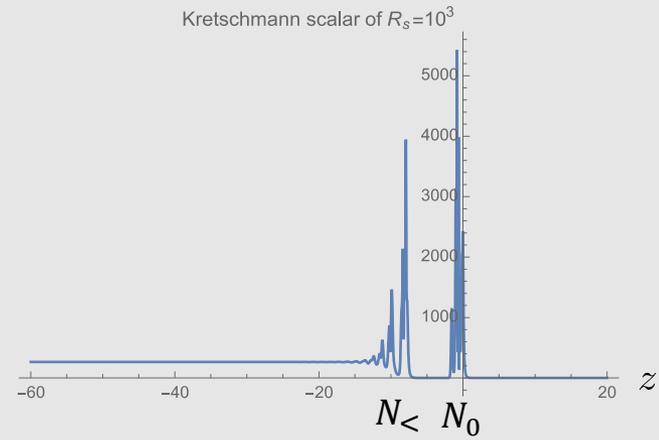
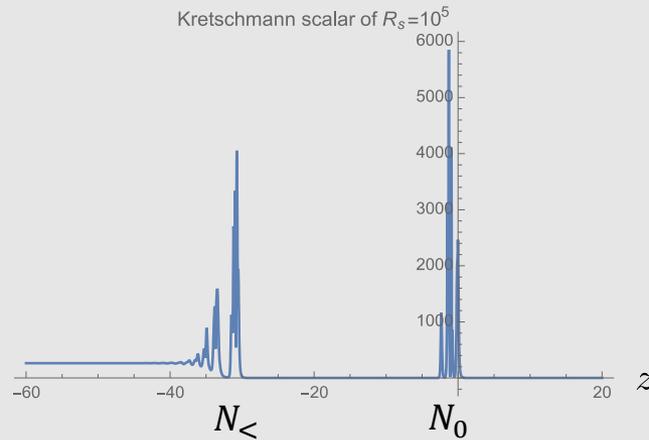
Average null energy condition

from classical effective energy momentum tensor is violated by LQG effect

# Black hole evaporation

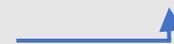


# Black hole evaporation



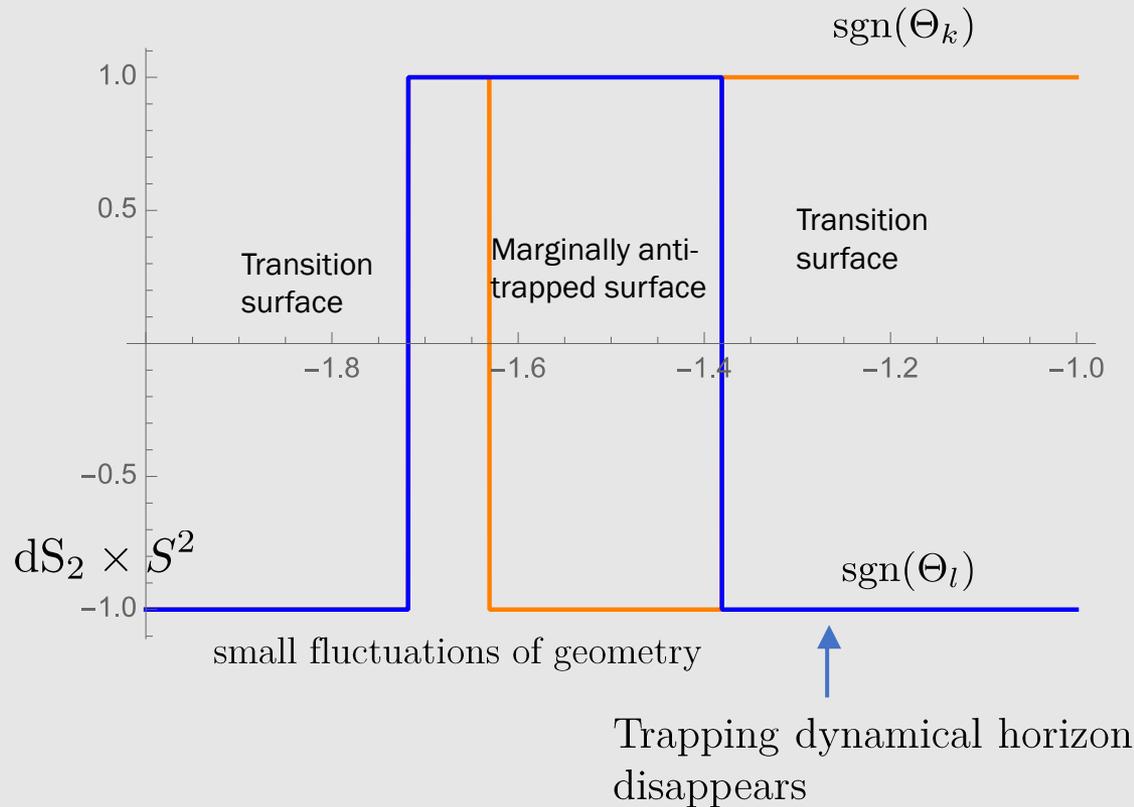
Asymptotic  $dS_2 \times S^2$  is invariant

