Theory of Relativity Seminar on

Changing Paradigms in Black hole Physics

Pankaj S. Joshi

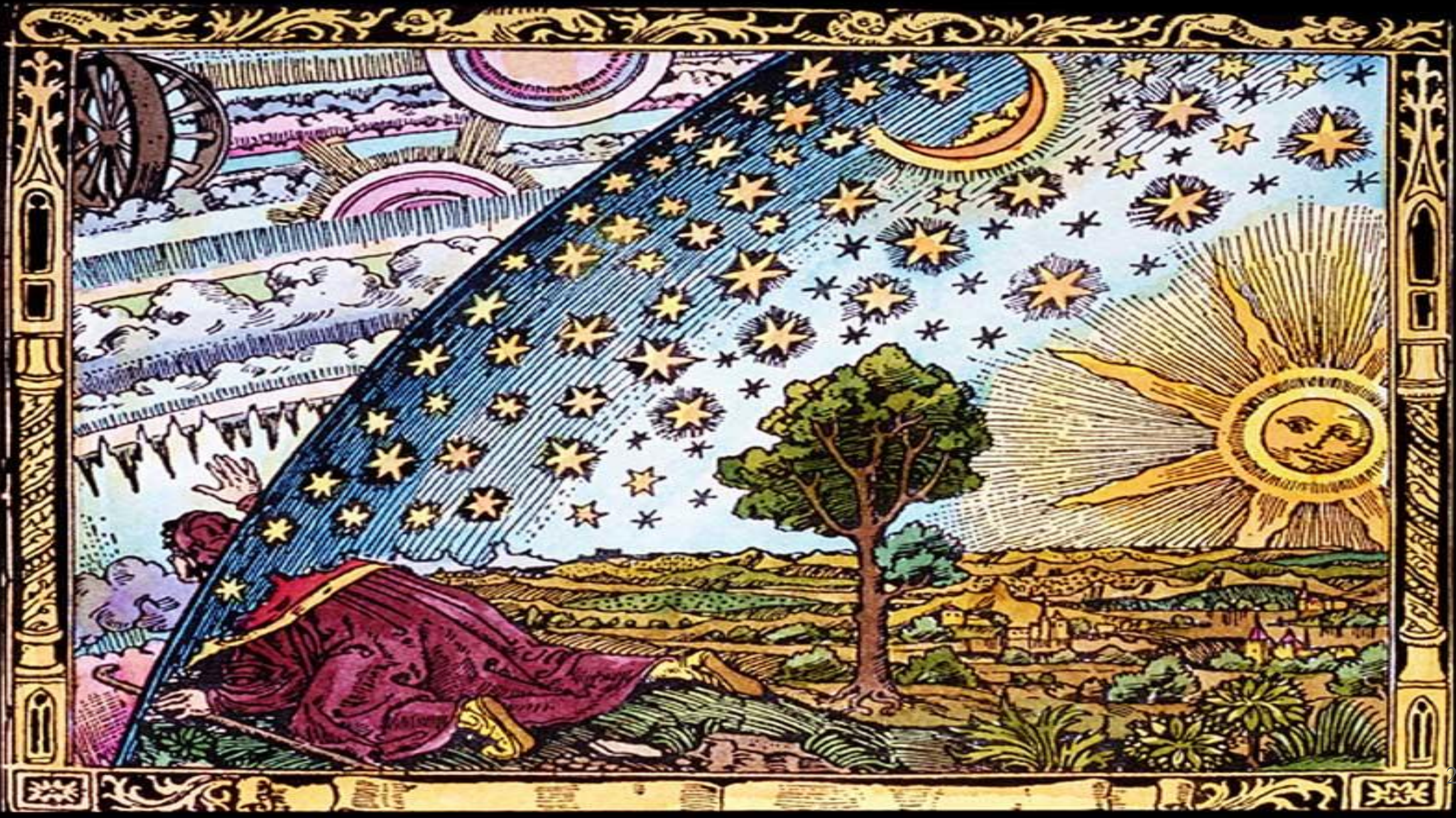
Institute of Nuclear Research, Warsaw, and Department of Fundamental Research

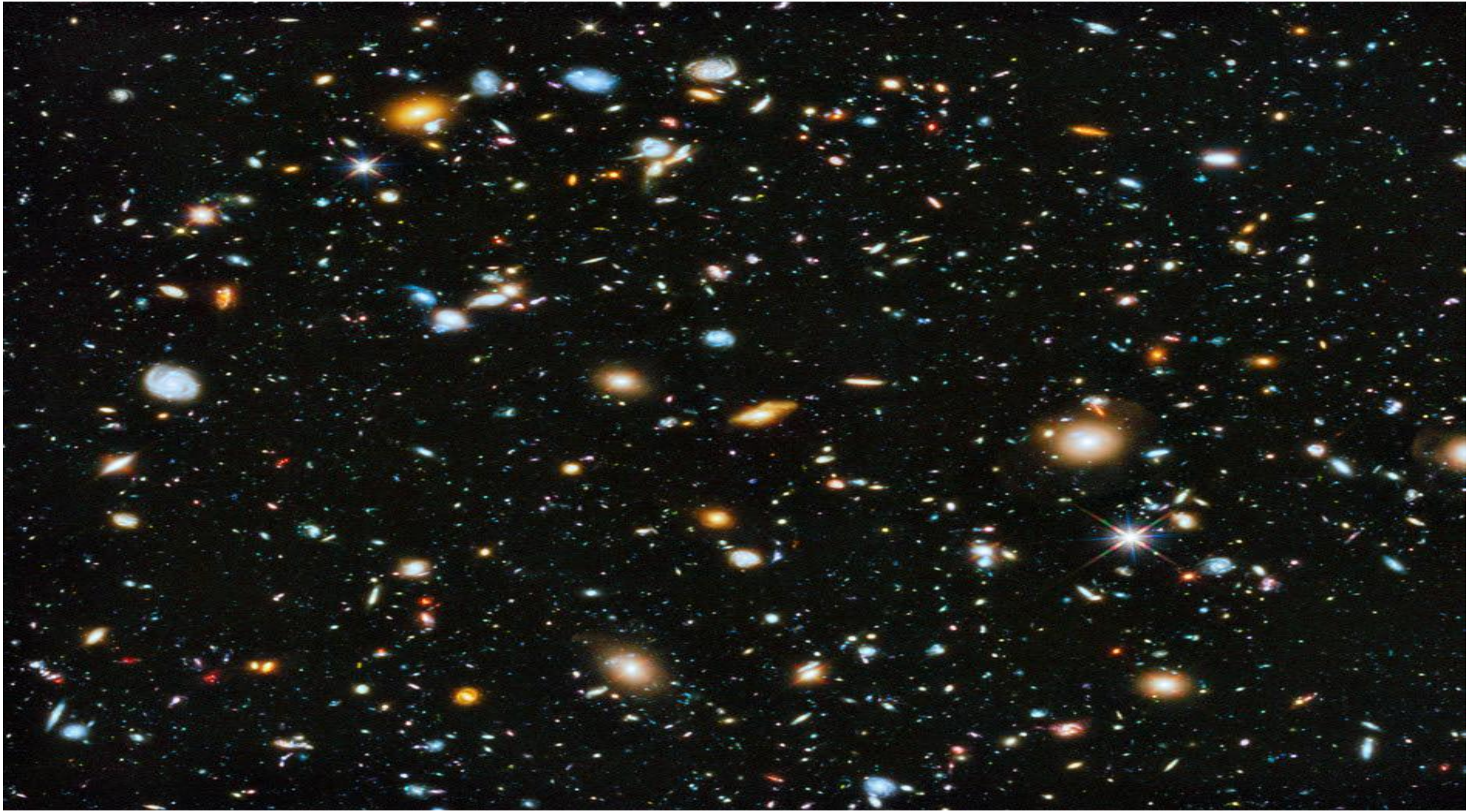
UNIVERSITY OF WARSAW

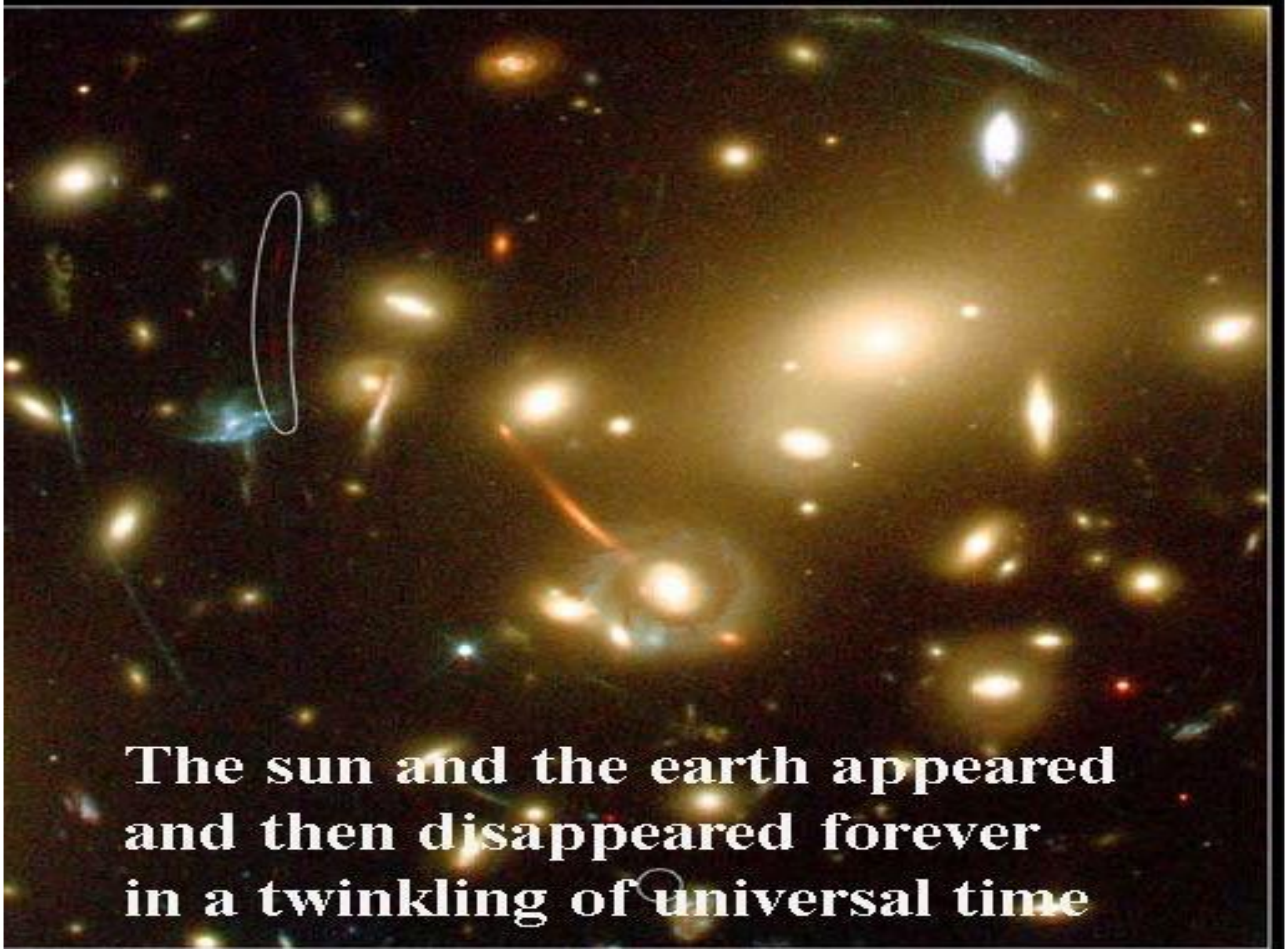
International Centre for Space and Cosmology, School of Arts and Sciences

Ahmedabad University

19th January 2024






The sun and the earth appeared
and then disappeared forever
in a twinkling of universal time

Major New Observational Missions and facilities coming up…

SKA, TMT, LIGO, LISA,
EVENT HORIZON TELESCOPE,
AND SUCH OTHERS…

We are on verge of probing Ultra- Strong Gravity Regions in Cosmos!!


NASA


Chandra

One of the Most Important Key Issues in Astrophysics and Cosmology

What Happens when a Massive Star dies?

Chandrasekhar's work: Star collapse and Stable Configuration Limit

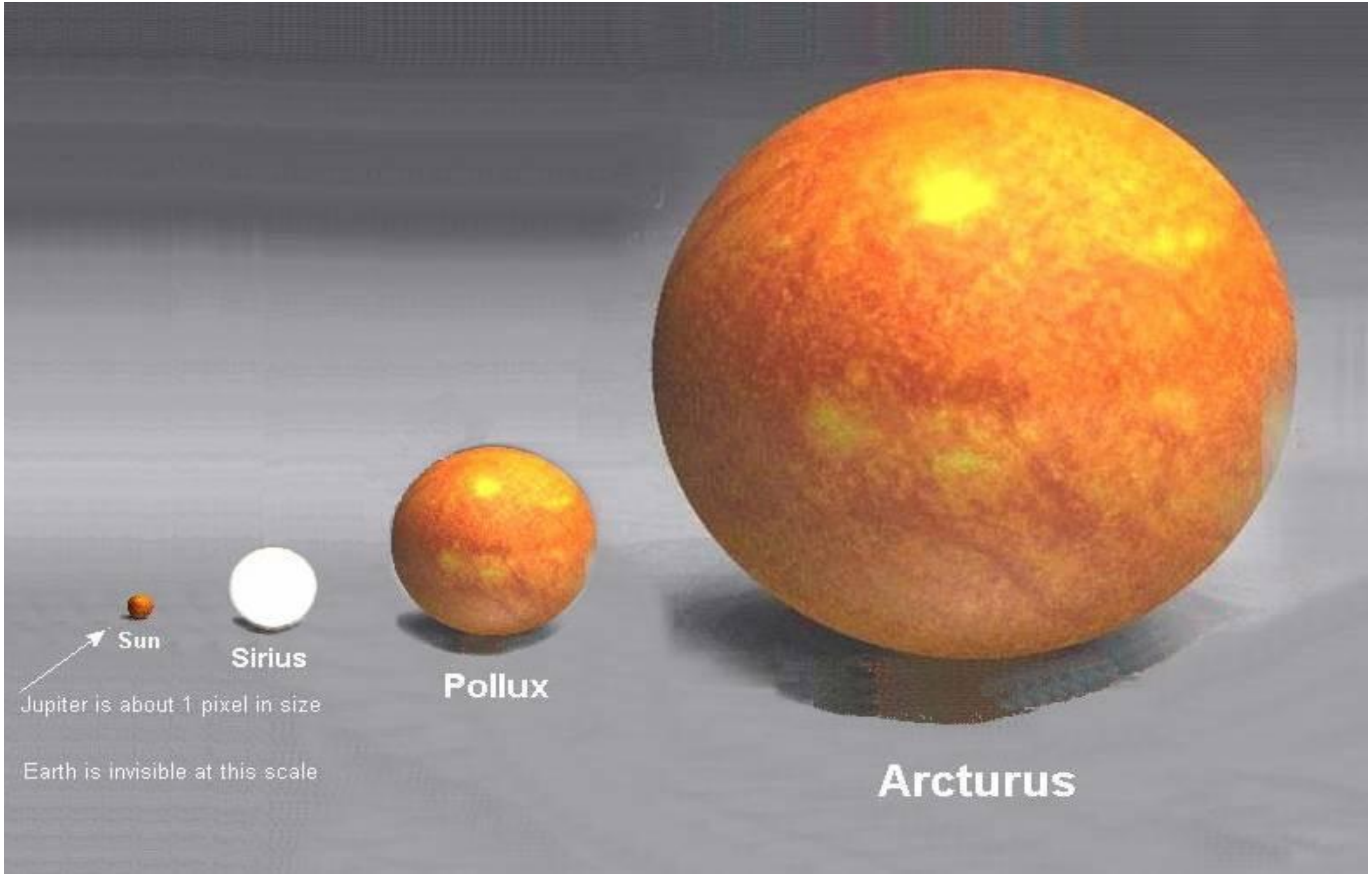
Continual Collapse for Massive Stars
What is the Final End state of such a Continual Collapse?

→ THE SUBJECT REALLY DATES BACK TO 1930s. IN FACT, CHANDRASEKHAR (1935) POINTED OUT:

Finally, it is necessary to emphasize one major result of the whole investigation, namely, that it must be taken as well established that the life-history of a star of small mass must be essentially different from the life-history of a star of large mass. For a star of small mass the natural white-dwarf stage is an initial step towards complete extinction. A star of large mass ($>\mathfrak{M}$) cannot pass into the white-dwarf stage, and one is left speculating on other possibilities.

→ THIS IS WHAT LED HIM TO AN EXTENSIVE STUDY OF BLACK HOLES IN GENERAL RELATIVITY.....

What is final fate of massive stars?



Gravitational Collapse

Massive Stars burn Much Faster

&

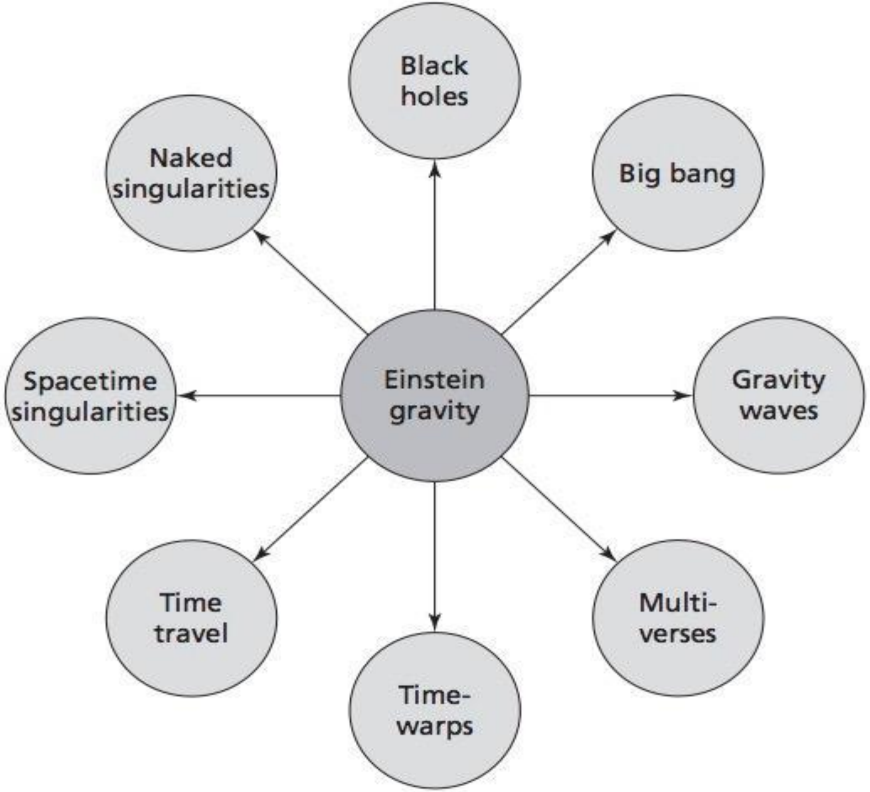
They are far More Luminous e.g a 10 Solar Masses Star does not endure More than Few Million Years

****

The Question of Final Fate of Massive Stars is of central important in Astronomy & Astrophysics

This is Fundamental to the Modern Theory and also the Astrophysical Applications of Black Holes Today…

GENERAL RELATIVITY NEEDED HERE

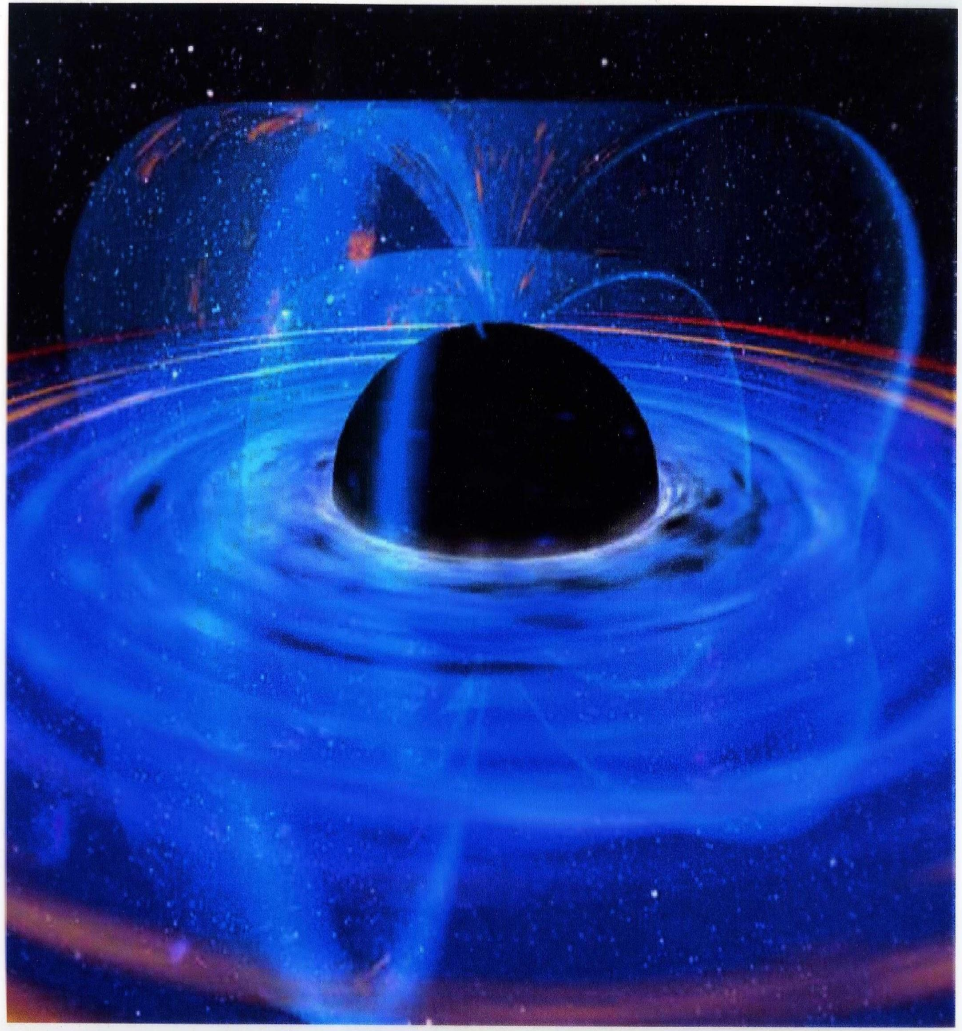

Black holes
Naked singularities
Big bang
Spacetime singularities
Einstein gravity
Gravity waves
Time travel
Time-warps
Multi-verses

A Black Hole is Born (1939) - but Immediately Discarded by Einstein!!

