Quantum gravity in the sky: cosmological entanglement

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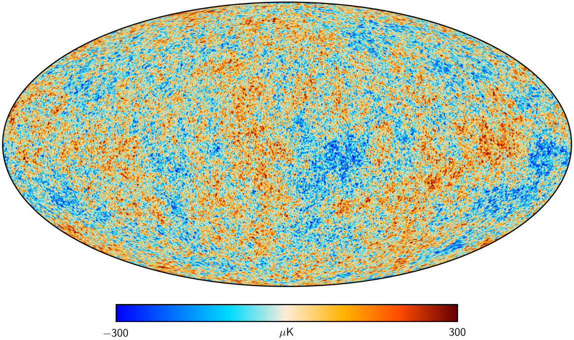


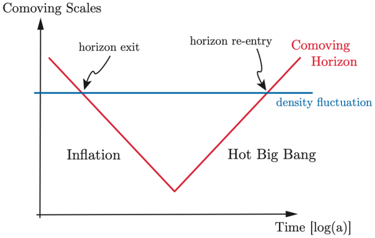
$$\frac{\Delta T}{T}(\mathbf{n}) = \left[\frac{1}{4}\delta_r + \psi + V_j^{(b)} n^j\right](\eta_{dec}, x_{dec}) + 2\int_i^f \dot{\psi}(\eta, x(\eta))\mathrm{d}\eta$$

The CMB temperature power spectrum depends on the primordial perturbation power spectrum $P_\psi(k)\delta^3(\mathbf{k}-\mathbf{k}') = \langle\psi(\mathbf{k})\bar{\psi}(\mathbf{k}')\rangle$. *Primordial perturbations are adiabatic, almost scale-invariant, gaussian, constant at superhorizon scales.*

In the FLRW universe, a primordial fluid or scalar field induces a gauge-invariant perturbation field v_k,

$$v_k'' + \left(c_s^2 k^2 - \mathcal{V}\right) v_k = 0,$$

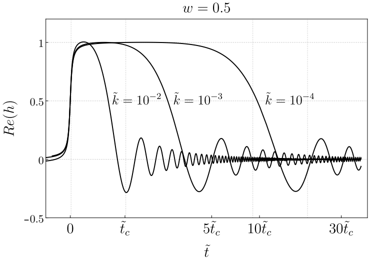
where $\mathcal{V} \sim \mathcal{H}^2$ (background expansion), $\frac{v_k}{a} \propto$ curvature perturbation. Two regimes $c_s k \ll \mathcal{H}$ or $c_s k \gg \mathcal{H}$. The amplification scenario for $\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{H}^2 > 0 \;\Rightarrow\;$ accelerated expansion (inflation) or contraction (big bounce):



Or, big bounce + inflation.

Amplitude amplification:

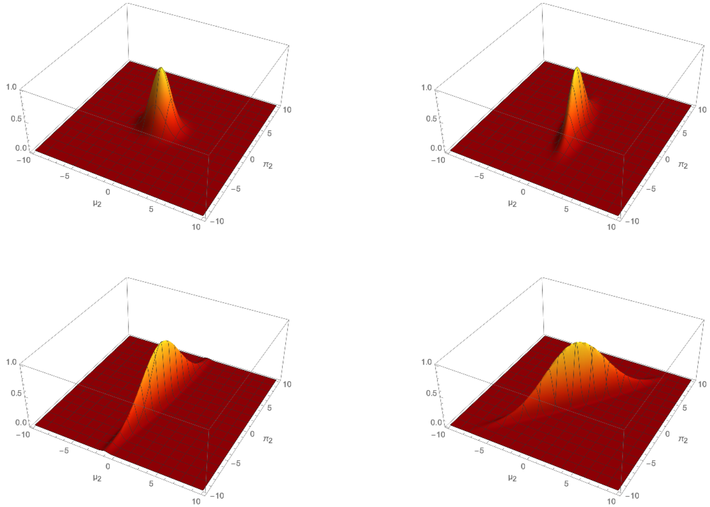
Asymptotic vacuum state $\eta_0 \to -\infty$: $v_k(\eta_0) = 1/\sqrt{k}$, $\acute{v}_k(\eta_0) = i\sqrt{k}$.



