Smooth extremal black holes and other tales

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Introduction:
Cold black holes

Black holes are simple (or are they?)

Well-known fact

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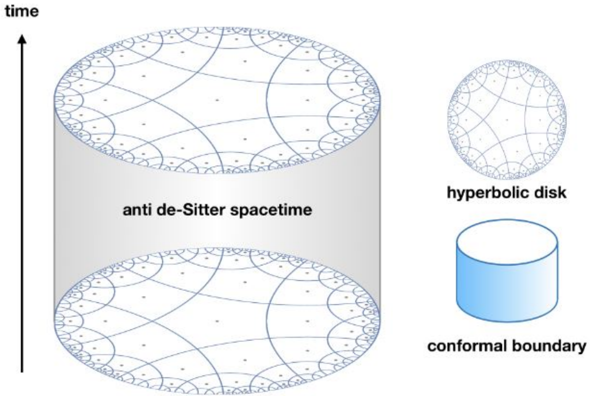
It's no longer true when we include Λ!

Physics in AdS



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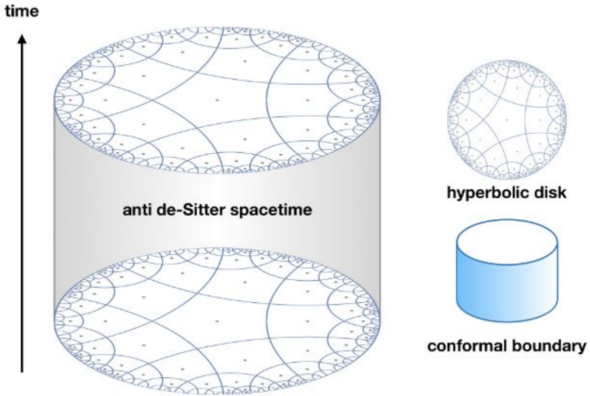

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AdS (even its covering space) is not a globally hyperbolic spacetime.

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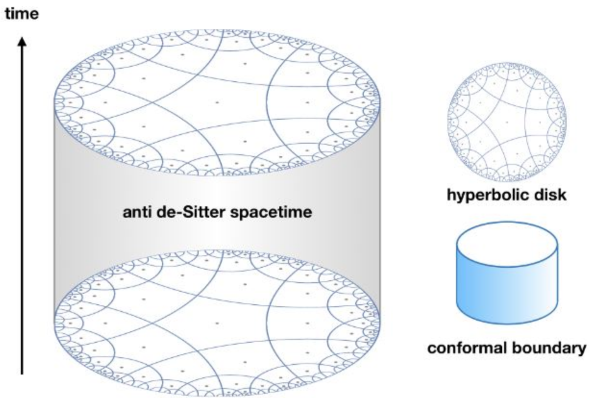

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$$\lim_{r\to\infty} g_{\mu\nu}(r, x^\mu) = h_{\mu\nu}(x^\mu)$$
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Clearly, if h, p are not axially symmetric, the whole spacetime cannot be described by Kerr-Newman-AdS!

Temperature

Let $\ell^a \partial_a$ be a null Killing vector field generating the horizon H. Then, under reasonable assumptions:

$$\nabla_\ell \ell^a |_H = \kappa^{(\ell)} \ell^a \quad (2)$$

and $\kappa^{(\ell)}$ is a constant. It may be identified with the Hawking temperature:

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If $\kappa^{(\ell)} = 0$, we call the horizon extremal.

Why should you care?

- No Hawking radiation – simplified quantum description?
- Motivation for Weak Gravity Conjecture
- In the AdS/CFT dictionary, extremal black holes' horizons describe IR fixed points
- All susy black holes are automatically extremal

Near horizon limit

Near the extremal horizon, we may write the fields as

$$
\begin{aligned}
g &= 2dv\left(dr + rh_a dx^a + \frac{1}{2}Fr^2 dv\right) + \gamma_{ab} dx^a dx^b \\
\mathcal{F} &= \Psi dv \wedge dr + rW_a dv \wedge dx^a + Z_a dr \wedge dx^a + \frac{1}{2}B_{ab} dx^a \wedge dx^b
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and consider a 1-parameter ($\epsilon > 0$) family of diffeomorphisms $\phi_\epsilon(v, r, x^a) = (\epsilon^{-1}v, \epsilon r, x^a)$. Surprisingly, $\phi_\epsilon^\star g, \phi_\epsilon^\star \mathcal{F}$ have good limits as $\epsilon \to 0$ and they define a new solution to the Einstein–Maxwell-(AdS) EOMs which describes only region very closely to the horizon.