Further to 1939, the subject was dormant…

1960s: Resurgence of Interest due to Discovery of Quasars, Radio Galaxies… No known Physics explains these ultra-high Energies!

ASSUMING ALL STARS COLLAPSE TO BLACK HOLES, HIDING THE SPACE-TIME SINGULARITY, *`COSMIC CENSORSHIP'*, HUGE DEVELOPMENTS IN PHYSICS-ASTROPHYSICS OF BLACK HOLES-HAWKING/PENROSE/WHEELER…



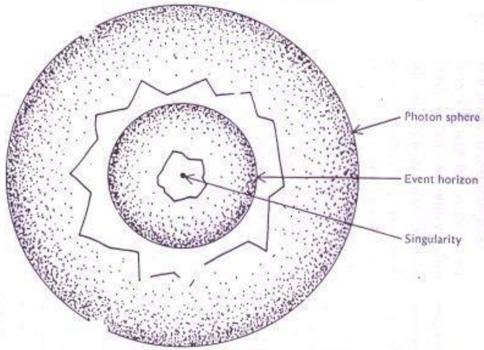
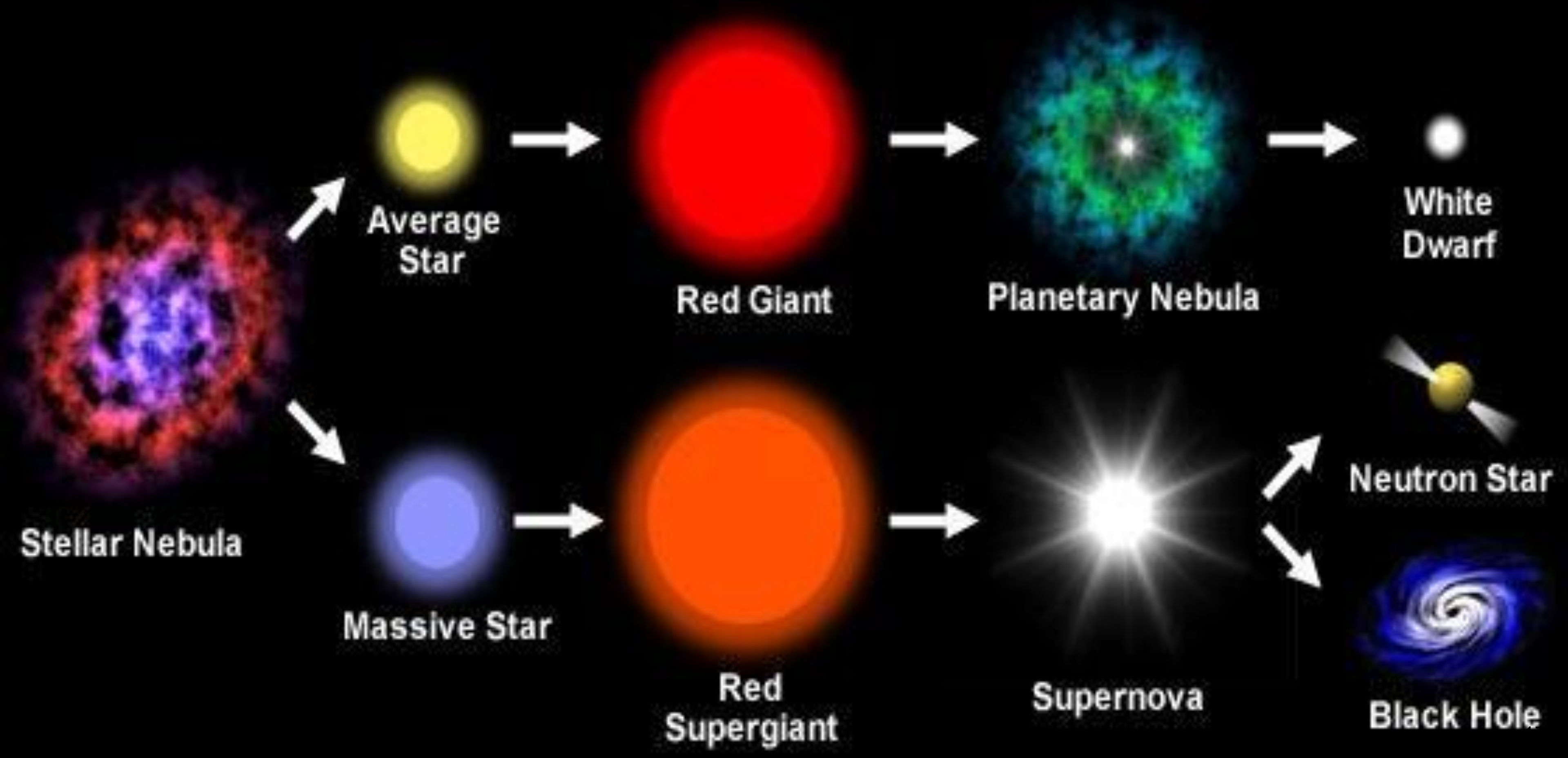


Figure 7-1. The black hole. *A schematic diagram of a black hole showing the photon sphere, the event horizon, and the singularity.*

Life Cycle of a Star



But Scientists did not Solve the Key Problem:

DO ALL MASSIVE STARS COLLAPSE TO BLACK HOLE?

Because, real stars are Inhomogeneous, have Internal pressure forces — as opposed to Idealised Models…

In recent years this big problem has been analyzed extensively

CONCLUSION

Both Black Holes and visible Naked Singularities develop as Collapse Final states

1916 **Karl Schwarzschild**

Vacuum solution of the Einstein field equation.

1939 **Oppenheimer-Snyder-Datt collapse**

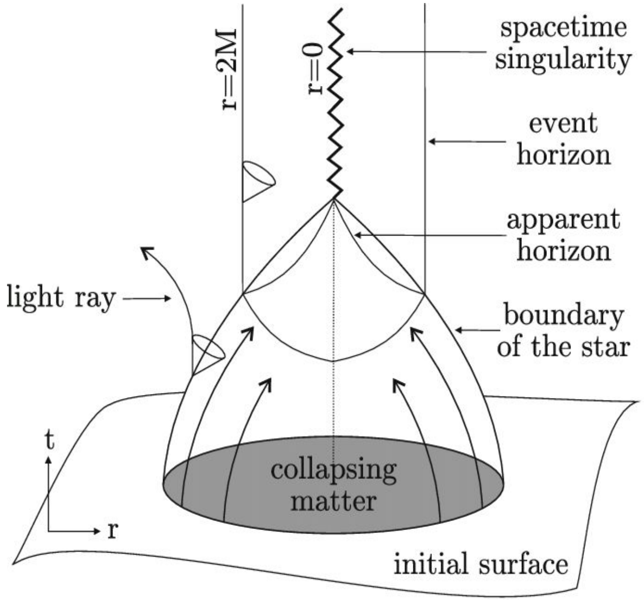
Homogeneous dust collapse

1969 **Roger Penrose**

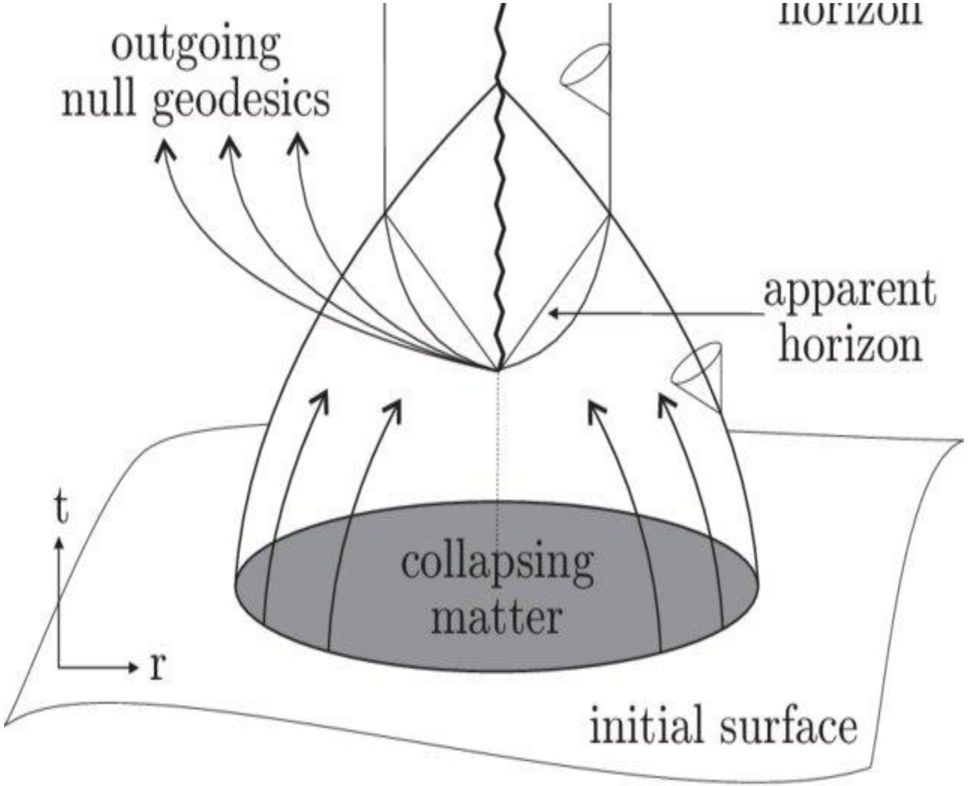
Cosmic Censorship Conjecture (CCC)

1993 **P. S. Joshi & I. H. Dwivedi**

Inhomogeneous dust collapse, Naked singularity



LTB Inhomogeneous collapse: Naked singularity



GRAVITATIONAL COLLAPSE => BLACK HOLE ONLY?

NO!

Take for example, the same model but with

$$\rho = \rho(t, r) = \rho_0 + \rho_2 r^2$$

with density peaked at centre, and decreasing away from centre. Here,

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t, r)$$

Then we have,

$$\nu = 0; e^{2\psi} = R'^2/(1 + f); \dot{R}^2 = f(r) + \frac{F(r)}{R}$$

The apparent horizon is then given by,

$$t_{\rm ah}(r) = t_0 - \frac{2}{3}F_0 r^3 - \frac{F_n}{3{F_0}^{3/2}} r^n + O(r^{n+1})$$

The effect of inhomogeneity is immediately seen…

Gravitational Collapse

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R(t,r)^2 d\Omega^2,$$

$$T^t_t = -\rho, \quad T^r_r = p_r, \quad T^\theta_\theta = T^\phi_\phi = p_\theta.$$

$$\begin{aligned}
p_r &= -\frac{\dot{F}}{R^2\dot{R}}, \quad \rho = \frac{F'}{R^2R'}, \\
\nu' &= 2\frac{p_\theta - p_r}{\rho + p_r}\frac{R'}{R} - \frac{p_r'}{\rho + p_r}, \\
2\dot{R}' &= R'\frac{\dot{G}}{G} + \dot{R}\frac{H'}{H}, \\
F &= R(1 - G + H),
\end{aligned}$$

The Singularity Curve and Apparent Horizon are given by

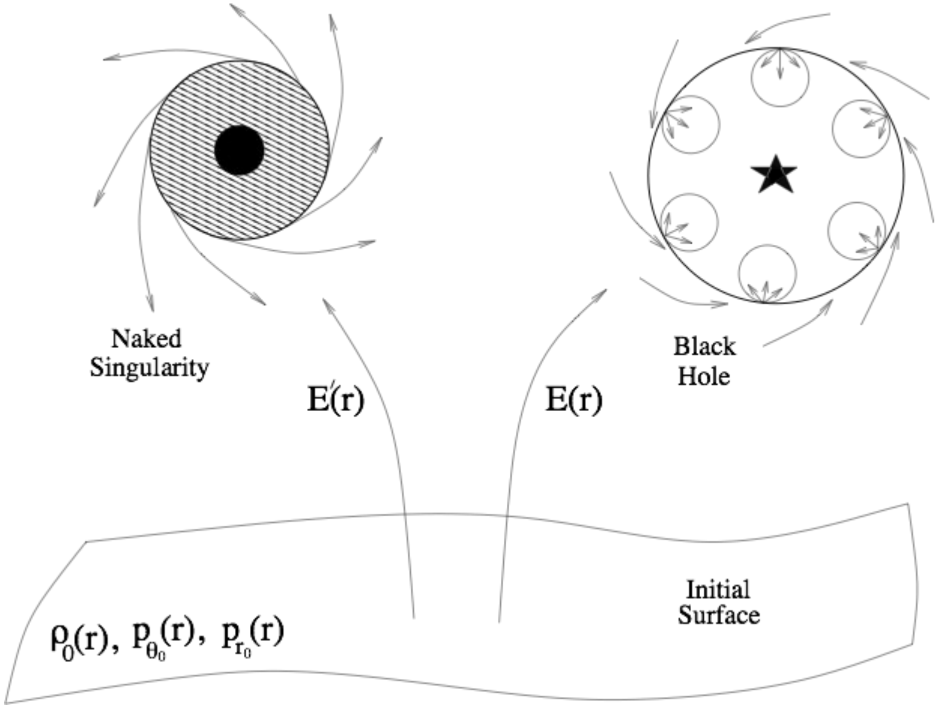
$$t_s(r) = t_s(0) + r\chi(0) + O(r^2)$$

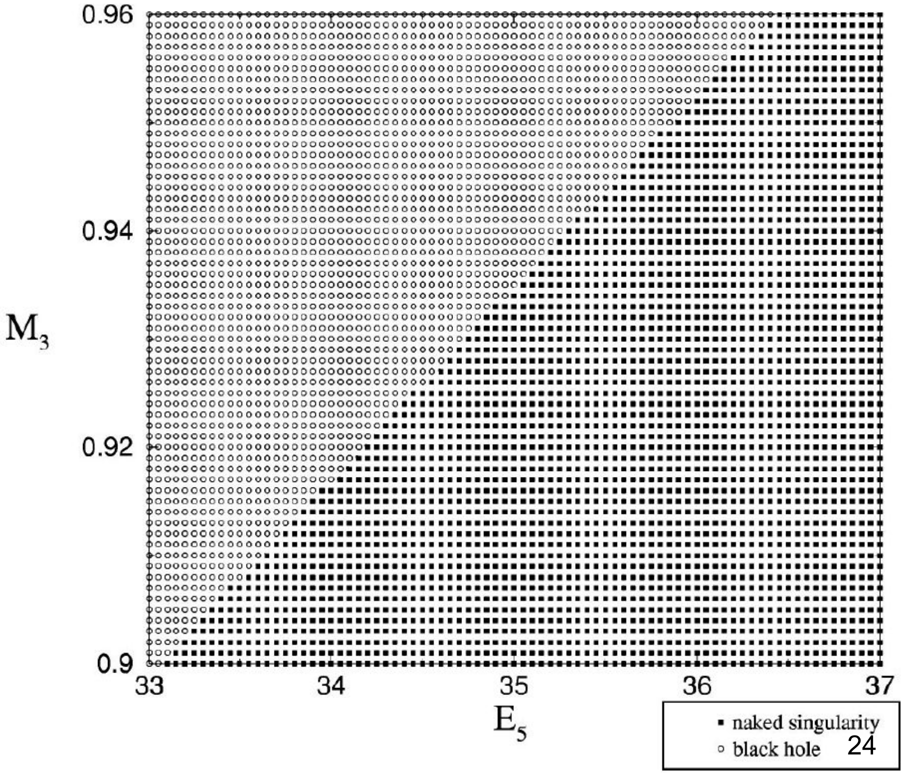
$$\chi(0) = -\int_0^1 \frac{v^{\frac{3}{2}} g_1(v)}{(M_0 + vk + 2vg_0(v))^{\frac{3}{2}}} dv$$

$$t_{ah}(r) = t_0 + \chi(0)r + o(r^2)$$

$$t_{ah}(r) = t_s(r) - \int_0^{v_{ah}} \frac{e^{-\nu}}{\sqrt{\frac{M_0}{v} + \frac{be^{2\nu} - 1}{r^2}}} dv$$

Visible Singularities - Structure & Implications





- Can we obtain equilibrium configurations From gravitational collapse? We consider Scalar field collapse with non-zero potentials And find classes of potentials that fulfill This purpose. The motivation of the problem Comes from galaxy and structure formation In the universe.

- We show formation Of strong curvature naked singularities from Gravitational collapse of scalar fields and vector fields with non-zero potentials.

- **Publication in collaboration with Dalhousie University, Halifax, Canada**

1. **Dipanjan Dey, Koushiki, Pankaj S. Joshi,** *Phys. Rev. D* **108** (2023) 10, 104045.
2. **Karim Mosani, Koushiki, Pankaj S. Joshi, Jay Verma Trivedi**, Tapobroto Bhanja, *Phys.Rev.D* **108** (2023) 4, 044049.

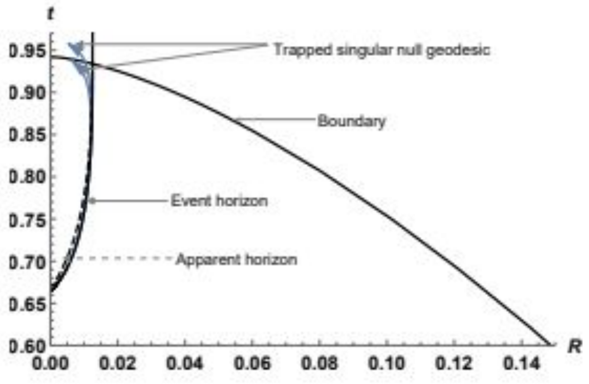


FIGURE: Formation of **Locally visible singularity** as an end state of a spatially inhomogeneous perfect fluid collapse with zero pressure.

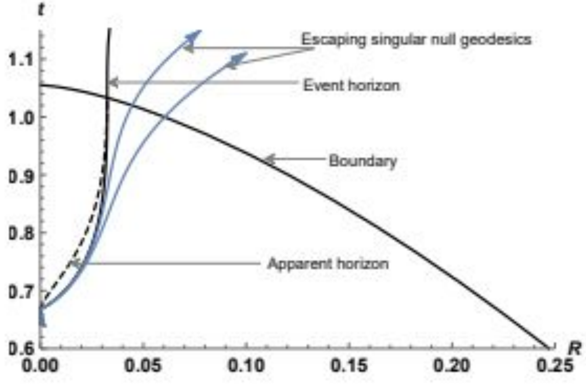


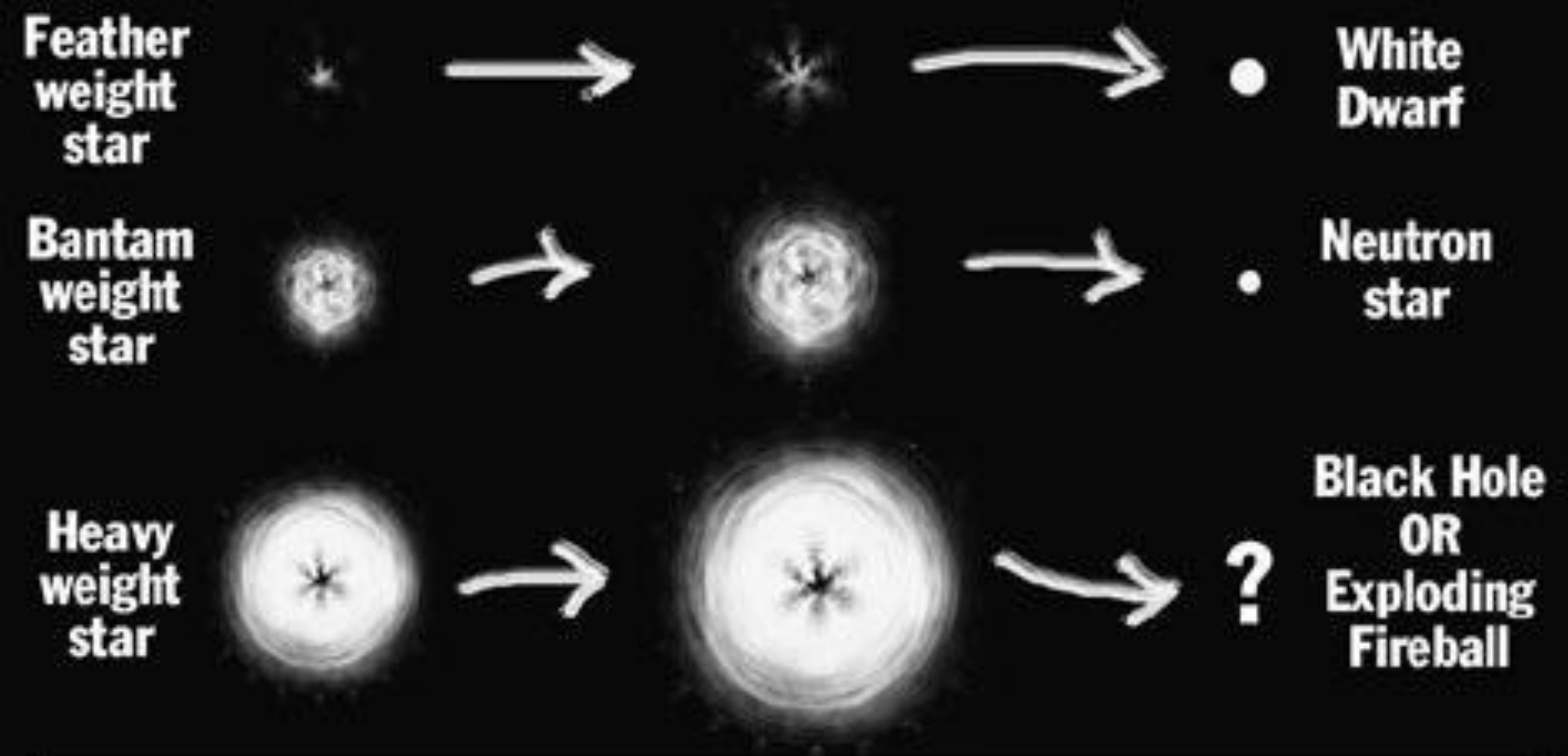
FIGURE: Formation of **Globally visible singularity** as an end state of a spatially inhomogeneous perfect fluid collapse with zero pressure.