In the Schrödinger picture:

$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{\acute{v}_{\boldsymbol{k}}}{v_{\boldsymbol{k}}}, \ \langle v_{\boldsymbol{k}}|\Psi\rangle = N_{\boldsymbol{k}}(\eta)e^{-\Omega_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}}^2}, \ N_{\boldsymbol{k}} = \left(\frac{2\Re(\Omega_{\boldsymbol{k}})}{\pi}\right)^{1/4}.$$

Squeezed vacuum:



The physical Hamiltonian:

$$H_T = H^{(0)} + \sum_{\boldsymbol{k}} H^{(2)}_{\boldsymbol{k}}$$

$$H^{(0)} = p^2, \quad H^{(2)}_{\boldsymbol{k}} = \frac{1}{2}\pi^2_{\boldsymbol{k}} + \frac{1}{2}\left(wk^2 - \mathcal{V}\right) v^2_{\boldsymbol{k}}$$

where $\mathcal{V} = \alpha_w p^2/q^2$, (q, p) - bg variables and $(v_{\boldsymbol{k}}, \pi_{\boldsymbol{k}})$ - pert variables.

Born-Oppenheimer quantization $H_T \mapsto \hat{H}^{(0)} + \sum_{\boldsymbol{k}} \hat{H}^{(2)}_{\boldsymbol{k}}$:

$$|\Psi\rangle_T = |\psi\rangle_{bg} \cdot |\psi\rangle_{pert} \in \mathcal{H}_{bg} \otimes \mathcal{H}_{pert}$$

with

$$i\partial_\eta |\psi\rangle_{bg} = \hat{H}^{(0)} |\psi\rangle_{bg}$$

and

$$\hat{H}^{(2)}_{\boldsymbol{k}} = \frac{1}{2}\hat{\pi}^2_{\boldsymbol{k}} + \frac{1}{2}\left(wk^2 - \langle\psi|\hat{\mathcal{V}}|\psi\rangle_{bg}\right)\hat{v}^2_{\boldsymbol{k}},$$

$$i\partial_\eta |\psi\rangle_{pert} = \hat{H}^{(2)}_{\boldsymbol{k}} |\psi\rangle_{pert}$$

In the Born-Oppenheimer quantization one obtains a trajectory universe ("effective" or "dressed" metric),

$$\mathcal{V}(\eta) \mapsto \langle \hat{\mathcal{V}} \rangle(\eta).$$

The M-S variable $v_{\boldsymbol{k}}$ in the flat-slicing gauge yields a quantum fluctuation of a matter mode $v_{\boldsymbol{k}} \sim \delta\phi_{\boldsymbol{k}}$ in a fixed background geometry.

No genuine quantum gravity effect is present! The trajectory universe could, in principle, be explained by a modified classical theory of gravity and a quantum scalar field.

General quantization:

$$i\partial_\eta|\Psi\rangle = \left(\widehat{H}^{(0)} + \widehat{H}^{(2)}\right)|\Psi\rangle$$

An entangled state emerges:

$$|\Psi\rangle_{BO} = |\psi\rangle_{bg}|\psi\rangle_{pert} \quad \rightarrow \quad |\Psi\rangle = \sum_n |\psi_n\rangle_{bg}|\psi_n\rangle_{pert}$$

as a genuine quantum gravity effect.