NHG equations

The limiting spacetime satisfy a simpler equation $(B = 0)$:

$$\begin{aligned} R_{ab} &= \frac{1}{2}h_a h_b - D_{(a}h_{b)} + \Lambda\gamma_{ab} + \frac{2}{D-2}\gamma_{ab}\Psi^2 \\ (D_a - h_a)\,\Psi &= 0 \\ F &= \frac{1}{2}h^2 - \frac{1}{2}D^a h_a + \Lambda - 2\left(\frac{D-3}{D-2}\right)\Psi^2 \end{aligned} \tag{5}$$

It can be further simplified if we assume that the spacetime is static – then

$$dF = Fh. \tag{6}$$

Extremal BHs – uniqueness

Uniqueness theorem [Chruściel, Tod '07, Kunduri, Lucietti '09]

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This suggests that perhaps extremal black holes are (in some sense) simpler?

Almost all extremal black holes in AdS are singular

Setting

On the background of the extremal RN-AdS

$$g = (\rho - r_+)^2 F(r) dv^2 + 2 dv d\rho + \rho^2 \mathring{\gamma}_{ab} dx^a dx^b, \tag{7}$$

let us consider a massless scalar field $g^{\mu\nu}\nabla_\mu \nabla_\nu \phi = 0$. We are interested in stationary solutions.

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let us consider a massless scalar field $g^{\mu\nu}\nabla_\mu \nabla_\nu \phi = 0$. We are interested in stationary solutions. Since the background is spherically symmetric, we can separate variables: $\phi = \sum_{\ell,m} Y_{\ell m} \phi_{\ell m}(\rho)$. EOMs read:

$$0 = (\rho - r_+)^2 r^2 \phi_{lm,\rho\rho} + ((\rho - r_+)^2 r^2 F)_{,\rho} \phi_{lm,\rho} + l(l+1)\phi_{lm}. \tag{8}$$

Equations near the horizon

$$0 = (\rho - r_+)^2 \rho^2 F \phi_{lm,\rho\rho} + ((\rho - r_+)^2 \rho^2 F)_{,\rho} \phi_{lm,\rho} + l(l+1)\phi_{lm} \quad (9)$$

This is a simple ODE with $\rho = r_+$ being its regular singular point. Thus, near $\rho = r_+$ we can approximate it by Euler equation:

$$0 = (\rho - r_+)^2 r_+^2 F(r_+)\phi''_{lm} + 2(\rho - r_+) r_+^2 F(r_+)\phi'_{lm} + l(l+1)\phi_{lm} \quad (10)$$

and so the leading order term is $\phi_{lm} \sim (\rho - r_+)^\gamma$, where

$$\gamma_\pm = \frac{\pm\sqrt{1 + \frac{4l(l+1)}{\sqrt{1-4Q^2\Lambda}}} - 1}{2} \quad (11)$$

and we of course choose positive solution.

Consequences

If we choose $\ell = 1$, we get $\gamma < 1$ as long as $Q^2\Lambda < 0$. As a result:

$$T_{\rho\rho} \sim (\phi_{,\rho})^2 \sim (\rho - r_+)^{2(\gamma-1)} \to \infty \tag{12}$$

so the energy-momentum tensor is divergent at the horizon.

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They are ALL finite. So maybe that's just a problem with coordinates?

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Result #1

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Result $\#1$

For almost all extremal black holes in AdS tidal forces in the null direction transversal to the horizon are infinite.