Visible Singularities - Structure & Implications

Work has taken place on many aspects of Collapse and Nature of Singularities:

* Collapse with different Equations of State
* Spherical Collapse analyzed extensively from the perspective of Black Hole and Naked Singularity Formation
* Few Non-spherical Collapse Models also available
* Self-similar and Scalar Field Collapse-Critical phenomena
* Numerical Models for Star Collapse developing well…
* Quantum Gravity Issues
* Genericity and Stability aspects
* Gravitational Waves from Collapsing Matter Clouds…
* Observational Implications of Naked Singularities

THE POINT IS, IF NAKED SINGULARITIES DEVELOP AS COLLAPSE FINAL STATES, THEN THEIR NATURE AND STRUCTURE IMPLY INTRIGUING THEORETICAL AND OBSERVATIONAL CONSEQUENCES…



The fate of a star depends on its mass

A Bet on a Cosmic Scale, And a Concession, Sort of

→ NEW YORK TIMES, FEB 12, 1997

By MALCOLM W. BROWNE

Dr. Stephen W. Hawking of Cambridge University in England — the brilliant theorist regarded as one of Albert Einstein's intellectual successors — has conceded defeat in a famous bet he made six years ago on a matter of cosmic significance.

The bet he made with two professors at the California Institute of Technology was that naked singularities could not exist, and now, it seems, they could — maybe.

During a visit to Caltech last week, Dr. Hawking, the author of "A Brief History of Time," a book that delves into the origins of the universe, conceded defeat "on a technicality" to Dr. John P. Preskill and Dr. Kip S. Thorne. The stake was for £100, plus clothing "embroidered with a suitable concessionary message."

A singularity is a mathematical point at which space and time are infinitely distorted, where matter is infinitely dense, and where the rules of relativistic physics and quantum mechanics break down. Singularities are believed to lurk at the hearts of black holes, which conceal th ir existence from the outer world. A naked singularity would be a singularity bereft of a concealing black-hole shell, and therefore visible, in principle, to outside observers.

Stephen Shames/Matrix

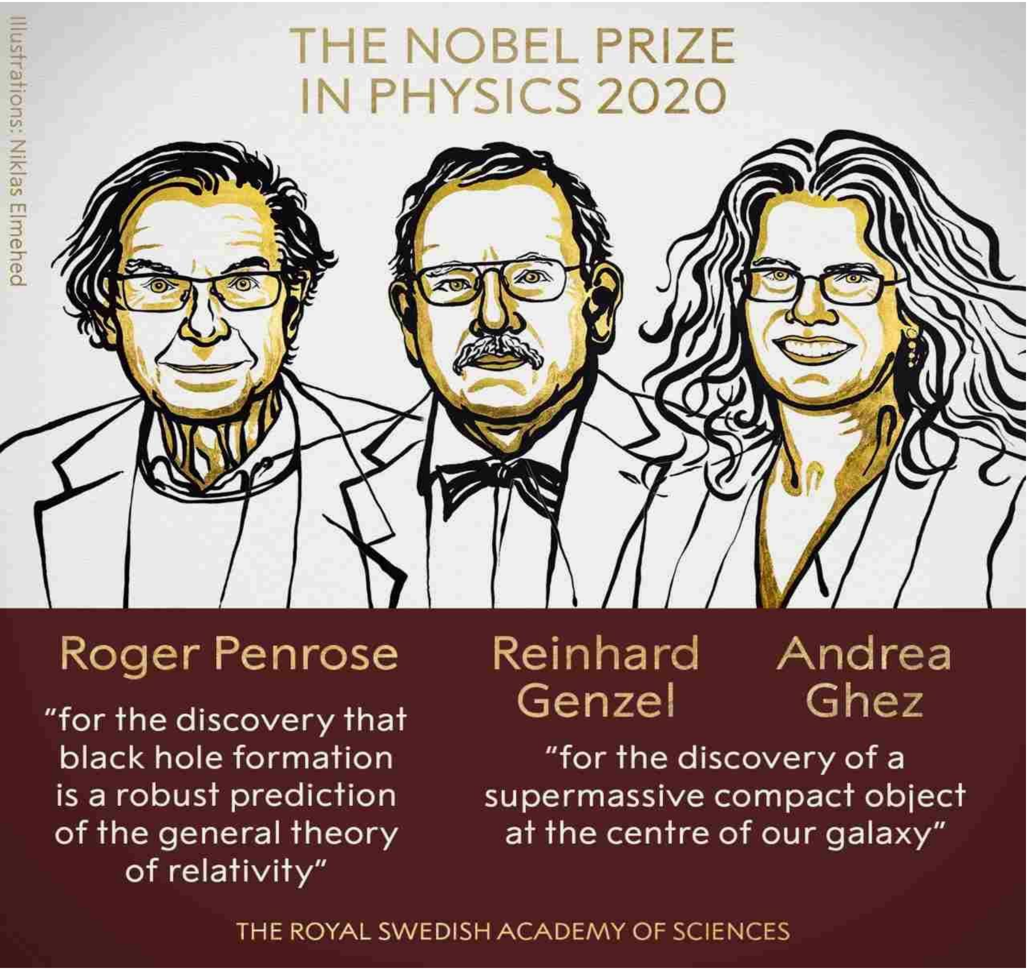
Bettor, and the bet (below).

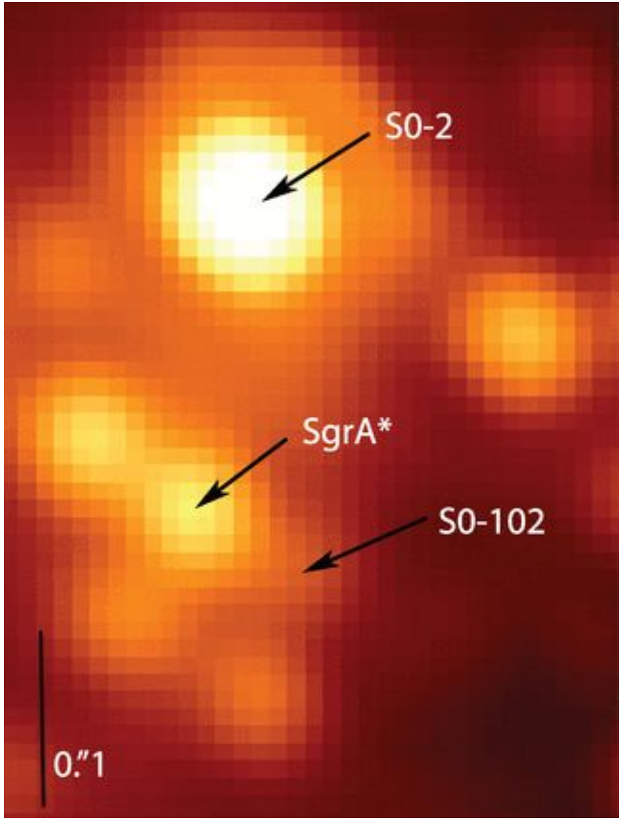
Although neither light nor any other kind of signal can escape from them, a half dozen or so black holes have been revealed by their gravitational effects on nearby stars. Black holes have also betrayed their presence by sucking in matter from nearby space. As it spirals toward the hole, the matter is heated to incandescence and the emission of

Continued on Page A22, Column 1

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

Stephen W. Hawking John P. Preskill & Kip S. Thorne
Pasadena, California, 24 September 1991





UCLA Galactic center group

https://www.youtube.com/watch?v=TfouEFuB-co

Recent Developments

* Recent Research Papers from Top Academic Places such as Cambridge, Perimeter, Princeton… on Formation of Naked singularities in Gravitational Collapse…
Currently Many Groups Internationally are working on these issues

* In fact, Many people now ask: OK, let them both exist, but then,

What are the Observational Signatures, What shall we see out there in the Skies?

The Important Question on Black Holes & Naked Singularities

What are Observational & Astrophysical Implications? Can we distinguish them in the Skies?

IT APPEARS THAT THE ANSWER IS INDEED IN AFFIRMATIVE!!

Several Ways have opened up; some of the Main Options are:

* Accretion Disk properties around BH and NS

* High Energy Particle Collisions & Consequences

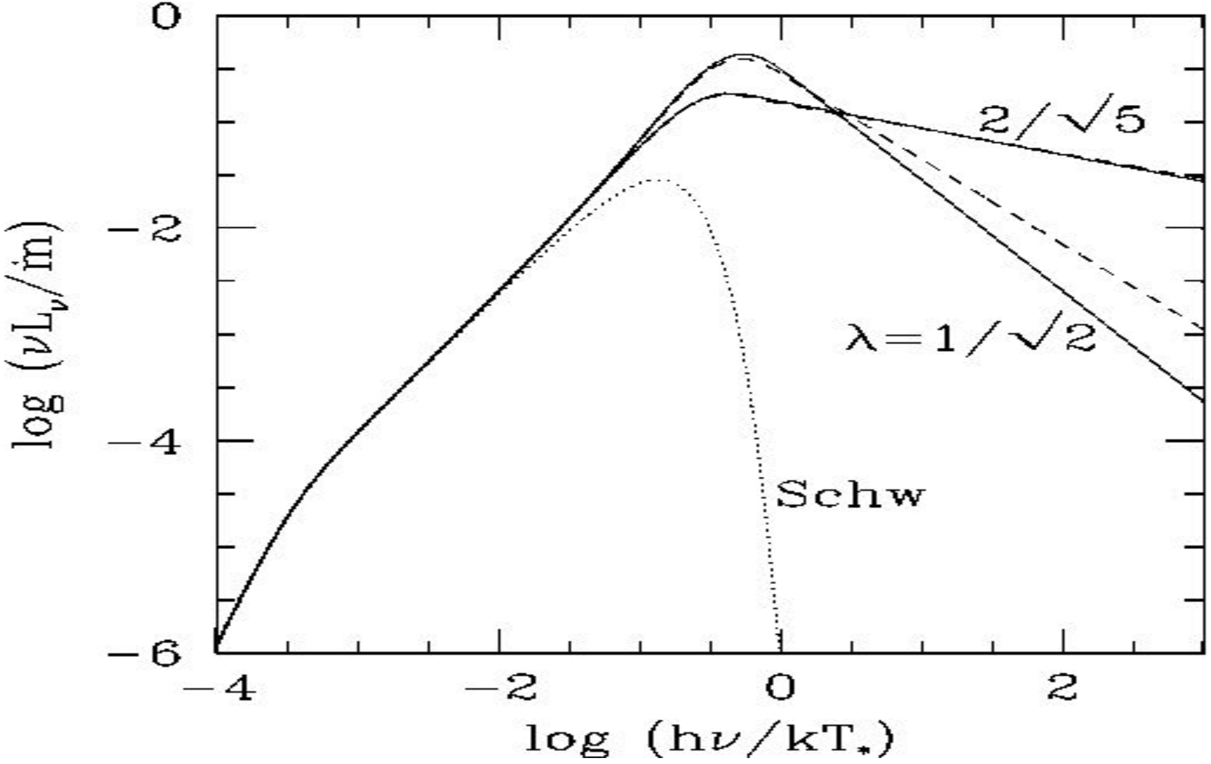
* Gravitational Lensing around these objects

* Shadows of BH & NS—Event Horizon Telescope

* Pulsar Behavior near BH & NS

* Gravitational Waves from BH & NS

* Quantum Gravity Lab??



Joshi, Malafarina and Narayan, *Class.Quant.Grav.* 31 (2014) 015002

High Energy Particle Collisions near Black holes and Singularities

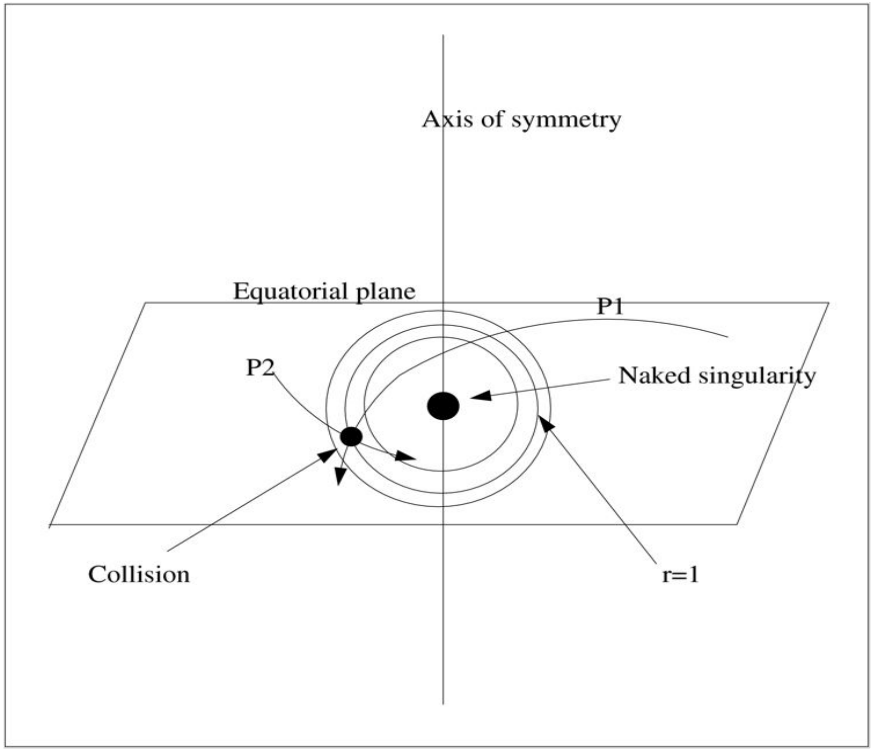
* Silk et al pioneered a study of particles colliding with very high energies in black hole geometries

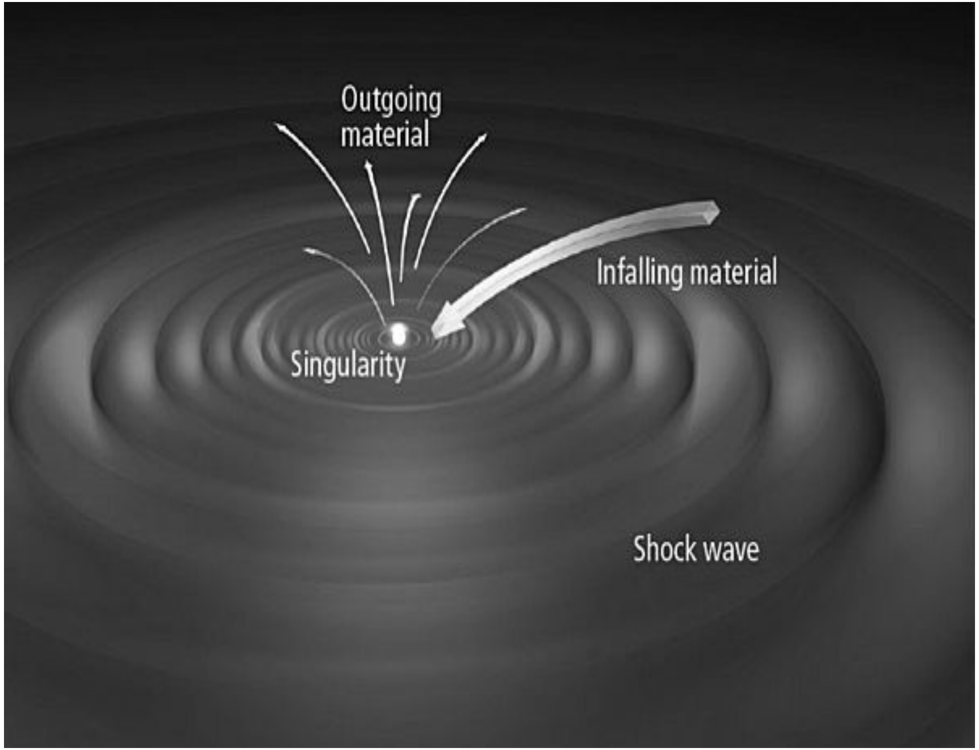
* The Basic Motivation: Terrestrial Labs limited; Can We have Astrophysical Particle Accelerators?

* While Black holes require various Fine Tunings, Naked Singularity geometries help avoid these.

* Harada, Kimura, Nakao, Patil, PSJ et al examined many spacetimes such as Kerr, Reissner-Nordstrom and others...

Patil et al (2011)





Energy extraction & naked singularity

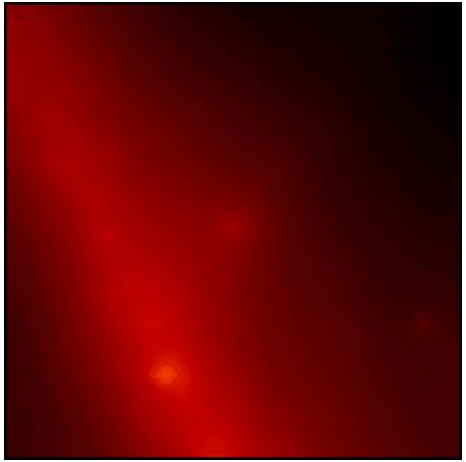
V. Patel, K. Acharya, P. Bambhaniya, P. S. Joshi, Universe **8,** 571 (2022).
V. Patel, K. Acharya, P. Bambhaniya, P. S. Joshi, Phys. Rev. D **107,** 064036 (2023).

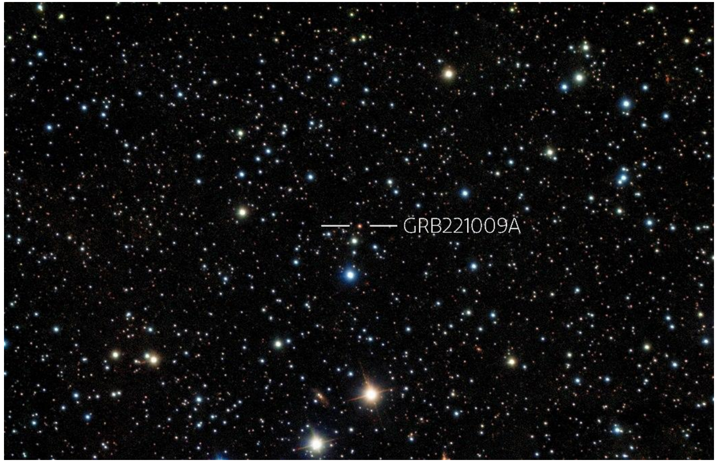
Energy extractions

Objective: Can we extract energy from ultra-strong gravitational fields near naked singularities?

Data collection: Fermi Gamma Ray Burst Monitor (GBM), Fermi Large Area Telescope (Fermi/LAT), Large High Altitude Air Shower Observatory (LHAASO) etc.

https://science.utah.edu/news/relativistic-jet/



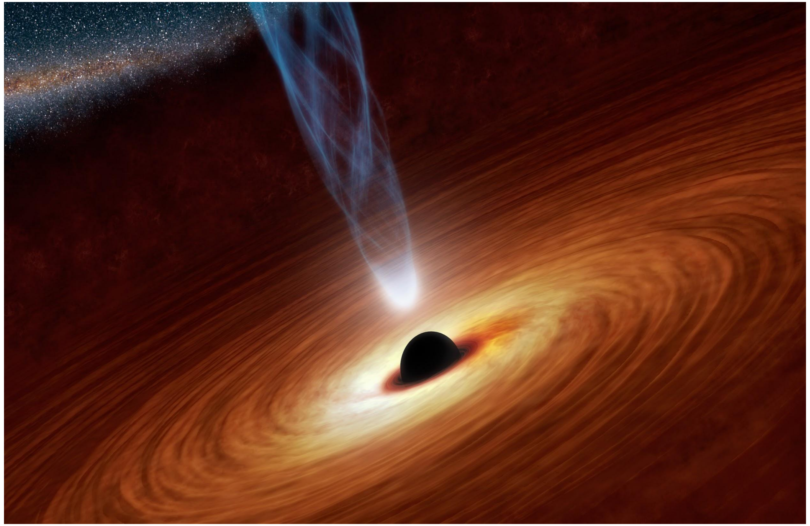


Ten-hour time lapse of GRB 221009A, as seen by the Fermi Gamma-ray Space Telescope

Energy extraction from the compact objects

The major concerns in high-energy astrophysics these days are the powering of active galactic nuclei, X-ray binaries, quasars, gamma ray & fast radio bursts, and the formation of jets.