Two neighborhoods  $N_<$ ,  $N_0$  become closer as  $R_s$  decreasing

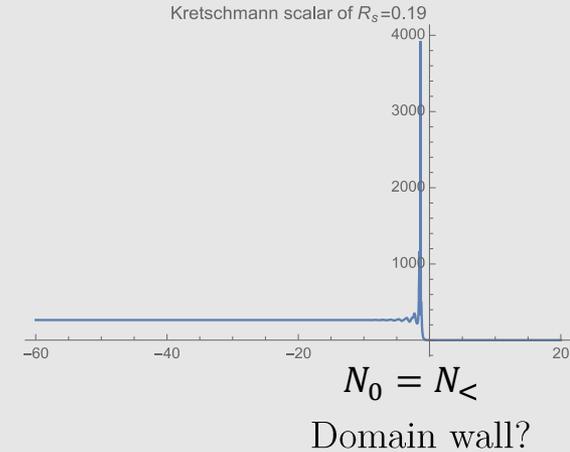


Domain wall?

# Black hole evaporation



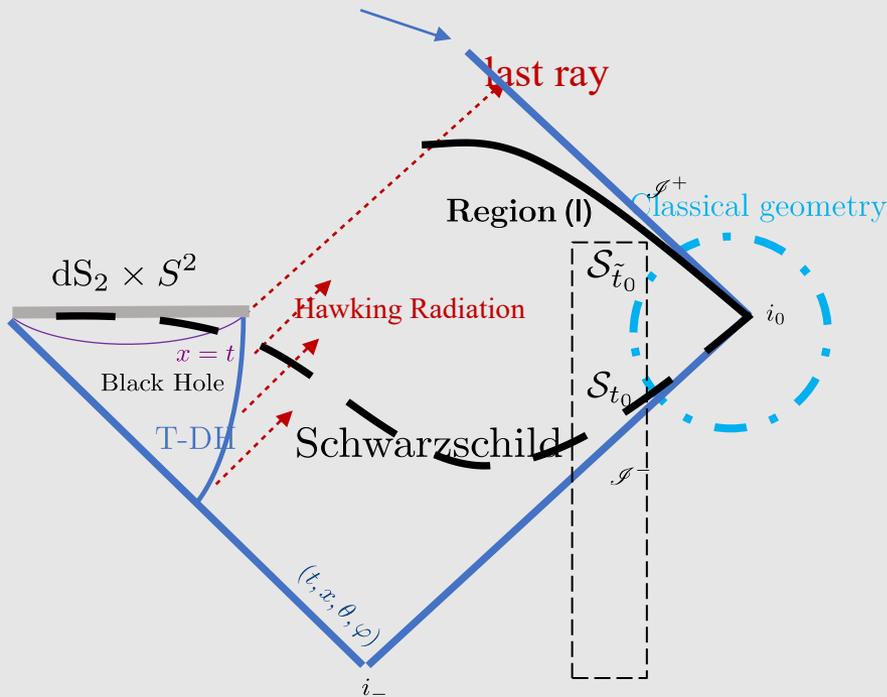
$4\pi E^x > \Delta, 2\pi E^\varphi > \Delta$  self-consistent with  $\bar{\mu}$  scheme