Notice that in this example (and in general) this is a statement independent of any asymptotic conditions, spacetime does not even need to be asymptotically AdS!

Setting

Since we are interested only in the near horizon behavior, we may write our (generic yet stationary) fields as $g = \mathring{g} + \delta g$, $F = \mathring{F} + \delta F$, where

$$\mathring{g} = 2\,\mathrm{d}v\left(\mathrm{d}\rho + \rho\, h_a\,\mathrm{d}x^a - \frac{1}{2}\rho^2\, C\mathrm{d}v\right) + q_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b \tag{14a}$$

$$\mathring{F} = E\,\mathrm{d}v\wedge\mathrm{d}\rho + \rho\, W_a\,\mathrm{d}v\wedge\mathrm{d}x^a + \frac{1}{2}B_{ab}\,\mathrm{d}x^a\wedge\mathrm{d}x^b, \tag{14b}$$

and $\delta g, \delta F$ vanish at the horizon (and thus by continuity are small nearby). Thus, it seems reasonable to expect that $(\delta g, \delta F)$ satisfies *linearized* Einstein-Maxwell equations on the background of $(\mathring{g}, \mathring{F})$.

Perturbations

Due to the symmetries, we may decompose our perturbations into eigenspaces of $\rho\partial_\rho - v\partial_v$. They are thus of the form

$$\delta g = \rho^\gamma \left(\delta F\, \rho^2\, \mathrm{d}v^2 + 2\, \rho\, \delta h_a\, \mathrm{d}v\, \mathrm{d}x^a + \delta q_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b\right)$$

$$\delta \mathcal{F} = \rho^\gamma \Big(\delta E\, \mathrm{d}v \wedge \mathrm{d}\rho + \rho\, \delta W_a\, \mathrm{d}v \wedge \mathrm{d}x^a + \rho^{-1} \delta Z_a\, \mathrm{d}\rho \wedge \mathrm{d}x^a + \frac{1}{2}\, \delta B_{ab}\, \mathrm{d}x^a \wedge \mathrm{d}$$

It is not hard to check that this implies

$$\delta C_{\rho a \rho b} \sim \gamma(\gamma - 1)\rho^{\gamma-2} \tag{15a}$$

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so the perturbations are singular, provided that $\gamma < 2$ (and $\gamma \neq 1$). This does not necessarily imply that those black holes are not weak solutions. This happens only when $\gamma \leq \frac{1}{2}$ [Christodoulou '09].

Static background

If $\mathring{g}$ is static, it follows that q_{ab} is maximally symmetric and we may further decompose perturbations with respect to the symmetries of q:

$$(\Delta_q + \frac{k^2}{r_+^2})\mathbb{S} = 0 \tag{16a}$$

$$\mathbb{S}_a = \frac{1}{k} D_a \mathbb{S} \tag{16b}$$

$$\mathbb{S}_{ab} = D_a D_b \mathbb{S} + \frac{q_{ab} k^2}{2r_+^2}\mathbb{S} \tag{16c}$$

and then $\delta q_{ab} = h_L \mathbb{S} + h_T \mathbb{S}_{ab}$ etc. For concreteness we will work only with scalar modes, vector ones in 4d have the same exponents.
In this way, we reduce to linearized Einstein-Maxwell equations into a system of homogeneous linear (algebraic) equations. Thus, non-trivial solutions exists only when appropriate determinant vanishes and this leads to conditions for γ. For each mode we find four possible values.

Spherical RN-AdS

If the cross-sections are spherical, we find

$$\gamma_{\pm\pm} = \frac{1}{2}\left[-1 \pm \sqrt{\frac{4\ell(\ell+1) + 5\sigma \pm 4\sqrt{\sigma^2 + 2\ell(\ell+1)(1+\sigma)}}{\sigma}}\right], \quad (17a)$$