- The Penrose process (PP).
- The magnetic Penrose process (MPP).
- Particle acceleration due to high curvature region.
- Blandford- Znajek (BZ) process.
- Blandford- Payne (BP) process.
- The collisional Penrose process.
- Banados-Silk-West (BSW) effect and the super Penrose process.
- Synchrotron radiation.



https://beta.nsf.gov/news/could-we-harness-energy-black-holes

Gravitational Lensing Studies

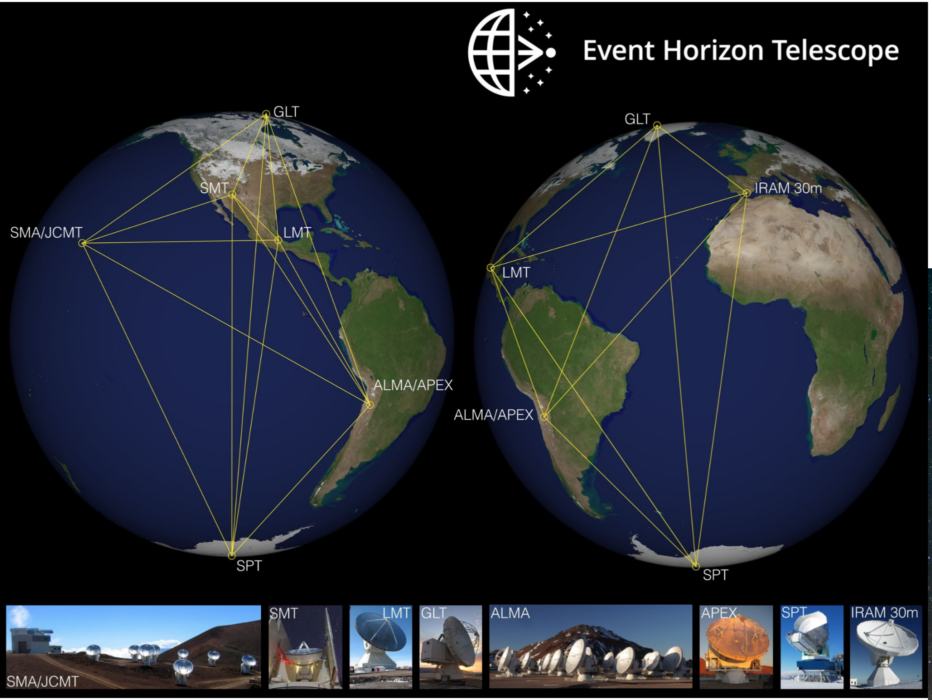
* Can we use Gravitational lensing to Distinguish the Black Holes and Naked singularities?

* Studies in this direction (Boza, Ellis,Virbhadra, Sahu, Narasimha, Patil, PSJ et al) show the two objects differ critically from each other for their Lensing Signatures in terms of Formation of Images, Einstein Rings etc.

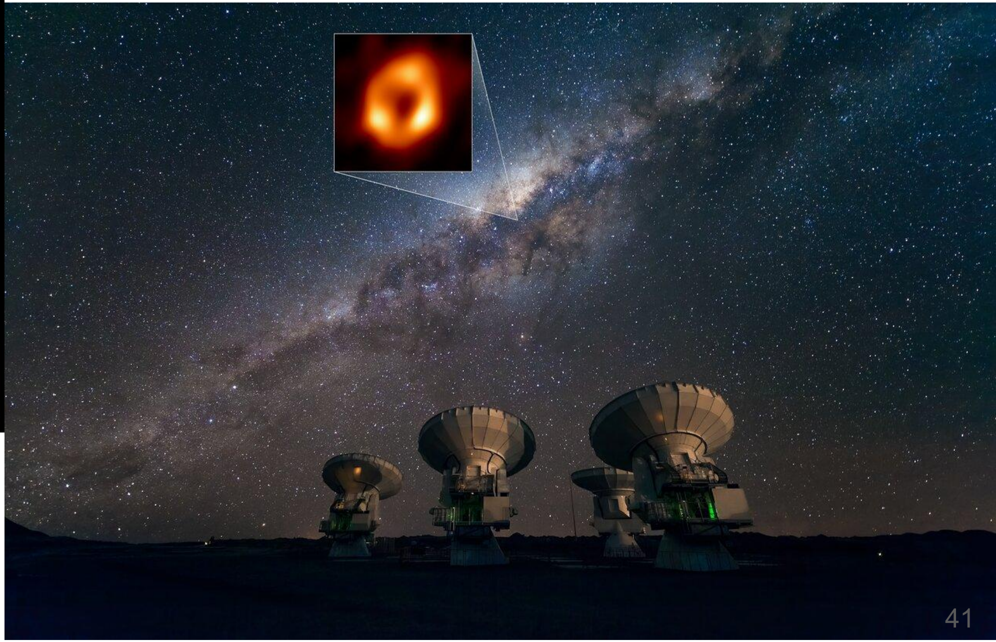
* The key difference is due to the Structure of Photon Spheres in each of these cases

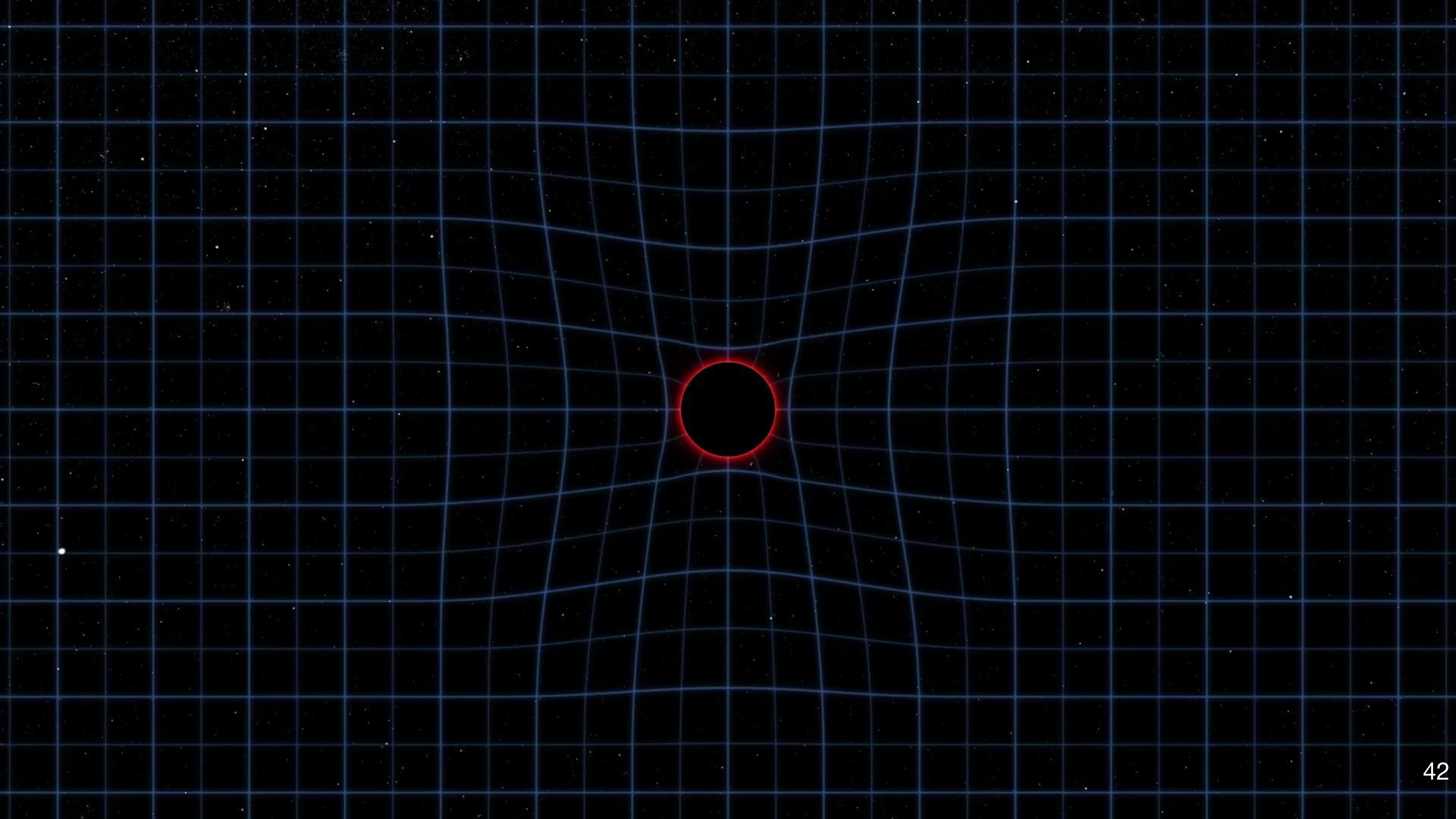
Shadow of Sgr A*:

The EHT Collaboration et al, Astro. Phys. Journal L, 930, 2022, L12



What should be the nature of Sgr A*?





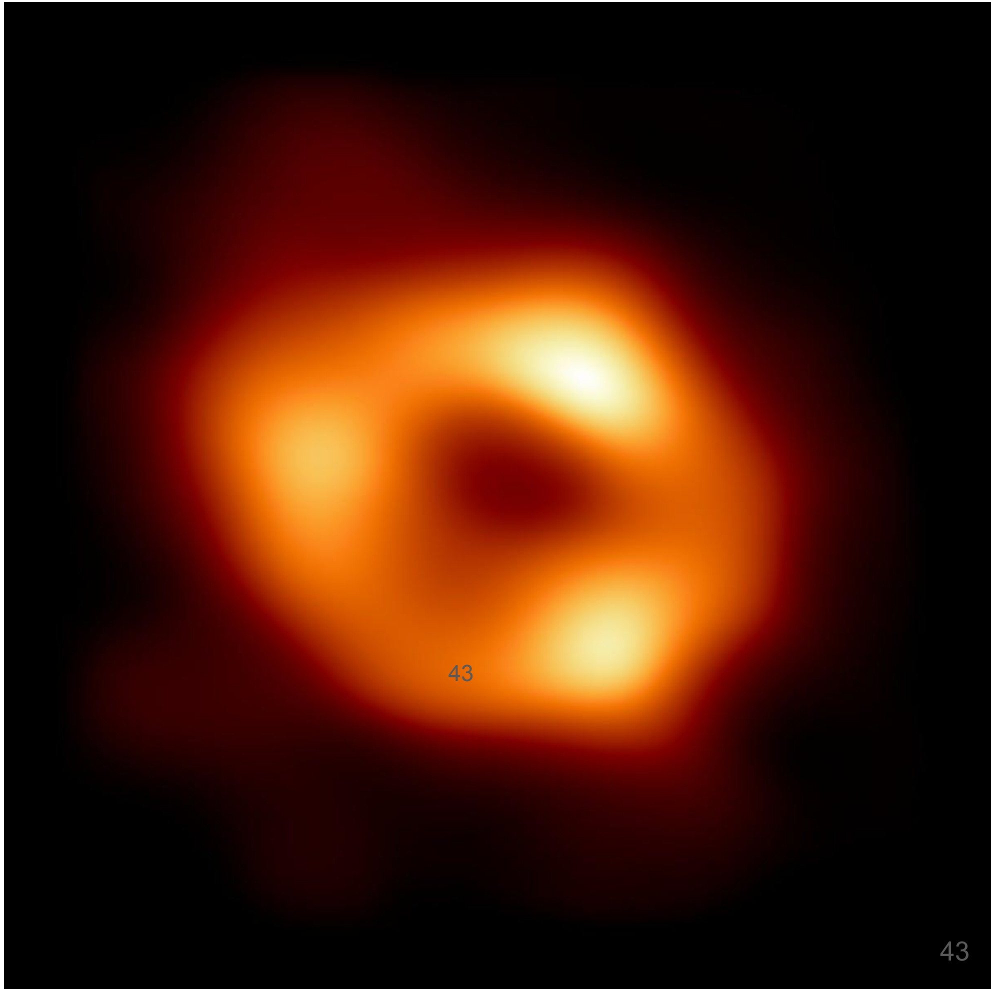
Shadow of Sgr A*:

The EHT Collaboration et al, Astro. Phys. Journal L, 930, 2022, L12

- **Mass:** 4.3 million solar mass
- **Inclination:** $i < 50$ degree
- **Distance:** 8.2 kpc
- **Diameter:** 51.8 ± 2.3 μas

- Model comparison disfavor:

- High Inclination ($i > 50$ degree)
- Static BH.
- Retrograde accretion disk

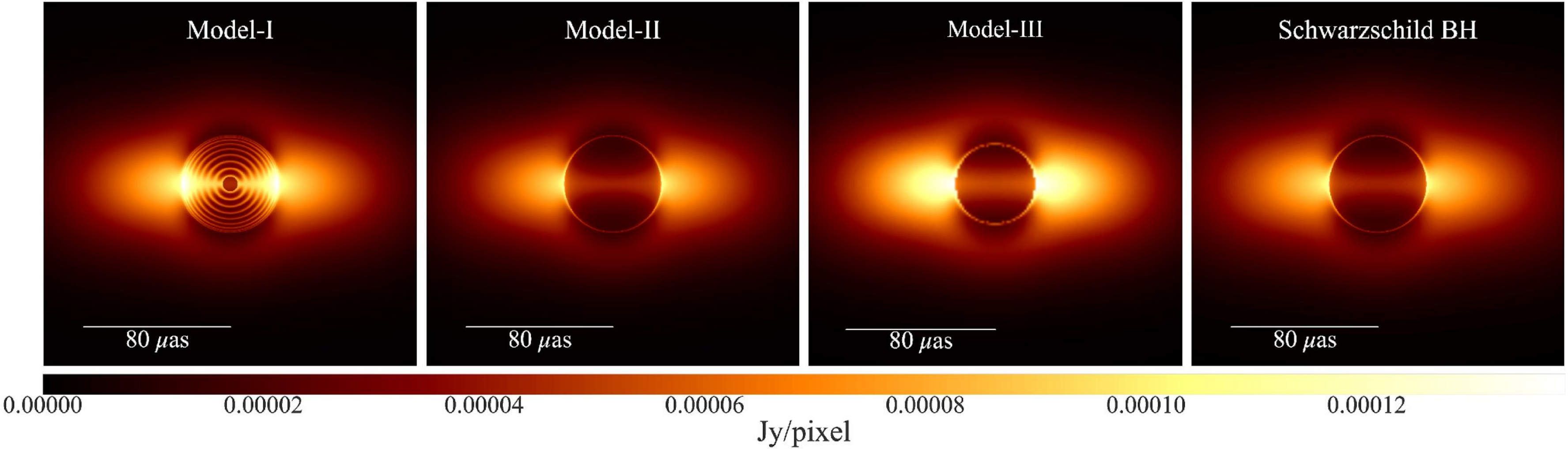


Shadows of the compact objects: Model Images for Sgr A*

JMN,
no photon sphere

JMN,
with photon sphere

JNW,
with photon sphere

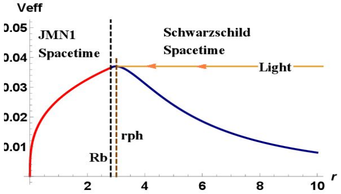


- We adopt the BH mass (M) for Sgr A* to be 4.3 million solar mass and source distance (D) from the Earth is 8200 pc

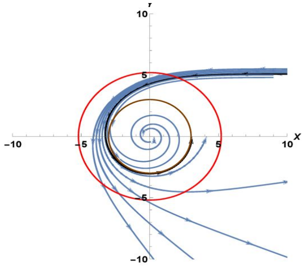
Saurabh, et al, (2023), accepted in Astronomy & Astrophysics.

A. B. Joshi at el, Phys. Rev. D **102**, 024022 (2020).

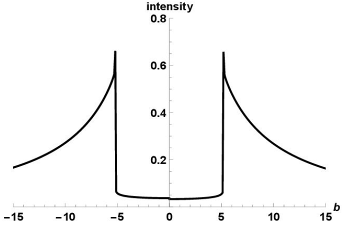
Shadow of a naked singularity without photon sphere



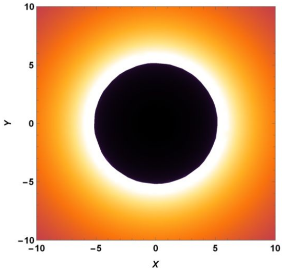
(a)Figure of effective potetial in JMN1 spacetime.



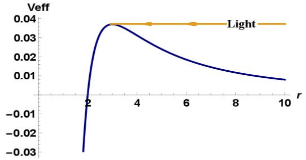
(b)Lensing effect in JMN1 spacetime.



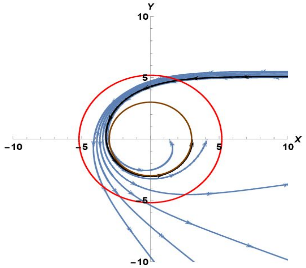
(a)Intensity distribution in Schwartzschild spacetime.



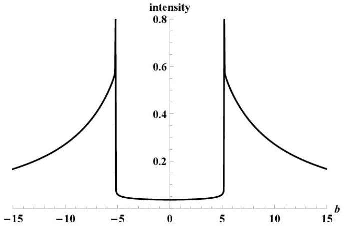
(b)Shadow in Schwartzschild spacetime.



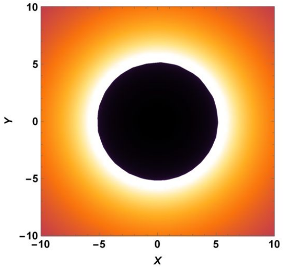
(c)Figure of effective potential in Schwartzschild spacetime.



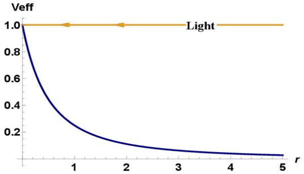
(d)Lensing effect in Schwartzschild spacetime.



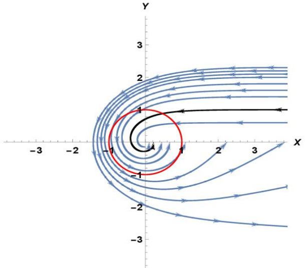
(c)Intensity distribution in JMN-1 spacetime.



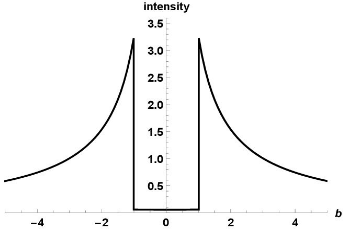
(d)Shadow in JMN-1 spacetime.



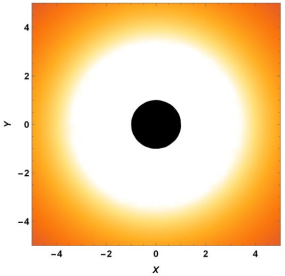
(e)Figure of effective potential in new spacetime.



(f)Lensing effect in new spacetime.



(e)Intensity distribution in new spacetime.

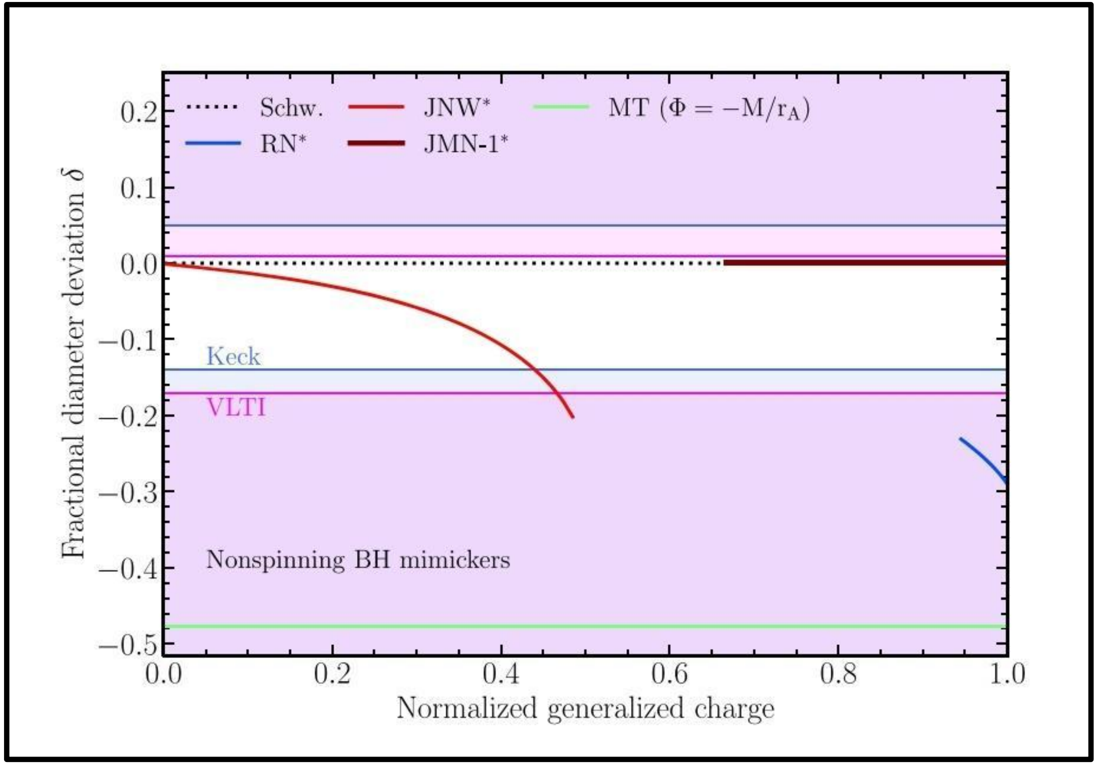


(f)Shadow in new spacetime.

Shadow of Sgr A*:

The EHT Collaboration et al, ApJL, 930, 2022, L17

- Constraints on specific compact object spacetimes:



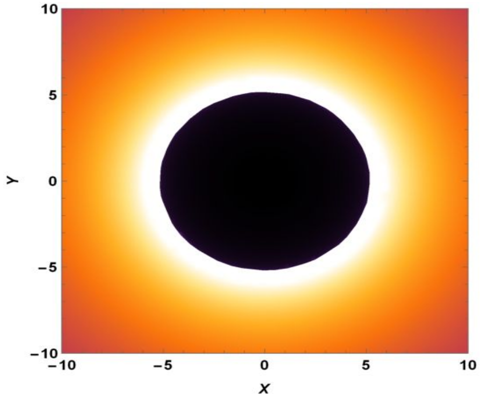
Quotes from EHT paper:

- "The white region correspond to shadow sizes that are consistent at the 68% level with EHT observation."
- "The JMN-1 naked singularity with a photon sphere may be one of the best possible black hole mimickers for Sgr A*."

