Progress in relational quantum dynamics

Philipp Höhn

Okinawa Institute of Science and Technology
&
University College London

Theory of Relativity Seminar, Warsaw,
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based on works with L. Chataignier, M. Lock, A. Smith, A. Vanrietvelde;
and older ones with B. Dittrich, M. Nelson, T. Koslowski,…

Problem of time in a nutshell

reparametrization (temporal diffeo) invariance implies *Hamiltonian constraint*:

$$\hat{C}_H \left|\psi_{\text{phys}}\right\rangle = 0$$

"timeless"? $\Rightarrow$ **background timeless, but not internally timeless**

$\Rightarrow$ **relational quantum dynamics**

Relational dynamics in a nutshell

[DeWitt ‘60s; Rovelli ‘90s+; Dittrich ‘00s; Page, Wootters ‘80s; Isham; Kuchar; …]

All measurements in real world relational:

$$\Downarrow$$

Premise: no external reference, all reference systems/frames are internal and physical

How do we describe physics relative to dynamical clock reference?

what is a temporal reference system?

- As non-invariant/asymmetric under C_H induced gauge symmetry as possible (invariants worst possible reference systems)

  $\Rightarrow$ want to parametrize orbits with clock DoFs

$\Rightarrow$ **reference DoFs are gauge DoFs**

Many faces of the problem of time

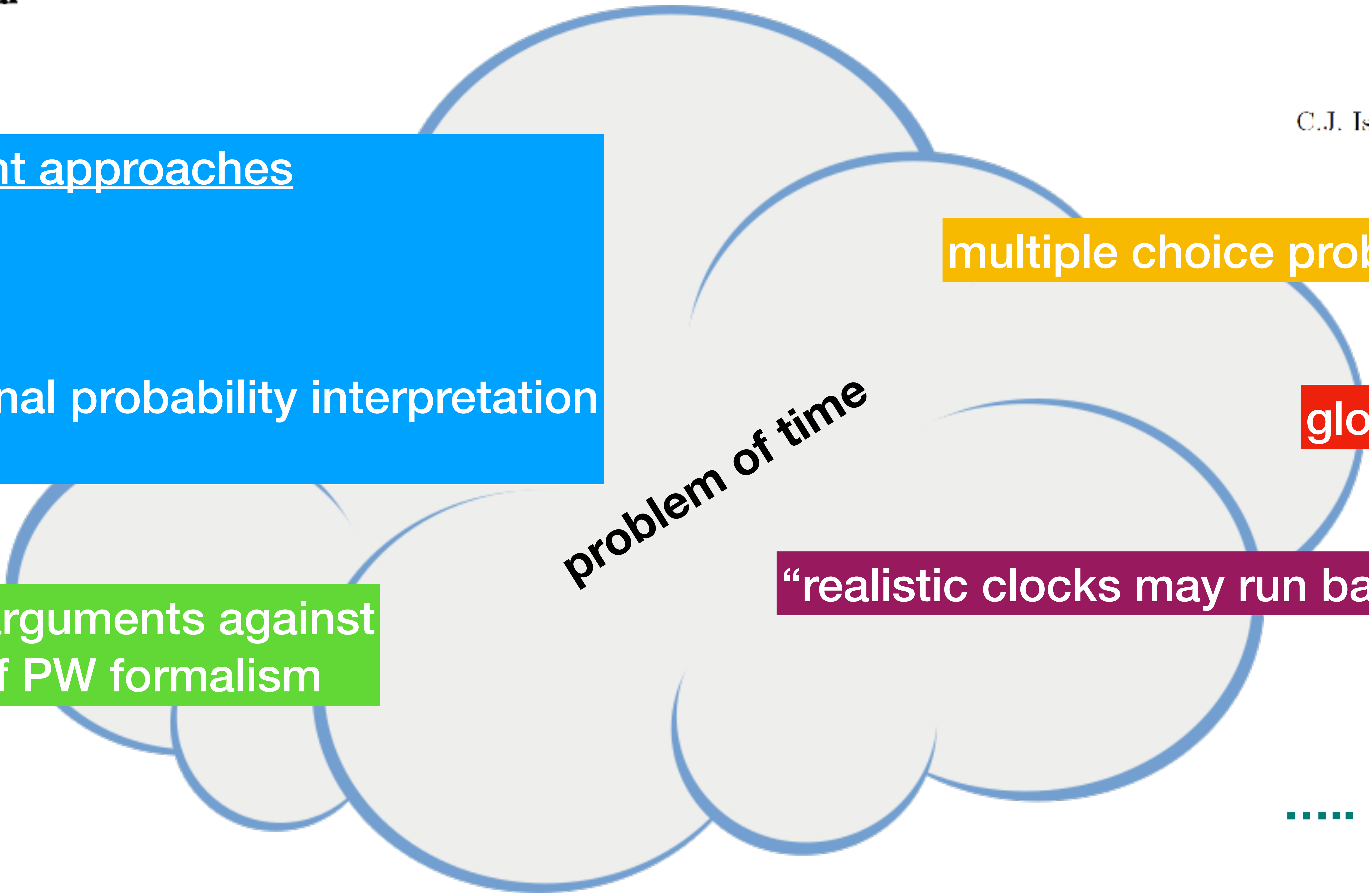
TIME AND INTERPRETATIONS OF QUANTUM GRAVITY

Karel V. Kuchař

Canonical Quantum Gravity and the Problem of Time [1] [2]

C.J. Isham

1992 reviews



<u>many independent approaches</u>

- ...
- Relational observables
- Deparametrizations
- Page-Wootters (PW) conditional probability interpretation
- ...

multiple choice problem

global time problem

"realistic clocks may run backward"

Kuchar's 3 arguments against viability of PW formalism

..... and many more

see also Anderson '17

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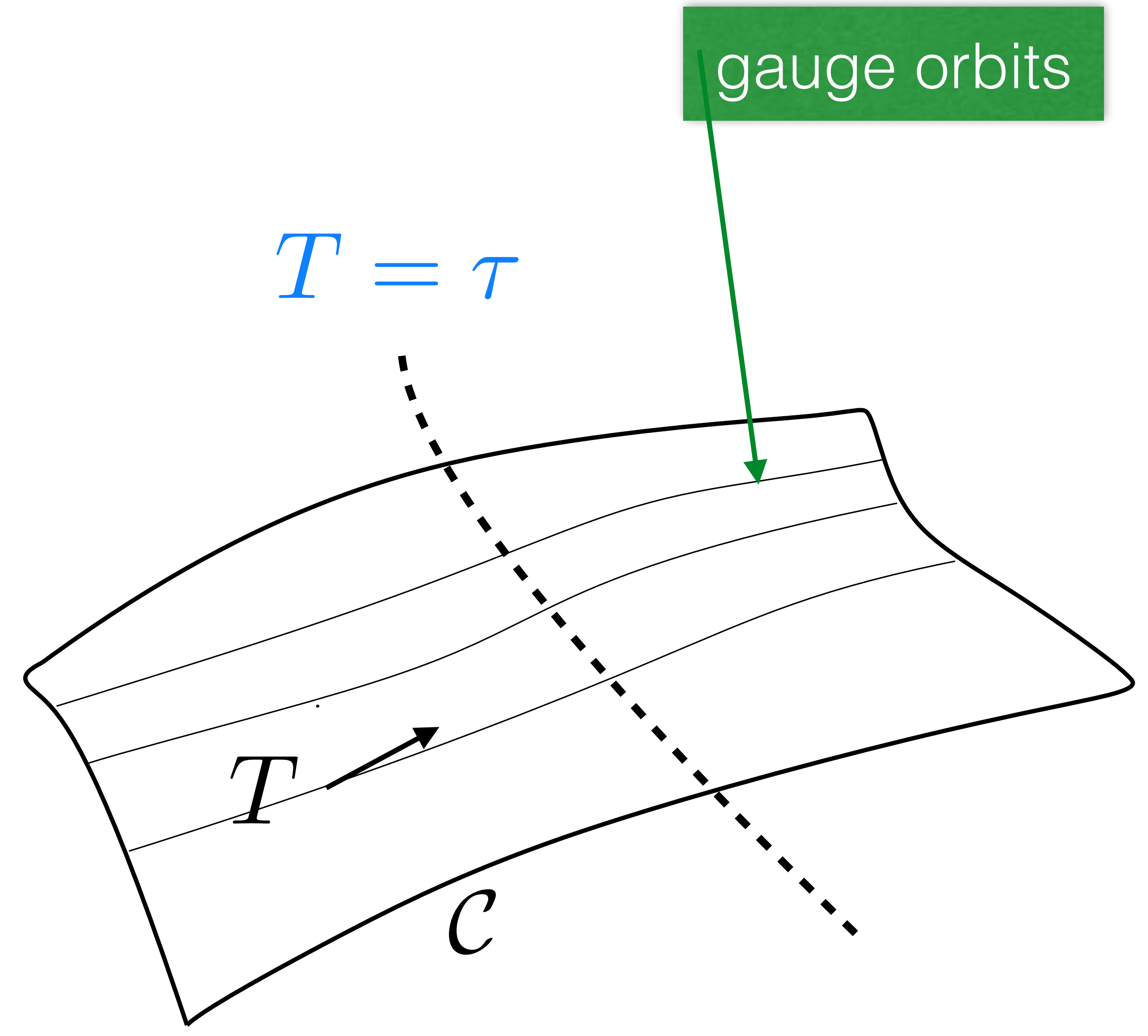
I. Relational observables

Rovelli, Dittrich, QG community

Observable that encodes how some observable f evolves relative to some dynamcial time variable T

What is value of f when clock T reads τ?



gauge-inv. evol. rel. to T
=
“scanning with T=const surfaces through constraint surface”

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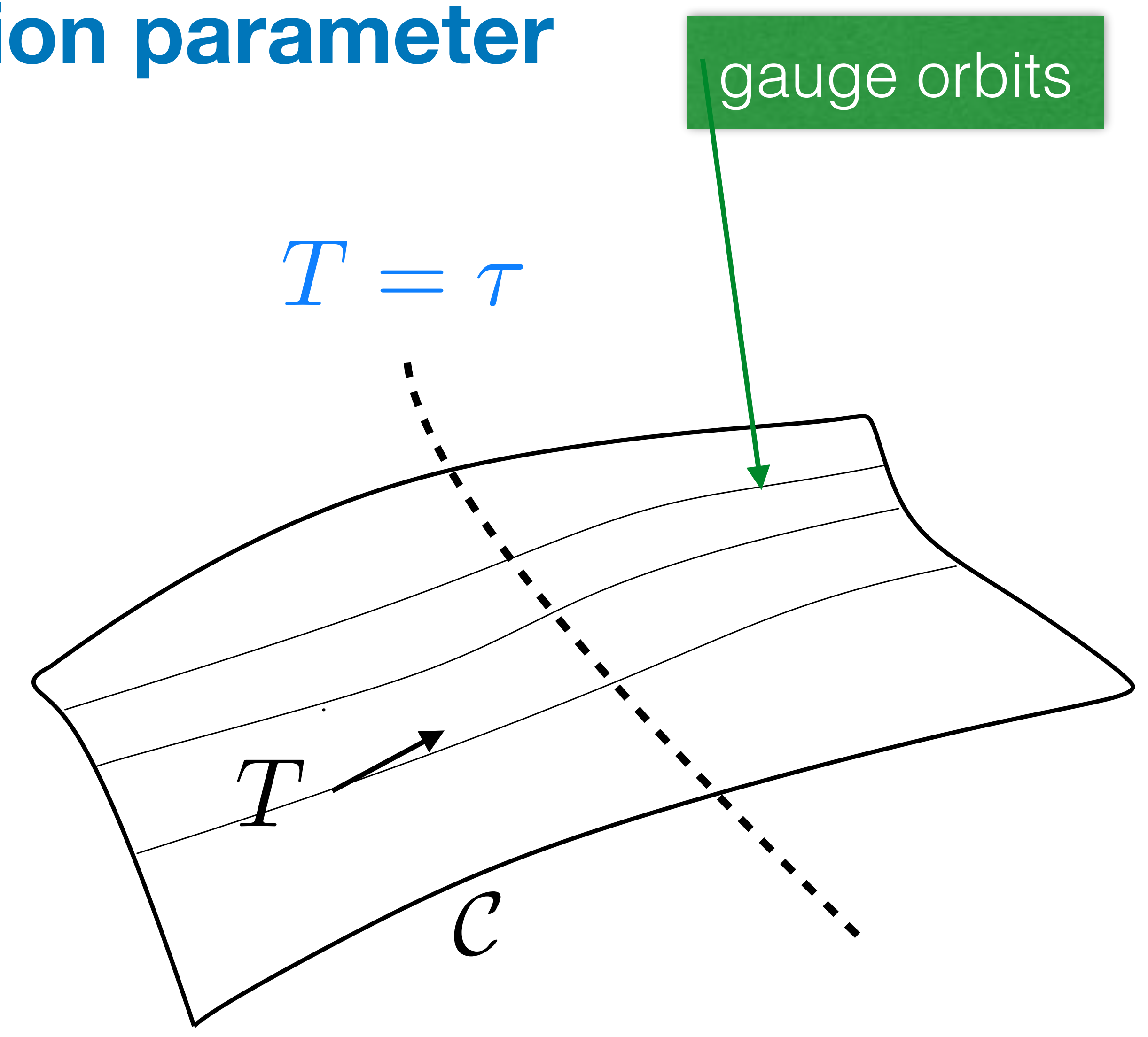
$$F_{f,T}(\tau) = \alpha^s_{C_H} \cdot f\Big|_{\alpha^s_{C_H} \cdot T = \tau}$$

$$\approx \sum_{n=0}^{\infty} \frac{(\tau - T)^n}{n!} \left\{ f, \frac{C_H}{\{T, C_H\}} \right\}_n$$

Dittrich '04; '05

reparametrization invariant $\{C_H, F_{f,T}(\tau)\} \approx 0$

τ is evolution parameter



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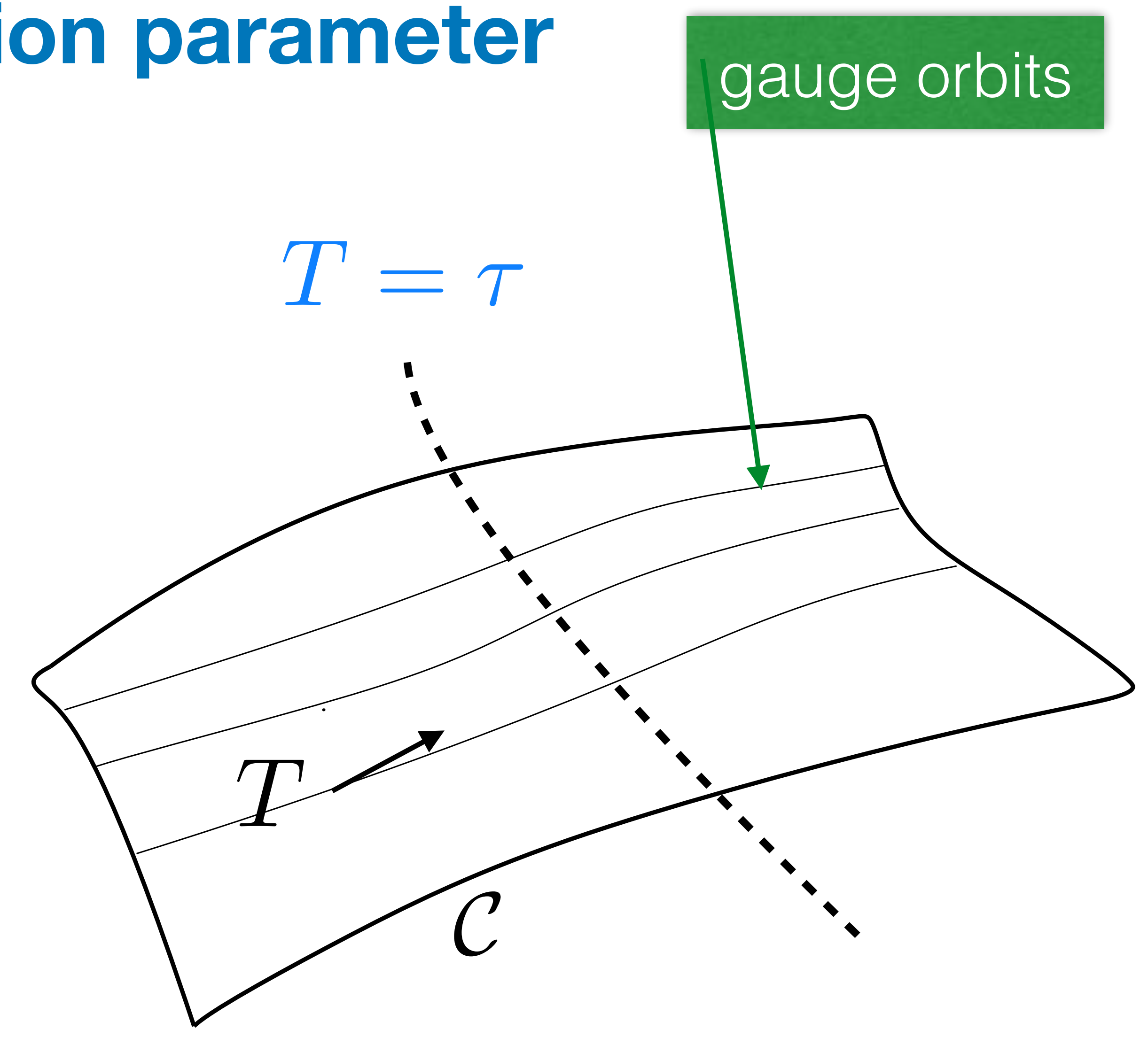

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Quantum dynamics? $\Rightarrow$ Promote $F_{f,T}(\tau)$ to operator on $\mathcal{H}_{\text{phys}}$

space of states satisfying $\hat{C}_H\,|\psi_{\text{phys}}\rangle = 0$

Example: parametrized particle

invariant under reparametrizations $s \to \tilde{s}(s)$

$$S = \frac{m}{2}\int dt\,\dot{q}^2 \xrightarrow{\text{Parametrize } t(s),\, q(s)} S_{\text{ext}} = \frac{m}{2}\int ds\, t' \left(\frac{q'}{t'}\right)^2$$

Legendre tr. $p_i = \dfrac{\partial L}{\partial q_i'}$

$$C_H = p_t + \frac{p^2}{2m} = 0$$

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relational observable: "what is position q of particle when clock t reads τ?"

$$F_{q,t}(\tau) = \sum_{n=0} \frac{(\tau - t)^n}{n!}\{q, C_H\}_n = \frac{p}{m}(\tau - t) + q$$

can be quantized and commutes with C_H

II. Deparametrization

Ashtekar, Bodendorfer, Bojowald, Dittrich, Giesel, PH, Husain, Kaminski, Lewandowski, Pawlowski, Rovelli, Singh, Thiemann, …

Usually, reduced phase space quantization

Multiple ways, e.g. deparametrize through symmetry reduction relative to chosen clock

1. canon. transf. splitting into gauge + gauge-inv. DoFs

$$\mathcal{T}_T : (t, p_t; q_i, p_i) \mapsto \left(T(t,p_t), P_T = \frac{C_H}{\{T, C_H\}}\right);\ (F_{q_i,T}(\tau), F_{p_i,T}(\tau))$$

constraint surface $C_H = 0$

$$\begin{array}{c} \mathcal{C} \\ \Big\downarrow \mathcal{T}_T \\ \mathcal{C} \end{array}$$

$P_T = 0$

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2. gauge fix to e.g. $T = 0$ **and solve constraint** $P_T = 0$

$\Rightarrow$ **Removes redundancy in description of constraint surface by removing reference system (i.e. clock) DoFs from among dyn. variables, without loss of info**

reduced relational observables $F_{f(q_i,p_i)}(\tau) := F_{f(q_i,p_i),T}(\tau)\big|_{T=P_T=0}$

satisfy EoMs $\dfrac{dF_f}{d\tau} = \{F_f, H_{\text{true}}\}_D + \dfrac{\partial H_{\text{true}}}{\partial \tau}$ **on reduced phase space** $\mathcal{P}_{\text{rest}|\text{T}}$

Dynamics from "perspective" of chosen clock

constraint surface $C_H = 0$

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Dynamics from "perspective" of chosen clock

$\Rightarrow$ **Quantize** $\mathcal{P}_{\text{rest|T}}$

constraint surface $C_H = 0$

$$\mathcal{C} \xrightarrow{\mathcal{T}_T} \mathcal{C} \xrightarrow{\pi_{T=0}} \mathcal{P}_{\text{rest|T}}$$

$P_T = 0$

III. Page-Wootters formalism

Page, Wootters '83

Dolby, Gambini, Giovanetti, Lloyd, Maccone, Marletto, Moreva, Pullin, Rossignoli, Smith, Vedral, …

Extract dynamics from physical states through conditional probabilities

split total system into "clock" C and "system" S

$$\hat{C}_H \left|\psi_{\text{phys}}\right\rangle = (\hat{H}_C + \hat{H}_S) \left|\psi_{\text{phys}}\right\rangle = 0$$

define clock states s.t. $$e^{-it\hat{H}_C} \left|\tau\right\rangle = \left|\tau + t\right\rangle$$

What is probability that $\hat{f}_S$ has outcome f_S given that clock reads τ?

$$P(f_S \text{ when } \tau) = \frac{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes |f_S\rangle\langle f_S|)|\psi_{\text{phys}}\rangle_{\text{kin}}}{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\text{phys}}\rangle_{\text{kin}}}.$$

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- conditional state of system when clock reads τ $\quad |\psi_S(\tau)\rangle := \langle\tau|\psi_{\text{phys}}\rangle$

solves **relational Schrödinger eq.** $\quad i\partial_\tau|\psi_S(\tau)\rangle = \hat{H}_S\,|\psi_S(\tau)\rangle$

evolution of S relative to C

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..... and many more

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Kuchar's 3 criticisms against PW formalism

Kuchar '92

1. incompatible with constraints

does not commute with $\hat{C}_H$ $\Rightarrow$ throws $|\psi_{\rm phys}\rangle$ out of $\mathcal{H}_{\rm phys}$

$$P(f_S \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|\boxed{(|\tau\rangle\langle\tau| \otimes |f_S\rangle\langle f_S|)}|\psi_{\rm phys}\rangle_{\rm kin}}{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\rm phys}\rangle_{\rm kin}}.$$

inner product on $\mathcal{H}_{\rm kin}$

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inner product on $\mathcal{H}_{\mathrm{kin}}$

2. wrong propagators for non-relativistic systems

$$P(q' \text{ when } \tau'|q \text{ when } \tau) = \frac{\langle\psi_{\mathrm{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}}{\langle\psi_{\mathrm{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\mathrm{phys}}\rangle_{\mathrm{kin}}}$$

$$= |\delta(\tau - \tau')\delta(q - q')|^2$$

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3. wrong localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } t) = |\psi(\vec{q},t)|^2 / \int d^3\vec{q}\,|\psi(\vec{q},t)|^2$$

(conditioning w.r.t. Minkowski time)

sol. to KG eqn

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inner product on $\mathcal{H}_{\text{kin}}$

effectively ended research on PW formalism for a while

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$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \ldots)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\text{phys}}\rangle_{\text{kin}}}{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)|\psi_{\text{phys}}\rangle_{\text{kin}}}$$

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Multiple choice problem

- many possible choices for relational clocks $\Rightarrow$ inequivalent quantum dynamics

e.g., 2 clocks variables T_1, T_2

$$T_1(T_2) \qquad \text{vs} \qquad T_2(T_1)$$

What if T_1, T_2 operators?

e.g. $T_1 = a, T_2 = \varphi$

in quantum cosmology

Kuchar (1992):

"The multiple choice problem is one of an embarrassment of riches: out of many inequivalent options, one does not know which one to select."

Isham (1993):

"Can these different quantum theories be seen to be part of an overall scheme that is covariant?… It seems most unlikely that a single Hilbert space can be used for all possible choices of an internal time function."

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No perfect clock for bounded Hamiltonians

For H bounded, NO self-adjoint T exists s.t.

1) $$[T, H] = i$$ Pauli, ‘58

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Other variables multivalued at given clock reading?