where

$$\sigma \equiv 1 + \frac{6\, r_+^2}{L^2}. \quad (17b)$$

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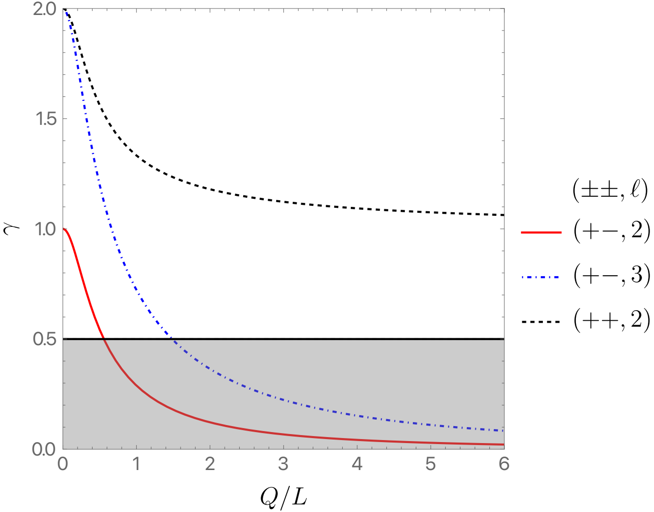
$\ell = 1$ mode is exceptional (since there are no gravitational dipoles) and there are only two possible values of γ:

$$\gamma_{\pm} = \frac{1}{2}\left(-1 \pm \sqrt{\frac{16 + 9\sigma}{\sigma}}\right). \quad (18)$$

Exponents



Exponent as a function of Q and $\Lambda = -\frac{3}{L^2}$

Toroidal RN-AdS

We normalize q in such a way that the cross-section area is $L_x L_y r_+^2$ with $x \sim x + L_x$ and $y \sim y + L_y$. Then,

$$\gamma_{\pm\pm} = \frac{1}{6}\left(\pm\sqrt{45 + 6\tilde{k}^2 \pm 36\sqrt{1 + \frac{\tilde{k}^2}{3}}} - 3\right), \tag{19}$$

where $\tilde{k} \equiv kL/r_+$. There is a qualitative difference: if we take r_+ sufficiently small, $\gamma_{+\pm}$ can become arbitrarily large and we avoid singularity (although we still deal with solutions of finite smoothness). For a torus obtained from a square with $L_x = L_y = 2\pi$, we have C^2 solutions if

$$\frac{r_+}{L} < \frac{1}{\sqrt{36 + 12\sqrt{3}}}. \tag{20}$$

Hyperbolic RN-AdS

We normalize the radius r_+ in such a way that $R = -\frac{2}{r_+^2}$. For simplicity, let us take $Q = 0$.

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$$\gamma = \frac{1}{2}\left(-1 + \sqrt{9 + 4k^2}\right), \tag{21}$$

so this leads to singularity if we may have $k^2 < 4$. The first non-zero eigenvalue of the Laplacian on a compact Riemann surface is bounded by

$$k^2 \leq \frac{2}{g-1}\lfloor\frac{g+3}{2}\rfloor \leq 4.$$

There is a better bound for $g = 2$: [Bonifacio '21]

$$k^2 \leq 3.8388977,$$

so generic perturbation is not C^2 as well.

Hyperbolic BHs cnd

When the black hole is charged, the situation is very different. Gravitational and Maxwell perturbations are then coupled to each other, and the two physical exponents become:

$$\gamma_{+\pm} = \frac{1}{2}\left[-1 + \sqrt{5 + \frac{4k^2 \pm 4\sqrt{\sigma^2 + 2(\sigma-1)(k^2+2)}}{\sigma}}\right], \qquad (22)$$

where $\sigma = 6\frac{r_+^2}{L^2} - 1$. The minimal radius of the hyperbolic extremal horizon is obtained with no charge, $r_+ = \frac{L}{\sqrt{3}}$, so $\sigma \geq 1$. Notice that when $\sigma > \frac{1}{4}\left(4 + 2k^2 + k^4\right)$, γ_{+-} becomes *negative*. Thus the perturbation blows up on the horizon and our perturbative scheme breaks down. It is likely that some curvature invariants will now diverge. This also signals an RG instability - a small change in the boundary conditions at asymptotic infinity (UV) would lead to a drastic change in the near horizon (IR) region.