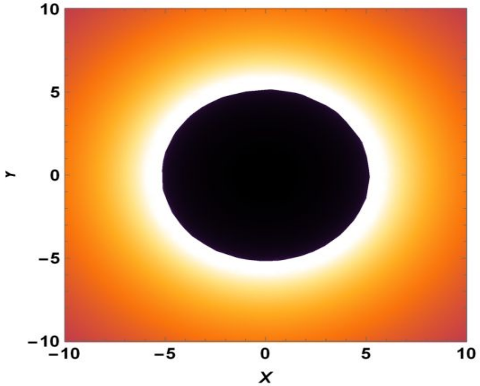
46

- "Therefore, the possibility that Sgr A* is a JMN-1 naked singularity cannot be ruled out based on the metric tests." EHT, ApJL, 930, 2022, L17

Intensity distribution in shadow



(b)Shadow in Schwartzschild spacetime.



(d)Shadow in JMN-1 spacetime.

- What if the two compact object have same shadow size as well as outside intensity distribution also same? How can we distinguish them?
- There are several theoretical models which predict that shadow size of a compact object can be same as the size of the equally massive Schwarzschild black hole's shadow
- Therefore, it is very difficult to understand the causal structure of the compact object using the shadow size.

It follows that: The existence and observations of a shadow is not sufficient, or need not ensure the existence of an event horizon necessarily in the galactic center

48

It follows that: The existence and observations of a shadow is not sufficient, or need not ensure the existence of an event horizon necessarily in the galactic center

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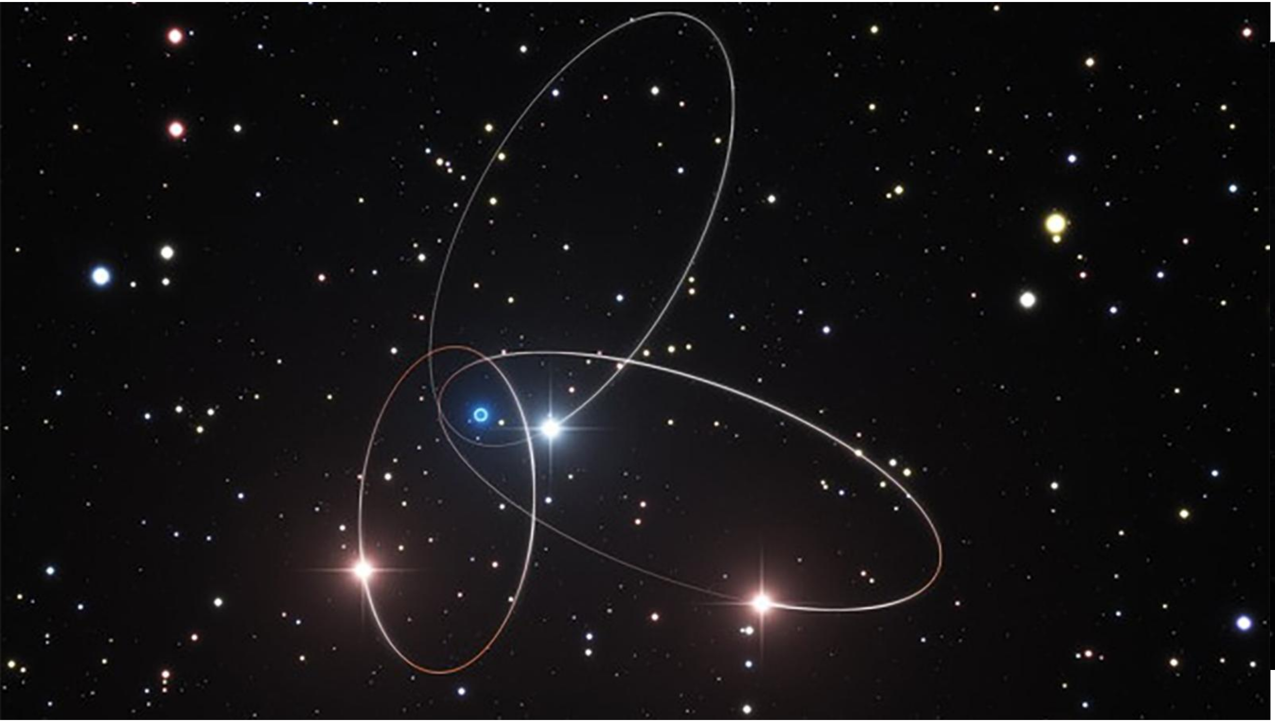
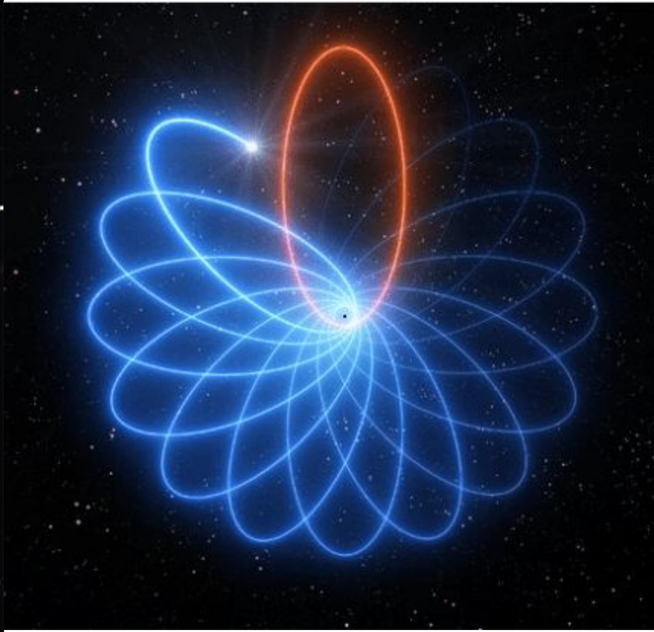
50

Relativistic orbits of S-stars:

Florian Peißker *et al, Astro. Phys. J.* 889 61 (2020).
M. Parsa *et al, Astro. Phys. J.* 845 22 (2017),
GRAVITY collaboration, Astronomy & Astrophysics, 636 L5 (2020).



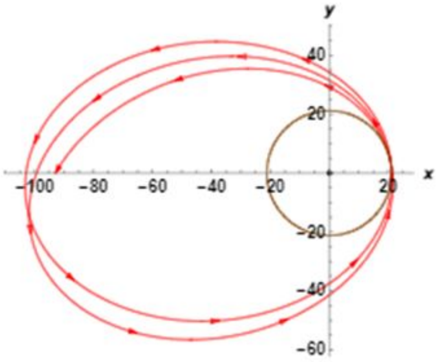
Credit: ESO/M. Parsa/L. Calçada

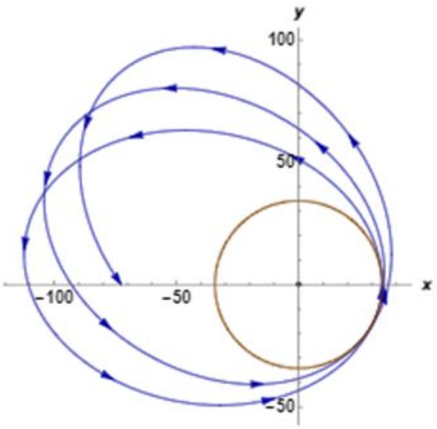


Precession of timelike bound orbits:

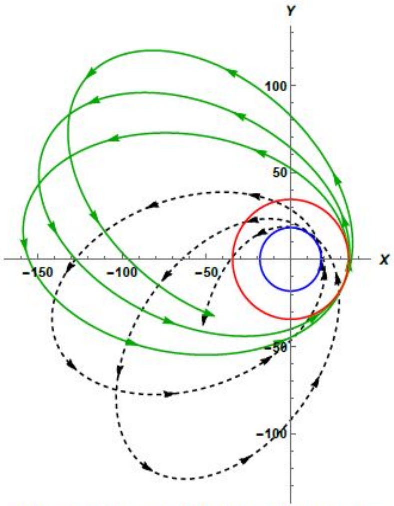
P. Bambhaniya et al, Phys. Dark Univ. 40, (2023).
D. N. Solanki et al, Eur. Phys. J. C 82, 77 (2022).

- Positive and Negative precession

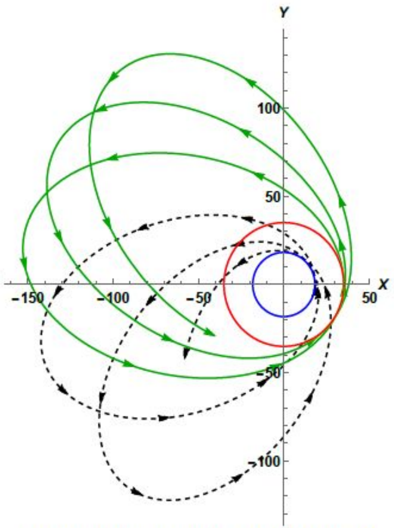




❖ Orbits in rotating JNW spacetime



(c)Timelike bound orbit in rotating JNW metric for $q = 12$ and $a = 0.3$.



(d)Timelike bound orbit in rotating JNW metric for $q = 12$ and $a = 0.5$.

Results:

- The JMN-1 and JNW naked singularities with a photon sphere can cast a similar shadow image as the black hole does and hence it is one of the best possible black hole mimickers for Sgr A*.

- The retrograde precession of timelike orbits is not possible in the SCH. and Kerr BHs spacetimes.

- The best fitting parameters of relativistic orbit of S2 star are estimated using the MCMC technique and obtained the lowest $\chi 2$ value is 4.71 for JNW spacetime.

  Therefore, the naked singularity could be a possible candidate which might represent the spacetime structure of the Sgr-A*.

WHY THIS IS SO IMPORTANT?

The issue of Black Holes vs Naked Singularities is Fundamental to Modern Theory of Black Holes; its Astrophysical Applications; in What Form; the Collapse Models would help us get a Fine-tuned Version of CCC; When Black Holes will form?

- On the Other hand if Naked Singularities Do arise in in Nature—in Star Collapse, then there could be Exciting Observational Implications!

- Very High Energy Explosive Cosmic Phenomena will be Typical Candidates, and theoretically there will be Most Intriguing Quantum Gravity Implications!!

- Any evidence of retrograde precession of any ‘S’ star can raise big question on the existence of a Kerr or SCH. BH at the Milky-way galaxy center.

The Key Point is: GR implies Existence of Strong Gravity Regions, where Both Quantum Gravity and General Relativity come into their Own & These could be Visible to External observers in Universe

****

As Wald pointed out: `If censorship fails, then in a literal sense, we would come face-to-face with the laws of Quantum Gravity whenever gravitational collapse to a naked singularity occurs in distant regions of our Universe..'

AN EXCITING POSSIBILITY!!

—*QUANTUM STARS?*—

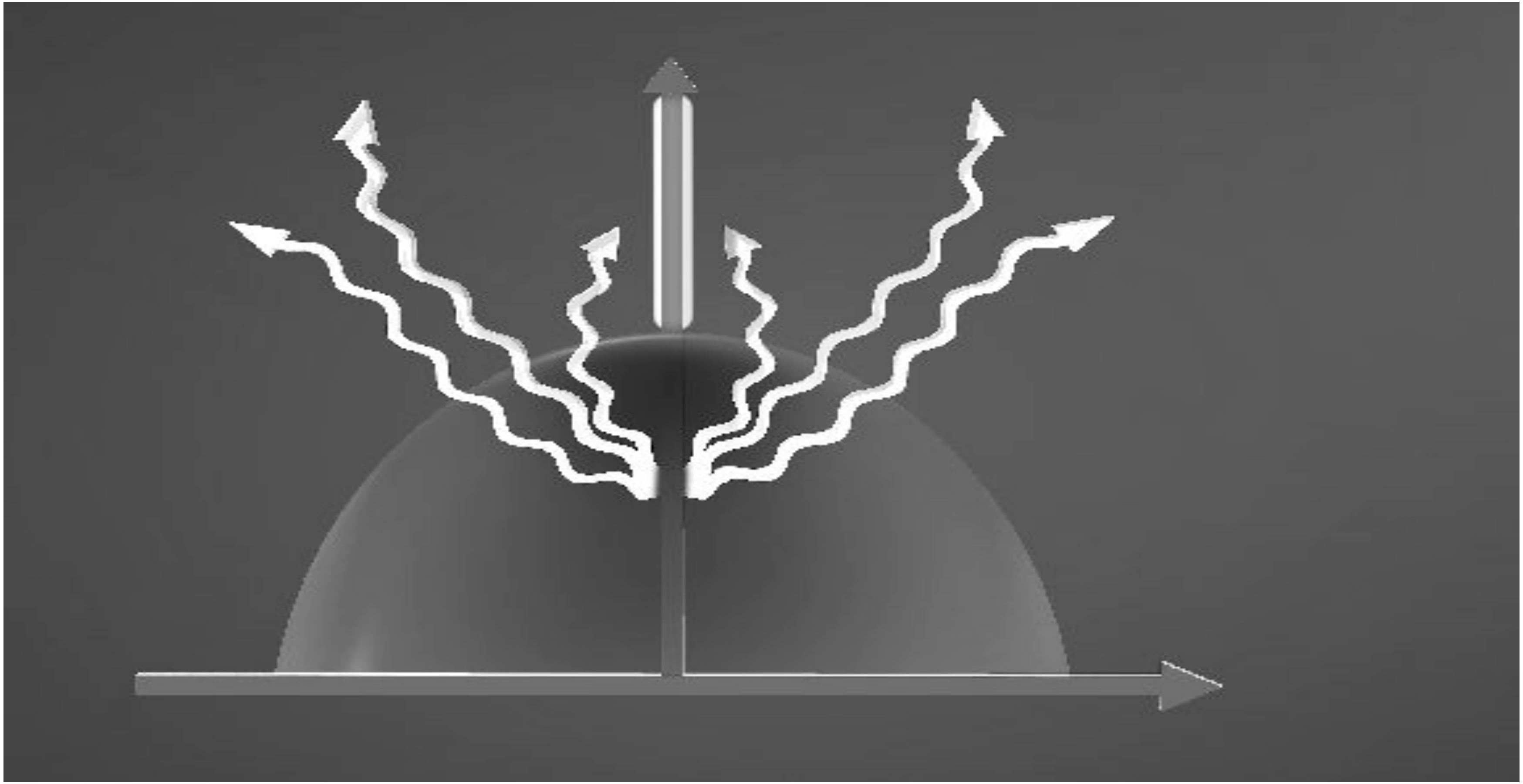
Naked Singularities in Star Collapse ==> Exciting Observational Implications—High Energy Explosive Cosmic Phenomena will be Typical Candidates…

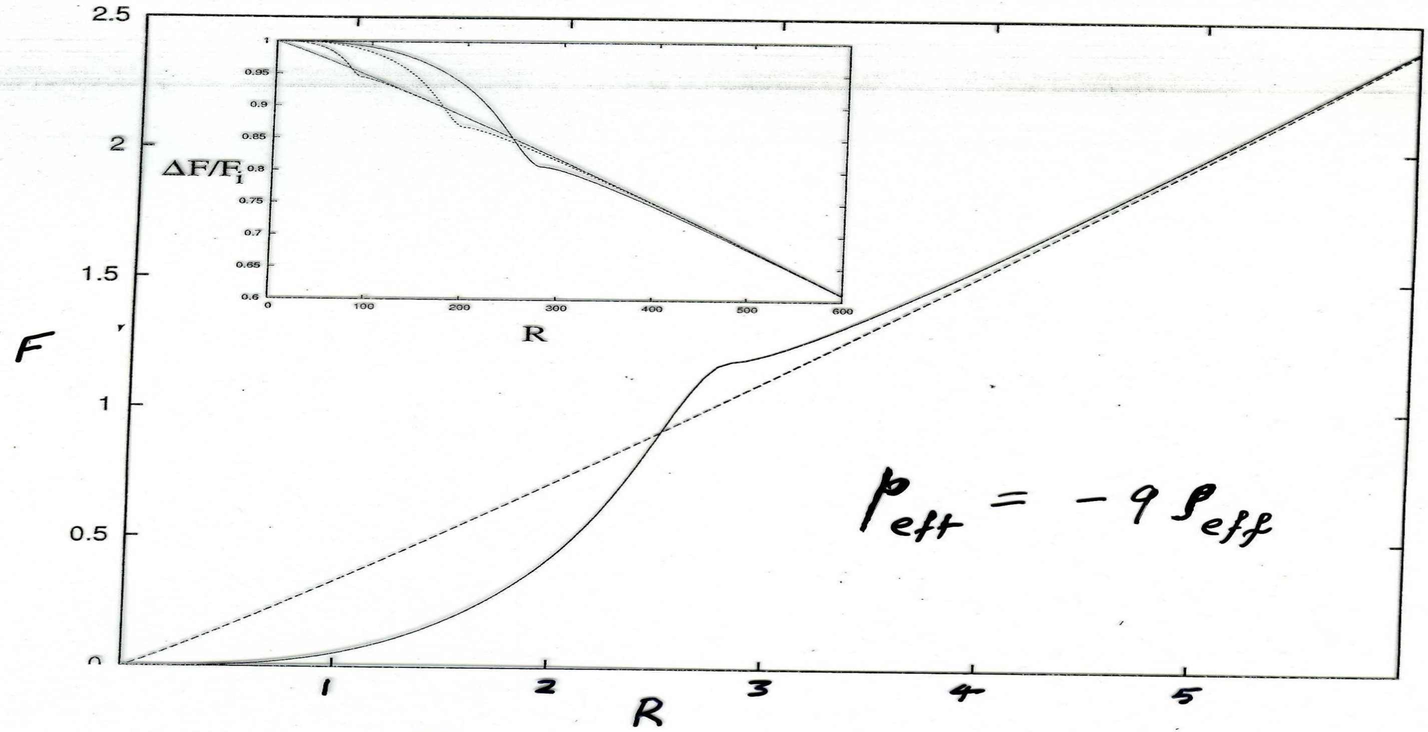
Can we Observe and See Quantum Gravity Effects taking place in their vicinity?

Does Nature produce a Quantum Star whenever a Massive Star Collapses??

THIS IS SO IMPORTANT FOR UNIFICATION OF PHYSICS

Naked Singularity is a Mini-Big bang!





→ The fact that LQG creates such a rapid mass loss is intriguing ...

-QUANTUM GRAVITY LAB?-

Naked Singularities in Star Collapse ==> Exciting Observational Implications—High Energy Explosive Cosmic Phenomena will be Typical Candidates…

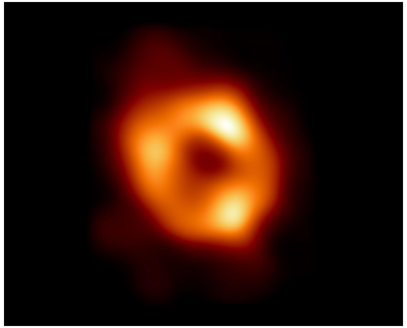
Can we Observe and See Quantum Gravity Effects taking place in their vicinity?

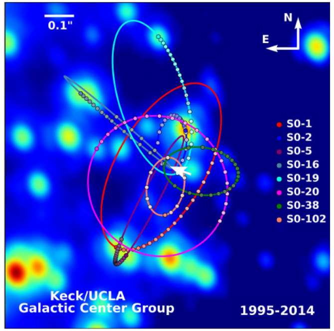
Does Nature produce a Quantum Gravity Lab whenever a Massive Star Collapses??

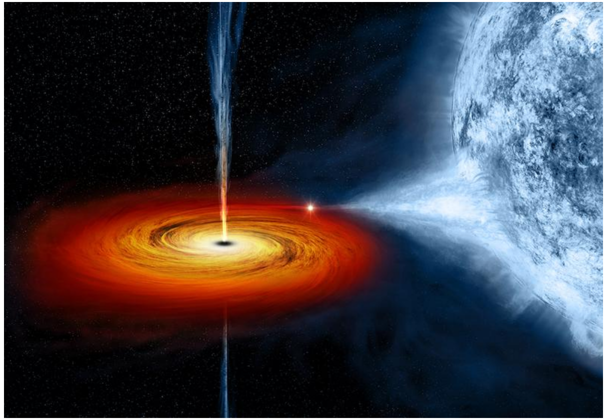
WHY IT'S SO IMPORTANT-UNIFICATION OF PHYSICS

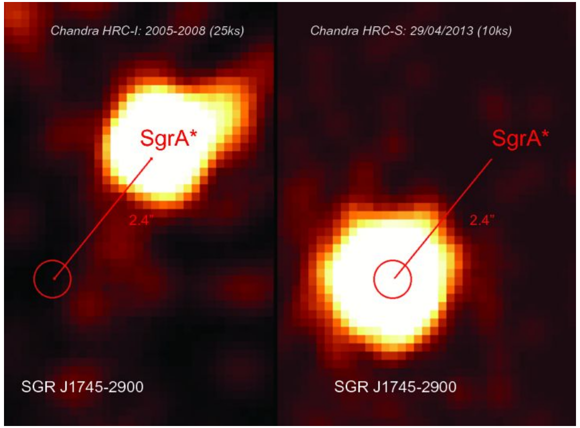
Astrophysical signatures of compact objects

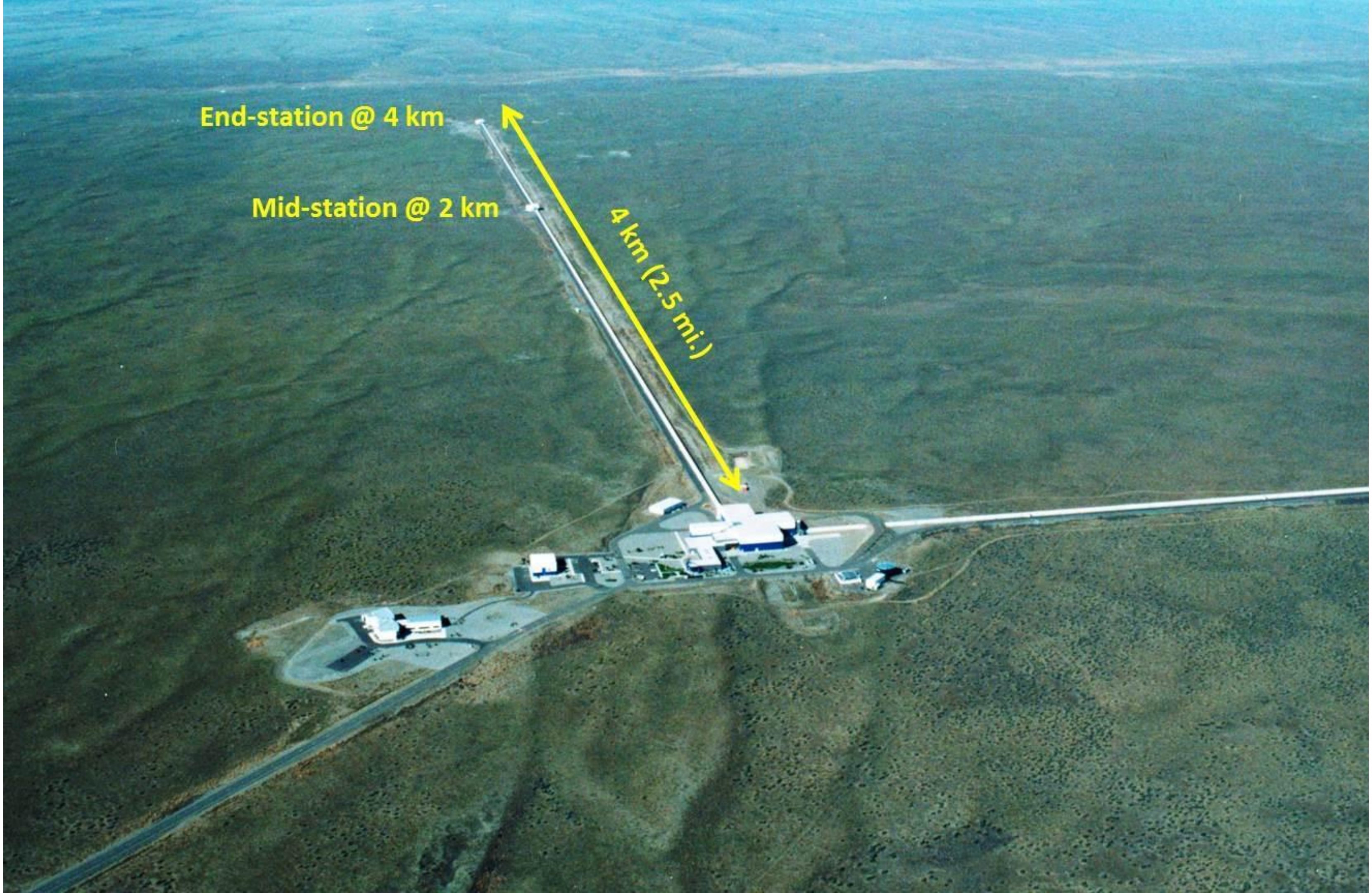
- **Astrophysical signatures of compact objects:** Accretion disk and Shadow properties, Energy Extraction, Relativistic orbits, Tidal disruption, Lens-Thirring effect, Gamma-ray burst theory, Relativistic Jets, Pulsar timing array.