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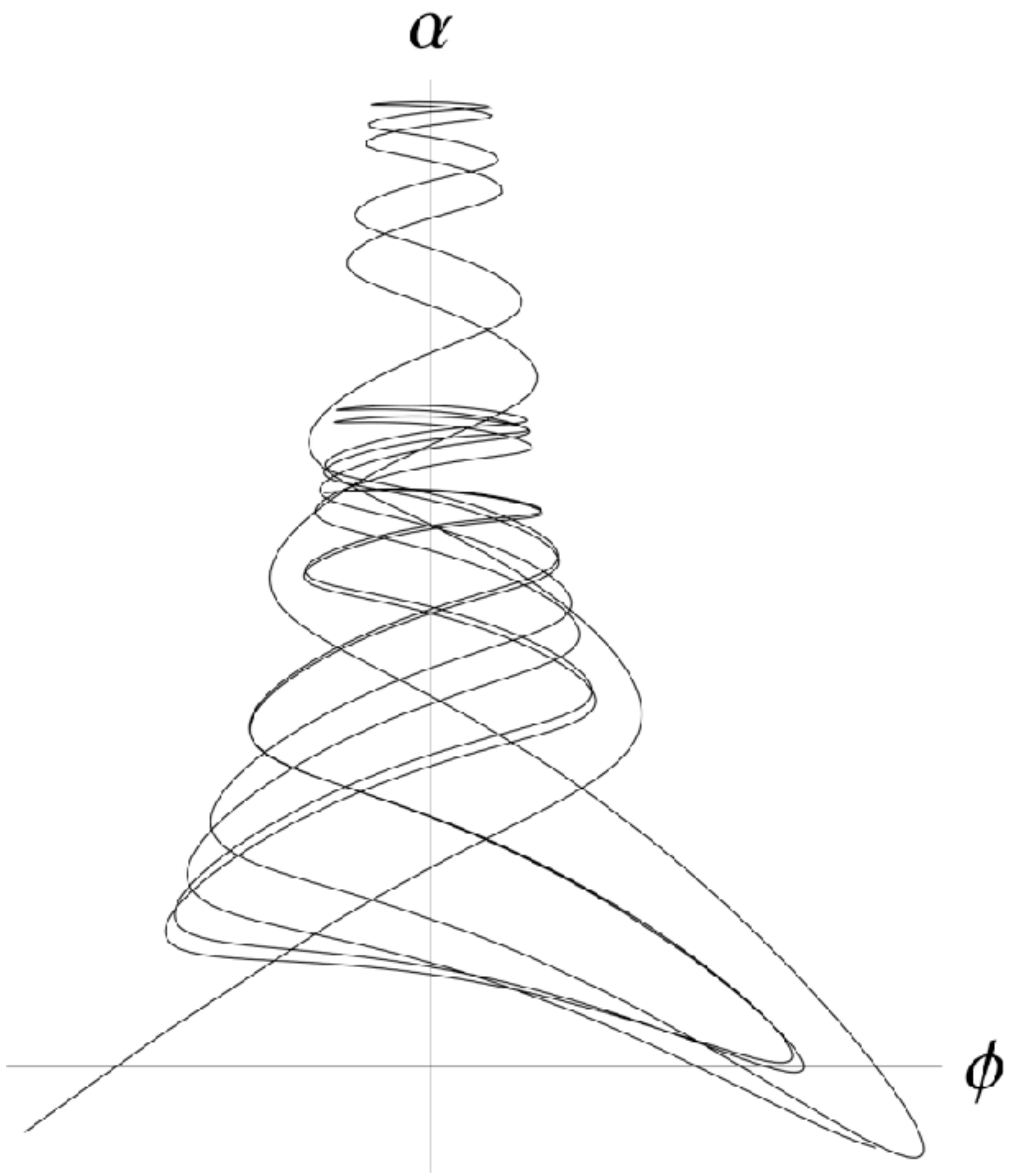

closed FRW with massive scalar field

What to do when clocks non-monotonic?

$$\Downarrow$$

multivaluedness of relations between evolving DoFs and clock

PH, Kubalova, Tsobanjan ‘12

Update on status of various faces of PoT

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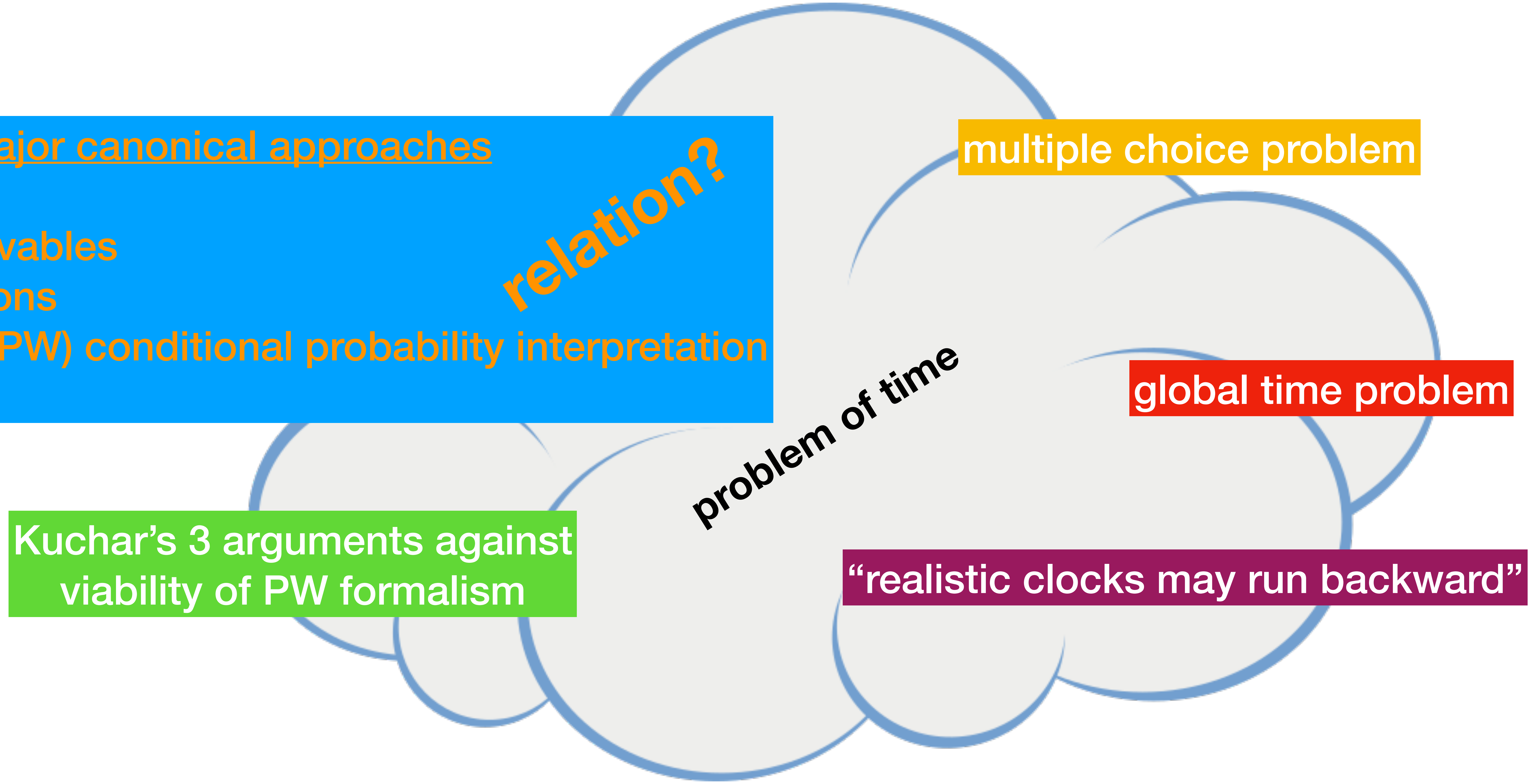

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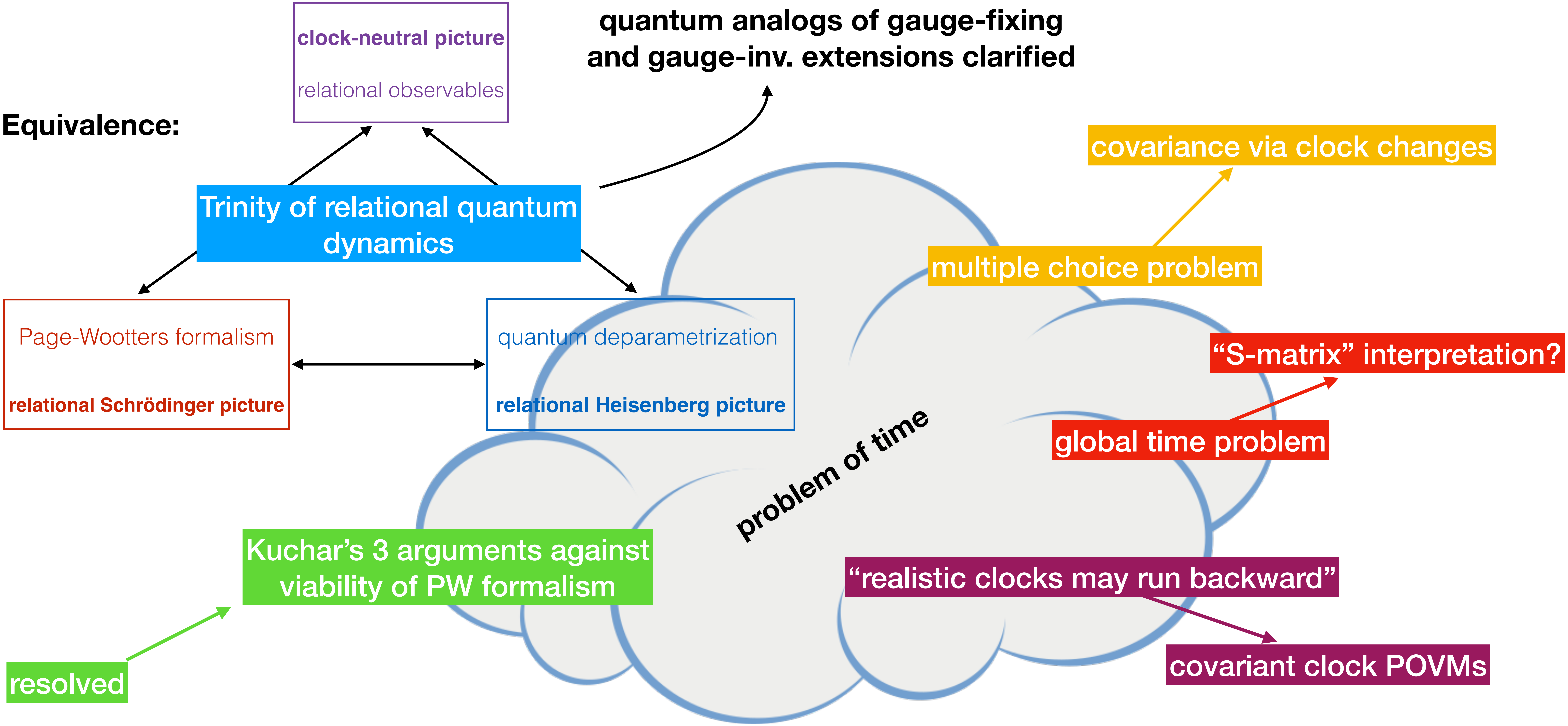

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Other variables multivalued at given clock reading?

“Realistic clocks may run backward”

No perfect clock for bounded Hamiltonians

For H bounded, NO self-adjoint T exists s.t.

give up idea that observables must be self-adjoint,
use generalized measurements
(positive operator-valued measures)

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Covariant clock POVMs

Holevo, Busch, Milburn, Caves, Braunstein, Brunetti,
Fredenhagen, Loveridge, Smith, PH, Lock,...

Probability measure for clock readings: $E_T(\Delta t) = \int_{\Delta t \subset \mathbb{R}} E_T(dt) \geq 0, \quad E_T(\mathbb{R}) = \mathbf{1}$

but $E_T(\Delta t)\, E_T(\Delta t') \neq 0$ possible if $\Delta t \cap \Delta t' = \emptyset$

covariance w.r.t. clock Hamiltonian $\hat{H}_C$ $\quad E_T(\Delta t + t) = U_C(t)\, E_T(\Delta t)\, U_C^\dagger(t)$

Effect operators run monotonically forward

how?

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Effect operators run monotonically forward

how? $E_T(dt) = \sum_\sigma |t,\sigma\rangle\langle t,\sigma|\,dt$ $\qquad |t,\sigma\rangle = \int d\varepsilon\, e^{-i\varepsilon t}\,|\varepsilon,\sigma\rangle$

σ : degeneracy label for $\hat{H}_C$

$\Rightarrow \quad U_C(t')\,|t,\sigma\rangle = |t+t',\sigma\rangle$

clock states are coherent states of group generated by $\hat{H}_C$

Covariant clock POVMs

Holevo, Busch, Milburn, Caves, Braunstein, Brunetti, Fredenhagen, Loveridge, Smith, PH, Lock,…

Probability measure for clock readings: $E_T(\Delta t) = \int_{\Delta t \subset \mathbb{R}} E_T(dt) \geq 0, \quad E_T(\mathbb{R}) = \mathbf{1}$

but $E_T(\Delta t)\, E_T(\Delta t') \neq 0$ possible if $\Delta t \cap \Delta t' = \emptyset$

covariance w.r.t. clock Hamiltonian $\hat{H}_C$ $\quad E_T(\Delta t + t) = U_C(t)\, E_T(\Delta t)\, U_C^\dagger(t)$

Effect operators run monotonically forward

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n-th moment operators $\hat{T}^{(n)} = \int_{\mathbb{R}} t^n\, E_T(dt)$ **satisfy generalization of canon. conjugacy** $\quad [\hat{T}^{(n)}, \hat{H}_C] = i\, n\, \hat{T}^{(n-1)}$

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Consistent probabilistic interpretation, prize to pay:

typically $\hat{T}^{(n)}$ not self-adjoint, $|t\rangle$ not orthogonal (perfectly distinguishable) and not eigenstates of $\hat{T}^{(1)}$

Covariant clock POVM example

PH, Smith, Lock 2007.00580
(see also Braunstein, Caves, Milburn '96)

as in relativistic constraints, clock Hamiltonian $\hat{H}_C = -\hat{p}_t^2$

clock states split into positive and negative frequency modes $|t,\sigma\rangle := \int_{\mathbb{R}} dp_t\, \sqrt{|p_t|}\, \theta(-\sigma\, p_t)\, e^{+i\, t\, p_t^2}\, |p_t\rangle$

Non-orthogonal $\langle t,\sigma|t',\sigma\rangle \neq \delta(t-t')$ but covariant $U_C(t')\,|t,\sigma\rangle = |t+t',\sigma\rangle$

1st moment of POVM
$$\hat{T}^{(1)} := \frac{1}{2\pi}\sum_{\sigma=\pm}\int_{\mathbb{R}} dt\, t\, |t,\sigma\rangle\langle t,\sigma|$$
$$= -\frac{1}{4}\left(\hat{t}\,\hat{p}_t^{-1} + \hat{p}_t^{-1}\,\hat{t}\right)$$

(not self-adjoint)

coincides with symmetric quantization of $T = -\dfrac{t}{2\,p_t}$ (which satisfies classically $\{T, H_C\} = 1$)

Many faces of the problem of time

TIME AND INTERPRETATIONS OF QUANTUM GRAVITY

Karel V. Kuchař

1992 reviews

Canonical Quantum Gravity and the Problem of Time [1] [2]

C.J. Isham

<u>3 major canonical approaches</u>

- ...
- Relational observables
- Deparametrizations
- Page-Wootters (PW) conditional probability interpretation
- ...

relation?

problem of time

multiple choice problem

global time problem

"realistic clocks may run backward"

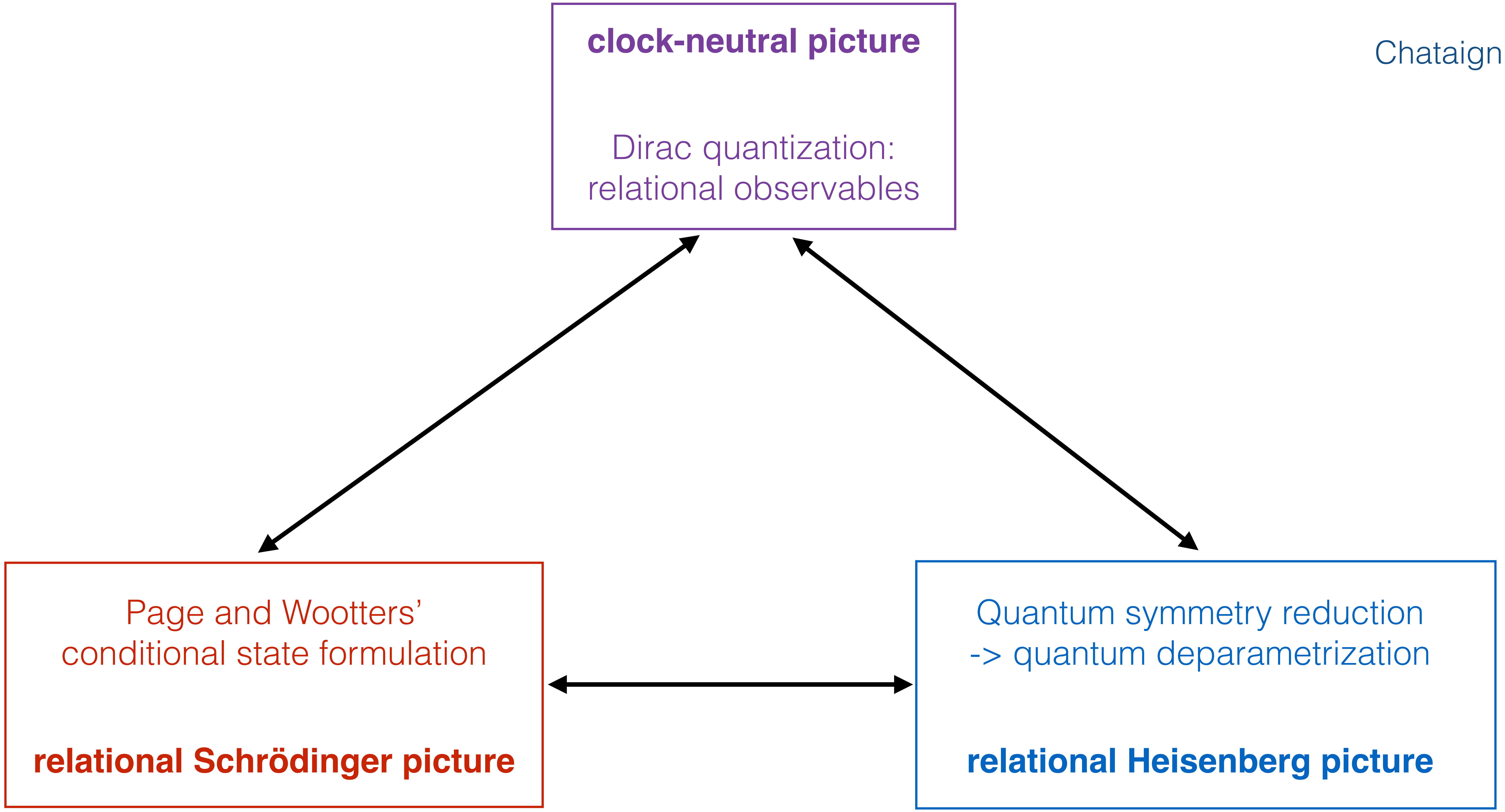
Kuchar's 3 arguments against viability of PW formalism

….. and many more

see also Anderson '17

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ 2007.00580;
Chataignier, PH, Lock to appear



Analogy with relativity

Tensors, are reference-frame-neutral objects: they encode physics as "experienced" in any local frame at once

e.g., stress-energy tensor at x: $$T_x : T_x\mathscr{M} \times T_x\mathscr{M} \to \mathbb{R}$$

$\Rightarrow$ contract with frame vectors, e_A, to produce energy-momentum numbers as "experienced" in frame

in abstract index notation:

frame-neutral description

$$T_{\mu\nu}$$

e^{μ}_{A} constitutes coordinate map into frame's perspective (inverse e^{A}_{μ})

$$T_{AB} := e^{\mu}_{A}\, T_{\mu\nu}\, e^{\nu}_{B} = T(e_A, e_B)$$

internal perspective of frame e_A on stress-energy tensor

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<u>**in abstract index notation:**</u>

frame-neutral description

$$T_{\mu\nu}$$

analog of Dirac observables on physical Hilbert space

e^μ_A constitutes coordinate map into frame’s perspective (inverse e^A_μ)

$$T_{AB} := e^\mu_A\, T_{\mu\nu}\, e^\nu_B = T(e_A, e_B)$$

analog of reduced quantum theories

internal perspective of frame e_A on stress-energy tensor

Restriction for now

As in Page-Wootters formalism, no interaction between clock and evolving DoFs

$$C_H = \underbrace{H_C}_{\text{clock}} + \underbrace{H_S}_{\text{system}}$$

H_C generator of group $G \simeq \mathbb{R}$ (clock monotonic) or $G \simeq \mathrm{U}(1)$ (periodic clock)

H_S arbitrary

- Vacuum Bianchi models
- FRW + m=0 scalar field
- Relativistic particle
- Many non-relativistic models
- periodic clock models

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ 2007.00580;
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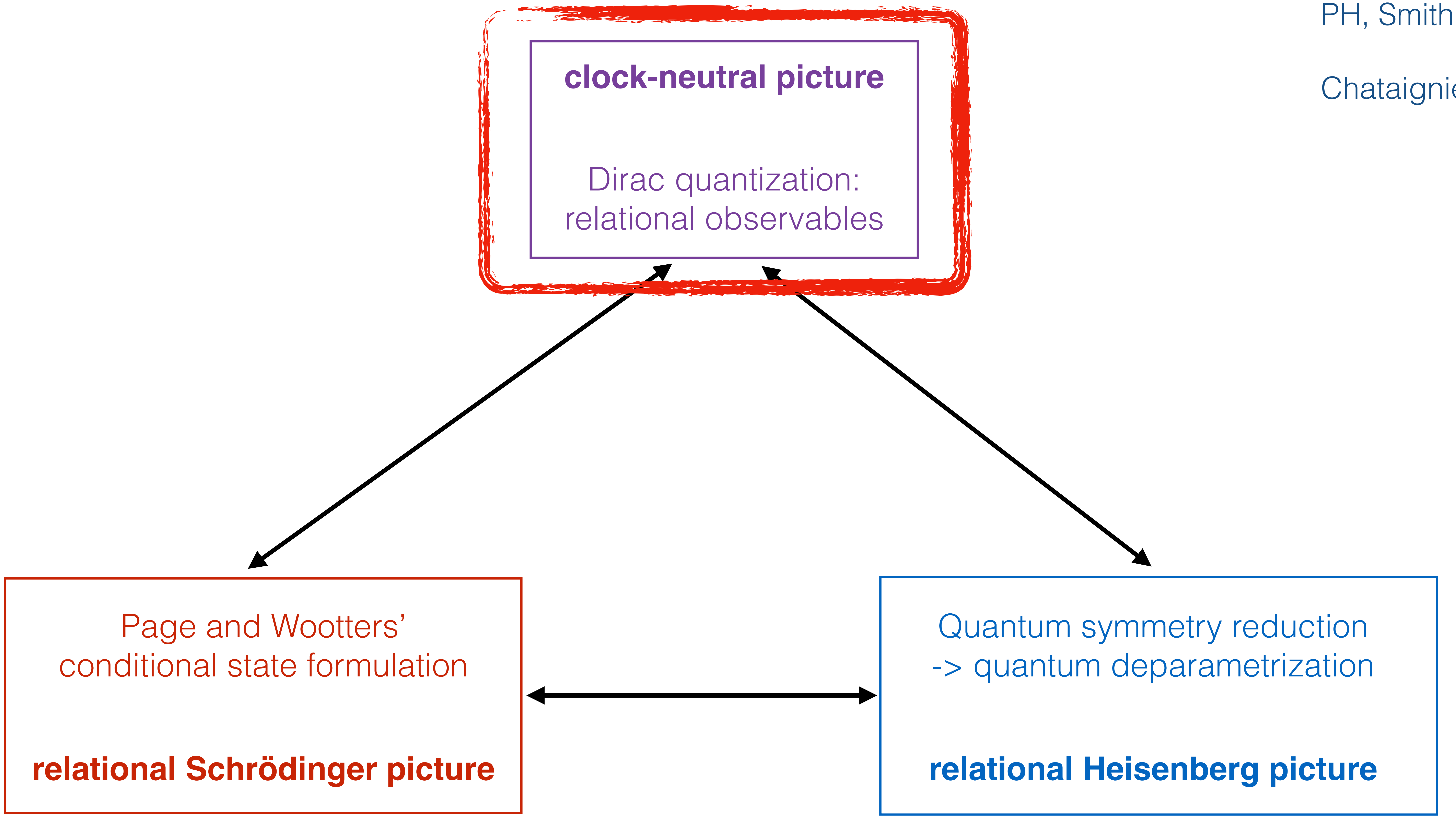


I. Quantize relational Dirac observables

Classically, choose time function T s.t. $\{T, H_C\} = 1$

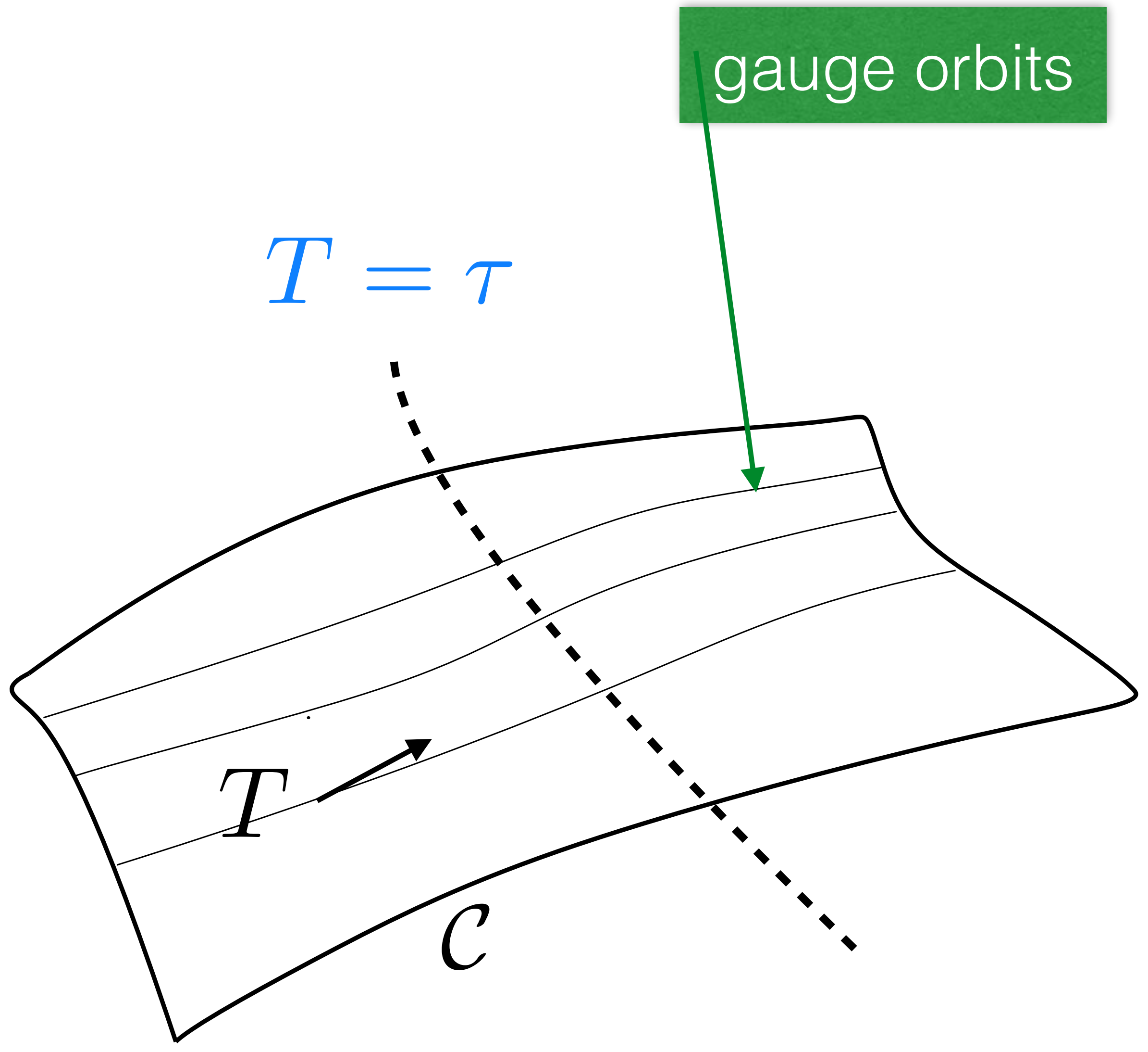
locally always possible

What is value of f_S when clock T reads τ?

$$F_{f_S,T}(\tau) \approx \sum_{n=0}^{\infty} \frac{(\tau - T)^n}{n!} \left\{ f_S, \frac{C_H}{\{T, C_H\}} \right\}_n$$

$$= \sum_{n=0}^{\infty} \frac{(\tau - T)^n}{n!} \{f_S, H_S\}_n$$

Dittrich '04, '05



Now quantize $F_{f_S,T}(\tau)$ $\Rightarrow$ need quantization of T^n $\Rightarrow$ covariant POVM

I. Quantum relational Dirac observables

PH, Smith, Lock 1912.00033 +2007.00580;
Chataignier, PH, Lock to appear;
[+ related work Chataignier 2006.05526]

'clock-neutral' states

$$\hat{C}_H \, |\psi_{\rm phys}\rangle = (\hat{H}_C + \hat{H}_S) \, |\psi_{\rm phys}\rangle = 0$$

What is value of $\hat{f}_S$ when clock reads τ?

$$\hat{F}_{f_S,T}(\tau) := \int E_T(dt) \otimes \sum_{n=0} \frac{i^n}{n!} (t-\tau)^n \left[\hat{f}_S, \hat{H}_S\right]_n$$

$$= \sum_\sigma \int dt \, e^{-i\hat{C}_H \, t} \Big(\underbrace{|\tau,\sigma\rangle\langle\tau,\sigma|}_{\text{'projector' onto clock time } \tau} \otimes \hat{f}_S \Big) \, e^{i\hat{C}_H \, t}$$

incoherent group averaging
or G-twirl

gauge-inv., strong Dirac observables $[\hat{F}_{f_S,T}, \hat{C}_H] = 0$

+ other nice algebraic properties (homomorphism, …)