Strong quantum effect  
 Hawking's derivation of evaporation fails  
 Full theory of LQG needed

# Black hole to white hole transition

$\mathcal{I}^+$  should be extended beyond



The extension is derived using effective dynamics

- The quantum dynamics in Region (I) near the spatial infinity  $i_0$  can be well approximated by the quantum field theory on classical background spacetime. The dynamical effect is weak near  $i_0$ .
- For all spatial slices Region (I), their asymptotic (internal and external) geometries near the spatial infinity  $i_0$  are classical and asymptotically flat. Their geometries are continuous extensions from geometries in the past.

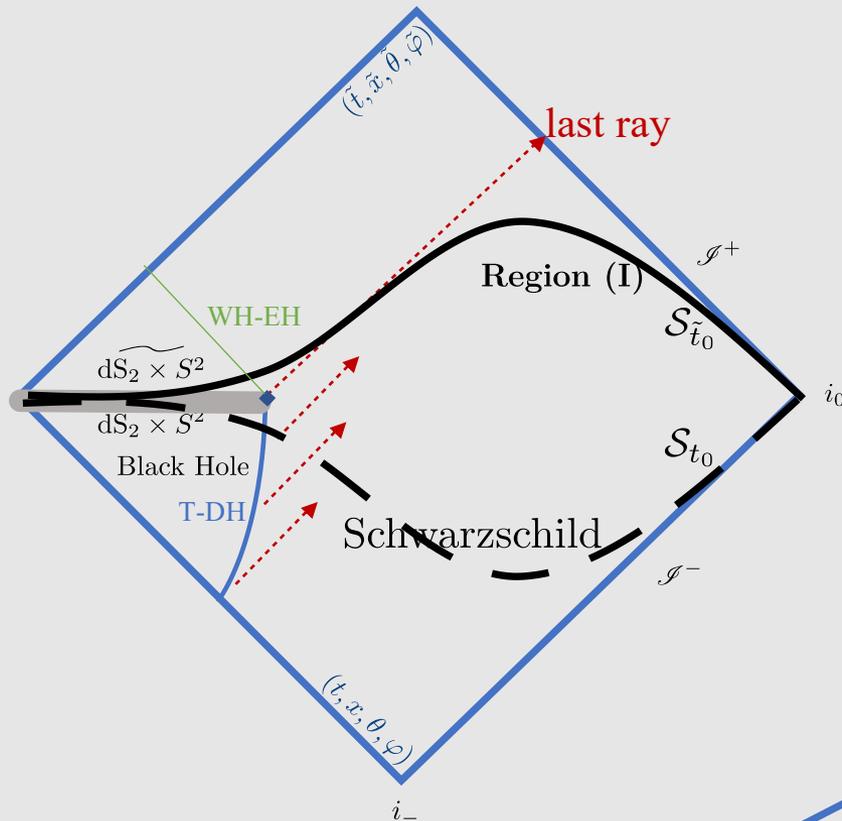
$\mathcal{S}_{t_0}, \tilde{\mathcal{S}}_{t_0}$  asymptotic geometries coincide with each other, their ADM mass are approximately the same: remnant blackhole mass before Hawking radiation stops