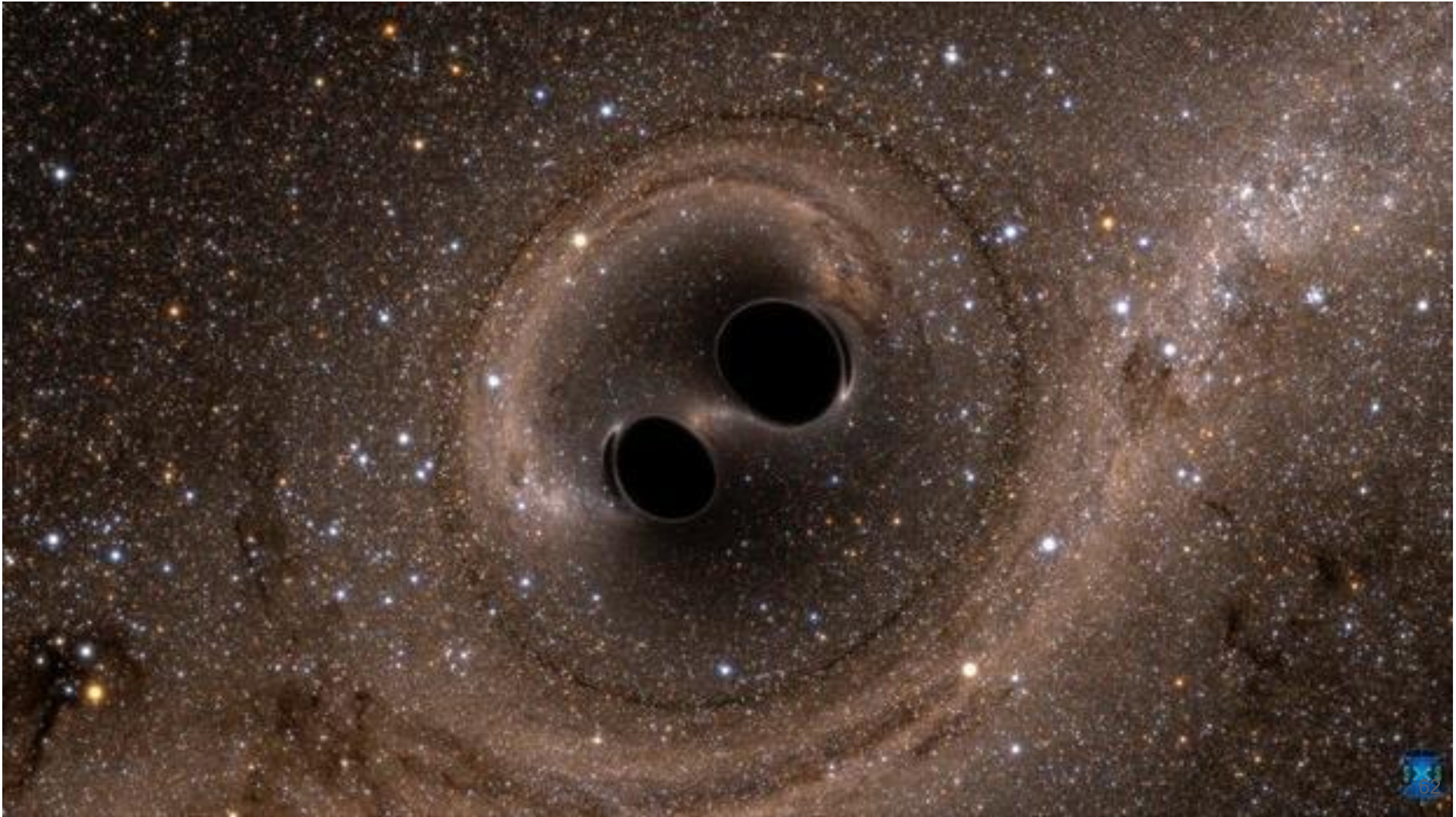


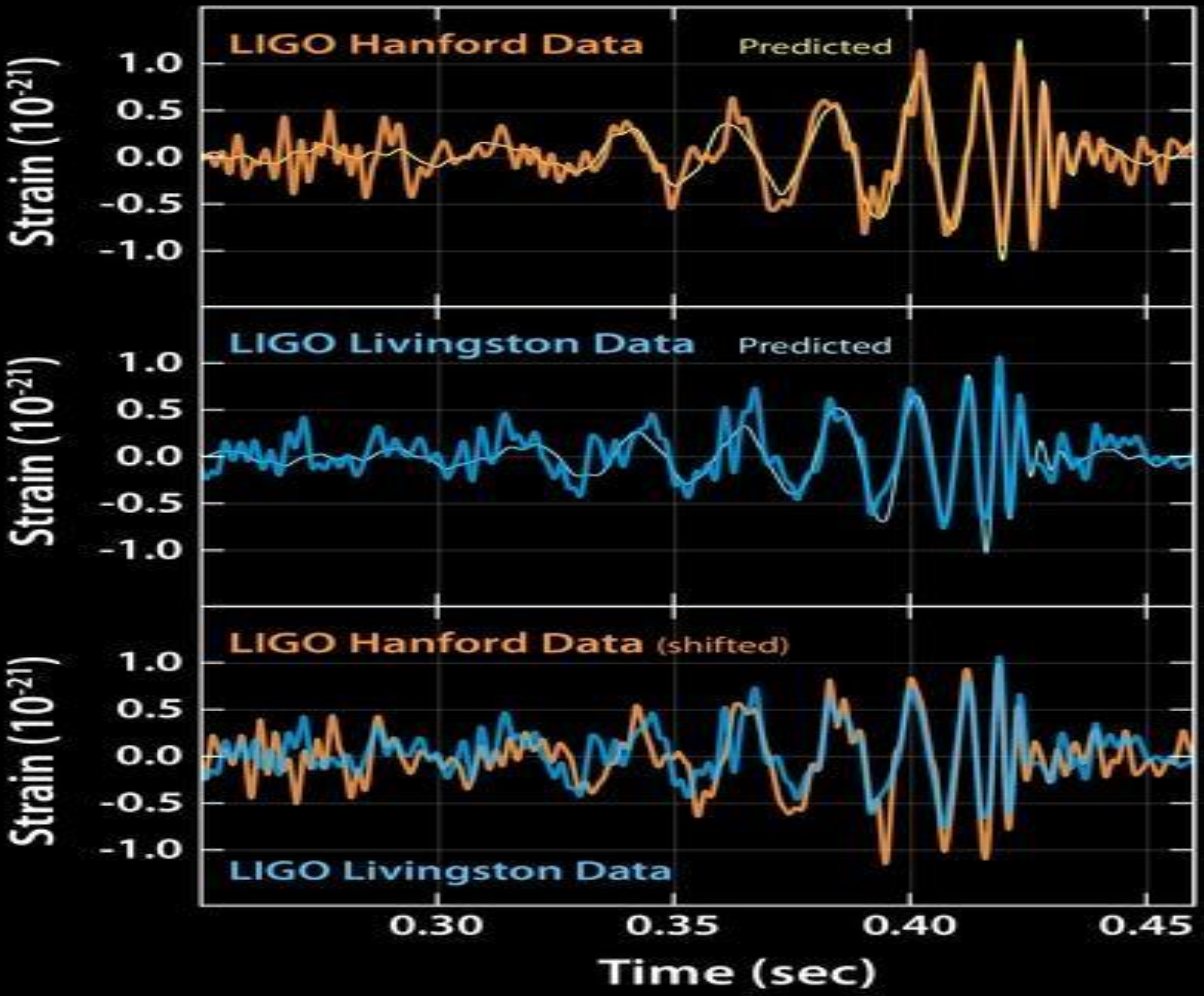


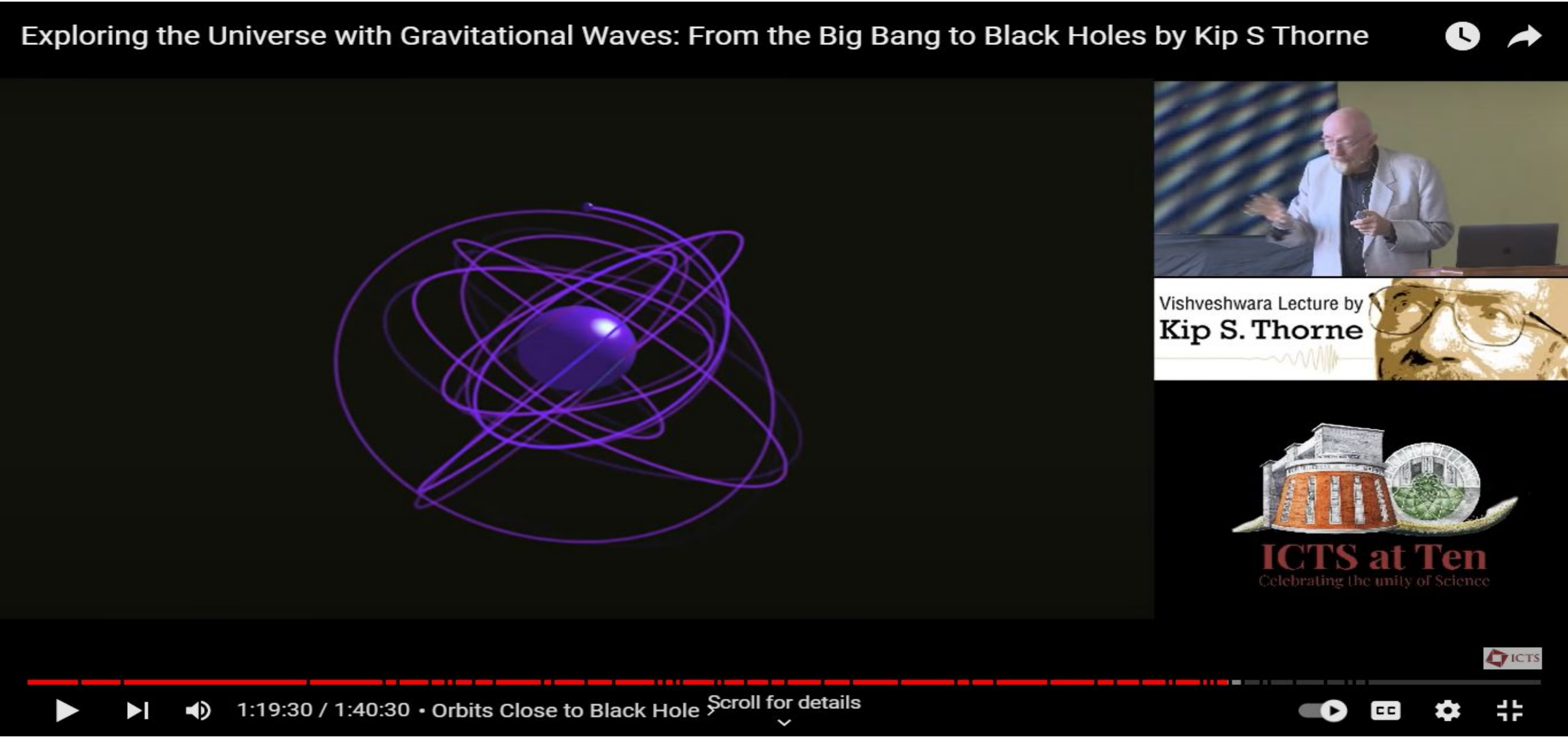


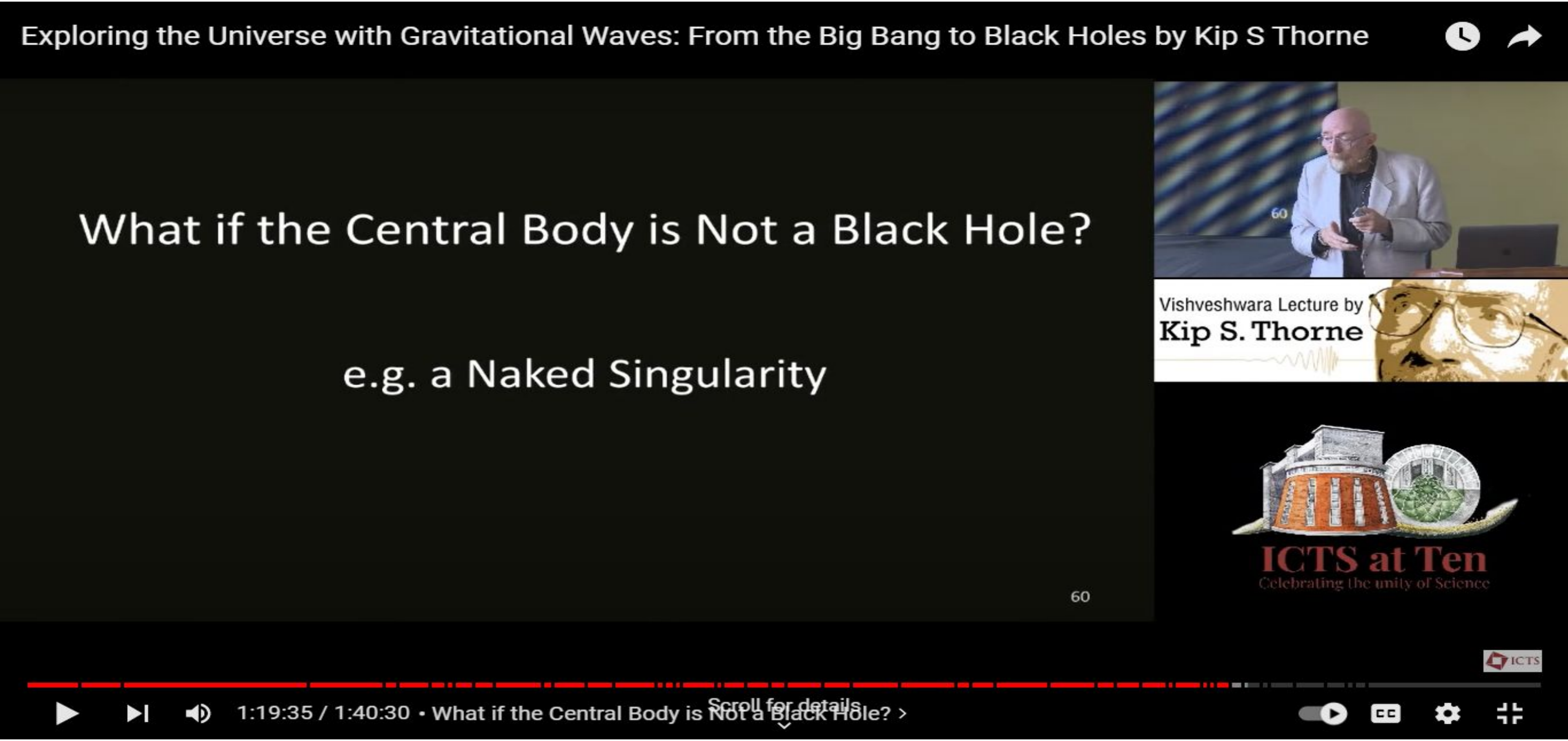



End-station @ 4 km
Mid-station @ 2 km
4 km (2.5 mi.)




LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)


Exploring the Universe with Gravitational Waves: From the Big Bang to Black Holes by Kip S Thorne
Vishveshwara Lecture by
Kip S. Thorne
ICTS at Ten
Celebrating the unity of Science
ICTS
1:19:30 / 1:40:30 • Orbits Close to Black Hole
Scroll for details


Exploring the Universe with Gravitational Waves: From the Big Bang to Black Holes by Kip S Thorne
What if the Central Body is Not a Black Hole?
e.g. a Naked Singularity
60
Vishveshwara Lecture by
Kip S. Thorne
ICTS at Ten
Celebrating the unity of Science
ICTS
1:19:35 / 1:40:30 • What if the Central Body is Not a Black Hole? >
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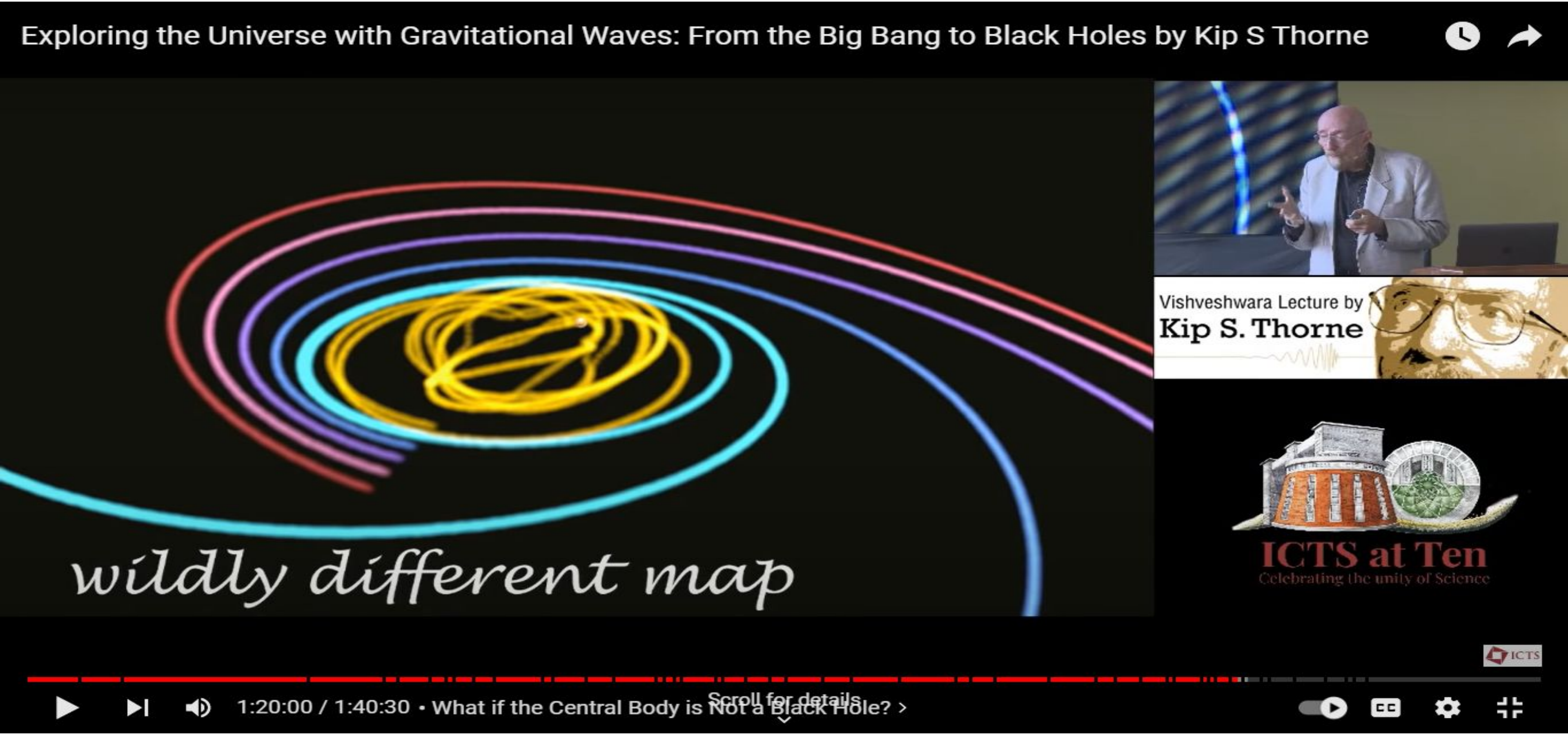
Exploring the Universe with Gravitational Waves: From the Big Bang to Black Holes by Kip S Thorne
wildly different map
Vishveshwara Lecture by
Kip S. Thorne
ICTS at Ten
Celebrating the unity of Science
ICTS
1:20:00 / 1:40:30 • What if the Central Body is Not a Black Hole? ›
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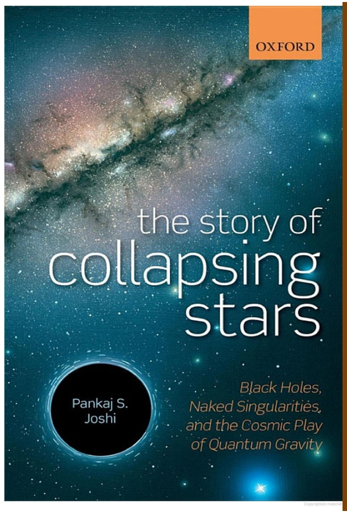
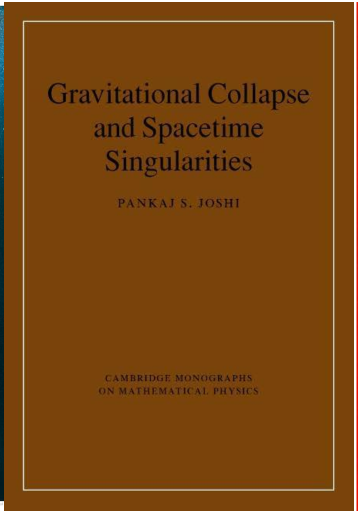
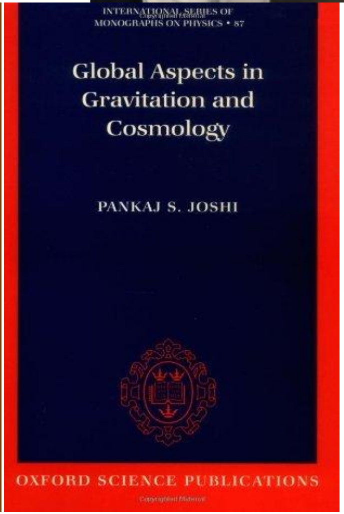
3M