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ 2007.00580;
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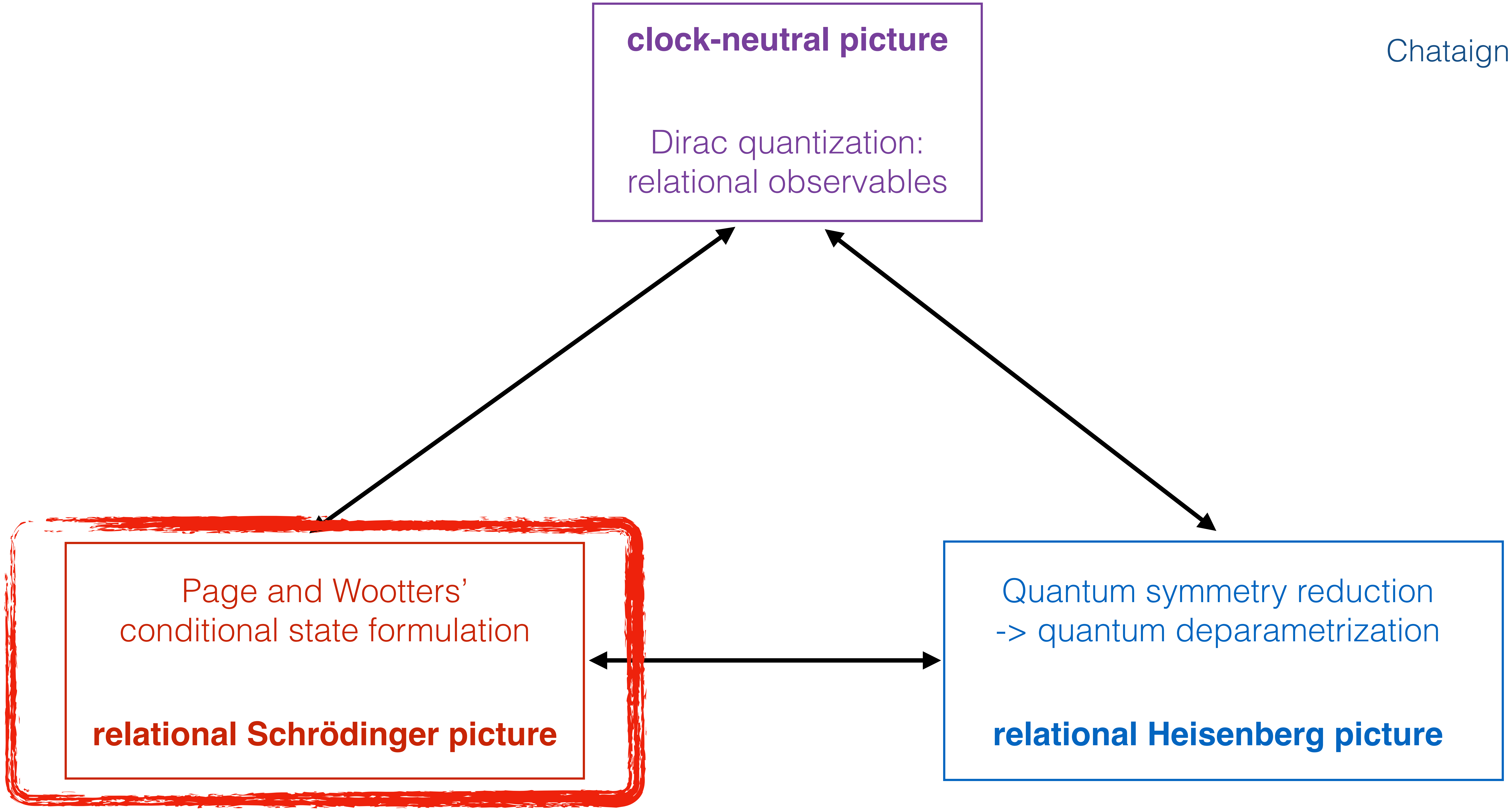

III. Page-Wootters formalism

PH, Smith, Lock 1912.00033
+ 2007.00580;
Chataignier, PH, Lock to appear

from covariant POVM

define reduction map by conditioning on clock reading $\mathcal{R}_{\mathbf{S}}^{\sigma}(\tau) := \langle \tau, \sigma | \otimes I_S$

analog of e^{μ}_{A} in relativity

$$\mathcal{R}_{\mathbf{S}}^{\sigma}(\tau) : \mathcal{H}_{\text{phys}} \rightarrow \mathcal{H}_{S,\sigma}$$

states satisfying $C_H \, |\psi_{\text{phys}}\rangle = 0$

reduced Hilbert space for S only ("clock perspective")

conditional state of system when clock reads τ $\quad |\psi_S^{\sigma}(\tau)\rangle := \mathcal{R}_{\mathbf{S}}^{\sigma}(\tau)\, |\psi_{\text{phys}}\rangle$

solves **relational Schrödinger eq.** $\quad i\partial_\tau \, |\psi_S^{\sigma}(\tau)\rangle = \hat{H}_S \, |\psi_S^{\sigma}(\tau)\rangle$

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key: reduction is invertible
(redundancy in kinematical description of physical states)

$\Rightarrow$ **reduction** $\mathcal{R}^{\sigma}_{S}(\tau)$ **is quantum analog of gauge fixing (removing redundancy)**

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Equivalence with relational observables

- **Rel. obs. reduce to Schröd. operators** $\mathcal{R}^{\sigma}_{\mathbf{S}}(\tau)\, F_{f_S,T}(\tau)\, \mathcal{R}^{\sigma}_{\mathbf{S}}(\tau)^{-1} = f_S$

- **expect. values (+ inner prod.) preserved** $\langle \psi_{\text{phys}} |\, F_{f_S,T}(\tau)\, | \psi_{\text{phys}} \rangle_{\text{phys}} = \langle \psi^{\sigma}_{S}(\tau) |\, f_S \, | \psi^{\sigma}_{S}(\tau) \rangle$

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physical inner product

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Equivalence with relational observables

- **Rel. obs. reduce to Schröd. operators**

$$\mathcal{R}_{\mathbf{S}}^{\sigma}(\tau)\boxed{F_{f_S,T}(\tau)}\mathcal{R}_{\mathbf{S}}^{\sigma}(\tau)^{-1} = \boxed{f_S}$$

manifestly gauge-inv. 'gauge-fixed'

- **expect. values (+ inner prod.) preserved**

$$\boxed{\langle \psi_{\text{phys}} | \, F_{f_S,T}(\tau) \, | \psi_{\text{phys}} \rangle_{\text{phys}}} = \boxed{\langle \psi_S^{\sigma}(\tau) | \, f_S \, | \psi_S^{\sigma}(\tau) \rangle}$$

The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
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Chataignier, PH, Lock to appear

clock-neutral picture

Dirac quantization:
relational observables

Page and Wootters'
conditional state formulation

relational Schrödinger picture

Quantum symmetry reduction
-> quantum deparametrization

relational Heisenberg picture

II. Recall classical parametrization

constraint surface $C_H = 0$

1. canon. transf. splitting into gauge + gauge-inv. DoFs

$$\mathcal{T}_T : (t, p_t; q_i, p_i) \mapsto \left(T(t, p_t), P_T = \frac{C_H}{\{T, C_H\}} \right); \ (F_{q_i,T}(\tau), F_{p_i,T}(\tau))$$

2. gauge fix to e.g. $T = 0$ **and solve constraint** $P_T = 0$

$\Longrightarrow$ **reduced relational observables** $F_{f(q_i,p_i)}(\tau) := F_{f(q_i,p_i),T}(\tau)\big|_{T=P_T=0}$

satisfy EoMs $\dfrac{dF_f}{d\tau} = \{F_f, H_{\text{true}}\}_D + \dfrac{\partial H_{\text{true}}}{\partial \tau}$ **on reduced phase space** $\mathcal{P}_{\text{rest}|\text{T}}$

Dynamics from "perspective" of chosen clock

$P_T = 0$

$$\begin{array}{c} \mathcal{C} \\ \big\downarrow \mathcal{T}_T \\ \mathcal{C} \\ \big\downarrow \pi_{T=0} \\ \mathcal{P}_{\text{rest}|\text{T}} \end{array}$$

II. Quantum deparametrization

PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033;
+ 2007.00580

Quantum analog of symmetry reduction relative to clock C

1. Transformation splitting DoFs into gauge + Dirac obs. across (kin.) tensor factorization

$$|\psi_{\mathrm{phys}}\rangle \in \mathcal{H}_{\mathrm{phys}} \xrightarrow{\mathcal{T}_T} \sum_\sigma |\varepsilon,\sigma\rangle_C \otimes |\psi_S^\sigma\rangle$$

$$\mathcal{T}_T = \int E_T(dt) \otimes e^{it(\hat{H}_S+\varepsilon)}$$

"disentangler"

Gauge: $\mathcal{T}_T\, \hat{T}^{(n)} \otimes I_S\, \mathcal{T}_T^{-1} = \underline{\hat{T}^{(n)} \otimes I_S}\,, \qquad \mathcal{T}_T\, \hat{C}_H\, \mathcal{T}_T^{-1} \approx \underline{(\hat{H}_C - \varepsilon) \otimes I_S}$

conjugate

Dirac observables: $\mathcal{T}_T\, \hat{F}_{f_S,T}(\tau)\, \mathcal{T}_T^{-1} \approx I_C \otimes \hat{f}_S(\tau)$

Heisenberg operator

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conjugate

$$\Big\downarrow \mathcal{T}_T$$

$$\sum_\sigma |\varepsilon,\sigma\rangle_C \otimes |\psi_S^\sigma\rangle$$

Dirac observables: $\mathcal{T}_T\, \hat{F}_{f_S,T}(\tau)\, \mathcal{T}_T^{-1} \approx I_C \otimes \hat{f}_S(\tau)$

Heisenberg operator

2. Condition on classical gauge $\langle\tau,\sigma|$ **(as in PW), get reduced Hilbert space** $\mathcal{H}_{S,\sigma}$ **for only** S

$$\Big\downarrow \langle\tau,\sigma| \otimes I_S$$

$$|\psi_S^\sigma\rangle \in \mathcal{H}_{S,\sigma}$$

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PH, Vanrietvelde 1810.04153
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conjugate

Dirac observables: $\mathcal{T}_T\,\hat{F}_{f_S,T}(\tau)\,\mathcal{T}_T^{-1}\approx I_C\otimes\hat{f}_S(\tau)$

Heisenberg operator

2. Condition on classical gauge $\langle\tau,\sigma|$ (as in PW), get reduced Hilbert space $\mathcal{H}_{S,\sigma}$ for only S

deparametrization map (isometry) $\quad\mathcal{R}^\sigma_{\mathbf{H}} := e^{-i\varepsilon\tau}\,(\langle\tau,\sigma|\otimes I_S)\,\mathcal{T}_T$

$$\mathcal{R}^\sigma_H\,\hat{F}_{f_S,T}(\tau)\,\mathcal{R}^{\sigma\,-1}_{\mathbf{H}} = \hat{f}_S(\tau)$$

yields relational Heisenberg picture relative to clock C

Overview: classical vs. quantum symmetry reduction

Classical structures		Quantum analogs	
Kinematical phase space	$\mathcal{P}_{\text{kin}}$	Kinematical Hilbert space	$\mathcal{H}_{\text{kin}}$
Constraint surface	$\mathcal{C}$	Physical Hilbert space	$\mathcal{H}_{\text{phys}}$
Gauge fixed reduced phase space(s) (rel. to clock C)	$\mathcal{P}_{S,\sigma}$	Reduced Hilbert space(s) (rel. to clock C)	$\mathcal{H}_{S,\sigma}$
Canon. transf. splitting gauge + gauge-inv. DoFs	$\mathcal{T}_T$	Trivialization ("disentangler")	$\mathcal{T}_T$
Gauge fixing clock function	$T = \tau'$	Conditioning on clock (POVM) states	$\langle \tau', \sigma \| \otimes I_S$
Gauge-fixed observables	$F_{f_S}(\tau) = f_S(\tau)$	Relational Heisenberg operators	$\hat{f}_S(\tau)$

Gauge-invariant extension of gauge-fixed quantity	Quantum analog
$\theta_\sigma \, F_{f_S,T}(\tau)$	${\mathcal{R}^\sigma_{\mathbf{H}}}^{-1} \, \hat{f}_S(\tau) \, \mathcal{R}^\sigma_{\mathbf{H}} \approx \Pi_\sigma \, \hat{F}_{f_S,T}(\tau)$

Projector onto σ-sector of H_C

Dirac vs. reduced quantization

$$
\begin{tikzcd}
\mathcal{P}_{\mathrm{kin}} \arrow[rr, "\text{Dirac quantization}"] \arrow[d, "C_H = 0"] & & \mathcal{H}_{\mathrm{kin}} \arrow[d, "\Pi_{\mathrm{phys}}"'] \\
\mathcal{C} \arrow[d, "\mathcal{R}^{\sigma}_{\mathrm{dep}}(\tau) := \pi_{T=\tau} \circ \mathcal{T}_T"] \arrow[rr, phantom, "\boxed{\text{Does diagram commute?}}"] & & \mathcal{H}_{\mathrm{phys}} \arrow[d, "\mathcal{R}^{\sigma}_{\mathbf{H}} := e^{-i\varepsilon\tau}\left(\langle\tau,\sigma| \otimes I_S\right)\mathcal{T}_T"'] \\
\mathcal{P}_{S,\sigma} \arrow[rr, "\text{reduced quantization}"'] & & \mathcal{H}_{S,\sigma}
\end{tikzcd}
$$

Dirac vs. reduced quantization

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\begin{tikzcd}[column sep=huge, row sep=large]
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\end{tikzcd}
$$

in some cases,

PH, Vanrietvelde 1810.04153
PH 1811.00611

but not in general

Vanrietvelde, PH, Giacomini 1809.05093
PH, Smith, Lock 1912.00033

“Constraining and quantizing don’t commute”

[Ashtekar, Horowitz, Kuchar, Kunstatter, Loll, Schleich, Romano, Tate, Giesel, Thiemann, Singh, Li, Husain, Lewandowski, Pawlowski, Dittrich, PH, Nelson, Koslowski,…]

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“Symmetry reduction and quantization don’t commute”

[Ashtekar, Horowitz, Kuchar, Kunstatter, Loll, Schleich, Romano, Tate, Giesel, Thiemann, Singh, Li, Husain, Lewandowski, Pawlowski, Dittrich, PH, Nelson, Koslowski,…]

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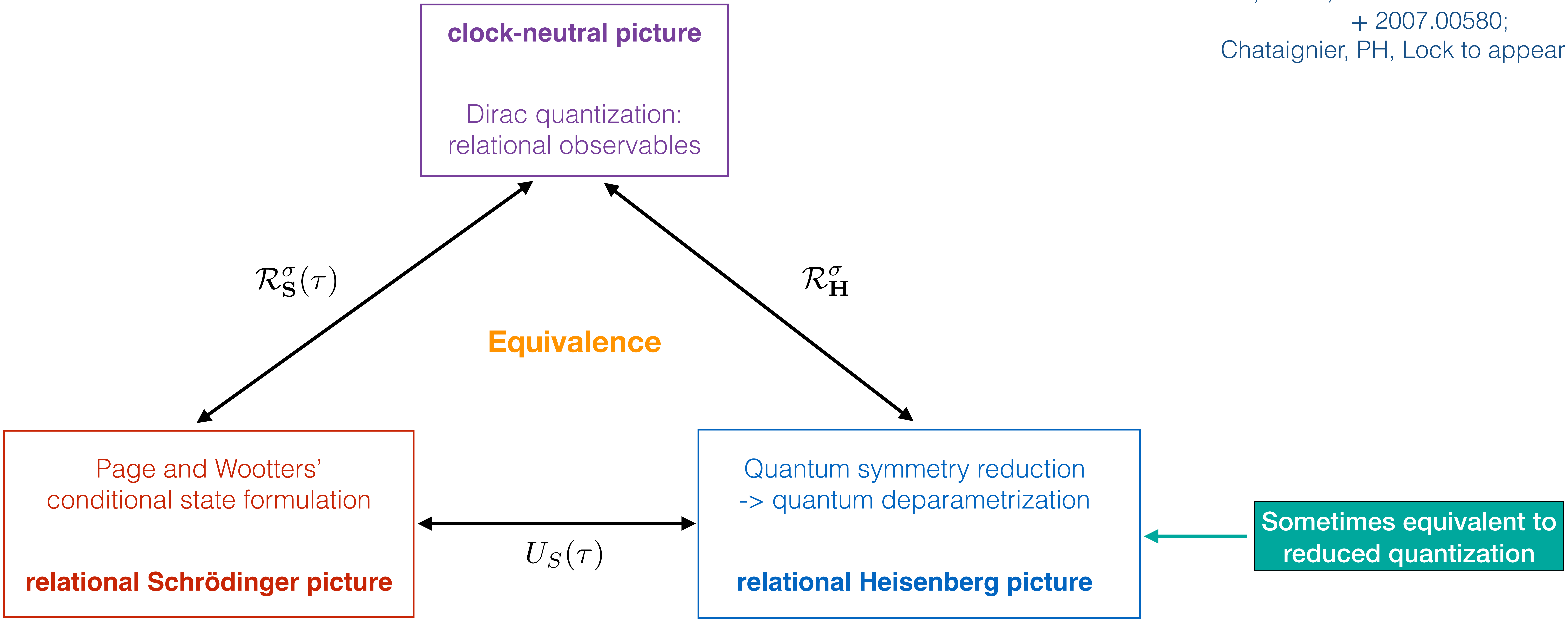
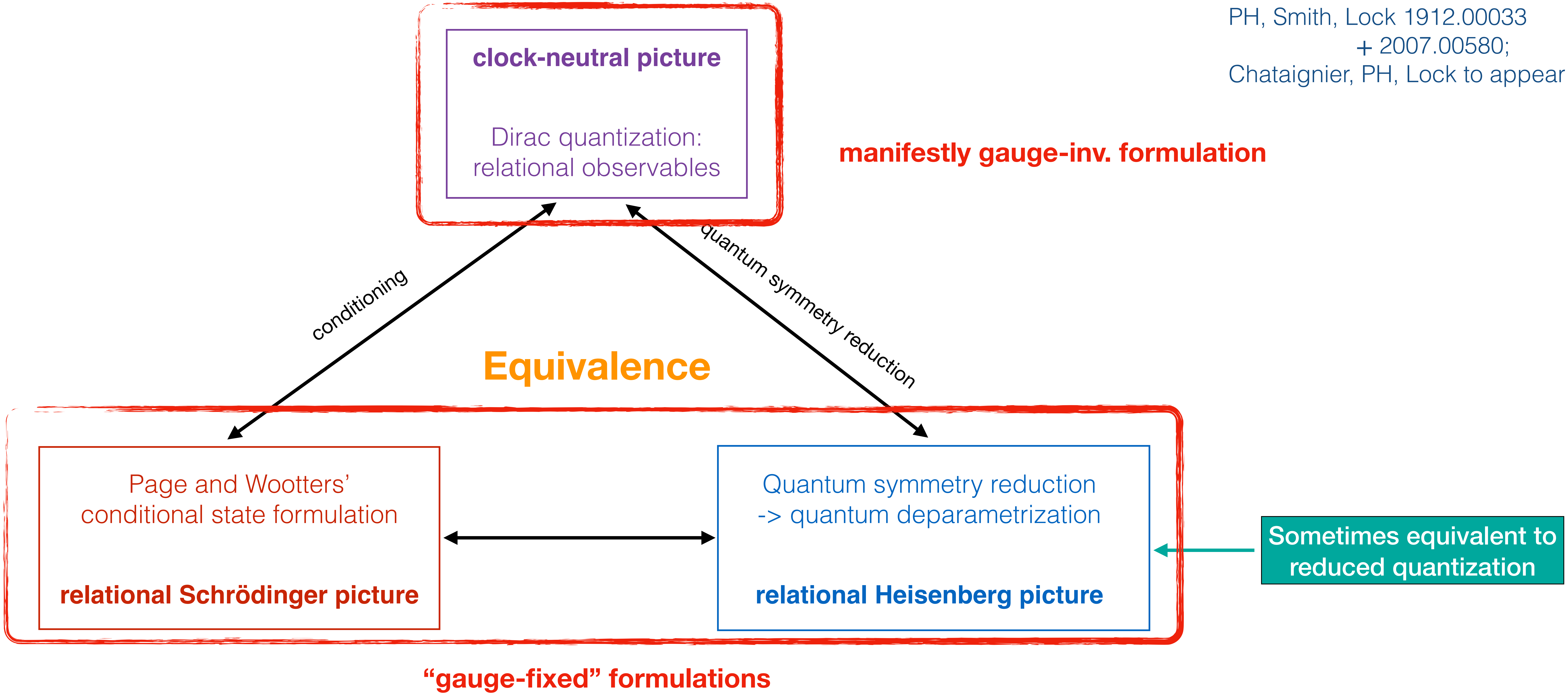


The trinity of relational quantum dynamics

PH, Smith, Lock 1912.00033
+ 2007.00580;
Chataignier, PH, Lock to appear



Periodic clocks as incomplete temporal frames

Chataignier, PH, Lock *to appear*

Hamiltonian constraint

$$C_H = H_C + H_S$$

clock Hamiltonian $\mathrm{U}(1)$-generator

system

⇒ periodic clock forces system to be also periodic!

$$F_{f_S,T}(\tau) \qquad |\psi_S(\tau)\rangle$$

periodic

Many faces of the problem of time

TIME AND INTERPRETATIONS OF QUANTUM GRAVITY

Karel V. Kuchař

1992 reviews

Canonical Quantum Gravity and the Problem of Time [1,2]

C.J. Isham

many independent approaches

- ...
- Relational observables
- Deparametrizations
- Page-Wootters (PW) conditional probability interpretation
- ...

Kuchar's 3 arguments against viability of PW formalism

problem of time

multiple choice problem

global time problem

"realistic clocks may run backward"

..... and many more

see also Anderson '17

Resolving Kuchar's 3 criticisms against PW formalism

1. <u>incompatible with constraints</u>

does not commute with $\hat{C}_H \quad \Rightarrow \quad$ throws $|\psi_{\rm phys}\rangle$ out of $\mathcal{H}_{\rm phys}$

$$P(f_S \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|\boxed{(|\tau\rangle\langle\tau| \otimes |f_S\rangle\langle f_S|)}|\psi_{\rm phys}\rangle_{\rm kin}}{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\rm phys}\rangle_{\rm kin}}.$$

inner product on $\mathcal{H}_{\rm kin}$

Resolving Kuchar's 3 criticisms against PW formalism

1. incompatible with constraints

does not commute with $\hat{C}_H$ $\Rightarrow$ throws $|\psi_{\text{phys}}\rangle$ out of $\mathcal{H}_{\text{phys}}$

$$P(f_S \text{ when } \tau) = \frac{\langle\psi_{\text{phys}}|\boxed{(|\tau\rangle\langle\tau| \otimes |f_S\rangle\langle f_S|)}|\psi_{\text{phys}}\rangle_{\text{kin}}}{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\text{phys}}\rangle_{\text{kin}}}.$$

inner product on $\mathcal{H}_{\text{kin}}$

Corollary from trinity (non-degenerate case):

PH, Smith, Lock 1912.00033

on $\mathcal{H}_{\text{phys}}$

$$\underbrace{\langle\psi_{\text{phys}}|\,\hat{F}_{f_S,T}(\tau)\,|\psi_{\text{phys}}\rangle_{\text{phys}}}_{\text{manifestly gauge-inv.}} = \underbrace{\langle\psi_S(\tau)|\,\hat{f}_S\,|\psi_S(\tau)\rangle}_{\text{`gauge-fixed'}}$$

on $\mathcal{H}_S$

Resolving Kuchar's 3 criticisms against PW formalism

1. ~~incompatible with constraints~~

does not commute with $\hat{C}_H$ $\Rightarrow$ throws $|\psi_{\text{phys}}\rangle$ out of $\mathcal{H}_{\text{phys}}$

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PH, Smith, Lock 1912.00033

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$$= \frac{\langle\psi_{\text{phys}}|\,\hat{F}_{|f_S\rangle\langle f_S|,T}(\tau)\,|\psi_{\text{phys}}\rangle_{\text{phys}}}{\langle\psi_{\text{phys}}|\psi_{\text{phys}}\rangle_{\text{phys}}}$$

Conditional probabilities are manifestly gauge-invariant $\Rightarrow$ provide relational observables with conditional probability interpretation

Upshot

Page-Wootters formalism is (quantum analog of) gauge-fixed formulation of manifestly gauge-invariant relational dynamics on physical Hilbert space

Resolving Kuchar's 3 criticisms against PW formalism

2. wrong propagators for non-relativistic systems

$$P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle \psi_{\text{phys}} | (|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|)(|\tau'\rangle\langle\tau'| \otimes |q'\rangle\langle q'|)(|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|) | \psi_{\text{phys}} \rangle_{\text{kin}}}{\langle \psi_{\text{phys}} | (|\tau\rangle\langle\tau| \otimes |q\rangle\langle q|) | \psi_{\text{phys}} \rangle_{\text{kin}}}$$

$$= |\delta(\tau - \tau')\delta(q - q')|^2$$

Resolving Kuchar's 3 criticisms against PW formalism

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PH, Smith, Lock 1912.00033

Given equivalence with relational Dirac observables, correct two-time conditioning for two observables A, B is

projectors on $\mathcal{H}_{\rm phys}$

cond. probab. on

$$P\left(B = b \text{ when } \tau' | A = a \text{ when } \tau\right) := \frac{\langle \psi_{\rm phys} | \boxed{\hat{F}_{\Pi_{A=a},T}(\tau) \cdot \hat{F}_{\Pi_{B=b},T}(\tau') \cdot \hat{F}_{\Pi_{A=a},T}(\tau)} |\psi_{\rm phys}\rangle_{\rm phys}}{\langle \psi_{\rm phys} | \hat{F}_{\Pi_{A=a},T}(\tau) |\psi_{\rm phys}\rangle_{\rm phys}}$$

$\mathcal{H}_{\rm phys}$

$$= \frac{\langle \psi_S(\tau) | \, \Pi_{A=a} \, U_S^\dagger(\tau' - \tau) \, \Pi_{B=b} \, U_S(\tau' - \tau) \, \Pi_{A=a} \, |\psi_S(\tau)\rangle}{\langle \psi_S(\tau) | \, \Pi_{A=a} \, |\psi_S(\tau)\rangle}.$$

$\mathcal{H}_S$

Correct transition probability in Schrödinger picture

Resolving Kuchar's 3 criticisms against PW formalism

2. wrong propagators for non-relativistic systems

$$\cancel{P(q' \text{ when } \tau' | q \text{ when } \tau) = \frac{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)(|\tau'\rangle\langle\tau'|\otimes|q'\rangle\langle q'|)(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)|\psi_{\rm phys}\rangle_{\rm kin}}{\langle\psi_{\rm phys}|(|\tau\rangle\langle\tau|\otimes|q\rangle\langle q|)|\psi_{\rm phys}\rangle_{\rm kin}}}$$

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$\mathcal{H}_{\rm phys}$

$$= \frac{\langle\psi_S(\tau)|\,\Pi_{A=a}\,U_S^\dagger(\tau'-\tau)\,\Pi_{B=b}\,U_S(\tau'-\tau)\,\Pi_{A=a}\,|\psi_S(\tau)\rangle}{\langle\psi_S(\tau)|\,\Pi_{A=a}\,|\psi_S(\tau)\rangle}.$$

$\mathcal{H}_S$

Non-rel. particle:
$A = B = \hat q$ &
outcomes $b = q'$ and $a = q$

$$= |\langle q'|\,U_S(\tau'-\tau)\,|q\rangle|^2$$

Previous proposals for getting transition probabilities

RAPID COMMUNICATIONS

PHYSICAL REVIEW D **79,** 041501(R) (2009)

Conditional probabilities with Dirac observables and the problem of time in quantum gravity

Rodolfo Gambini,[1] Rafael A. Porto,[2] Jorge Pullin,[3] and Sebastián Torterolo[1]
[1]*Instituto de Física, Facultad de Ciencias, Iguá 4225, esq. Mataojo, Montevideo, Uruguay*
[2]*Department of Physics, University of California, Santa Barbara, California 93106, USA*
[3]*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803-4001, USA*
(Received 25 September 2008; published 13 February 2009)

combines PW + rel. observables
(but no equivalence)