Backreaction from Hawking radiation are ignored



# Black hole to white hole transition



EoMs can be solved by reflection symmetry given before

New foliation need to be introduced  $(\tilde{t}, \tilde{x}, \tilde{\theta}, \tilde{\varphi})$  by time reflection symmetry

$$t \rightarrow -t, \quad x \rightarrow -x$$

$$E^{\tilde{x}}(p) = -E^x(\mathcal{R}^{-1}(p)), \quad \mathcal{R} : \mathcal{S}_t \rightarrow \mathcal{S}_{\tilde{t}}$$

$$E^{\tilde{\varphi}}(p) = E^\varphi(\mathcal{R}^{-1}(p))$$

New boundary conditions by classical EoMs

White hole solution: anti-trapped region

$$dS_2 \times S^2 \longrightarrow \widetilde{dS_2 \times S^2}$$

Same  $E_a^j$  but opposite  $K_j^a$

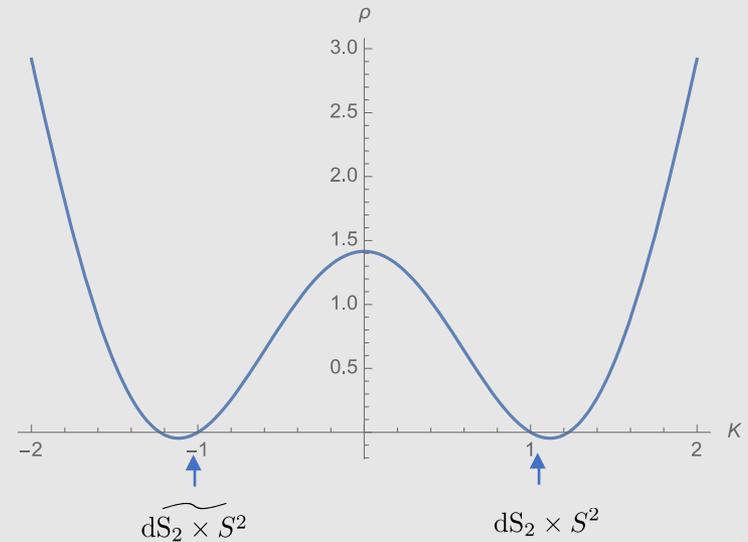
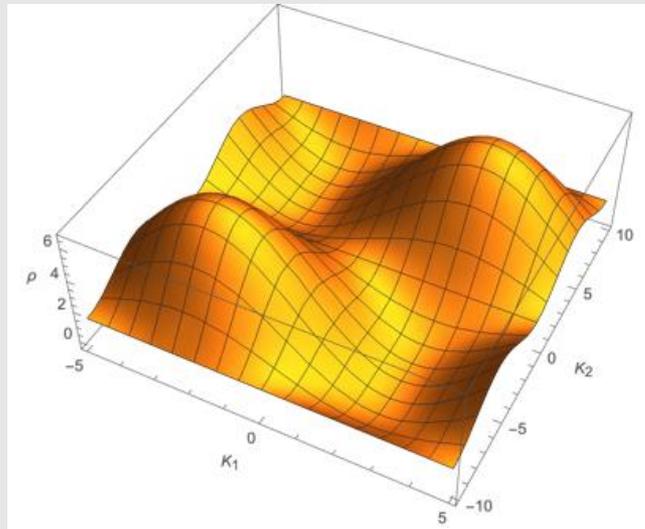
Non-classical transition allowed: quantum tunnelling?