A few remarks

- The bigger r_+, the bigger divergence we get and $\gamma \to 0$ as $Q^2 \to \infty$
- For r_+ large enough, also modes with higher ℓs are divergent
- Although we have restricted ourselves to the electrically charged black holes, the same holds for magnetic ones
- and for $\Lambda > 0$ although the results are less spectacular

Kerr AdS

We can analyze also stationary Kerr-AdS perturbations. It's easier to use the Teukolsky equations rather than deal with metric perturbations. Spin $s = +2$ field describes at the horizon exactly $C_{\rho a \rho b}$ which is our key observable.

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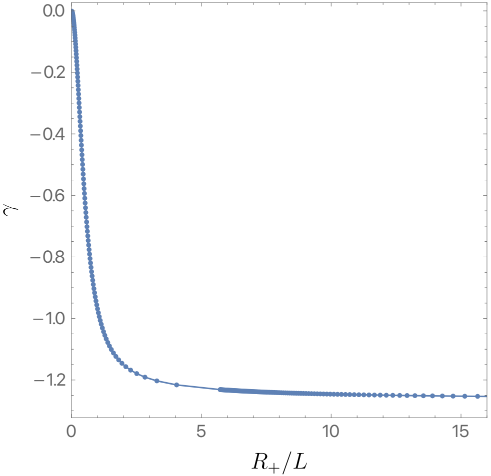
We can investigate it almost analytically - the angular equation (or rather: eigenproblem) must be solved numerically. From the radial one we get $C_{\rho a \rho b} \sim (\rho - r_+)^\gamma$ with γ being the function of the angular eigenvalue.

Kerr AdS

For $\ell = 2$, $m = 0$ we find



Exponent as a function of the area radius

Kerr AdS

As we saw, $s = 2, \ell = 2, m = 0$ mode is always singular at the horizon. However, this singularity is milder than for RN-AdS ($\gamma_{metric} > \frac{1}{2}$ for Kerr-AdS).
However, if we additionally break $U(1)$ symmetry ($m \neq 0$), then certain modes have complex γ with $\mathrm{Re}\,\gamma = -\frac{5}{2}$. This is a sign of superradiant instability. We still don't know what's the endpoint of this instability.

Linear vs non-linear analysis

We presented our reasoning as a linearized analysis. However, it is a little more than that. Have we considered instead a metric $g^{RN} + (r - r_+)^n \delta g$, and solved full Einstein equations near the horizons, we would arrive at the same solutions with the same conditions for n.
Thus, our results are exact *as long as* perturbations exhibit power-law behavior near the horizon. We are unable to account for non-perturbative corrections. To this end, we need to solve numerically Einstein equations with appropriate ($SO(3)$-breaking) boundary conditions at infinity.

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For $T \approx 0$, there is a huge near horizon region in which the spacetime looks like $T = 0$. In particular, we have AdS_2 factor. Thus, we expect that in this region tidal forces should have power-like growth. However, this region ends shortly before the horizon and then we go to the horizon which is located at $\rho - r_+ \sim T$.

Finite temperature

One can argue that $C_{rarb} \sim T^{\gamma-2}$ with the same exponent γ as at $T = 0$ we have $C_{rarb} \sim (\rho - r_+)^{\gamma-2}$. This is the best way to probe these exponents numerically with a high accuracy.