⇒ additional inv. clock

⇒ approx. recovery
(decoherence induced modifications)

PHYSICAL REVIEW D **92,** 045033 (2015)

Quantum time

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]
[1]*NEST-INFM and Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[2]*RLE and Department of Mechanical Engineering, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*
[3]*Dipartimento Fisica "A. Volta" and INFN Sezione Pavia, Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*

⇒ recovers correct result via time measurements
using ancilla systems and ideal clocks

⇒ both proposals modify Hamiltonian with additional DoFs compared to Kuchar's setup

Resolving Kuchar's 3 criticisms against PW formalism

3. wrong localization probability for Klein-Gordon systems

$$P(\vec{q} \text{ when } t) = |\psi(\vec{q},t)|^2 / \int d^3\vec{q}\, \underbrace{|\psi(\vec{q},t)|^2}_{\text{sol. to KG eqn}}$$

(conditioning w.r.t. Minkowski time)

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(conditioning w.r.t. Minkowski time)

sol. to KG eqn

conditioning instead w.r.t. covariant clock POVM:

Newton-Wigner localization probability for Klein-Gordon systems (separate ± modes)

PH, Smith, Lock 2007.00580

$$P(\vec{q}\ \text{when}\ \tau,\sigma) = |\psi_S^\sigma(\tau,\vec{q})|^2$$

$\sim t/p_t$

sol. to Schrödinger eqn, Newton-Wigner wave function

approximate, but best possible localization notion in relativistic particle dynamics

$$\langle\psi^\sigma_{\text{phys}}|\psi^\sigma_{\text{phys}}\rangle_{\text{phys}} = \sigma\,(\psi^\sigma_{\text{phys}},\psi^\sigma_{\text{phys}})_{\text{KG}} \equiv \int d\vec{q}\,|\psi^\sigma_S(\tau,\vec{q})|^2$$

$\Rightarrow$ Page-Wootters formalism IS a viable approach to relational dynamics

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….. and many more

see also Anderson '17

Analogy with covariance in relativity

Tensors, are reference-frame-neutral objects: they encode physics as "experienced" in any local frame at once

e.g., stress-energy tensor at x: $$T_x : T_x\mathscr{M} \times T_x\mathscr{M} \to \mathbb{R}$$

$\Rightarrow$ contract with frame vectors, e_A, to produce energy-momentum numbers as "experienced" in frame

in abstract index notation:

frame-neutral description

$$T_{\mu\nu}$$

analog of Dirac observables on physical Hilbert space

e^μ_A constitutes coordinate map into frame's perspective (inverse e^A_μ)

$$T_{AB} := e^\mu_A \, T_{\mu\nu} \, e^\nu_B = T(e_A, e_B)$$

analog of reduced quantum theories

internal perspective of frame e_A on stress-energy tensor

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$$\xrightarrow{e^\mu_A \,\bullet\, e^\nu_B}$$

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internal perspective of frame e_A

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2nd frame

$\tilde{e}^{\mu}_{A'} \bullet \tilde{e}^{\nu}_{B'}$

$e^{\mu}_{A} \bullet e^{\nu}_{B}$

e^{μ}_{A} constitutes coordinate map into frame’s perspective (inverse e^{A}_{μ})

$$\tilde{T}_{A'B'} = \tilde{e}^{\mu}_{A'} T_{\mu\nu} \tilde{e}^{\nu}_{B'}$$

$$T_{AB} := e^{\mu}_{A}\, T_{\mu\nu}\, e^{\nu}_{B} = T(e_A, e_B)$$

internal perspective of frame $\tilde{e}_{A'}$ on same object

internal perspective of frame e_A

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$\tilde{e}^{A'}_{\mu} \bullet \tilde{e}^{B'}_{\nu}$

$e^{\mu}_{A} \bullet e^{\nu}_{B}$

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internal perspective of frame $\tilde{e}_{A'}$

internal perspective of frame e_A

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2nd frame

$\tilde{e}^{A'}_{\mu} \bullet \tilde{e}^{B'}_{\nu}$

$e^{\mu}_{A} \bullet e^{\nu}_{B}$

$\Lambda_A{}^{A'} \bullet \Lambda^{B'}{}_B$

$$\tilde{T}_{A'B'} = \tilde{e}^{\mu}_{A'} T_{\mu\nu} \tilde{e}^{\nu}_{B'} \longrightarrow T_{AB} := e^{\mu}_{A} T_{\mu\nu} e^{\nu}_{B} = T(e_A, e_B)$$

internal perspective of frame $\tilde{e}_{A'}$

$$\Lambda_A{}^{A'} = e^{\mu}_{A} \tilde{e}^{A'}_{\mu}$$

local Lorentz transformation

internal perspective of frame e_A

Analogy with covariance in relativity

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Change of internal frame perspective *<u>via</u>* frame-neutral structure and coordinate transformation $\Lambda_A{}^{A'}$

2nd frame

$$
\begin{tikzcd}
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$$

internal perspective of frame $\tilde{e}_{A'}$

$$\Lambda_A{}^{A'} = e^\mu_A \tilde{e}^{A'}_\mu$$

local Lorentz transformation

internal perspective of frame e_A

Quantum clock covariance

basic idea: $|\psi_{\text{phys}}\rangle$ **is not a 'timeless', but clock-neutral state, i.e. description of physics prior to having chosen temporal reference system relative to which dynamics of remaining DoFs is described**

Why possible?

$\Rightarrow$ **symmetry/constraint induced redundancy in description of** $\mathcal{H}_{\text{phys}}$

$\Rightarrow$ **many different ways in describing same invariant** $|\psi_{\text{phys}}\rangle$

$\Rightarrow$ **associate with different clock choices**

$\Rightarrow$ **reduction maps (removing redundancy) relative to different clock choices as "quantum coordinate maps" into "clock perspective"**

Quantum clock changes

PH, Vanrietvelde 1810.04153
PH 1811.00611
PH, Smith, Lock 1912.00033 + 2007.00580

E.g. suppose

$$\hat{C}_H = \hat{H}_1 + \hat{H}_2 + \hat{H}_S$$

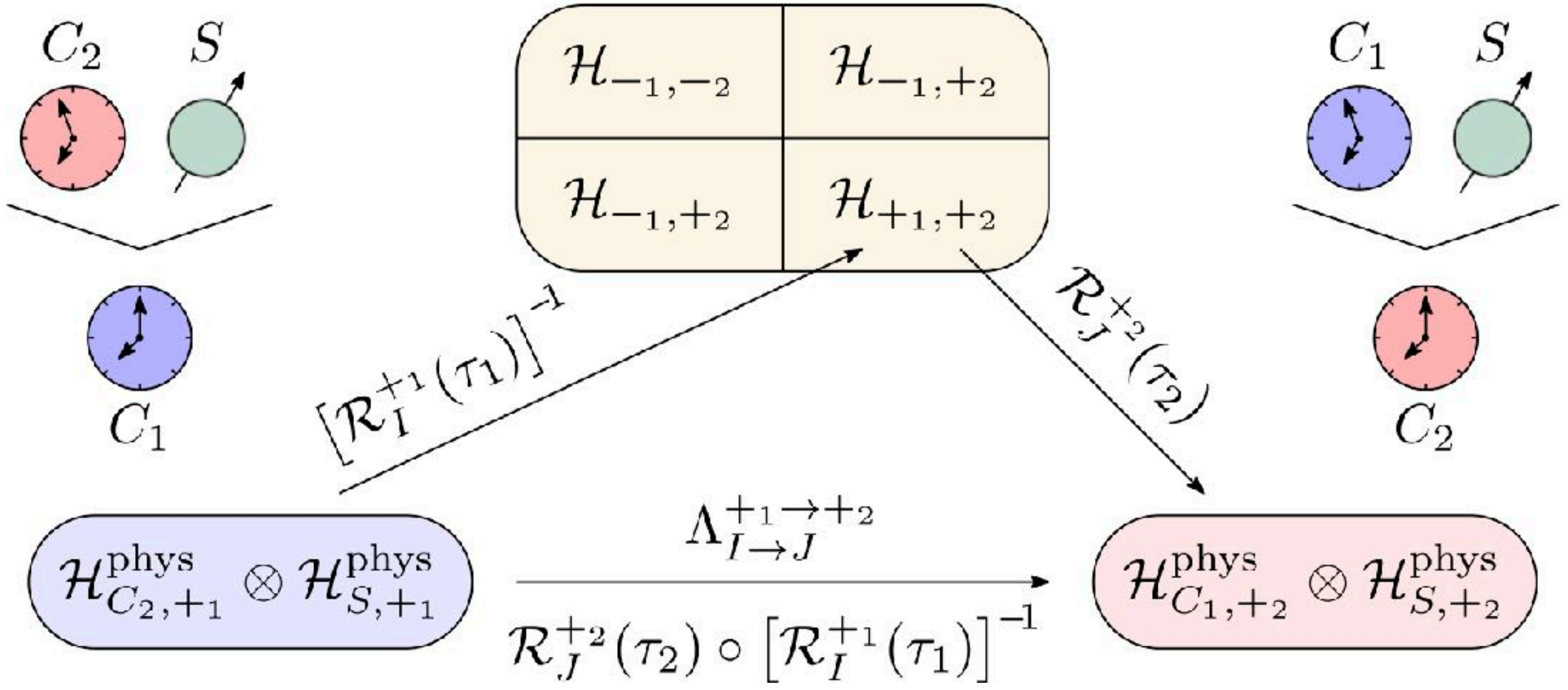
clock 1 clock 2

“quantum coordinate transformations” (frequency-sector-wise), schematic:

$$\Lambda_{C_1 \to C_2} = \mathcal{R}^{\sigma_2}(\tau_2) \circ \mathcal{R}^{\sigma_1}(\tau_1)^{-1}$$

analog of $\Lambda_A{}^{A'} = e_A^\mu \, \tilde{e}_\mu^{A'}$ from relativity

clock-neutral $\mathcal{H}_{\text{phys}}$



“perspective” of clock C_1 **“perspective” of clock C_2**

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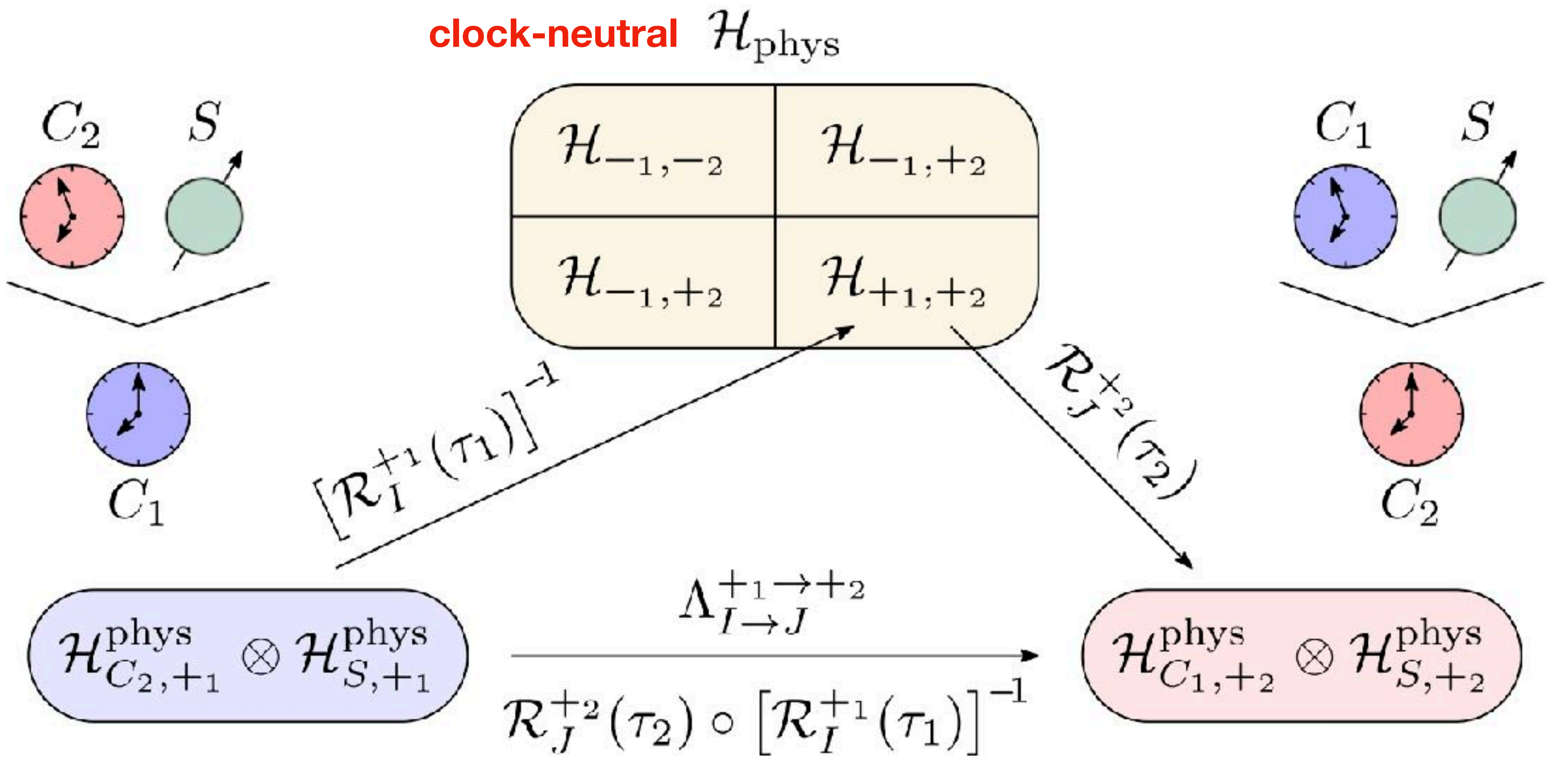


"perspective" of clock C_1 **"perspective" of clock C_2**

State transf. $$|\psi^{\sigma_2}_{C_1 S|C_2}\rangle = \Lambda_{C_1\to C_2} |\psi^{\sigma_1}_{C_2 S|C_1}\rangle$$

Observable transf. $$\hat{O}_{C_1 S|C_2} := \Lambda_{C_1\to C_2}\, \hat{O}_{C_2 S|C_1}\, \Lambda_{C_2\to C_1}$$

Always describe **same** physics, but relative to different perspectives

Temporal frame dependence of physics

- **entanglement depends on the quantum frame**

  PH, Lock, Ahmad, Smith, Galley 2103.01232; de la Hamette, Galley, PH, Loveridge, Müller 2110.13824; Castro-Ruiz, Oreshkov 2110.13199; Giacomini et al 1712.07207; Vanrietvelde, PH, Giacomini, Castro-Ruiz 1809.00556

- **“quantum relativity” of comparing readings of and synchronizing different quantum clocks**

  PH, Vanrietvelde, 1810.04153; PH, Smith, Lock 2007.00580; Bojowald, PH, Tsobanjan 1011.3040

- **Temporally local time evolution relative to one clock may appear as superposition of time evolutions relative to another**

  Castro-Ruiz, Giacomini, Belenchia, Brukner 1908.10165; PH, Smith, Lock 1912.00033

- **Indirect clock self-reference effects**

  PH, Smith, Lock 1912.00033 + 2007.00580

earlier approaches to quantum clock covariance:

- semiclassical Bojowald, PH, Tsobanjan 1011.3040; PH, Kubalova, Tsobanjan 1111.5193
- reduced quantization Malkiewicz 1407.3457; 1601.04857

Quantum relativity of subsystems

PH, Lock, Ahmad, Smith, Galley
2103.01232

3 kinematical subsystems subject to constraint

$$\hat{C} = \hat{C}_A + \hat{C}_B + \hat{C}_C$$

either can be degenerate and $\mathrm{U}(1)$ or $(\mathbb{R}, +)$ generator

relational observables of C relative to B

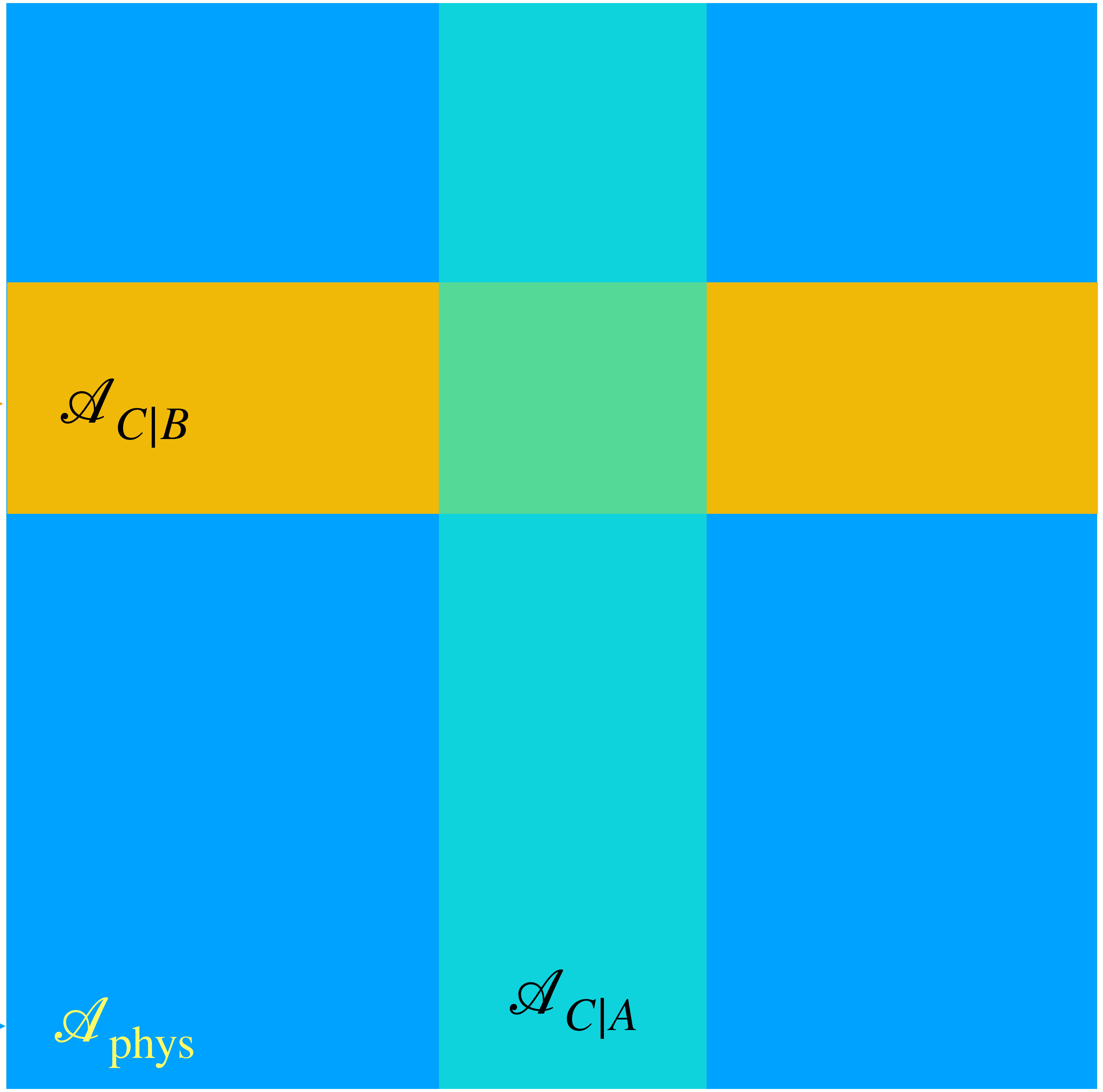
$\Rightarrow$ different relational (inv.) ways to refer to a kinematical subsystem

$\Rightarrow$ different relational observable subalgebras inside total invariant algebra

$\Rightarrow$ induce different inv. tensor factorizations

observables on $\mathscr{H}_{\text{phys}}$

$\Rightarrow$ different appearance of same physics



relational observables of C relative to A

Many faces of the problem of time

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Periodic clocks as incomplete temporal frames

Chataignier, PH, Lock *to appear*

Hamiltonian constraint

what if S does not have "enough" periodic states and observables?

$$C_H = H_C + H_S$$

clock Hamiltonian $\mathrm{U}(1)$-generator

system

$\Rightarrow$ **periodic clock forces system to be also periodic!**

$$F_{f_S,T}(\tau) \qquad |\psi_S(\tau)\rangle$$

periodic

Periodic clocks as incomplete temporal frames

Chataignier, PH, Lock *to appear*

Example: incommensurate oscillators

what if S does not have "enough" periodic states and observables?

$$C_H = H_C - H_S$$

Oscillator 1 with frequency ω_1

Oscillator 2 with frequency ω_2

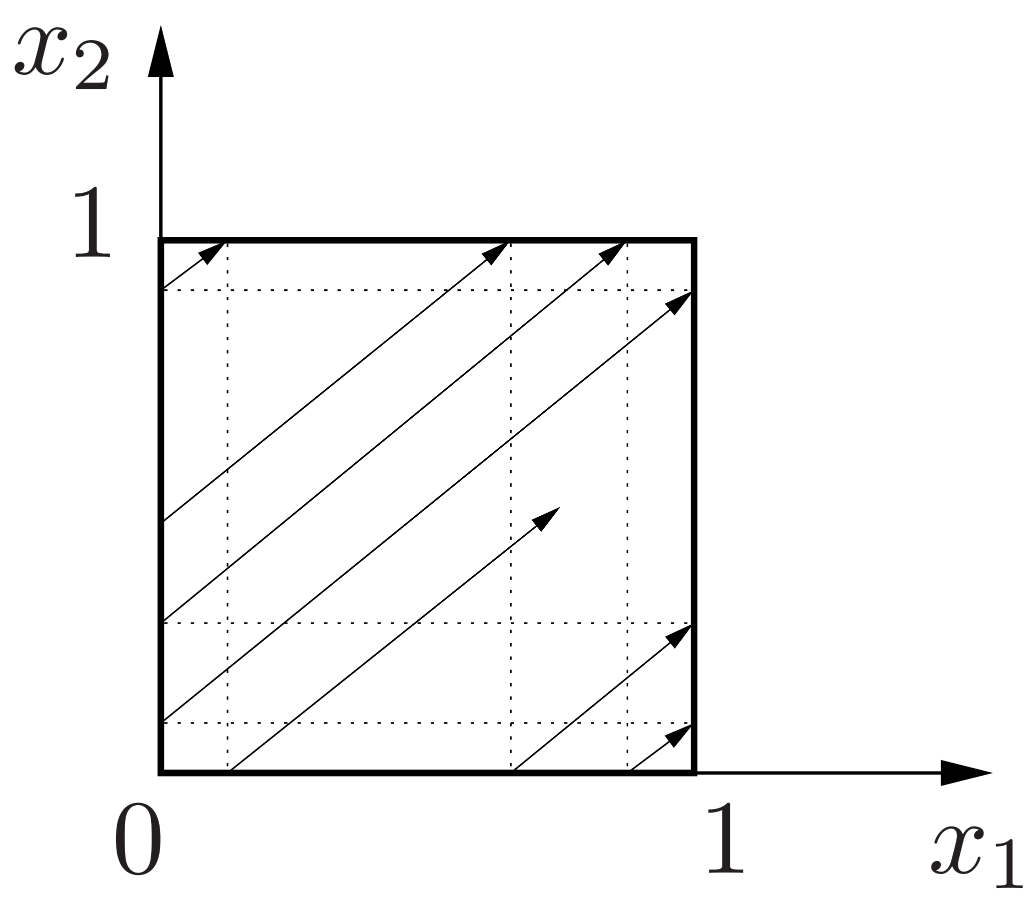
if $\omega_1/\omega_2 \notin \mathbb{Q}$ then S does not feature non-trivial states/observables periodic in ω_1 and $\dim \mathscr{H}_{\text{phys}} = 1$ or 0

$\Rightarrow$ challenge for a classical limit!

Global time problem and “S-matrix interpretation” of relational dynamics

Non-trivial interplay of quantum and classical relational dynamics
in semiclassical regime of non-global clocks consistent with earlier findings

PH, Kubalova, Tsobanjan ’12
Dittrich, PH, Nelson, Koslowski ‘16



⇒ semi-integrable models prevent existence of semiclassical limit in standard quantization

⇒ polymer quantization saves winding numbers in QT, can it come to the rescue for periodic clocks as well?

“adapt quantization to observables” Dittrich, PH, Nelson, Koslowski ‘16

in any case: in absence of relational observables defined in full QT, still have transition amplitudes

kinematical observables

$$S(\tau_2, O_2; \tau_1, O_1) := \langle \tau_2, O_2 | \, \Pi_{\text{phys}} \, | \tau_1, O_1 \rangle$$

Interpret dynamics of full QG theory rather in terms of transition amplitudes?