# Evidence of quantum tunneling

Improved Hamiltonian density at  $dS_2 \times S^2$  and  $\widetilde{dS_2} \times S^2$

Reflection symmetry of  $K \rightarrow -K$

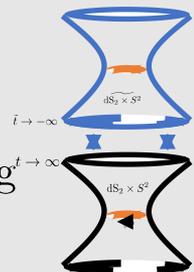
$$\rho = -\frac{C_\Delta}{E^\varphi \sqrt{E^x}} = \frac{1}{2E^x} + \frac{\sin(2\beta\sqrt{\Delta}K_1) \sin(\beta\sqrt{\Delta}K_2)}{\beta^2 \Delta} + \frac{\sin^2(\beta\sqrt{\Delta}K_2)}{2\beta^2 \Delta}$$



double-well effective potential of  $\rho$

$K_x$  fluctuations are large since  $E_x$  is small

Possible quantum tunneling  $t \rightarrow \infty$



# Infinitely many infrared states

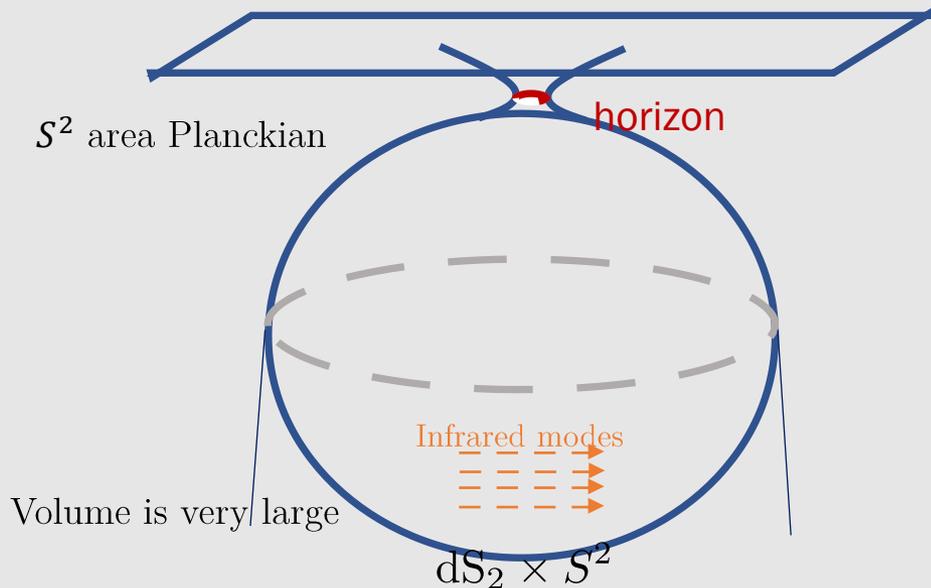
$$\rho = \mathbf{H}_\Delta / \sqrt{\det(q)} = 0 \quad \text{in } dS_2 \times S^2$$

In terms of states  $|r_0, \alpha_0; \alpha_1\rangle \in \mathcal{H}_{dS_2 \times S^2}$

$$\langle r_0, \alpha_0; \alpha_1 | \hat{\rho} | r_0, \alpha_0; \alpha_1 \rangle = 0$$

All states are infrared states, soft modes, and they are infinitely many

An example of Wheeler's bag of gold



Action of diffeomorphisms

$$e^{\widehat{i\varepsilon\mathcal{V}(N)}} |r_0, \alpha_0; \alpha_1\rangle = |r_0, \alpha_0; \alpha'_1\rangle,$$

$$\alpha'_1(x) = \alpha_1(x) + \varepsilon\alpha_0^{-1}N(x) + \varepsilon\partial_x N(x)$$

$\mathcal{H}_{dS_2 \times S^2}$  A representation space of the group of 1-dimensional diffeomorphisms

e.g. Witt algebra

$$[L_m, L_n] = (m - n)L_{m+n}, \quad L_n = -ie^{in\theta} \frac{\partial}{\partial\theta}$$

# Outlook

- From black hole to the Nariai limit
  - Thermalization predicted from  $H_\Delta$
  - Relation with Hawking radiation
- Near the Nariai limit
  - Detailed analysis of quantum tunneling
  - Detailed analysis of infrared modes
- From the Nariai limit to white hole. Quantization of  $H_\Delta$ 
  - Quantum chaos and transition beyond effective theory
- Large dust energy/momentum density and dynamical collapse

Thank you