How Eating Meat Contributes to Global Warming
SCIENTIFIC AMERICAN
Nanomedicine
Revolutionizing the Fight against Disease
Black holes may have even stranger siblings that violate known laws of physics
NAKED SINGULARITIES
Fold the Brain
How Forces That Shape It Link to Autism and More
Submarine Lava
The Mysterious, Red-Hot Origins of the Ocean Floor
Electric Rockets
Efficient Travel to Deep Space


OXFORD
the story of collapsing stars
Black Holes, Naked Singularities, and the Cosmic Play of Quantum Gravity
Pankaj S. Joshi


Gravitational Collapse and Spacetime Singularities
PANKAJ S. JOSHI
CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS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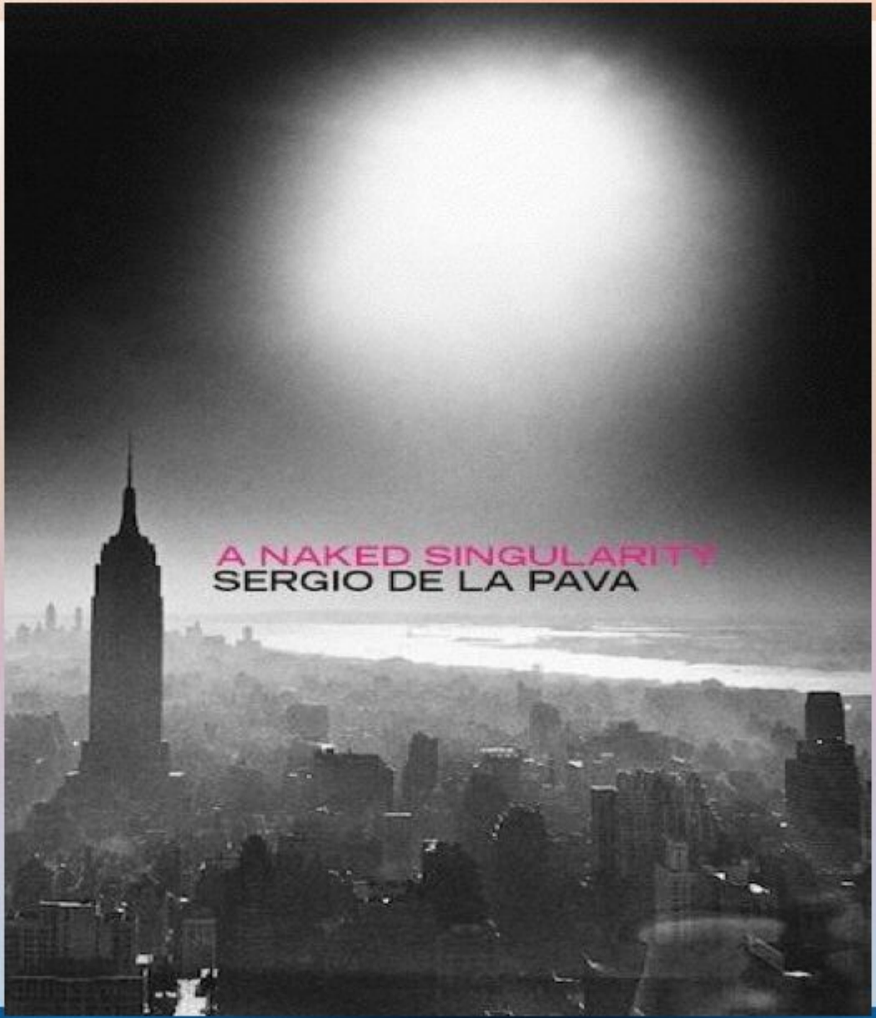

A NAKED SINGULARITY
SERGIO DE LA PAVA

International Centre for Space and Cosmology (ICSC)

Ahmedabad University

ICSC office, Hub-2, School of Arts and Science, Ahmedabad University

Dipanjan Dey

Tapobroto Bhanja

Kanwar Preet
Kaur

Rucha Desai

Karim Mosani

Ashok Joshi

Divyesh
Vitthani

Koushiki
Bhattacharya

Jay Verma
Trivedi

Rajibul Sheikh

Vishva Patel

Kauntey Acharya

Parth Bambhaniya

Saurabh

Siddharth Madan

Kshitij Pandey

Aadarsh Mehta

Divyesh Solanki

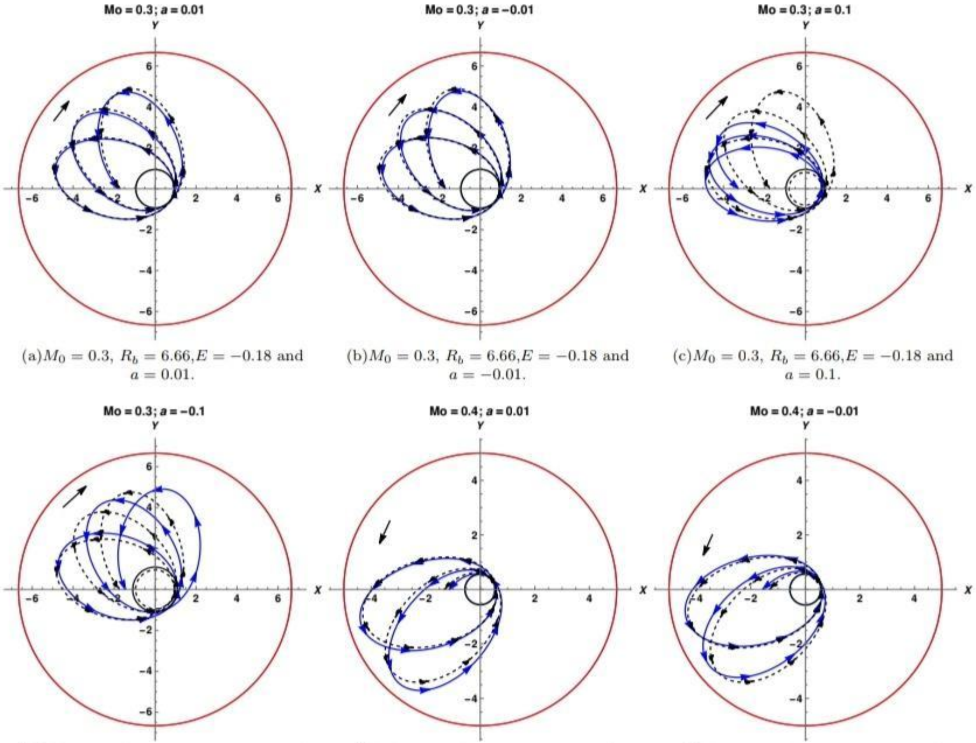
Divya Tahelyani

Shailee Shah

Thank you for your attention…

P. Bambhaniya et al, Phys. Dark Univ. 40, (2023).
D. N. Solanki et al, Eur. Phys. J. C 82, 77 (2022).

- Orbits in Slowly rotating JMN1 spacetime



(a)$M_0 = 0.3,\ R_b = 6.66, E = -0.18$ and $a = 0.01$.

(b)$M_0 = 0.3,\ R_b = 6.66, E = -0.18$ and $a = -0.01$.

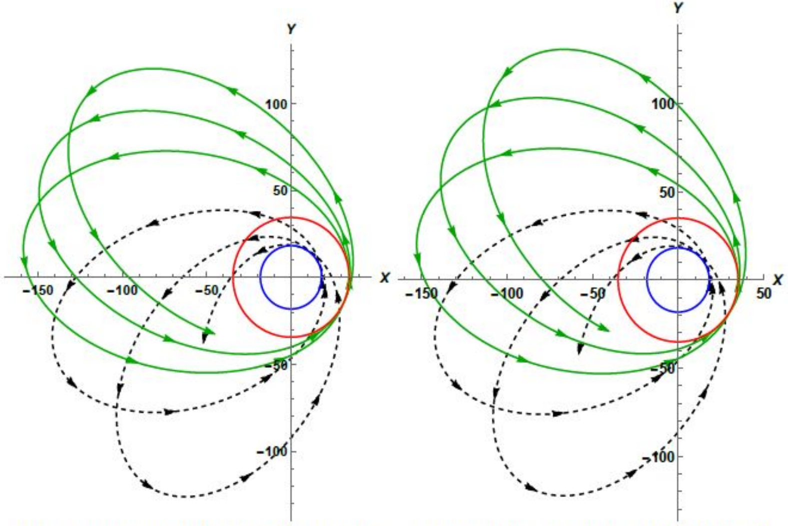
(c)$M_0 = 0.3,\ R_b = 6.66, E = -0.18$ and $a = 0.1$.

(d)$M_0 = 0.3,\ R_b = 6.66, E = -0.18$ and $a = -0.1$.

(e)$M_0 = 0.4,\ R_b = 5, E = -0.21$ and $a = 0.01$.

(f)$M_0 = 0.4,\ R_b = 5, E = -0.21$ and $a = -0.01$.

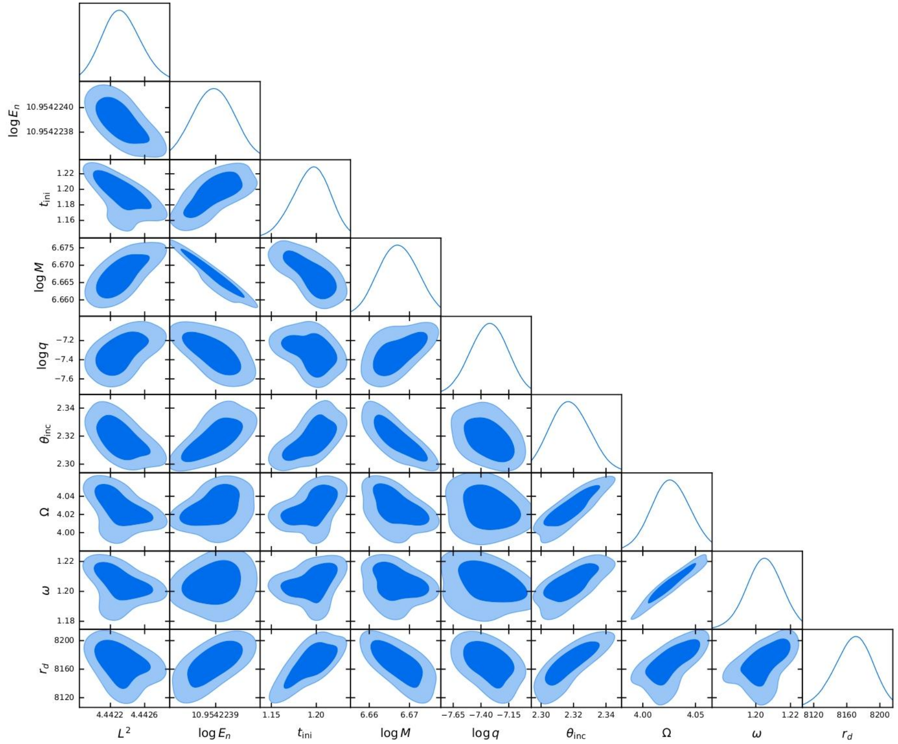
- Orbits in rotating JNW spacetime



(c)Timelike bound orbit in rotating JNW metric for $q = 12$ and $a = 0.3$.

(d)Timelike bound orbit in rotating JNW metric for $q = 12$ and $a = 0.5$.

Minimal Chi-square method and Monte-Carlo-Markov-Chain (MCMC) technique.



Parameter	JNW (95% limits)
L^2 $(pc^2(km/s)^2)$	$4.44216^{+0.00075}_{-0.00075}$
$\log E_n$ $(km/s)^2$	$10.95422391^{+0.00000034}_{-0.00000033}$
t_{ini} (year)	$1.199^{+0.038}_{-0.040}$
$\log M$ $(M\odot)$	$6.666^{+0.010}_{-0.012}$
$\log q$ $(M\odot)$	$-7.46^{+0.58}_{-0.57}$
θ_{inc} (radian)	$2.316^{+0.025}_{-0.025}$
Ω (radian)	$4.017^{+0.035}_{-0.033}$
ω (radian)	$1.199^{+0.029}_{-0.029}$
Distance (parsec), r_d	8169^{+34}_{-39}
Time period (yr), T	16.1379
Minimal χ^2	4.71

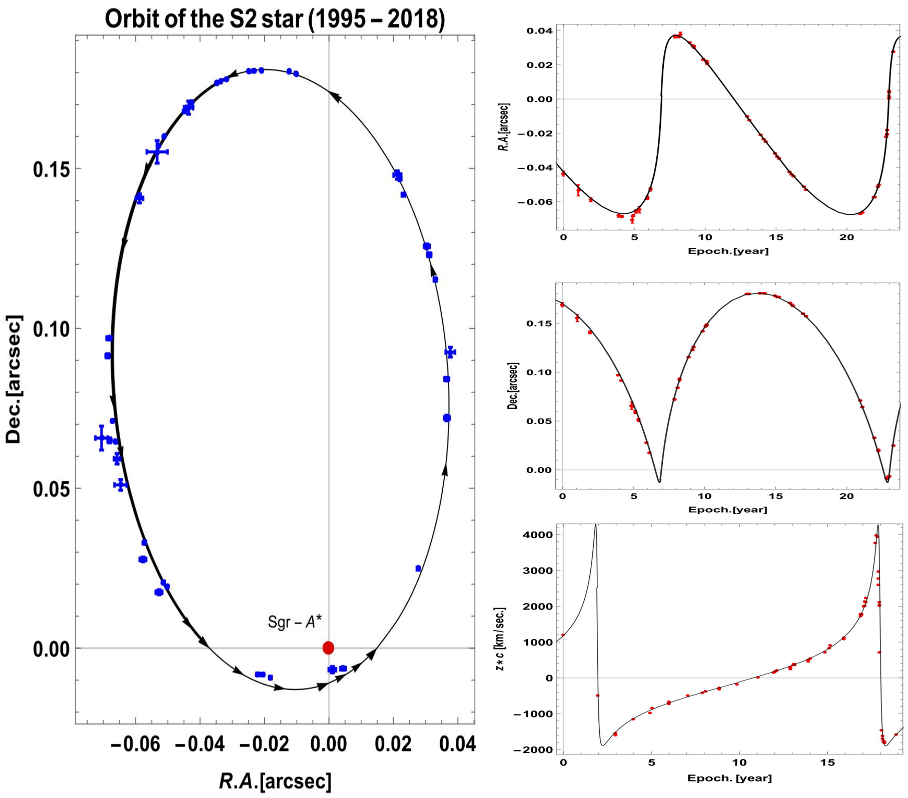
TABLE I: Estimated best-fit values of the parameters for the JNW metric.

1-σ and 2-σ bestfit regions and the posterior distributions: We estimate the best fitting parameters for the JNW metric using the MCMC technique and obtain the lowest Chi-square value is 4.71

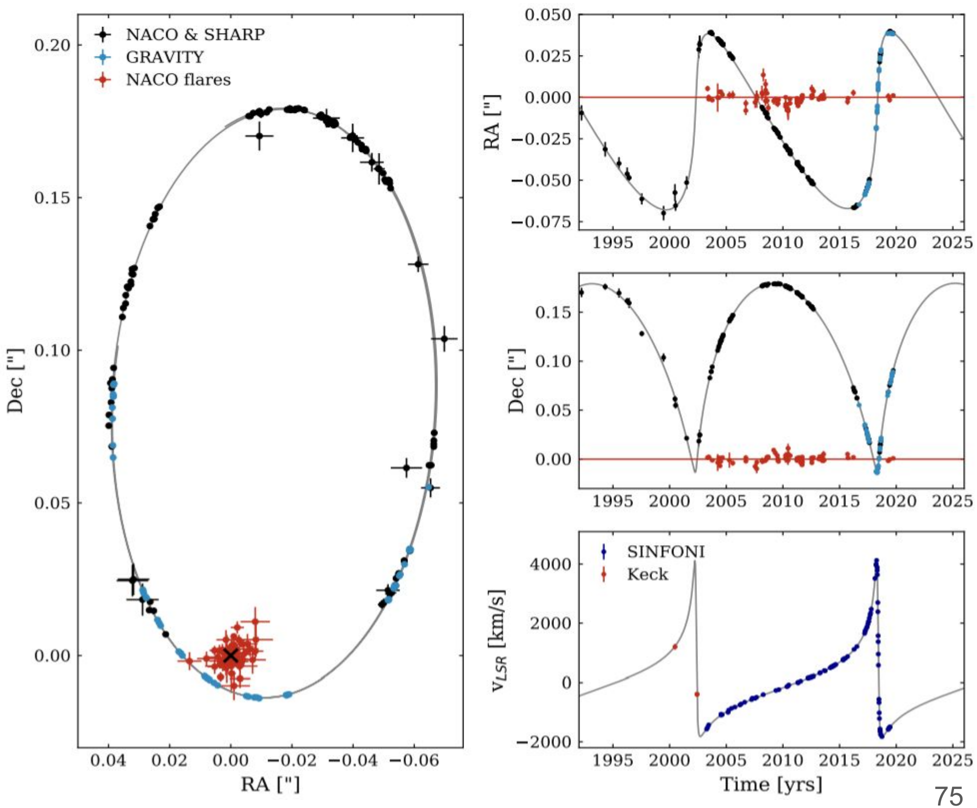
Relativistic orbits of S2 star in JNW:

P. Bambhaniya et al, arXiv:2209.12610. Accepted in Eur. Phys. J. C
GRAVITY collaboration, Astronomy & Astrophysics, 636 L5 (2020).

JNW naked singularity

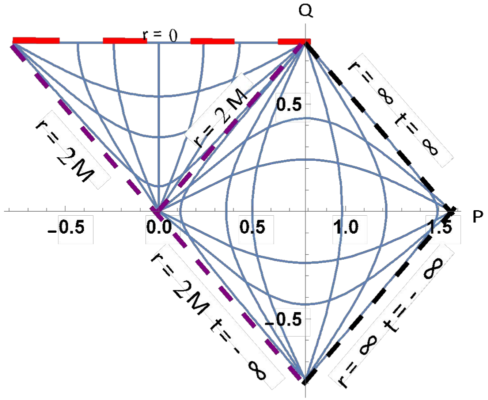


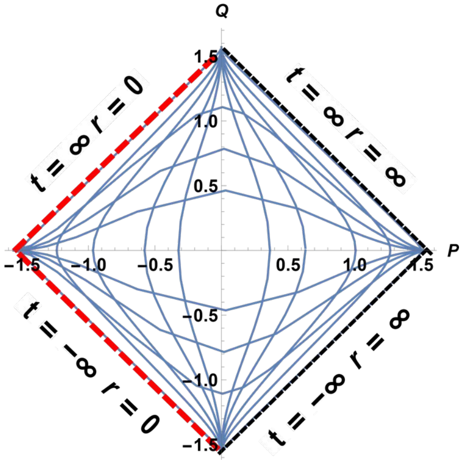
SCH. BH.

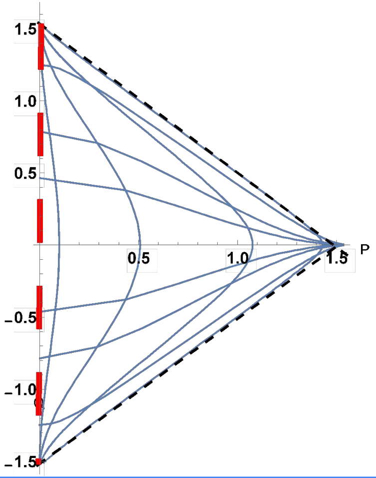


Compact Objects

- Concept of singularity in galactic center is widely accepted and people are believe that central singularity should be hidden for the asymptotically observer. In 1969 Penrose propose weak cosmic censorship conjecture (WCCC) state that, singularity produce during gravitational collapse can not seen by asymptotic observer and this singularity must always hidden within cover called event horizon.
- **Is physically such a singularity exist, which can also be visible for asymptotic observer?**
- **Is general theory of relativity is correct in strong field region?**
- One can show that a spherically symmetric, homogeneous, dust collapse always terminates into a black hole. Depending upon initial conditions, the final spacetime can be a black hole or a naked singularity.
- In[1], it is shown that **Joshi-Malafarina-Narayan (JMN)** spacetimes with central timelike singularity are obtained as the asymptotic equilibrium state of a quasi-static gravitational collapse.