Outlook: quantum reference frames for general symmetry groups

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Change of internal frame perspective *<u>via</u>* frame-neutral structure and coordinate transformation $\Lambda_A{}^{A'}$

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\end{tikzcd}$$

internal perspective of frame $\tilde{e}_{A'}$

$$\Lambda_A{}^{A'} = e^{\mu}_{A} \tilde{e}^{A'}_{\mu}$$

local Lorentz transformation

internal perspective of frame e_A

Perspective-neutral approach to quantum frame covariance for general symmetry groups

Anne-Catherine de la Hamette*[1,2], Thomas D. Galley†[3], Philipp A. Höhn‡[4,5], Leon Loveridge§[6], and Markus P. Müller¶[1,2,3]

[1]Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria
[2]Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria
[3]Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo ON N2L 2Y5, Canada
[4]Okinawa Institute of Science and Technology Graduate University, Onna, Okinawa 904 0495, Japan
[5]Department of Physics and Astronomy, University College London, London, United Kingdom
[6]Quantum Technology Group, Department of Science and Industry Systems, University of South-Eastern Norway, 3616 Kongsberg, Norway

October 26, 2021

arXiv:2110.13824

nt-ph] 26 Oct 2021

Perspective-neutral approach for general groups

de la Hamette, Galley, PH, Loveridge, Müller 2110.13824

Suppose $\mathscr{H}_{\text{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$ **carries unitary prod. rep. of (unimodular)** G

G-frame

system

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \qquad g \in G$$

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<u>**complete G-frame 'orientation states':**</u>

coherent states: $|\phi(g)\rangle$ $\Rightarrow$ **give rise to a covariant POVM:** $U_R(g')\,|\phi(g)\rangle = |\phi(g'g)\rangle$

orientation states typically not orthogonal $\langle\phi(g)\,|\,\phi(g')\rangle \nsim \delta(g, g')$

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generalization of clock states:

$$|t,\sigma\rangle = \int d\varepsilon\, e^{-i\varepsilon t}\,|\varepsilon,\sigma\rangle$$

$$\Rightarrow \quad U_C(t')\,|t,\sigma\rangle = |t+t',\sigma\rangle$$

clock states are coherent states of group generated by $\hat{H}_C$

Perspective-neutral approach for general groups

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Relational observables for general groups through G-twirl:

"what's the value of f_S when R is in orientation g?"

$$F_{f_S,R}(g) = \int_G d\tilde{g}\; U_{RS}(\tilde{g})\big(\,|\,\phi(g)\rangle\langle\phi(g)\,|\otimes f_S\big)\;U^\dagger_{RS}(\tilde{g})$$

The trinity generalizes to G-frames

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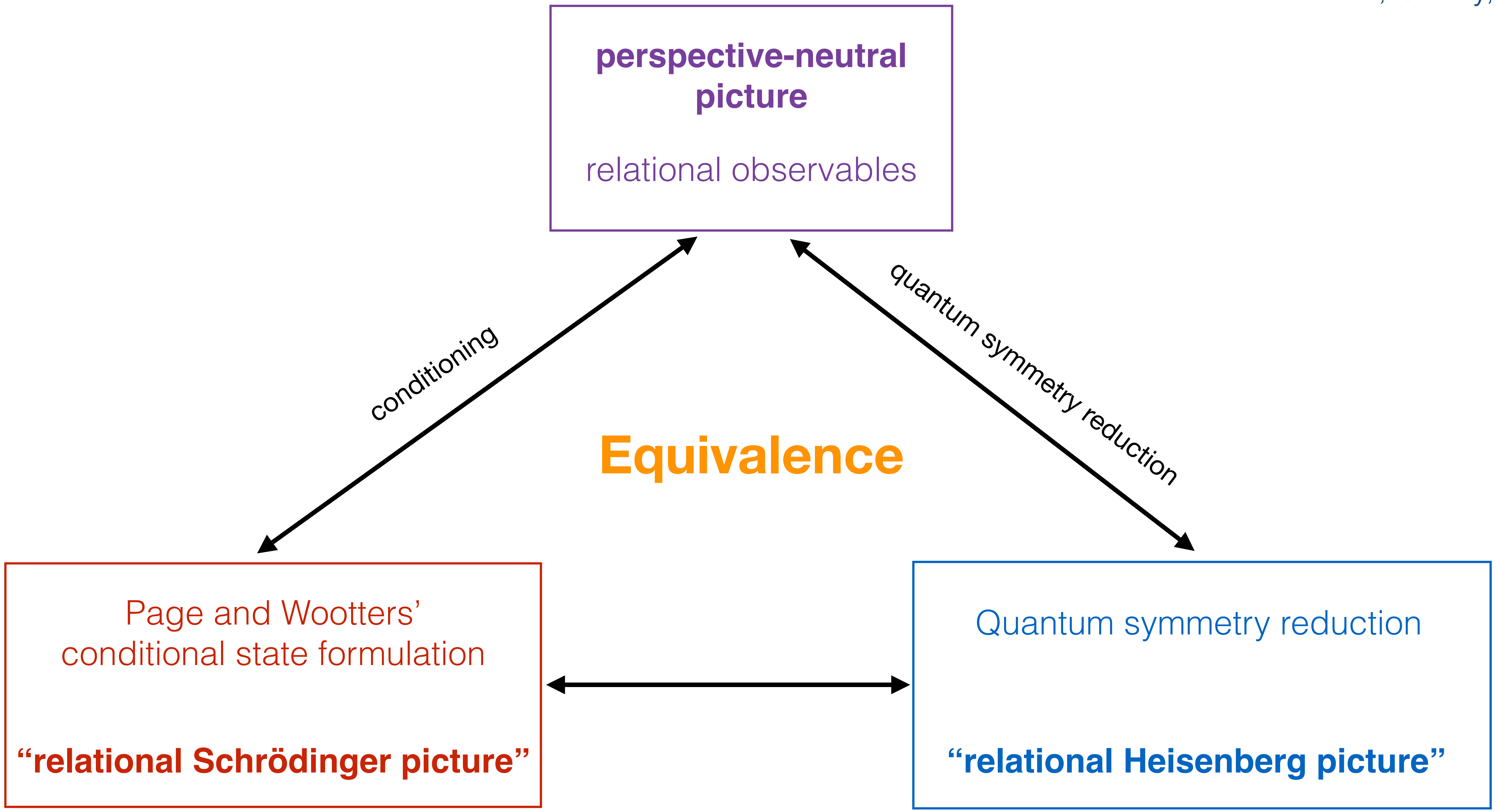

QRF changes

(includes quantum clock changes)

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$$\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$

frame 1 frame 2 system

Perspective-neutral physical Hilbert space

$$\mathscr{H}_{\text{phys}}$$

$$U_{R_1R_2S}(g)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

some new effects for non-Abelian groups

$$\varphi_{R_1}(g_1) = \langle\phi(g_1)|_{R_1} \otimes \mathbf{1}_{R_2S}$$

$$\varphi_{R_2}(g_2) = \langle\phi(g_2)|_{R_2} \otimes \mathbf{1}_{R_1S}$$

States relative to internal perspective of R_1

$$\varphi_{R_2} \circ \varphi_{R_1}^{-1}$$

states relative to internal perspective of R_2

generalizes previous efforts:

Giacomini, Castro-Ruiz, Brukner '17
Vanrietvelde, PH, Giacomini, Castro-Ruiz '18
Vanrietvelde, PH, Giacomini '18
PH, Vanrietvelde '18
PH '18
Castro-Ruiz, Giacomini, Belenchia, Brukner '19
PH, Smith, Lock '19 + '20
de la Hamette, Galley '20
Krumm, PH, Müller '20
PH, Lock, Ahmad, Smith, Galley '21
Giacomini '21

New perspective on “wave function of the universe”

$$\hat{H}(N)\,|\psi_{\rm phys}\rangle = 0\,, \qquad \hat{H}_a(\vec{M})\,|\psi_{\rm phys}\rangle = 0$$

plenty of redundancy here

<u>Proposal: wave function of the universe as</u>

- perspective-neutral quantum state of universe

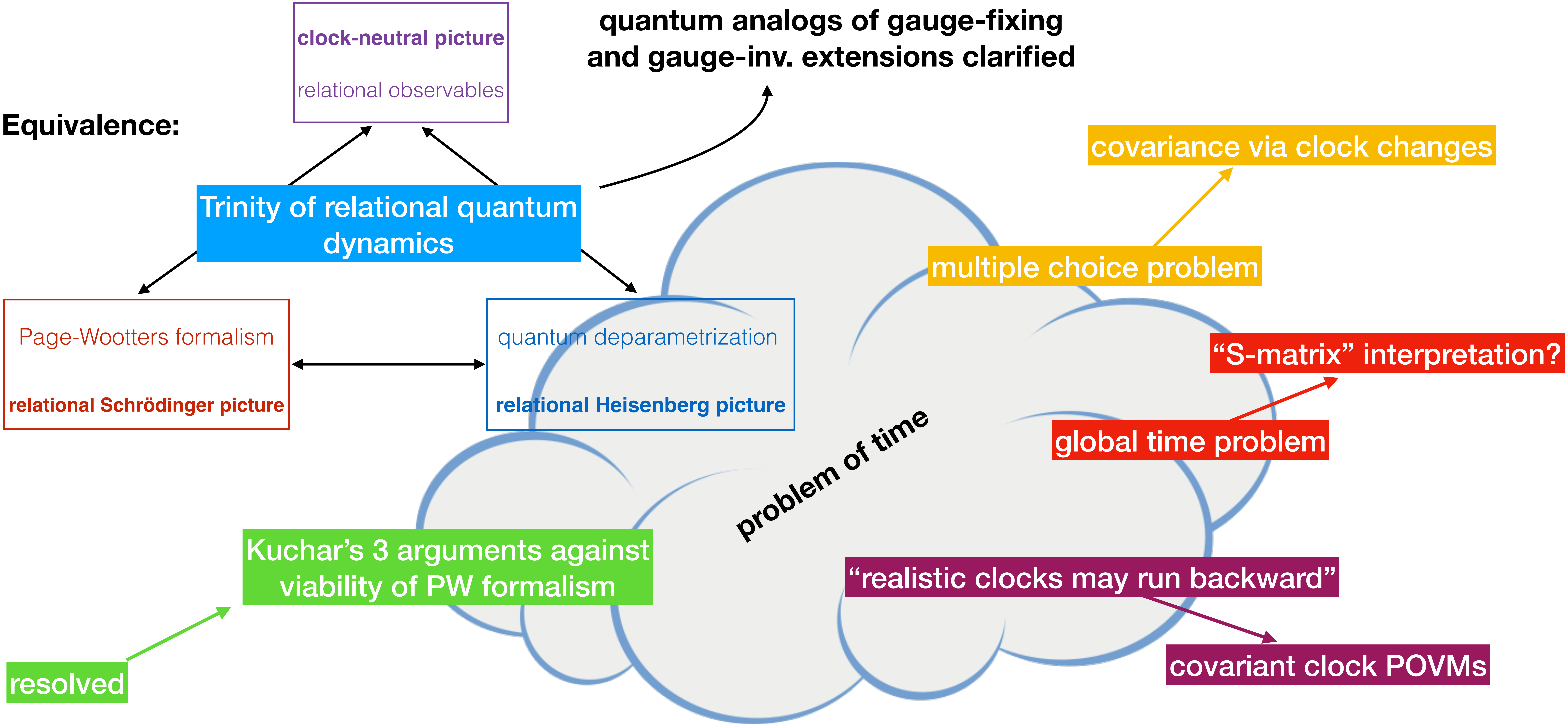
  $\Rightarrow$ global description prior to choice of QRF
- Link between all internal QRF perspectives on the universe

PH 1811.00611
(see also PH 1706.06882
+ 1412.8323)

Proposal to render “wave function of the universe” compatible with Carlo’s “Relational Quantum Mechanics”

Rovelli quant-ph/9609002

Conclusion: some updates on status of various faces of PoT



Appendix

Quantum relativity of subsystems

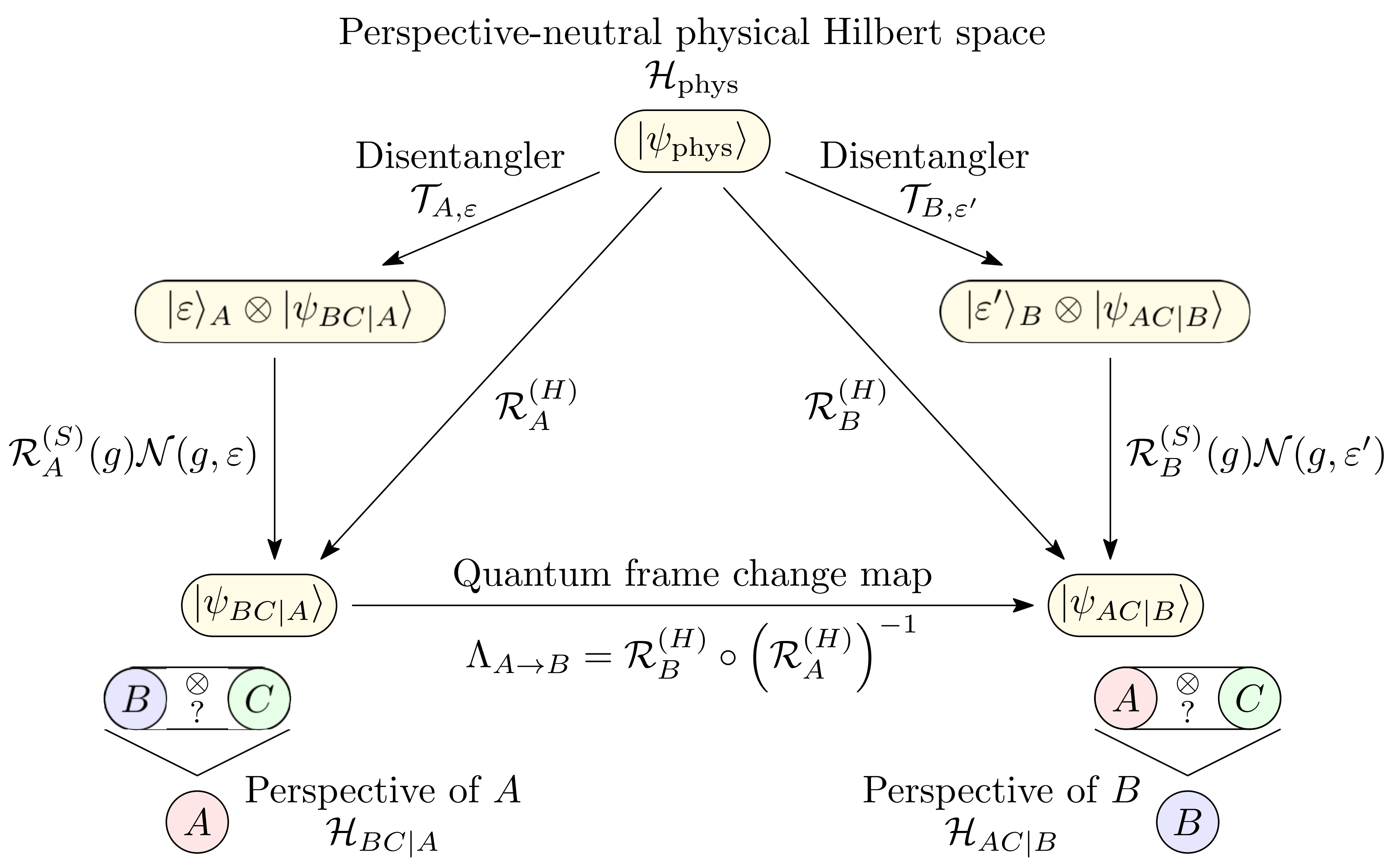
PH, Lock, Ahmad, Smith, Galley '21

more explicitly in QM/QC:

3 kinematical subsystems subject to constraint

$$\hat{C} = \hat{C}_A + \hat{C}_B + \hat{C}_C$$

either can be degenerate and $\mathrm{U}(1)$ or $\mathbb{R}$ generator



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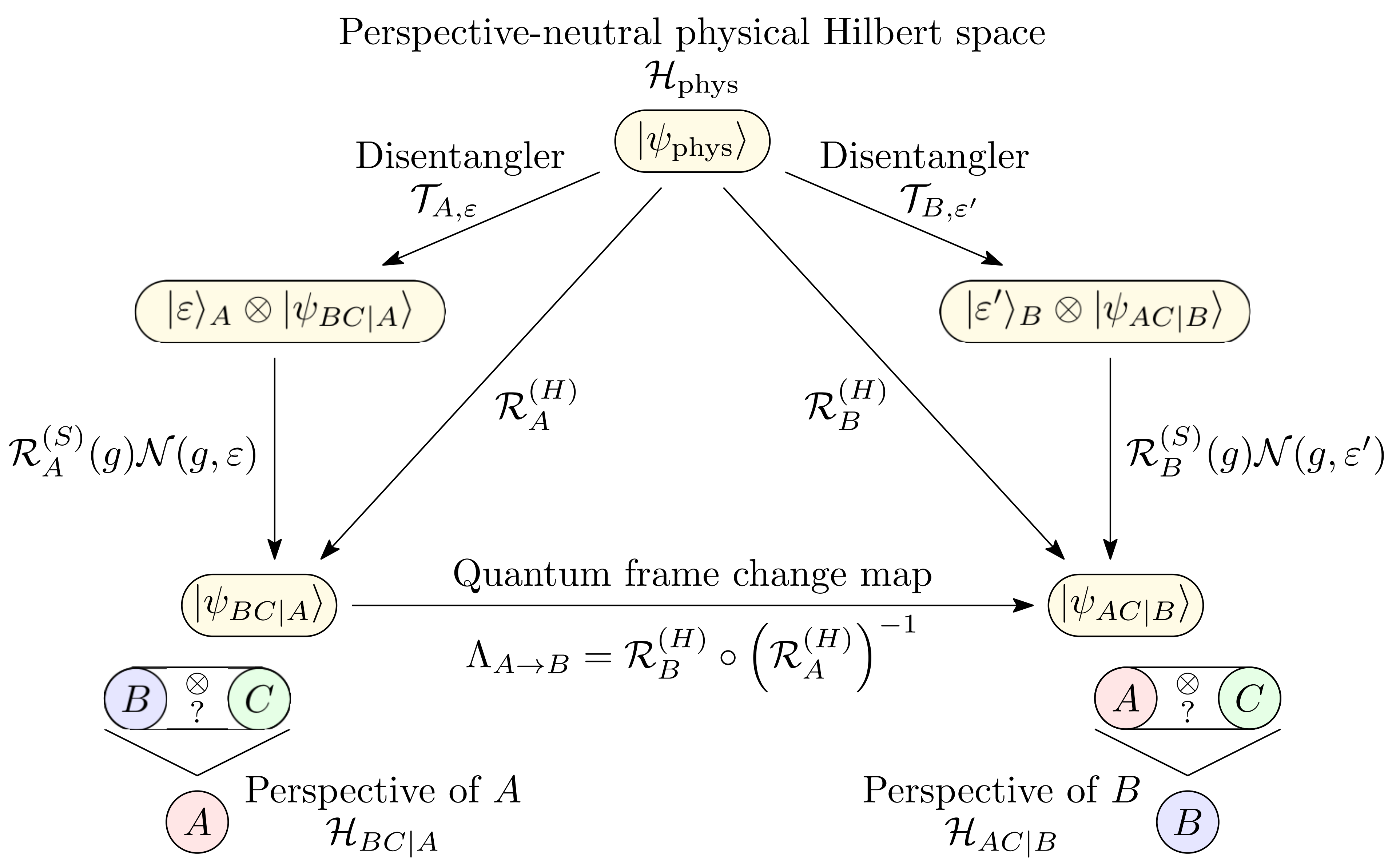
frame dependent gauge-invariant tensor factorizations:



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1. necessary and sufficient condition for $\mathscr{A}_{\text{phys}}$ to factorize

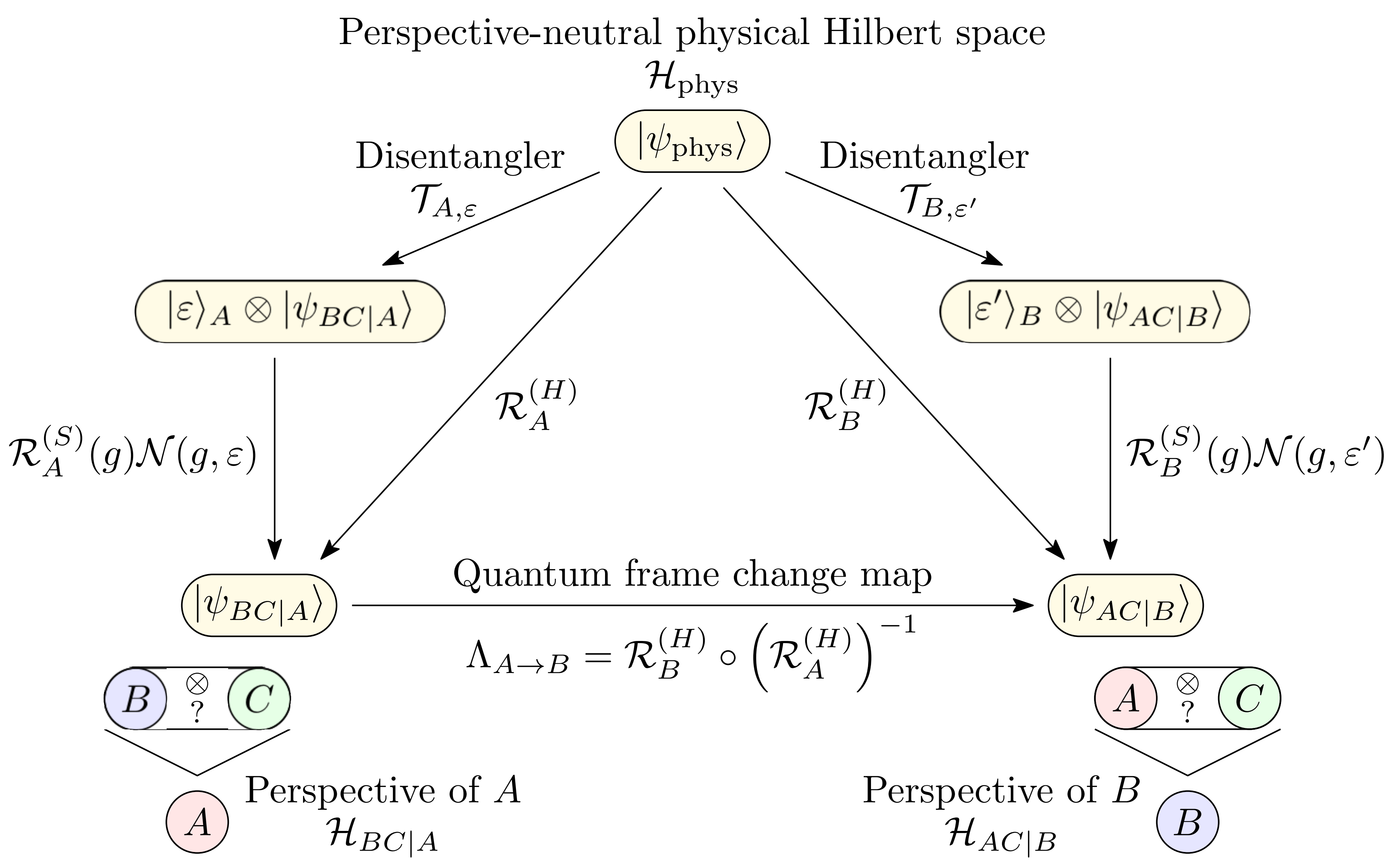
Minkowski sum

$$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \qquad \Leftrightarrow \qquad \sigma_{AB|C} = M(\sigma_{A|BC}, \sigma_{B|AC}) \qquad (*)$$

relational observables of A relative to C

$\sigma_{AB|C} = \mathrm{spec}(\hat{\mathrm{C}}_\mathrm{A} + \hat{\mathrm{C}}_\mathrm{B}) \cap \mathrm{spec}(-\hat{\mathrm{C}}_\mathrm{C})$

$\sigma_{A|BC} = \mathrm{spec}(\hat{\mathrm{C}}_\mathrm{A}) \cap \mathrm{spec}(-\hat{\mathrm{C}}_\mathrm{B} - \hat{\mathrm{C}}_\mathrm{C})$



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2. factorizability frame dependent: e.g. possible that

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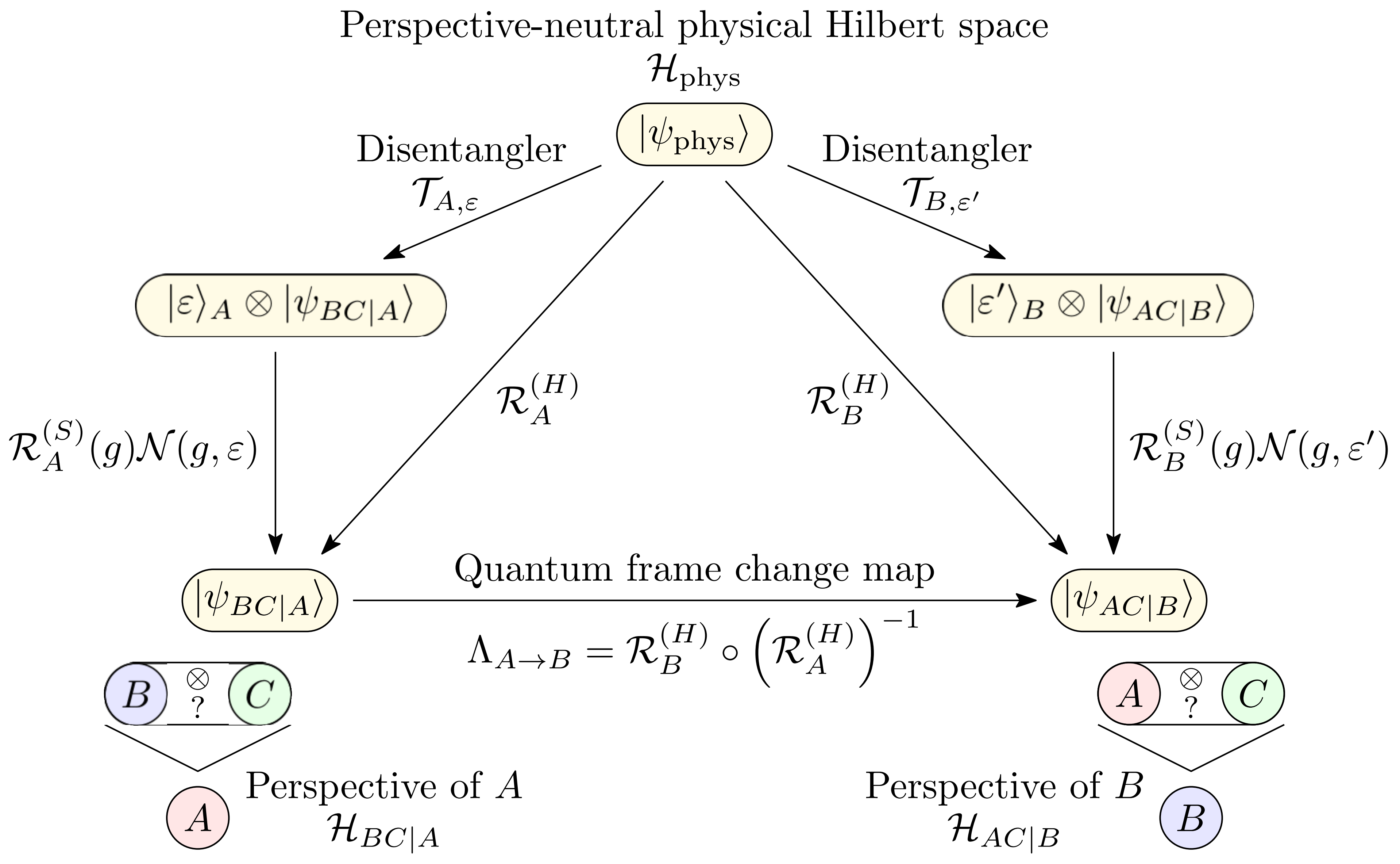

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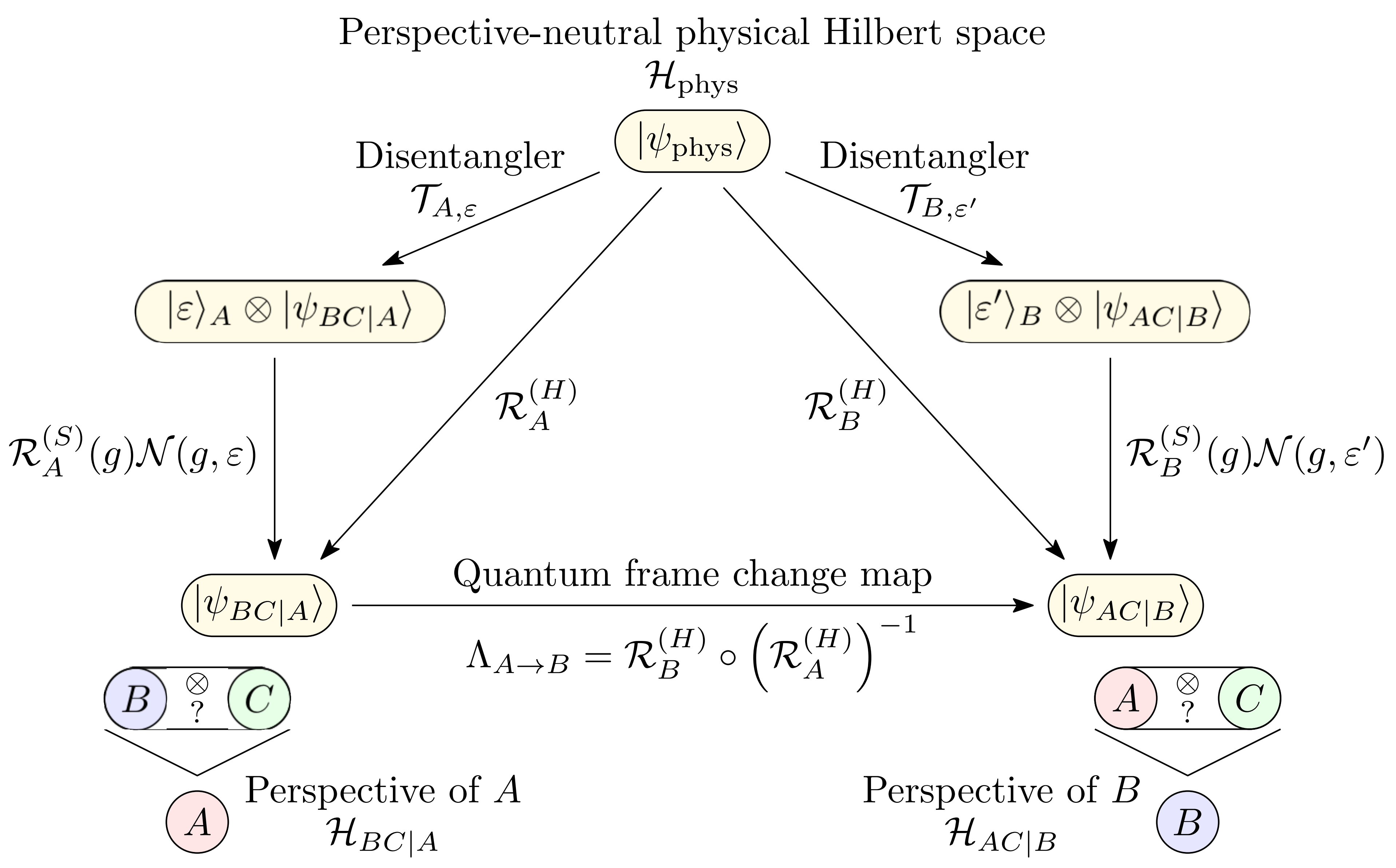
3. even if (*) satisfied in two frames, factorization necessarily frame-dependent

$$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{A|C} \otimes \mathscr{A}_{B|C} \simeq \mathscr{A}_{A|B} \otimes \mathscr{A}_{C|B} \quad \text{but} \quad \mathscr{A}_{A|B} \neq \mathscr{A}_{A|C}$$



Upshot: frame-dependent gauge-inv. entanglement

PH, Lock, Ahmad, Smith, Galley '21

"frames B and C mean different inv. DoFs when they refer to subsystem A"

if factorizability in two frame perspectives, i.e.