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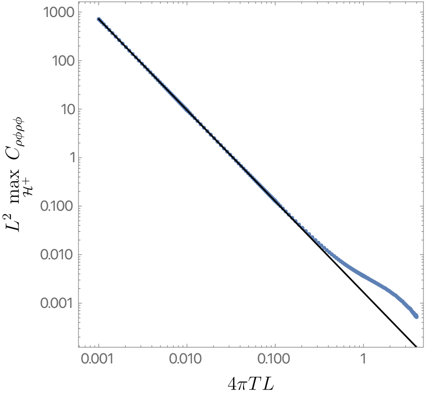
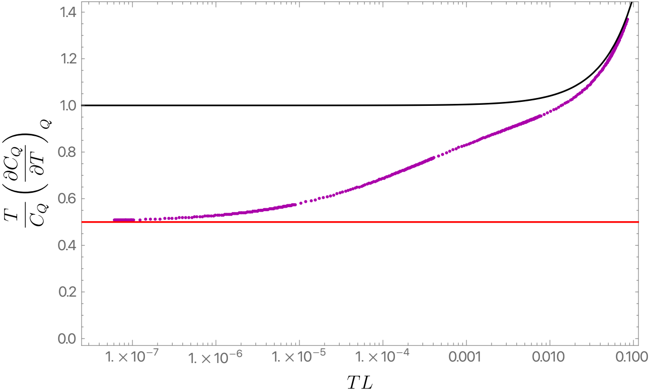


$\gamma_{numerical} \approx 0.122401$ vs $\gamma_{analytical} \approx 0.122025$

Finite temperature

By going to the second order in the perturbation theory, we can argue that $\delta S \sim T^{2\gamma}$. If $\gamma < \frac{1}{2}$, then, this is dominating over the usual relation $S^{RN-AdS} = S_0 + S_1 T + O(T^2)$. In particular, we would have $C_Q = 2\gamma T^{2\gamma} + O(T)$ – this is a clear sign for the holographic theory!



A (very) short introduction to EFTs

There are many reasons to believe that GR is in fact only a low-energy description of whatever the fundamental theory is. Instead of constructing UV completion, we could parametrize it as a series in derivatives (that could be apriori derived from that theory). The first non-trivial terms are

$$\mathcal{L}_6 = \frac{\eta\kappa^2}{2} R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab} \tag{23a}$$

and

$$\mathcal{L}_8 = \frac{\lambda\kappa^4}{2}(R_{abcd}R^{abcd})^2 + \frac{\tilde{\lambda}\kappa^4}{2}(\tilde{R}_{abcd}R^{abcd})^2 \tag{23b}$$

We can ask what happens with the extremal Kerr for such a theory (keeping only terms linear in $\eta, \lambda, \tilde{\lambda}$).

Strategy

It is possible to find the EFT-corrected near-horizon limit of Kerr. Then, on this background, we may look for transversal deformations. The whole thing can be written down as sourced linearized Einstein equations. At the end of the day, we find that the leading exponent is

$$\gamma = 2 + \frac{24\kappa^4\eta}{7J^4} - \frac{21(32+45\pi)\kappa^6\lambda}{5J^3} - \frac{12(736+315\pi)\kappa^6\tilde{\lambda}}{5J^3} + \dots \tag{24}$$

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Note that this describes 's-wave' transversal deformation, which comes just from the fact that our Kerr black hole is asymptotically flat and not only near-horizon!

We thus see that whether we have a singularity depends highly on signs of η, λ and $\tilde{\lambda}$.

Signs

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One can prove (using the analyticity and unitarity of the S matrix) that $\lambda, \tilde{\lambda} > 0$ in any consistent theory. That's good because these Wilson coefficients come with a minus sign to the exponent.

On the other hand, η has a logarithmic divergence. Close to the UV cutoff, it behaves like

$$\eta \sim \sum \left(\frac{1}{m_s^2} - \frac{4}{m_f^2} + \frac{3}{m_v^2} \right). \tag{25}$$

Signs

One can prove (using the analyticity and unitarity of the S matrix) that $\lambda, \tilde{\lambda} > 0$ in any consistent theory. That's good because these Wilson coefficients come with a minus sign to the exponent.
On the other hand, η has a logarithmic divergence. Close to the UV cutoff, it behaves like

$$\eta \sim \sum \left(\frac{1}{m_s^2} - \frac{4}{m_f^2} + \frac{3}{m_v^2} \right) . \tag{25}$$

In particular, it is negative for the Standard matter content! That is good, because it enters the exponent with a plus sign. On the hand, the (two-loop) log divergences, make it positive in the infrared. Somehow surprisingly, this is actually negligible even for supermassive black holes.

Further directions

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THANK YOU
FOR YOUR ATTENTION!