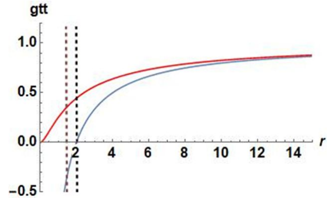
[1] P. S. Joshi *et al*, Equilibrium configurations from gravitational collapse, *Class. Quantum Grav.* 28 235018 (2011).

Schwarzschild black hole and Null singularity spacetime: The spacetimes are static, spherically symmetric asymptotically flat solution of Einstein equations.

- Spherically symmetric, homogeneous, dust collapse always terminates into a Schwarzschild black hole.
- When the collapsing matter is homogeneous and dustlike, the final fate of the gravitational collapse is necessarily a Schwarzschild black hole.
- Null naked singularity solution of Einstein equations which resembles with a Schwarzschild spacetime at large distances.

$$ds^2_{sch} = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$ds^2_{null} = -\left(1 + \frac{M}{r}\right)^{-2} dt^2 + \left(1 + \frac{M}{r}\right)^2 dr^2 + r^2 d\Omega^2$$



Where, G = C = 1 and $M = ADM\ Mass,\ d\Omega^2 = d\theta^2 + \sin^2\theta\ d\phi^2$

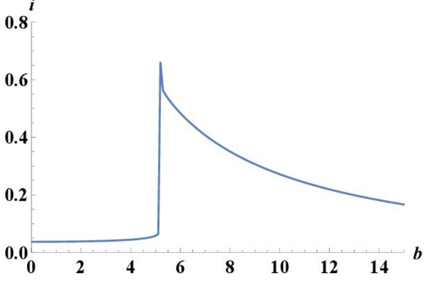
Joshi-Malafarina-Narayan-1 (JMN-1) spacetime : In P. S. Joshi *et al*, it is shown that in asymptotic time, JMN-1 spacetime can formed as an end state of the gravitational collapse of matter cloud with zero radial pressure and non-zero tangential pressure.

- This spacetime has a timelike strong singularity at the centre. JMN-1 spacetime matches with the external Schwarzschild spacetime at $r = R_b$.

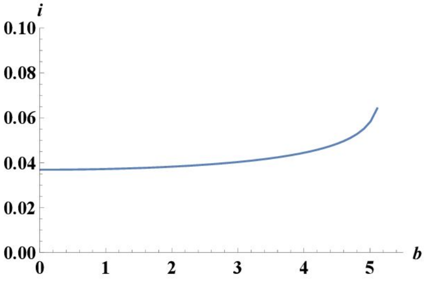
$$ds^2 = -(1 - M_o)\left(\frac{r}{R_b}\right)^{\left(\frac{M_o}{1-M_o}\right)} dt^2 + \frac{dr^2}{(1 - M_o)} + r^2 d\Omega^2$$

Where, M_o and R_b are positive constant, $M = ADM\ Mass = 0.5\ M_0 R_b$.

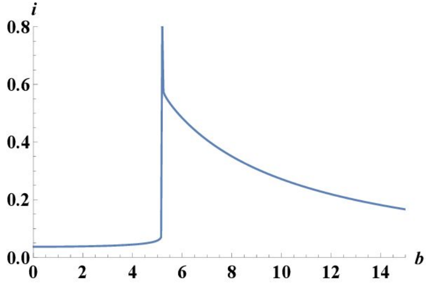
P. S. Joshi *et al*, Equilibrium configurations from gravitational collapse, *Class. Quantum Grav.* 28 235018 (2011).



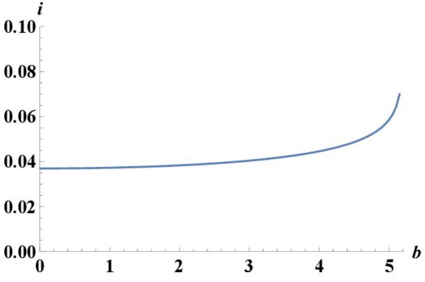
(a)



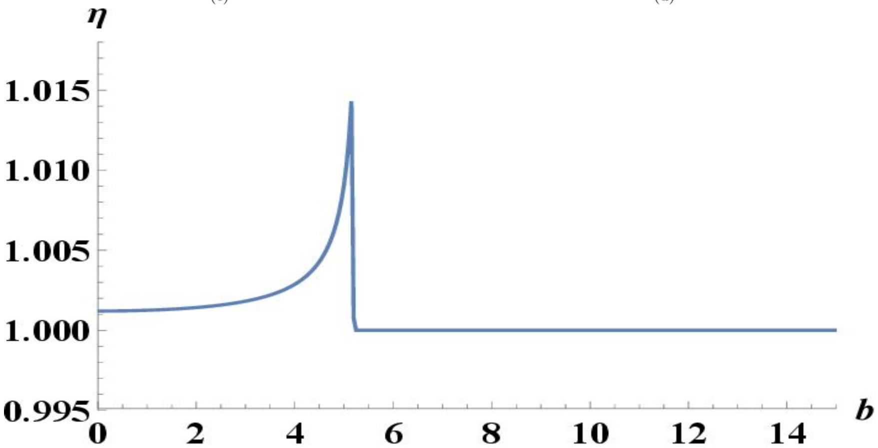
(b)



(c)



(d)



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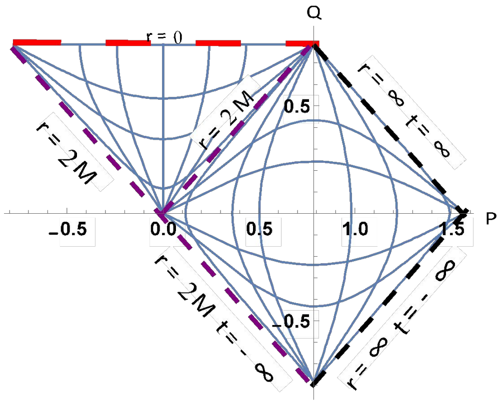
Where, M_o *and* R_b *are positive constant*, $M = ADM\ Mass = 0.5\ M_0 R_b$.

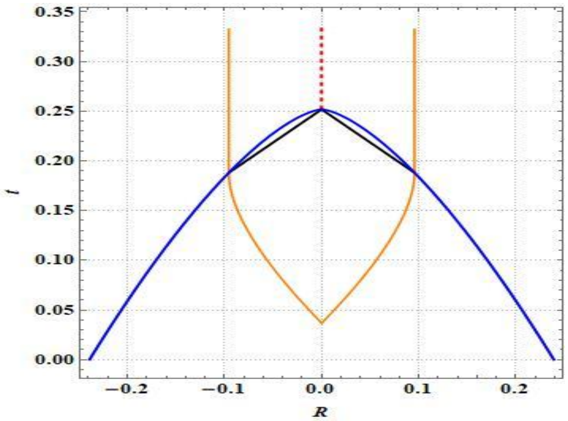
For comparative study of intensity distribution between Schwarzschild and JMN1 spacetimes, we introduce new parameter η which can be written as

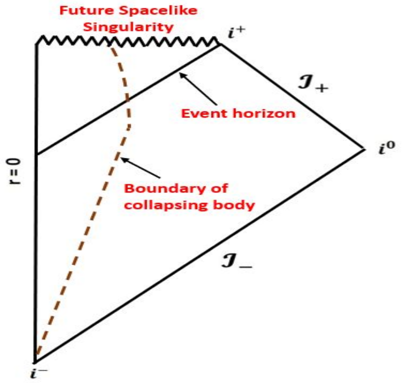
$$\eta = \frac{I_{JMN}}{I_{SCH}}$$

where I_{JMN} and I_{SCH} are the observed intensities of light in JMN1 and Schwarzschild spacetimes at a particular value of impact parameter b.

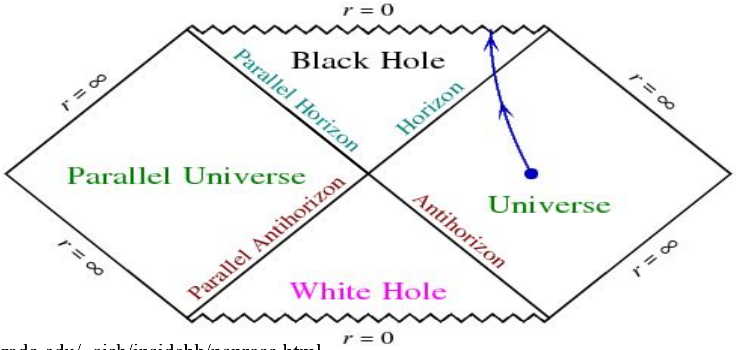
Comparing Shadows of Black hole and Naked Singularity, arXiv:2106.13175v1.







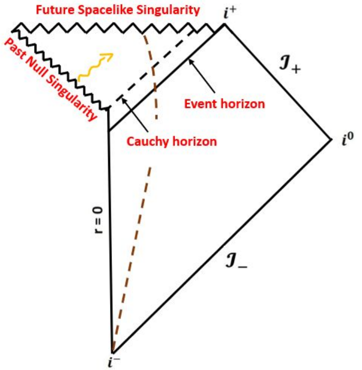
Future spacelike singularity: The spacetime is timelike and null geodesically future incomplete and globally hyperbolic; e.g. Schwarzschild black hole singularity. Schwarzschild singularity formed due to spatially homogeneous gravitationally collapsing dust glued to exterior Schwarzschild spacetime.

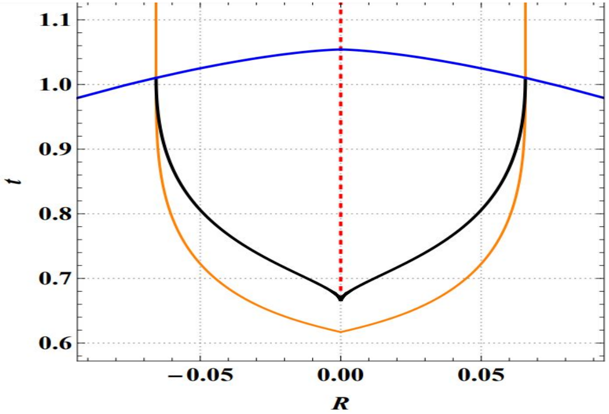


https://jila.colorado.edu/~ajsh/insidebh/penrose.html

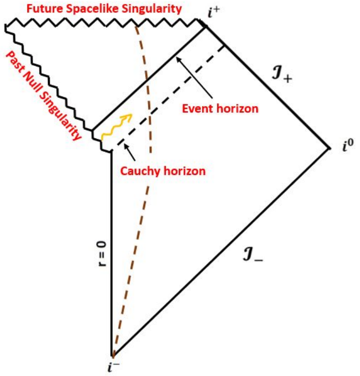
Past spacelike singularity: The spacetime is timelike as well as null geodesically past incomplete and globally hyperbolic; e.g. Schwarzschild white hole singularity.

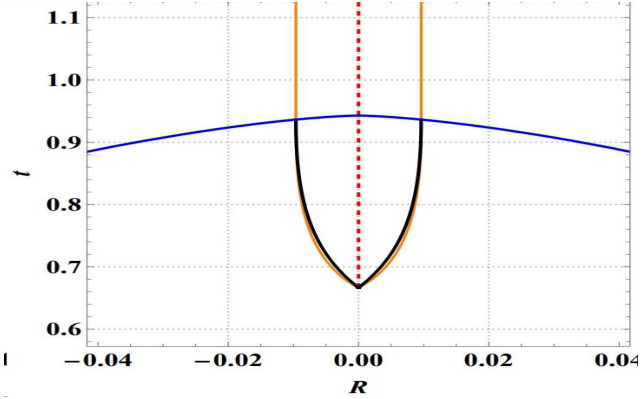
Locally naked singularity formed due to spatially inhomogeneous gravitationally collapsing dust glued to exterior Schwarzschild spacetime



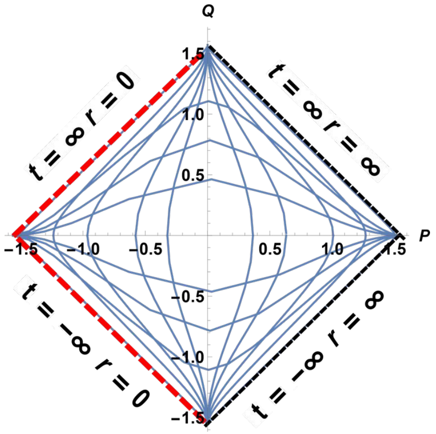


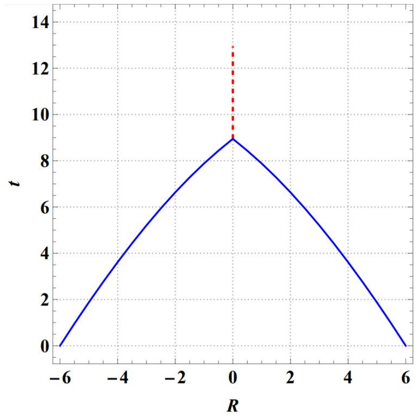
Globally naked singularity formed due to spatially inhomogeneous gravitationally collapsing dust glued to exterior Schwarzschild spacetime

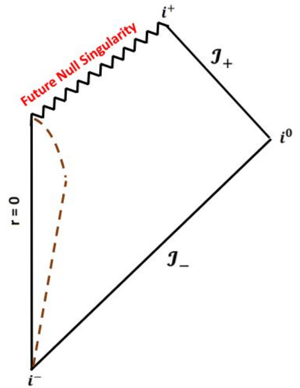




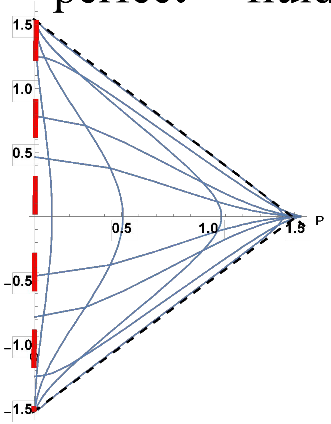
Past null singularity: The spacetime is timelike as well as null geodesically past incomplete and not globally hyperbolic; e.g. locally/globally visible singularities in LTB spacetime.







Future null singularity: The spacetime is timelike as well as null geodesically future incomplete and globally hyperbolic. Future null singularity formed due to spatially homogeneous gravitationally collapsing perfect fluid glued to exterior asymptotically-flat non-vacuum spacetime



Timelike singularity: the spacetime is future and past timelike and null geodesically incomplete. Additionally, the spacetime is not globally hyperbolic.

NAKED SINGULARITY OBTAINED FROM GRAVITATIONAL COLLAPSE

- Spherically symmetric perfect fluid collapsing cloud in comoving coordinates (t, r, θ, ϕ):

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\omega^2.$$

- R: Physical radius of the collapsing cloud.
  Scaling function: $v(t,r) = R(t,r)/r$. $v(0,r) = 1$.

  Misner-Sharp mass function (F): Mass inside a shell of radial coordinate r and time t:

  $\boxed{F = R(1 - G + H)}$; $\quad G(t,r) = e^{-2\psi}R'^2, \quad$ and $\quad H(t,r) = e^{-2\nu}\dot{R}^2.$
- **Time curve** $t(r, v)$: Solution of the boxed equation.
- **Singularity curve**: $t_s(r) = \lim_{v\to 0} t(r, v)$. Also $v(t = t_s(r), r) = 0$

- **Strong singularity (Tipler [2])**: Any object hitting the singularity is crushed to zero volume.

> For a four-dimensional spacetime manifold $(\mathcal{M}, g)$, consider a causal geodesic $\gamma : [t_0, 0) \to \mathcal{M}$. Let $\eta_{(i)} : [t_0, 0) \ni \lambda \mapsto \eta_{(i)}(\lambda) \in T_{\gamma(\lambda)}\mathcal{M}$ be the Jacobi fields. The volume element defined by the wedge product of the independent Jacobi fields along γ, should approach to zero as $\lambda \to 0$.

A sufficient condition for strong singularity (Clarke and Krolak [3]):

- Consider an unhindered gravitational collapse of a matter cloud ending up in a spacetime "singularity".
- Tangent to the ORNG (K): $K^i = \frac{dx^i}{d\lambda}$ (components in comoving spherical coordinate $x^i = (t, r, \theta, \phi)$ basis).
- At least along one null geodesic with the affine parameter λ, with $\lambda = 0$ at the singularity, the following inequality should be satisfied:

$$\boxed{\lim_{\lambda \to 0} \lambda^2 R_{ij} K^i K^j > 0.}$$

[2] F. J. Tipler, Physics Letters A, **64**, 1 (1977).

[3] C. J. S. Clarke and A. Krolak, J. Geom. Phys. **2**, 127 (1985).

STRONG NAKED SINGULARITY OBTAINED FROM THE GRAVITATIONAL COLLAPSE OF A SPHERICALLY SYMMETRIC SPATIALLY INHOMOGENEOUS PERFECT FLUID

- A locally naked singularity, formed due to the gravitational collapse of a spherically symmetric perfect fluid, is strong if [4]

$$\alpha \in \left\{ \frac{2n+1}{3};\ \ n \geq 4;\ \ n \in \mathbb{N} \right\},\quad \alpha = 3, 11/3, 13/3, ...$$

[4] K. Mosani, D. Dey, and P. S. Joshi, Phys. Rev. D **101**, 044052 (2020).

- If the singularity is at least locally naked, then the **Outgoing Radial Null Geodesic (ORNG)** equation:

$$\frac{dt}{dr} = e^{\psi-\nu}.$$

At $(R, r) \to (0, 0)$ the ORNG is governed by $R = X_0 r^{\alpha}, \qquad X_0 > 0$.

Statement: Necessary and sufficient condition for the singularity formed due to sp. sy. perfect fluid collapse to be at least locally naked is the existence of positive real root (i.e. $X_0 \in \mathbb{R}^+$) of $V(X)$, where

$$V(X) = X - \frac{1}{\alpha}\left(X + \sqrt{\frac{F_0(0)}{X}}\left(\chi_1(0) + 2r\chi_2(0) + 3r^2\chi_3(0)\right) r^{\frac{5-3\alpha}{2}}\right)\left(1 - \sqrt{\frac{F_0(0)}{X}} r^{\frac{3-\alpha}{2}}\right),$$ [1]

where

$$\alpha \in \left\{\frac{2n}{3} + 1; \quad n \in N\right\}, \qquad \chi_i(v) = \frac{1}{i!}\frac{d^i t(r,v)}{dr^i}\bigg|_{r=0}, \qquad F(r,v) = \sum_{i=0}^{\infty} F_i(v) r^{i+3}.$$

[1] P. S. Joshi and I. H. Dwivedi, Phys. Rev. D **47**, 5357 (1993).

INTRODUCTION

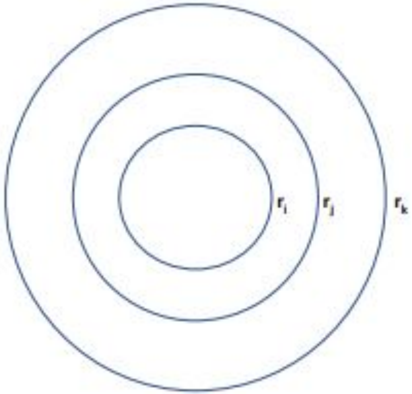


FIGURE: Contraction of concentric shells, each identified by comoving radial coordinate r_i, $i \in \mathbb{R}^+$. Singularity curve ($t_s(r)$): comoving time at which the 'r' shell collapses to singularity (zero physical radius). **Spatially Homogeneous collapse**: $t_s(r) = \text{const}\ \forall r$. **Spatially inhomogeneous collapse**: $t_s(r_i) < t_s(r_j)$ for $i < j$.

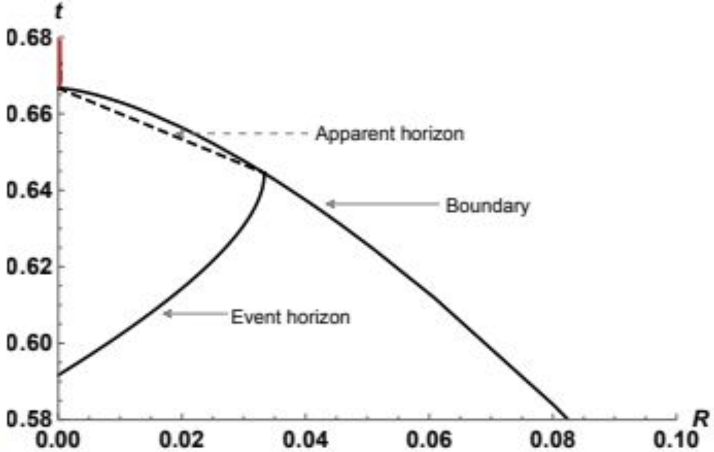


FIGURE: Spatially homogeneous perfect fluid collapse with zero pressure (Oppenheimer-Snyder-Datt collapse-1938/1939).

Conclusions

- Local nakedness is linked to inhomogeneity in the density of the collapsing cloud.
- The problem of finding whether a singularity (formed due to gravitational collapse of sp. sy. perfect fluid) is naked or not can be reduced to the problem of finding if positive real roots exist for a certain function $V(X)$.
- One can obtain a naked singularity that is strong in the sense of Tipler.
- Existence or otherwise of a naked singularity is coordinate independent since its existence corresponds to non-existence of Cauchy surfaces, and vice versa (Existence or otherwise of Cauchy surfaces does not depend on coordinate choice).

[4] K. Mosani, D. Dey, and P. S. Joshi, Phys. Rev. D **101**, 044052 (2020).