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then correlations/entanglement of A with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '19)

$\Rightarrow$ gauge-inv. entanglement entropy in general $S(\rho_{A|B}) \neq S(\rho_{A|C})$ for same global physical state

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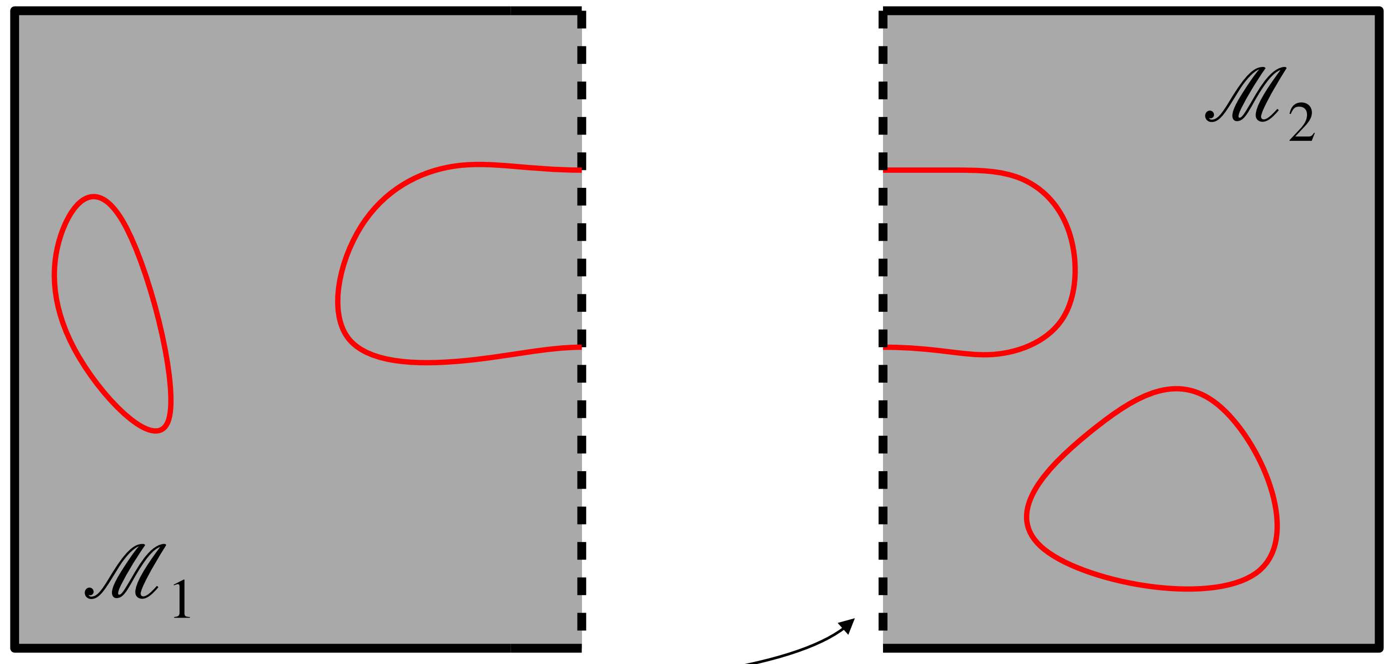
emphasize: gauge-inv./relational notion of subsystem
$\Rightarrow$ extend to field theory via edge modes?

a priori not a gauge-inv. notion of subsystems



Reference frames provide context for interpreting invariant observables

Relational & dressed observables

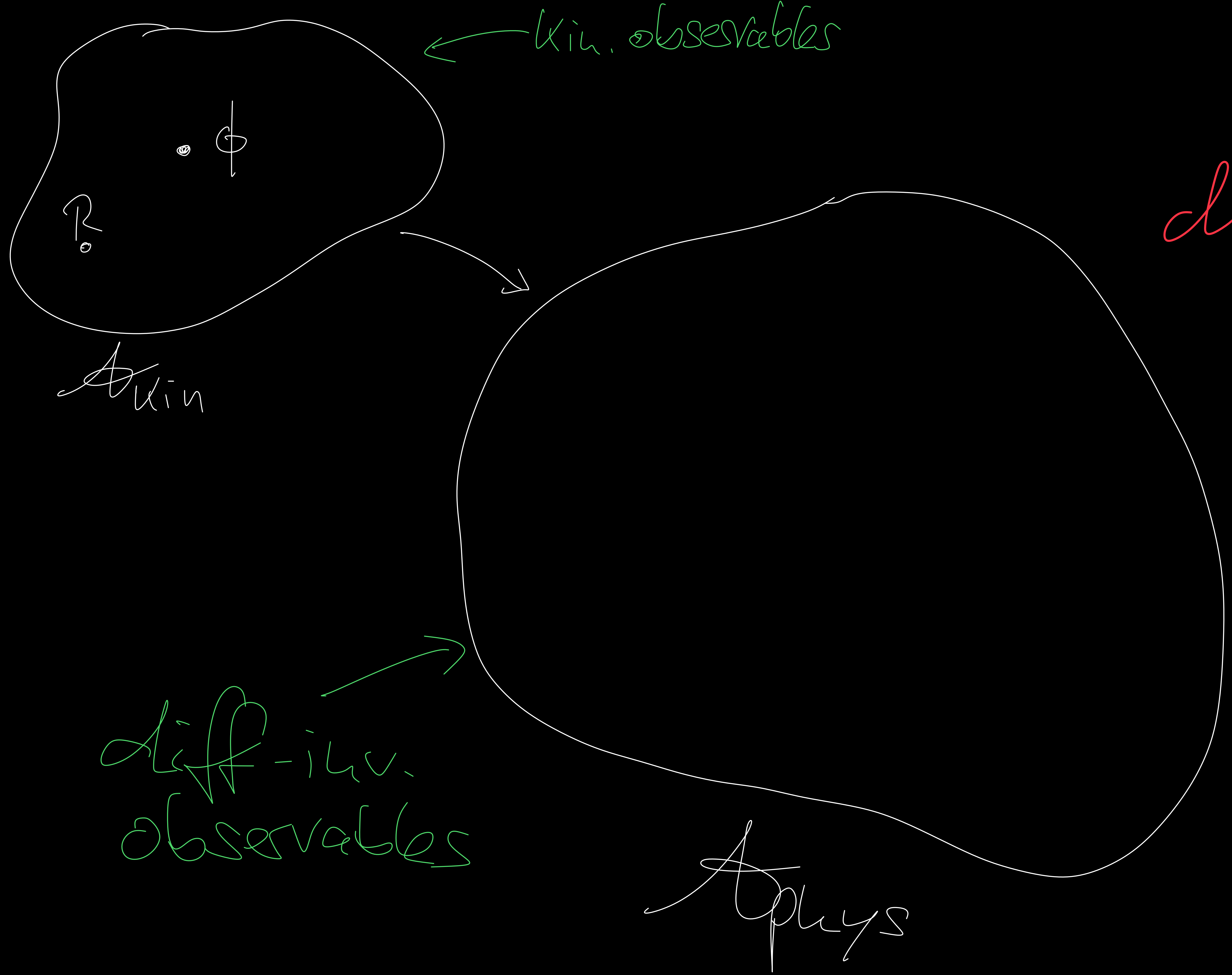
how to interpret diff-inv observables?

Relational & dressed observables

how to interpret diff-inv observables?

$\Longrightarrow$ in terms of some phys. reference frame

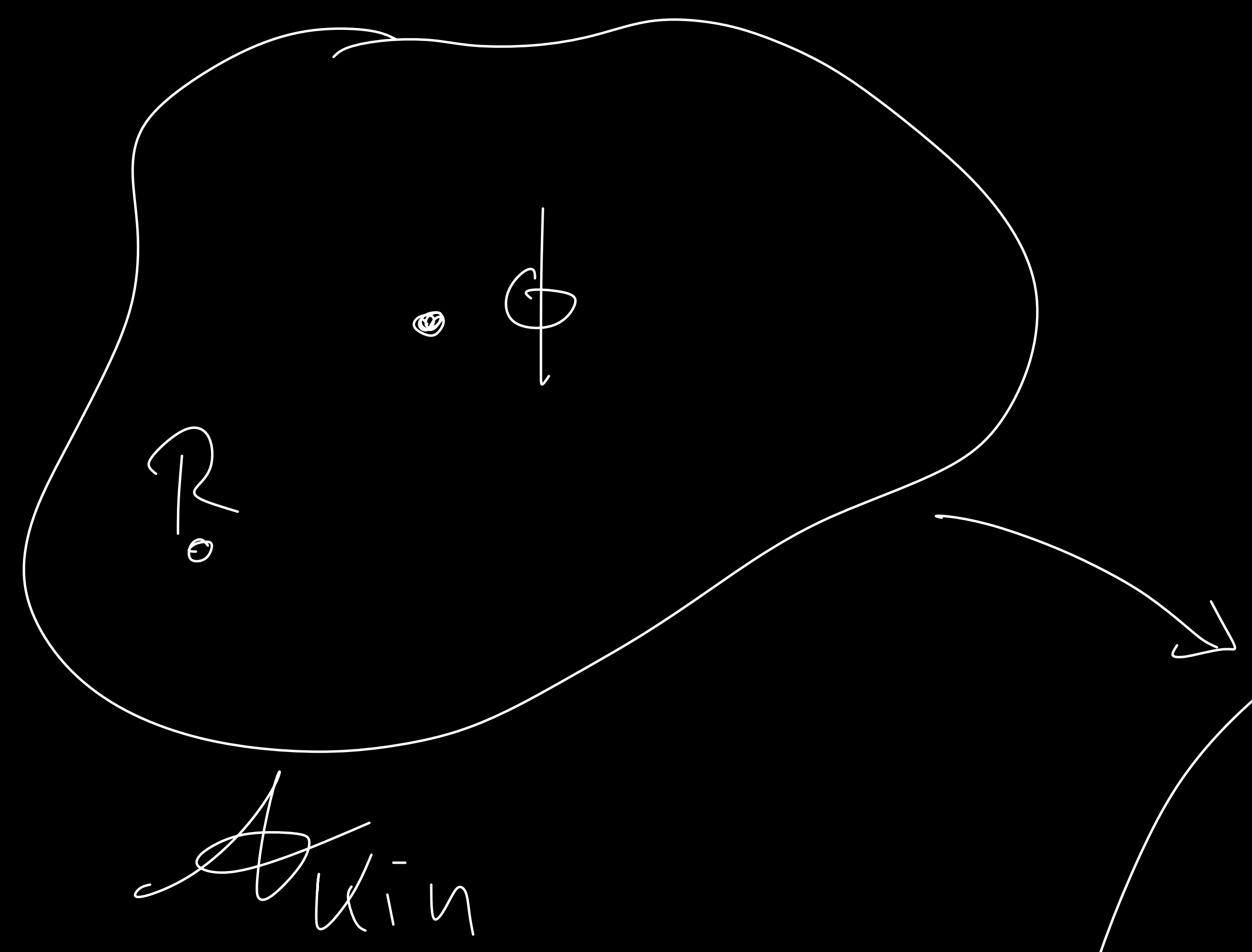
Relational & dressed observables

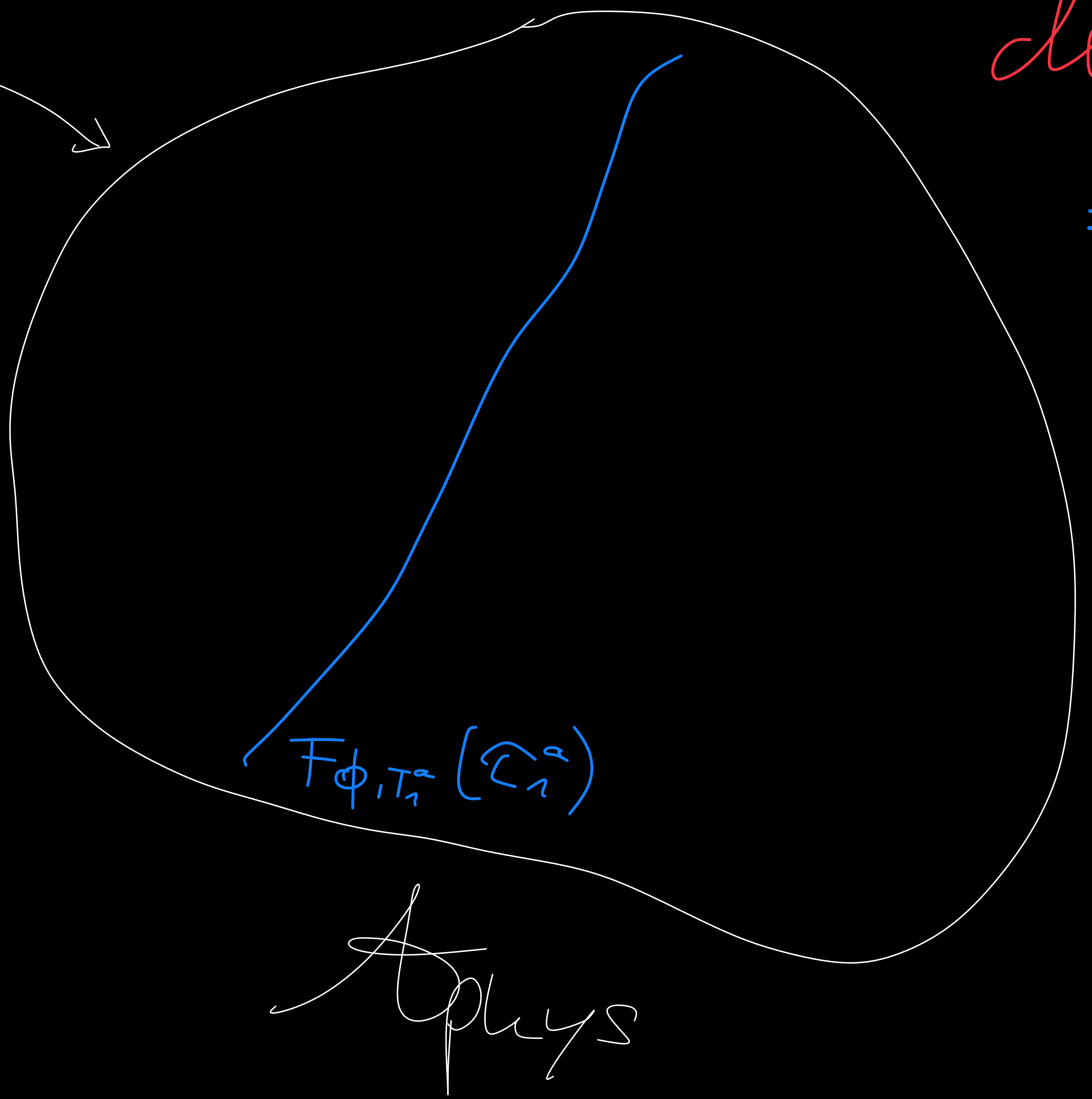


how to interpret diff-inv observables?

$\Rightarrow$ in terms of some phys. reference frame

Relational & dressed observables stratify invariant algebra





how to interpret diff-inv observables?

$\Longrightarrow$ in terms of some phys. reference frame

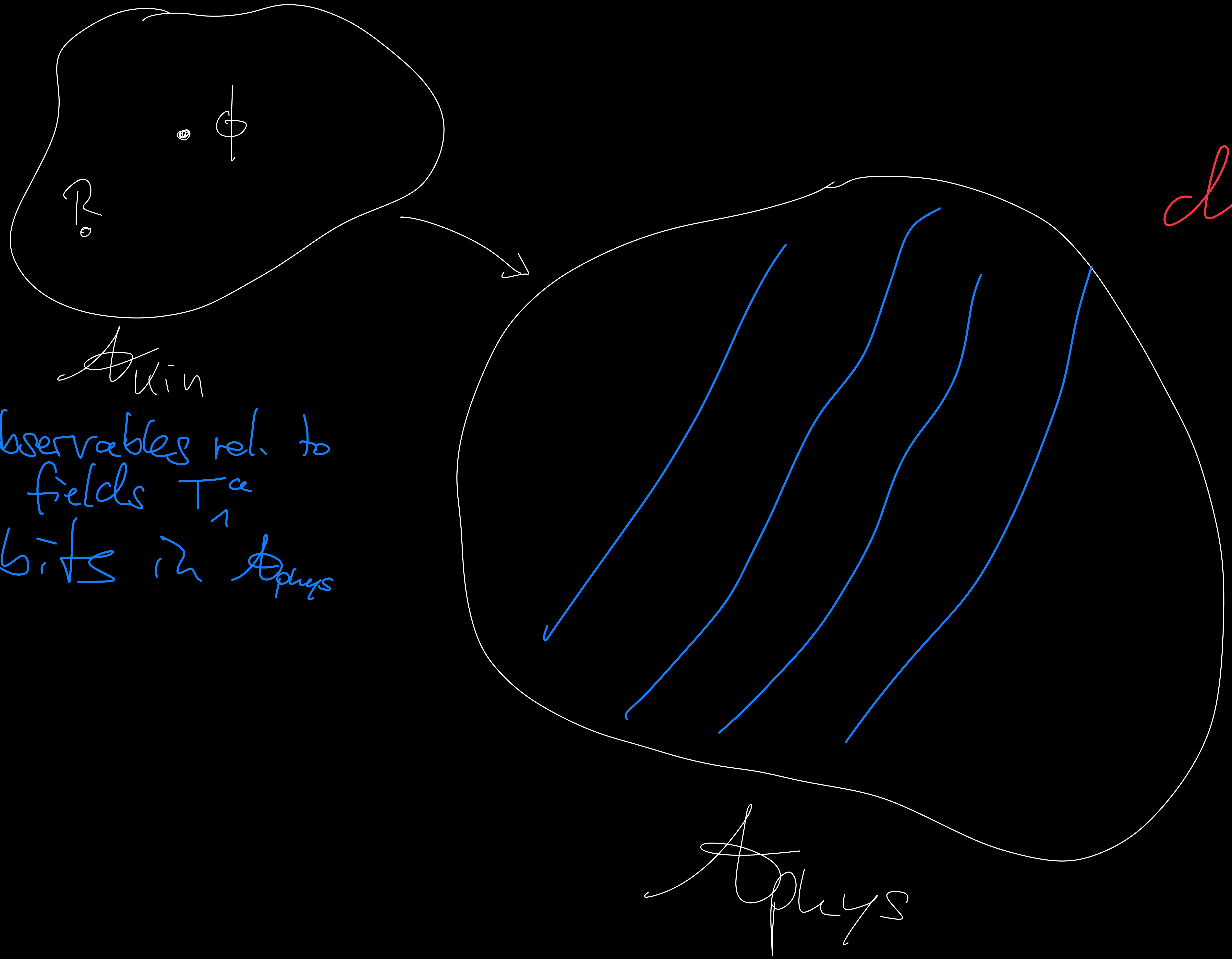
relational observables rel. to reference fields T_1^a

e.g. $F_{\phi, T_1^a}(\tau_1^a)$

"scalar field when reference reads $T_1^a = \tau_1^a$"

$\Longrightarrow$ 4-parameter family of obs.

Relational & dressed observables stratify invariant algebra



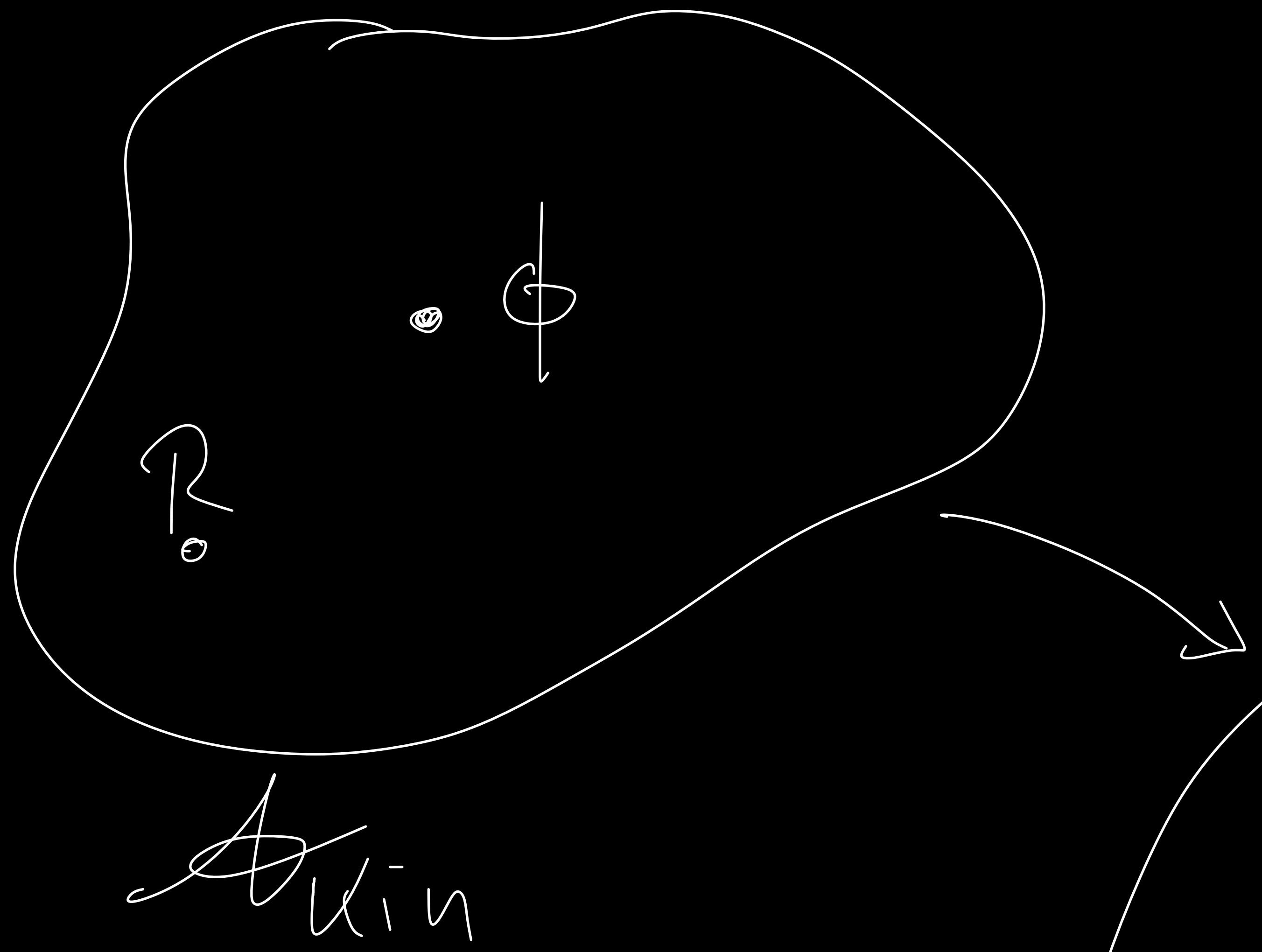
relational observables rel. to reference fields T^α_1

$\longrightarrow$ orbits in $\mathcal{A}_{phys}$

how to interpret diff-inv observables?

$\Rightarrow$ in terms of some phys. reference frame

Relational & dressed observables stratify invariant algebra



relational observables rel. to reference fields T_1^a

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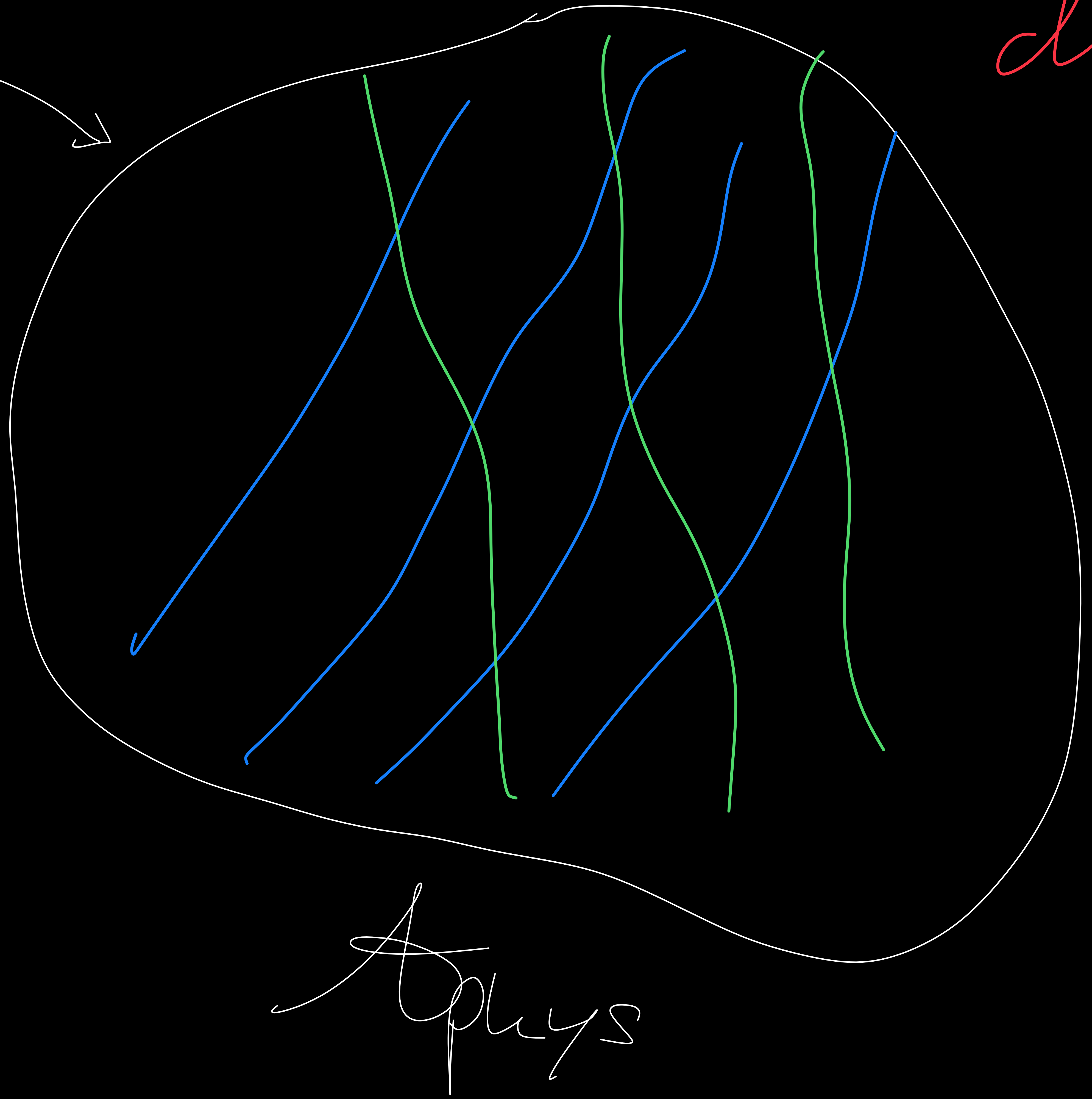
rel. obs. rel. to ref. fields T_2^a



how to interpret diff-inv observables?

$\Longrightarrow$ in terms of some phys. reference frame

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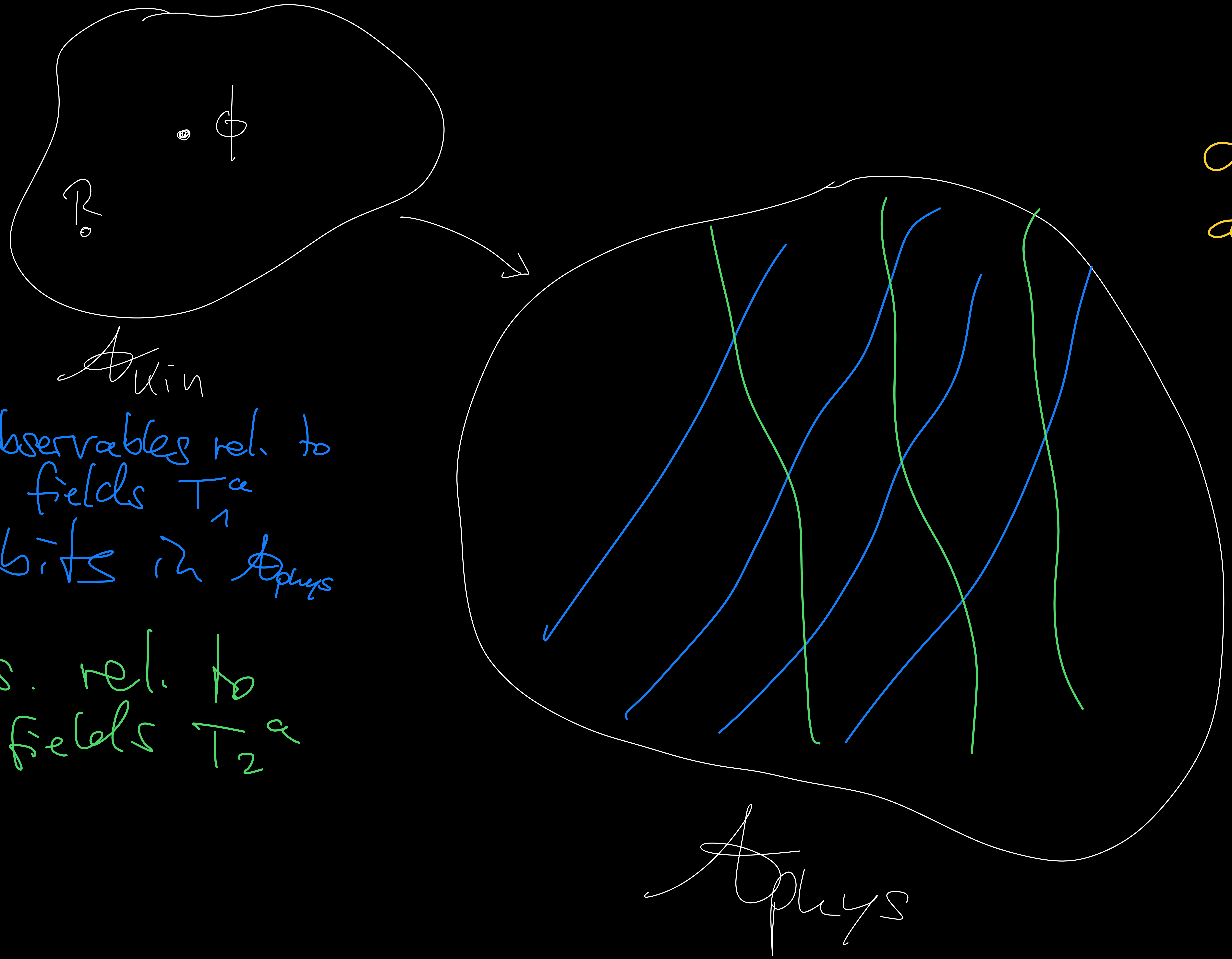

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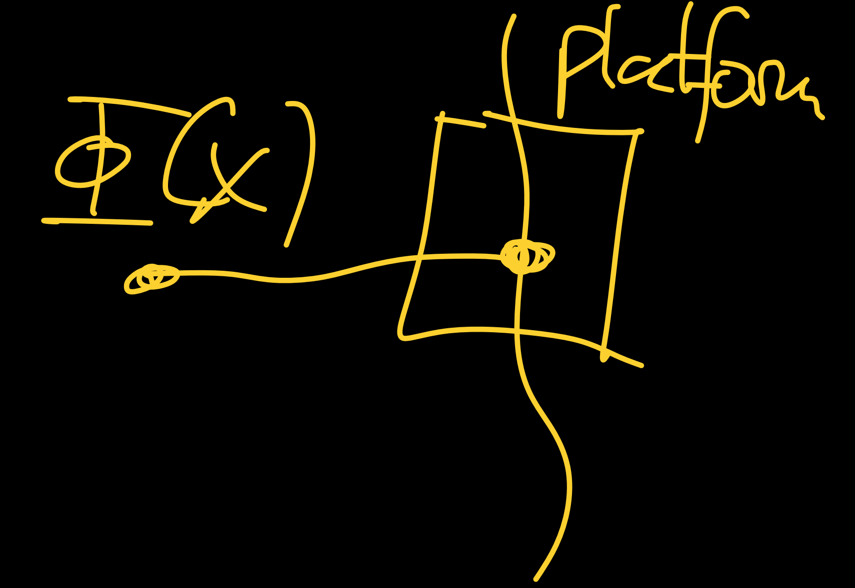
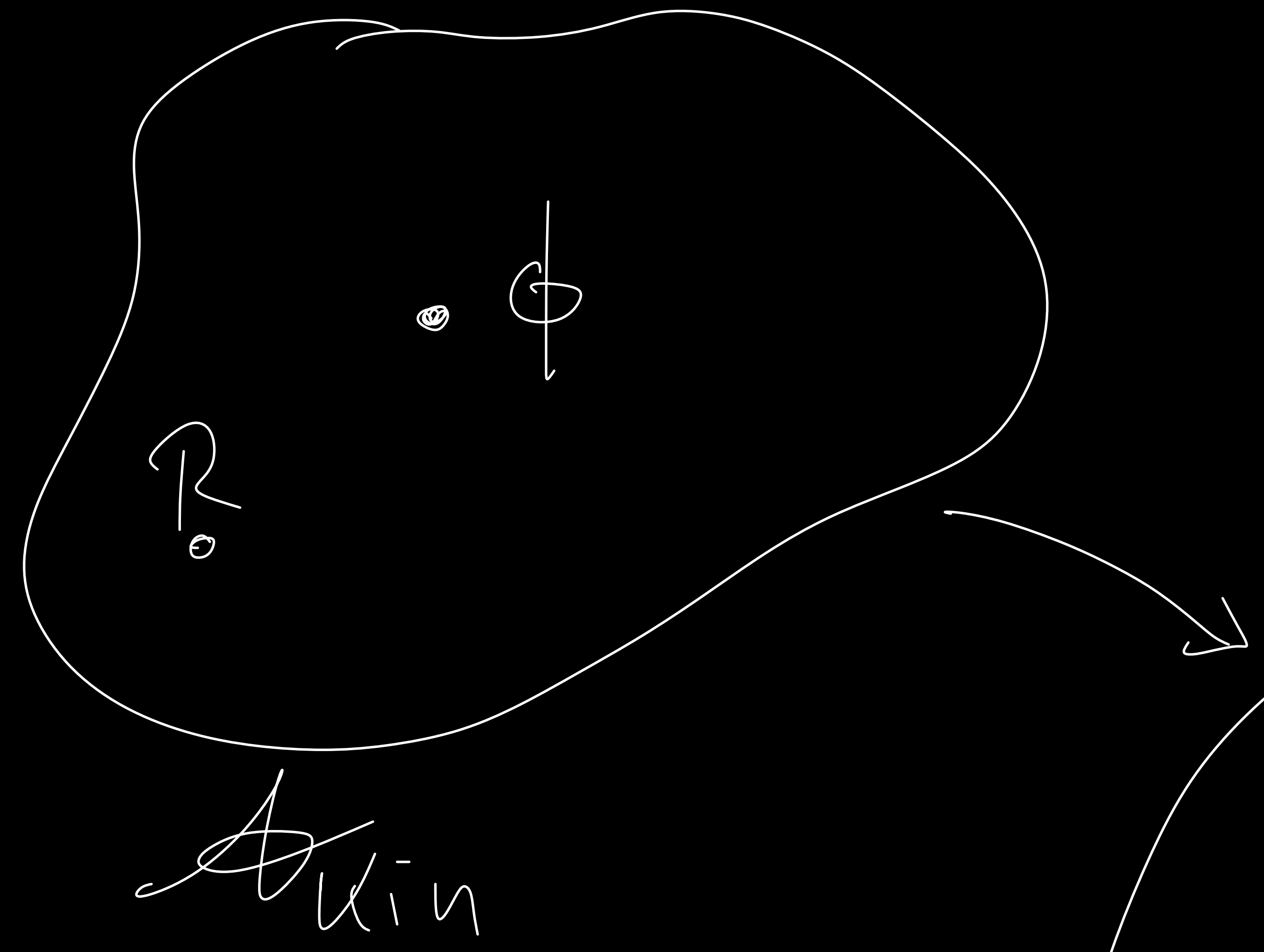
dressed observables accord. to prescription 1



$\Longrightarrow$ 4-parameter family of obs

Relational & dressed observables stratify invariant algebra



relational observables rel. to reference fields T_1^a

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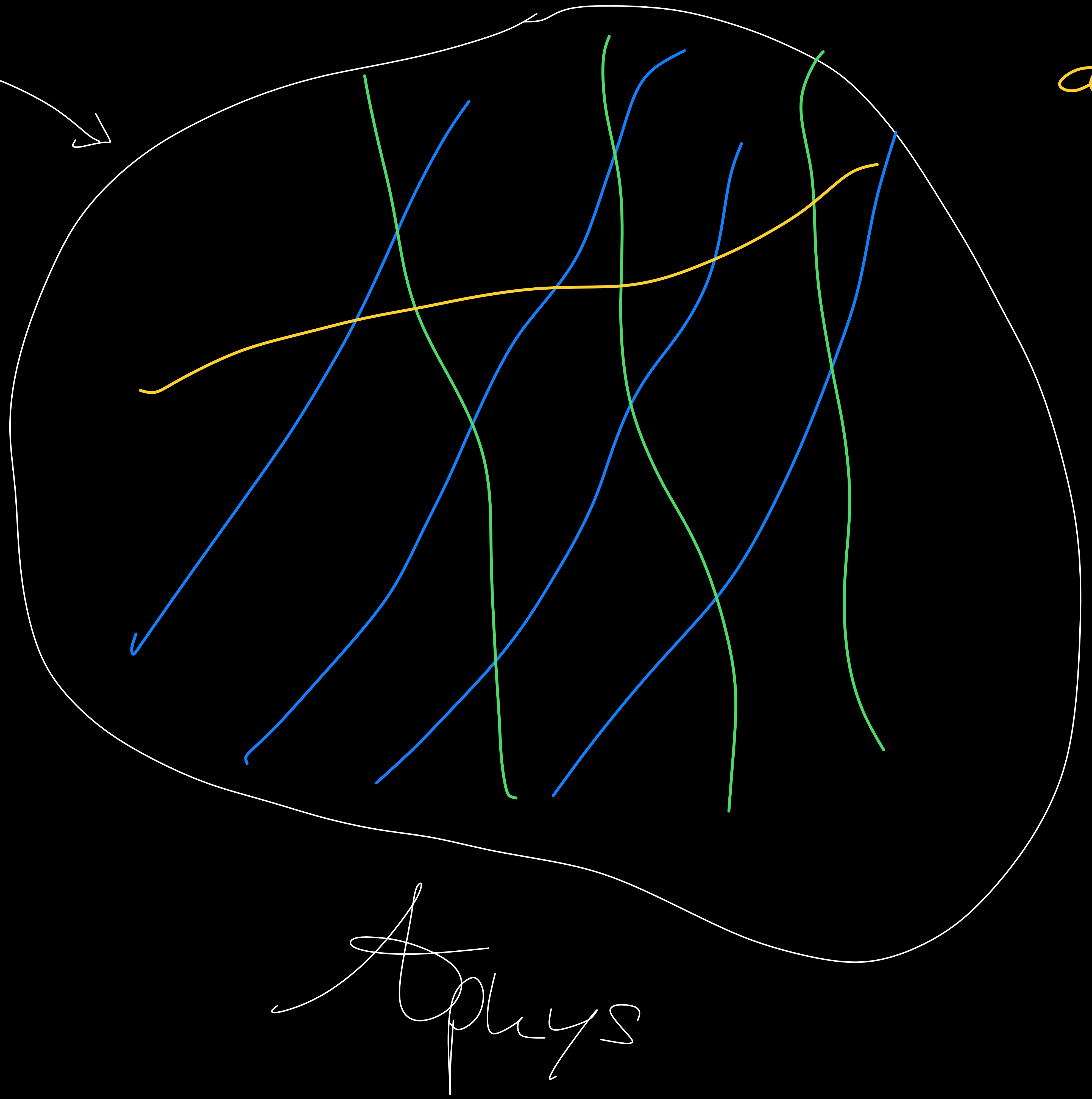
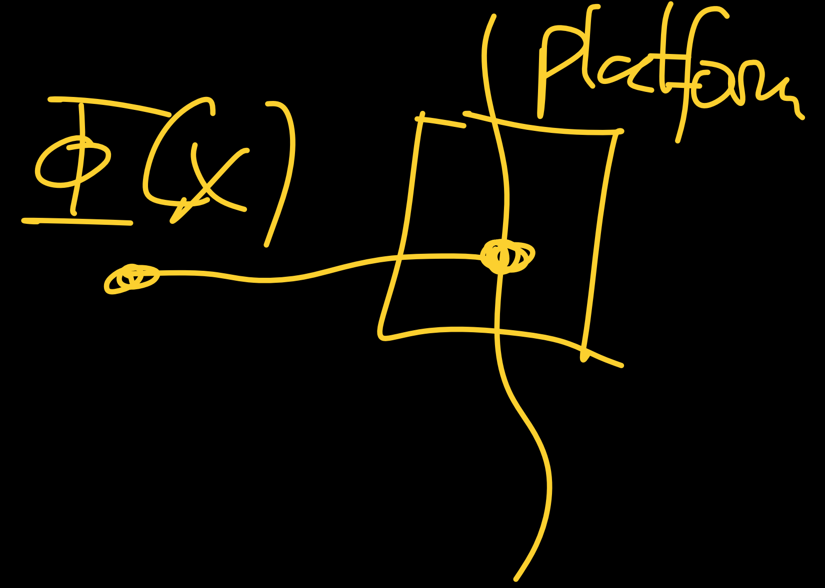


dressed observables accord. to prescription 1



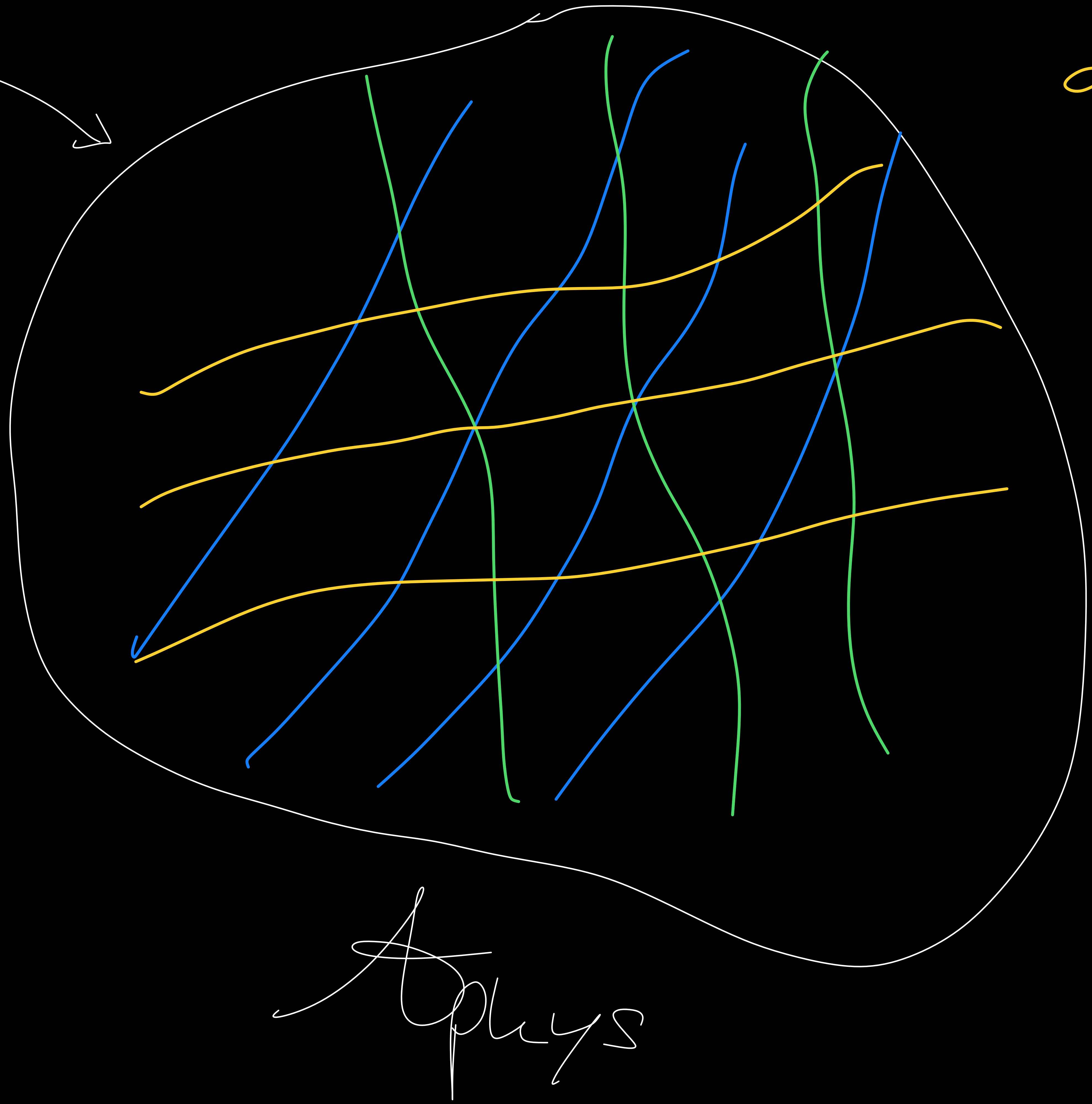
$\Longrightarrow$ 4-parameter family of obs.

Relational & dressed observables stratify invariant algebra

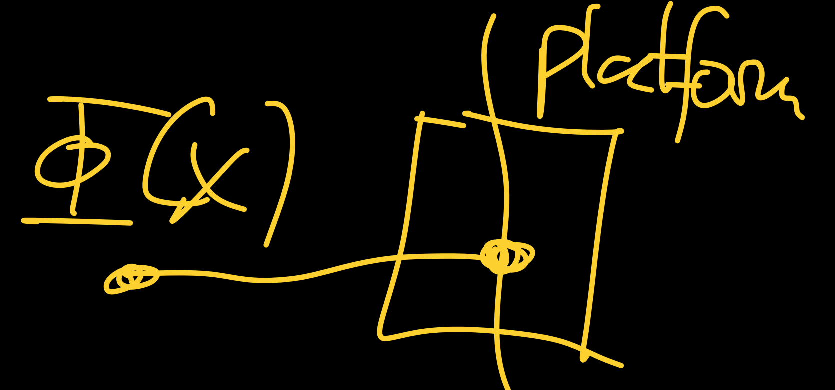
relational observables rel. to reference fields T_1^a

$\longrightarrow$ orbits in $\mathcal{A}_{phys}$

rel. obs. rel. to ref. fields T_2^a



dressed observables accord. to prescription 1



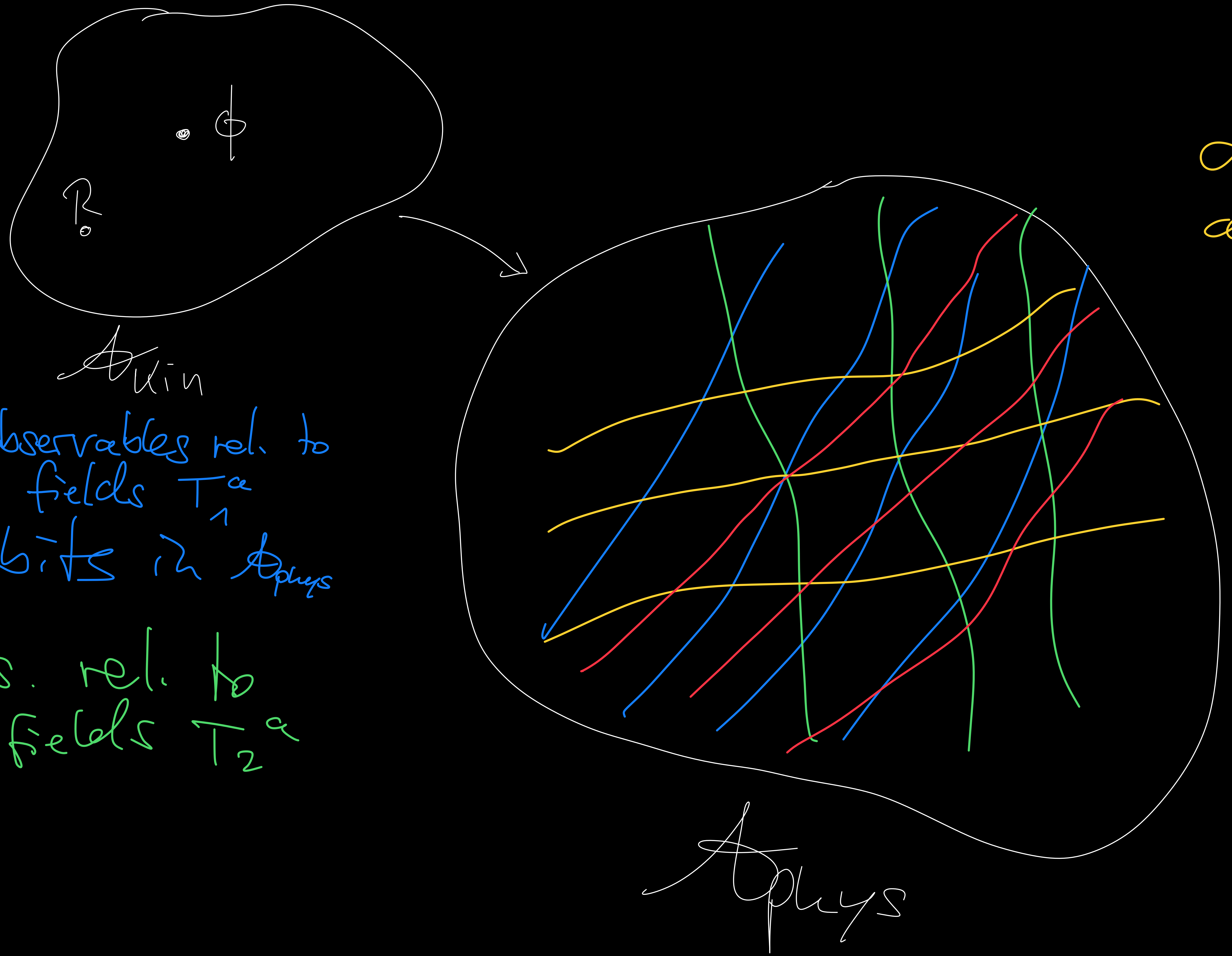
$\Longrightarrow$ orbits

Relational & dressed observables stratify invariant algebra

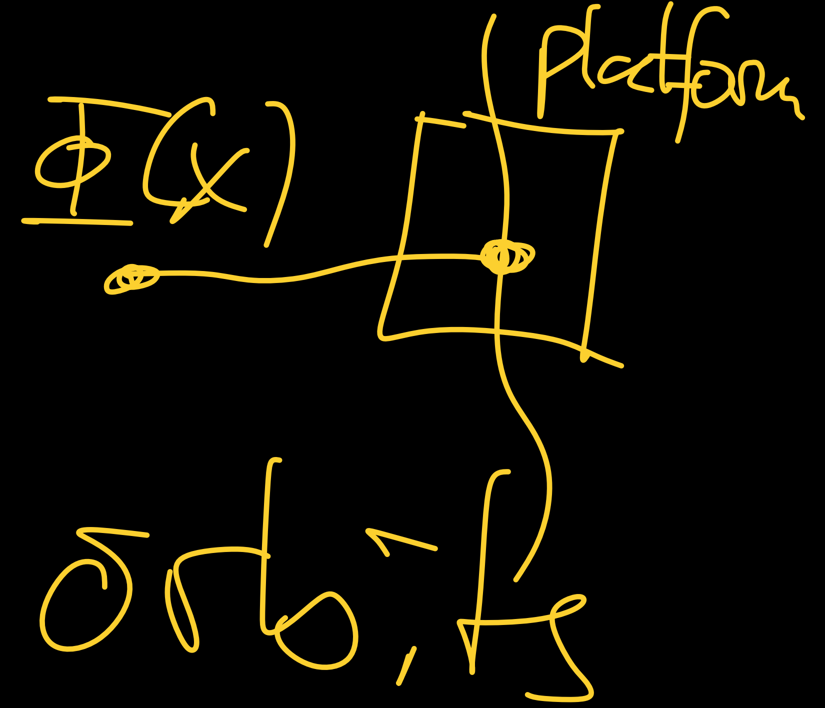
relational observables rel. to reference fields T_1^α

$\longrightarrow$ orbits in $\mathcal{A}_{phys}$

rel. obs. rel. to ref. fields T_2^α



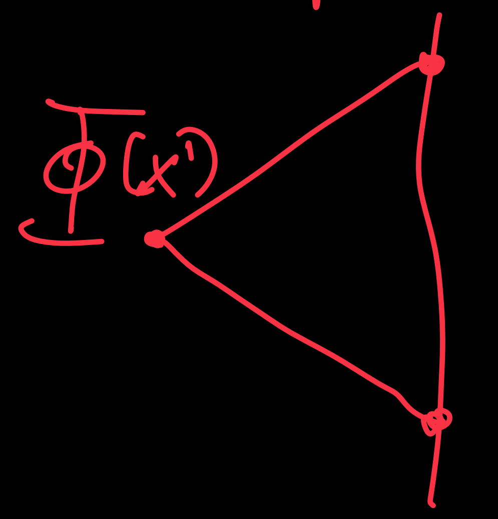
dressed observables accord. to prescription 1



$\Longrightarrow$ orbits

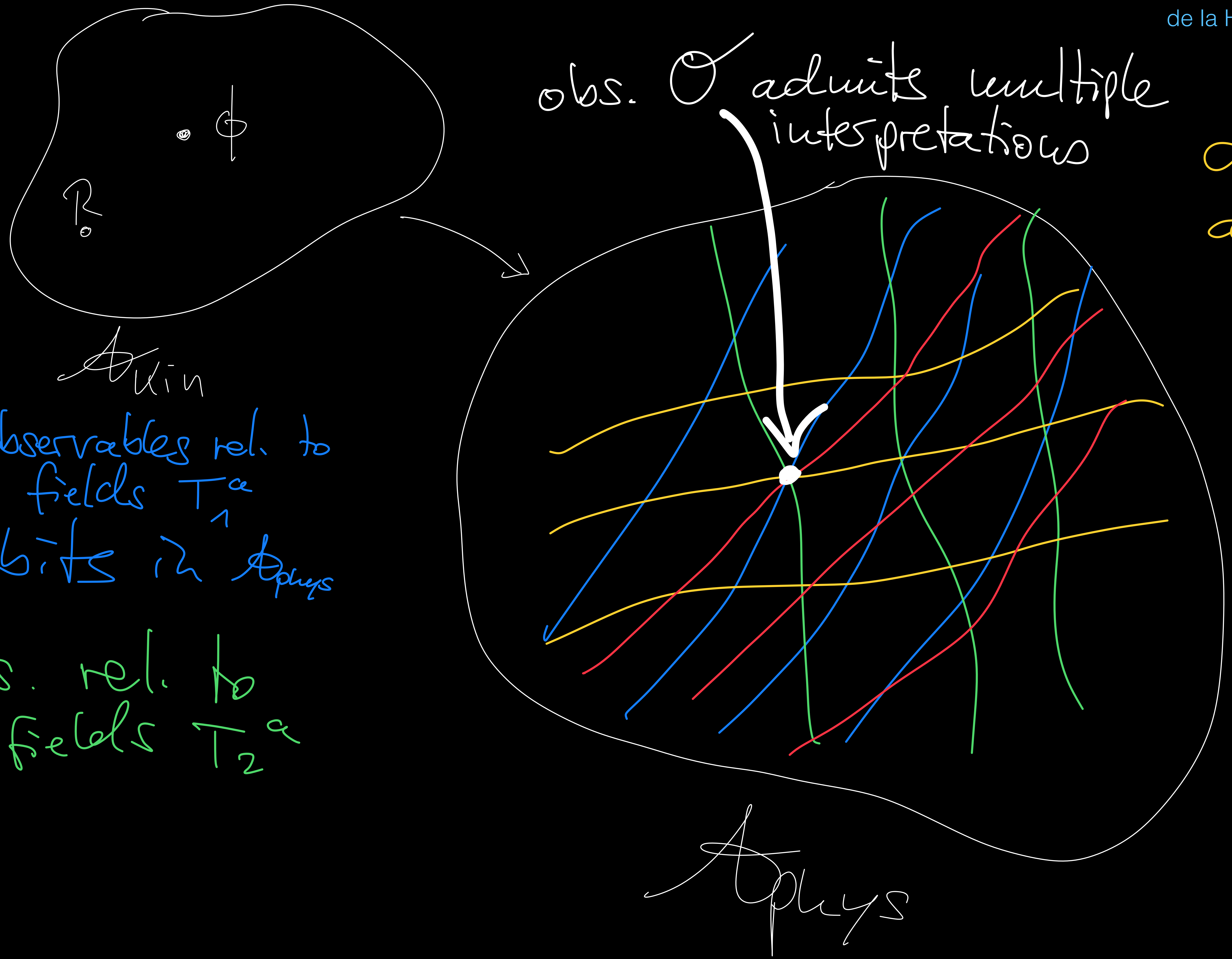
dressed obs. accord. to prescription 2

e.g.



Relational & dressed observables stratify invariant algebra

de la Hamette, Galley, PH, Loveridge, Müller 2110.13824



relational observables rel. to reference fields T_1^a

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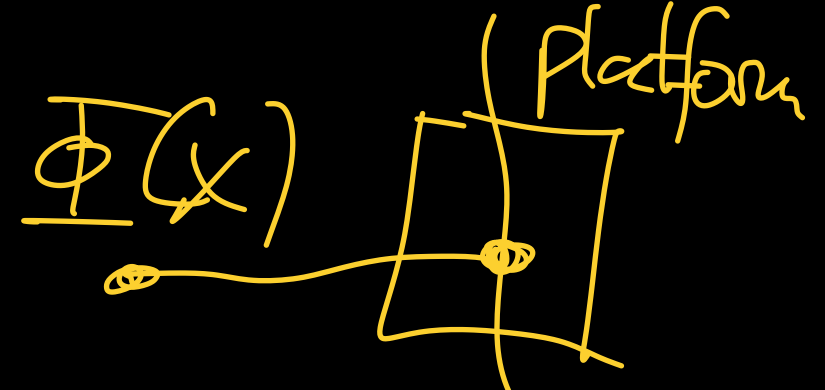
rel. obs. rel. to ref. fields T_2^a

dressed observables accord. to prescription 1



$\Longrightarrow$ orbits

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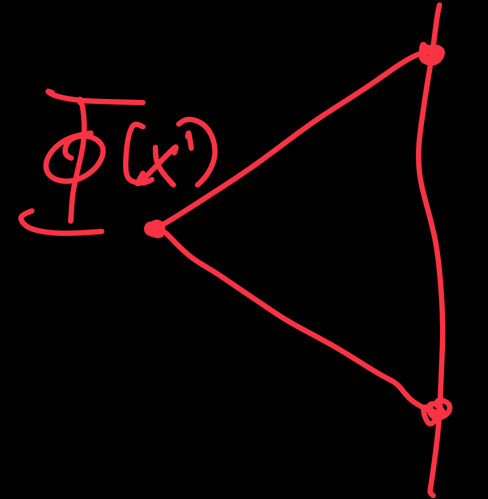

RFs provide "context" for interpreting invariant observables

if $\exists$ global ideal RF, then every element of $\mathscr{A}_{\text{phys}}$ can be written as relational/dressed observable relative to it

de la Hamette, Galley, PH, Loveridge, Müller 2110.13824

(otherwise argument local in $\mathscr{A}_{\text{phys}}$)

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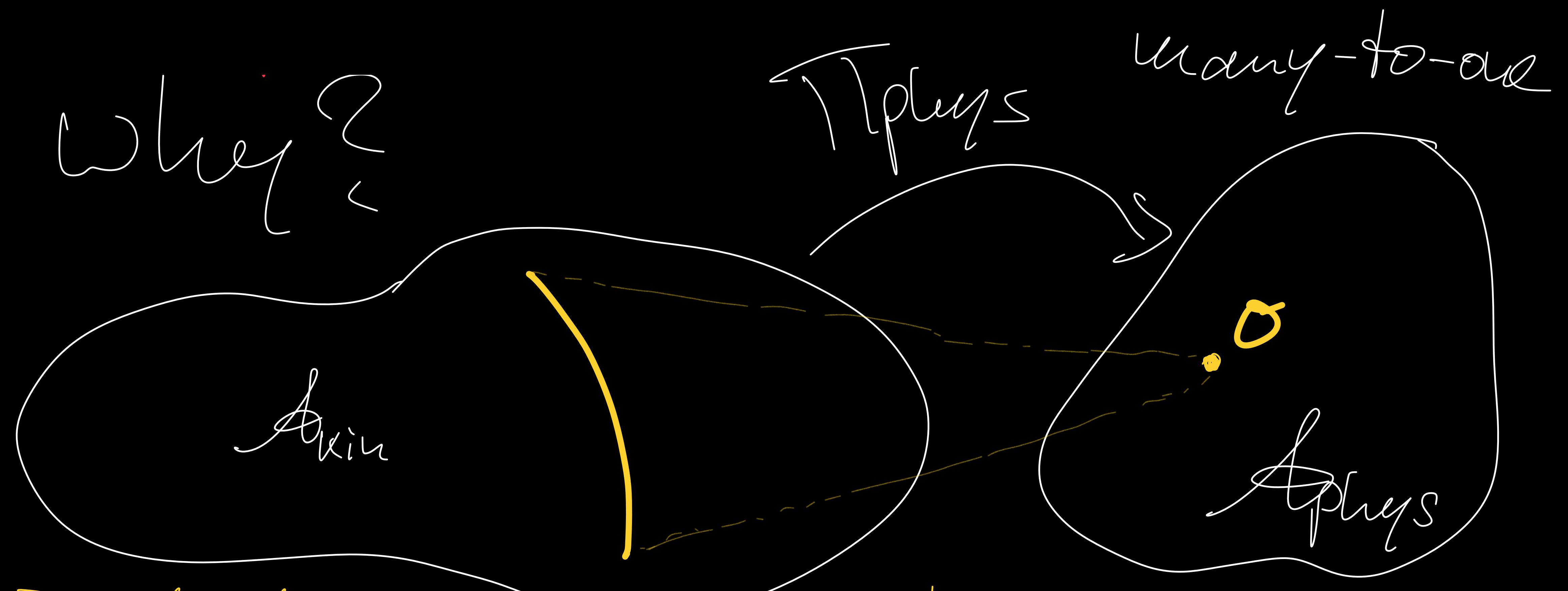
(otherwise argument local in $\mathscr{A}_{\rm phys}$)

same diff-inv. observable $O \in \mathscr{A}_{\rm phys}$ can be relational or dressed observable in multiple ways

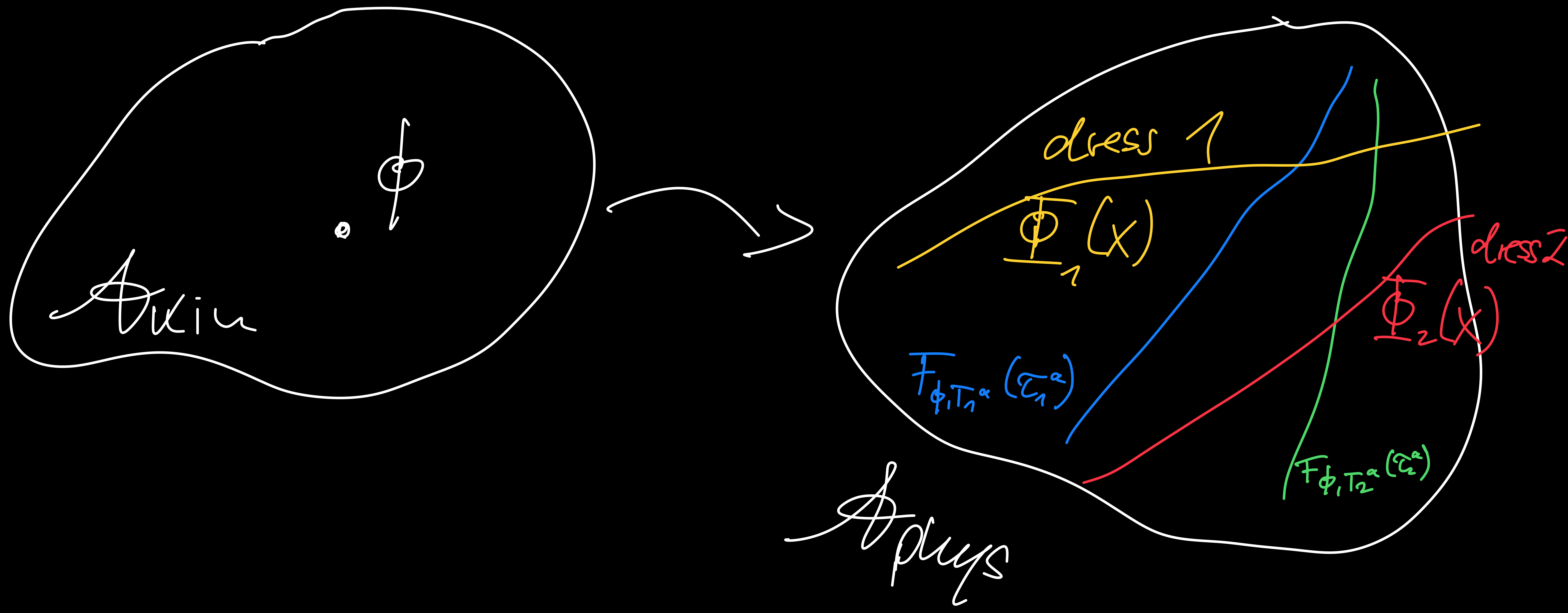
given abstract diff-inv. observable admits multiple physical interpretations in terms of RFs
⇒ RFs provide the context in which to interpret gauge-inv. observables



⇒ redundancy: many ways to write O in terms of kin. data

Quantum relativity of subsystems

Converse: many ways to "invariantize" same kinem. observable



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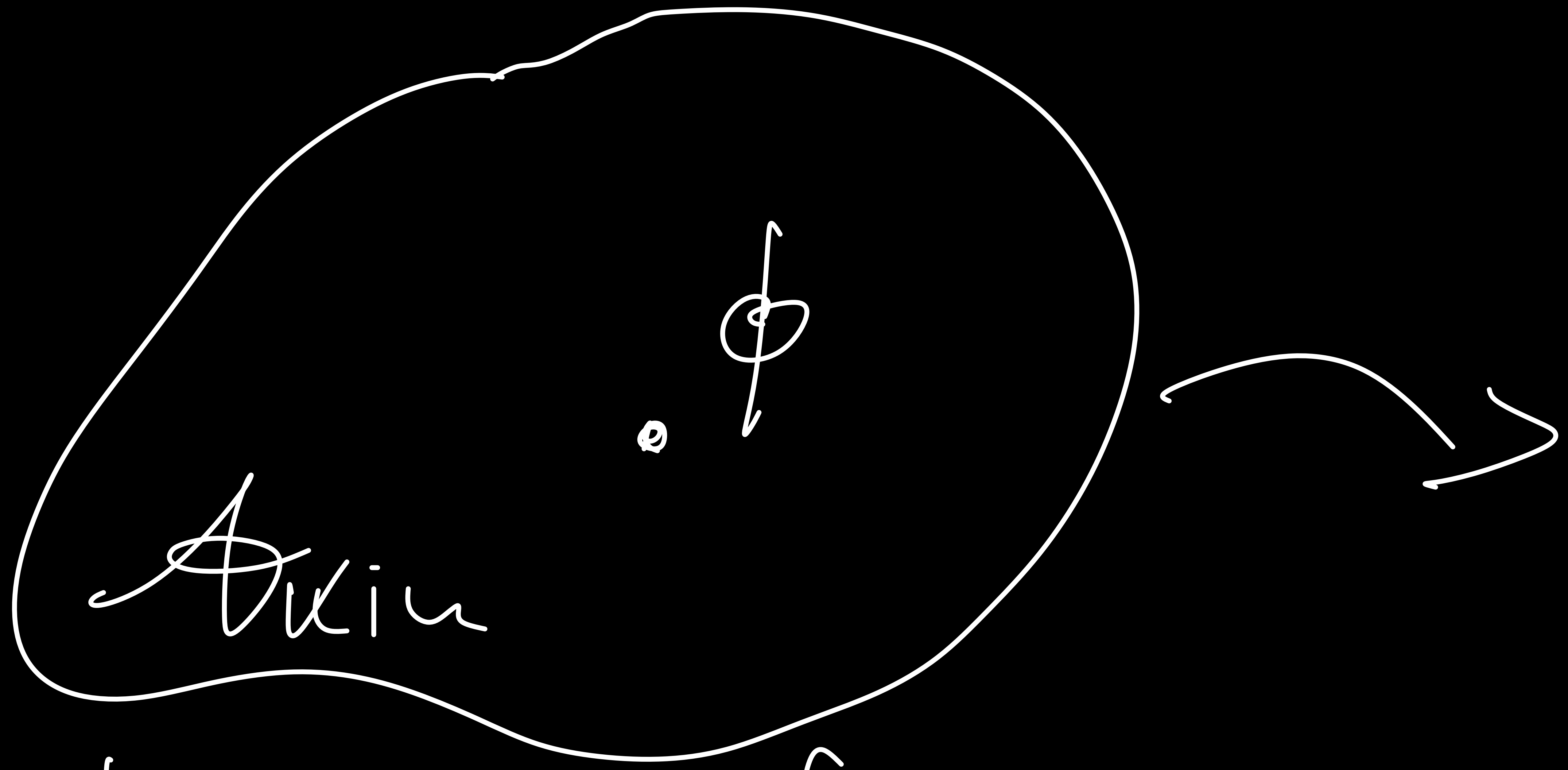
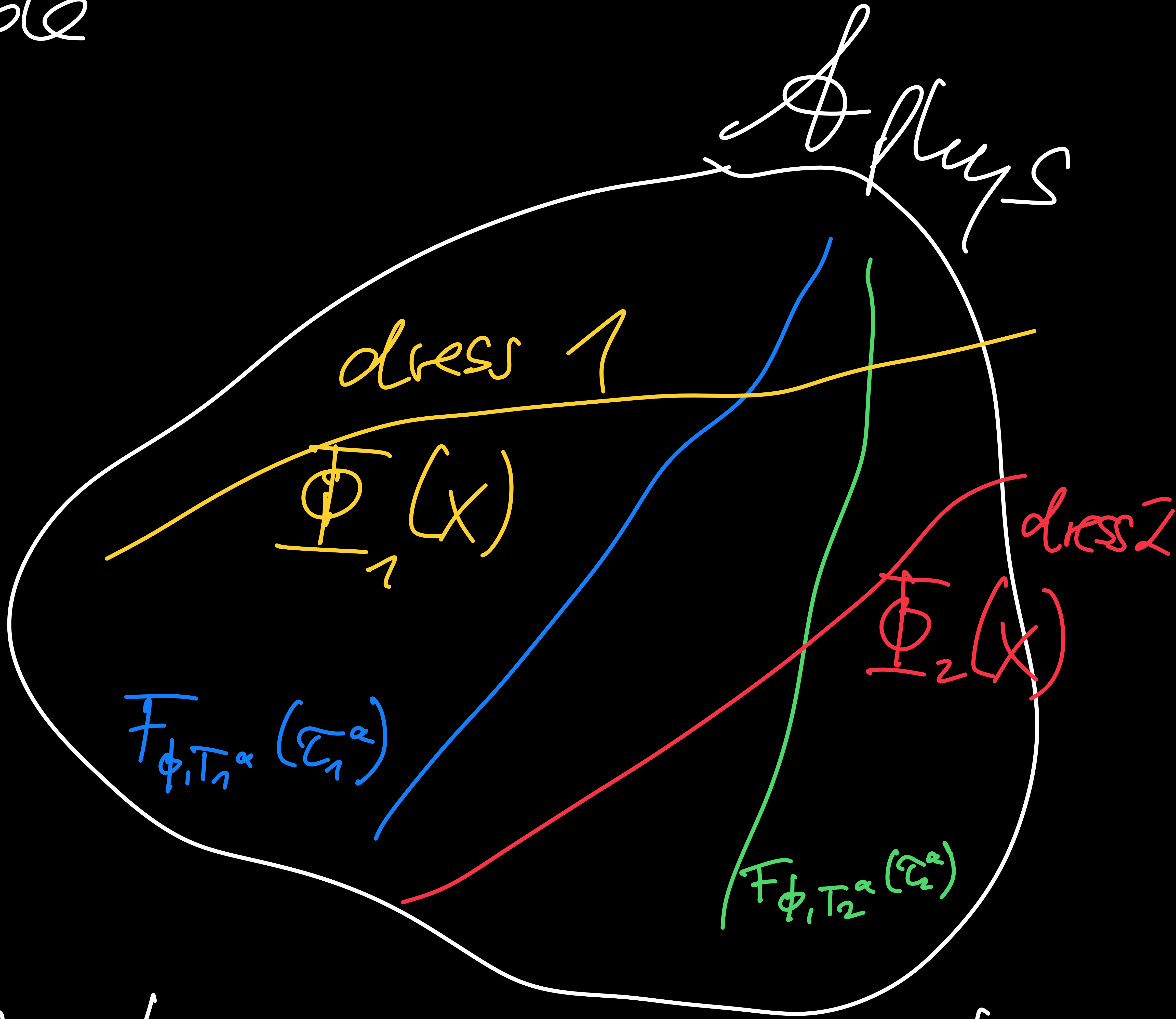
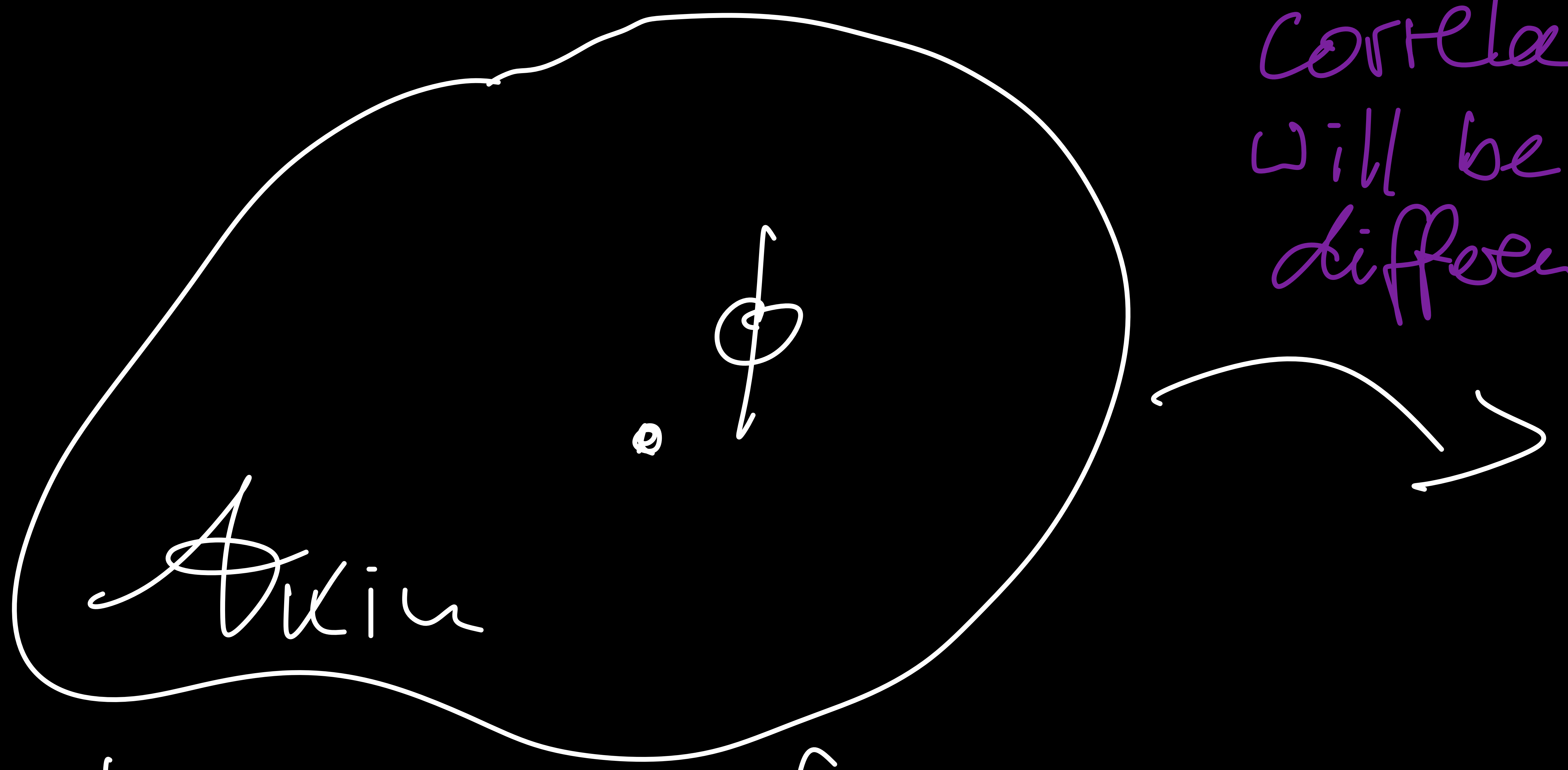


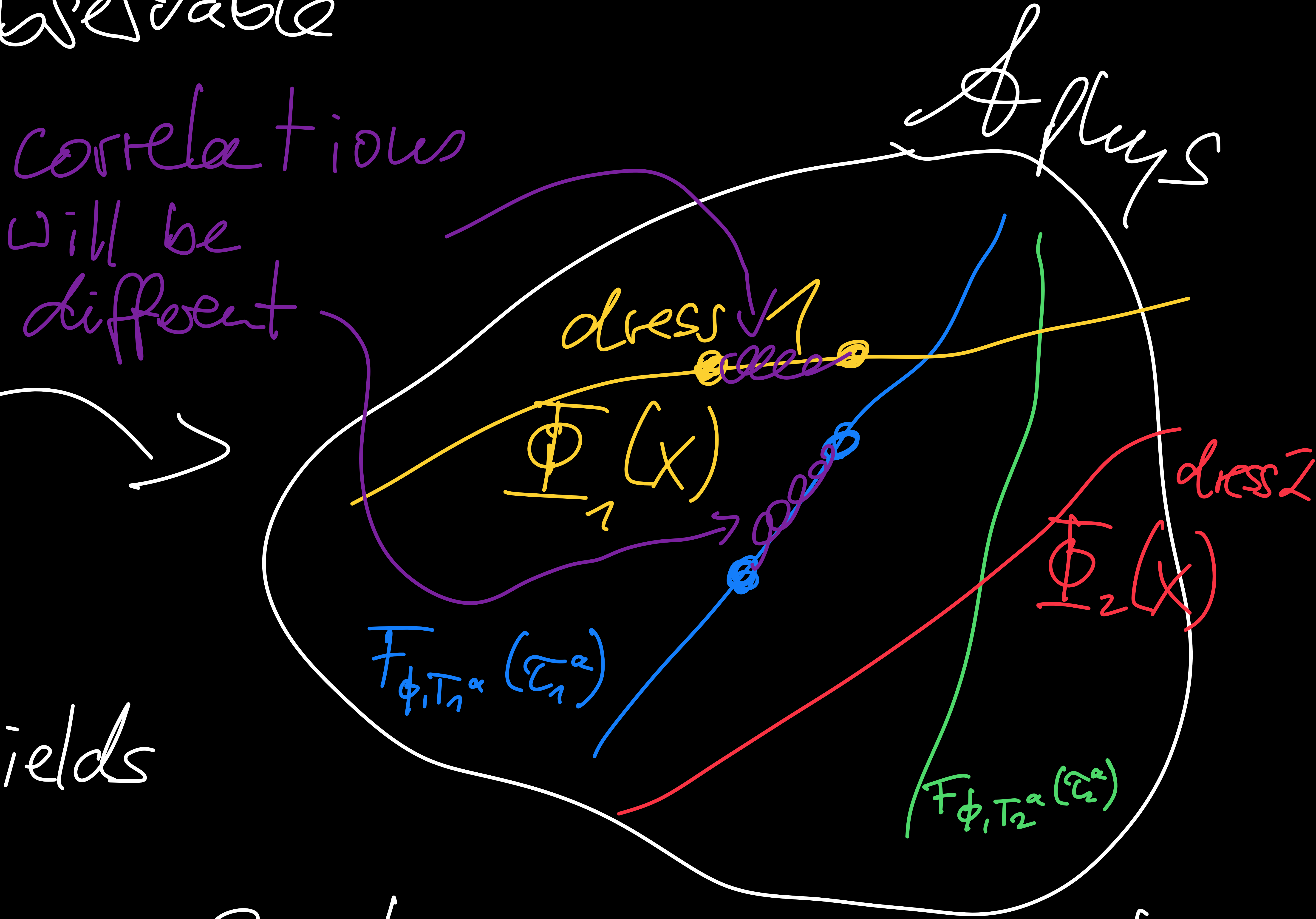


$\Longrightarrow$ ϕ rel. to diff. references yields diff. inv. observ.

$\Longrightarrow$ reference frame dep. of physics, e.g. correlations

Converse: many ways to "invariantize" same kinem. observable





$\Longrightarrow$ ϕ rel. to diff. references yields diff. inv. observ.

$\Longrightarrow$ reference frame dep. of physics, e.g. correlations