Solving strategy:

1. Fix a basis of bg vectors $\{|\psi_n\rangle\}_{bg}$ such that

$$i\partial_\eta|\psi_n\rangle_{bg} = \widehat{H}^{(0)}|\psi_n\rangle_{bg}$$

and determine Born-Oppenheimer solutions

$$i\partial_\eta|\psi_n\rangle_{pert} = \langle\psi_n|\widehat{H}^{(2)}|\psi_n\rangle_{bg}|\psi_n\rangle_{pert}$$

2. Expand the solution in terms of B-O universes:

$$|\Psi\rangle = \sum_{m,n} \alpha_{nm} |\psi_n\rangle_{bg} |\psi_m\rangle_{pert}$$

and solve numerically

$$i\alpha'_{nm} = M^{-1}_{nl} S^{-1}_{mk} H^{(2)}_{lmkp} \alpha_{mp} - S^{-1}_{mk} H^{(2)}_{ppkp} \alpha_{np}$$

where $H^{(2)}_{lmkp} = \langle\psi_k|\langle\psi_l|\widehat{H}^{(2)}|\psi_m\rangle_{bg}|\psi_p\rangle_{pert}$, $M_{nm} = \langle\psi_n|\psi_m\rangle_{bg}$ and $S_{kp} = \langle\psi_k|\psi_p\rangle_{pert}$.

To determine $H^{(2)}_{lmkp}$ we shall need to solve analytically the background wavefunction and calculate $\langle\psi_l|\hat{\mathcal{V}}|\psi_m\rangle_{bg}$ with $\hat{\mathcal{V}} \propto \widehat{D}\widehat{Q}^{-4}\widehat{D}$.

Example: BIVERSE

The coherent states

$$\langle x \,|\, q, p \rangle_{bg} = \sqrt{\frac{2}{\Gamma(\nu+1)}} \left(\frac{\xi_\nu - iqp}{\xi_\nu + iqp} \right)^{\frac{\nu+1}{2}} \xi_\nu^{\frac{\nu+1}{2}} \; \frac{x^{\nu+1/2}}{q^{\nu+1}} \exp \left[-\frac{1}{2} (\xi_\nu - i\, qp) \frac{x^2}{q^2} \right]$$

for $H_{sem} = \langle q, p | \widehat{H}^{(0)} | q, p \rangle_{bg} = p^2 + \frac{K'}{q^2}$ and

$$q' = \frac{\partial H_{sem}}{\partial q}, \quad p' = -\frac{\partial H_{sem}}{\partial p}$$

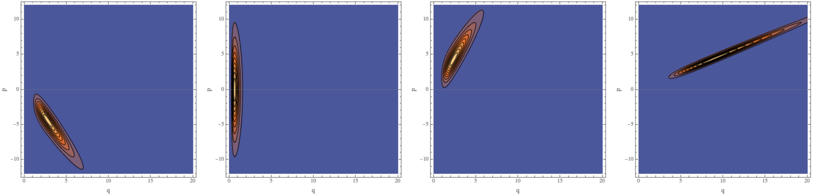
solve **exactly**

$$i\partial_\eta |q, p\rangle_{bg} = \left(\hat{P}^2 + \frac{K}{\hat{Q}^2} \right) |q, p\rangle_{bg}$$

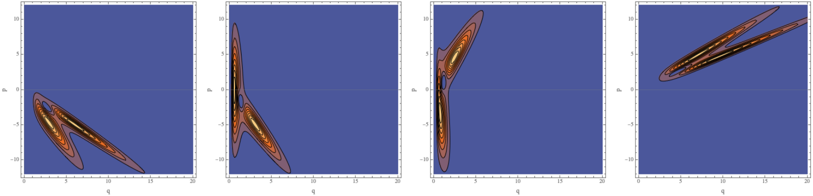
We pick two random trajectories $\eta \mapsto (q, p)$ to define $|\psi_0\rangle_{bg} = |q_0, p_0\rangle_{bg}$ and $|\psi_1\rangle_{bg} = |q_1, p_1\rangle_{bg}$, and solve for two B-O universes:

$$|\Psi_0\rangle = |\psi_0\rangle_{bg} \cdot |\psi_0\rangle_{pert} \quad \text{and} \quad |\Psi_1\rangle = |\psi_1\rangle_{bg} \cdot |\psi_1\rangle_{pert}$$

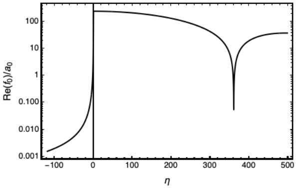
Single universe:

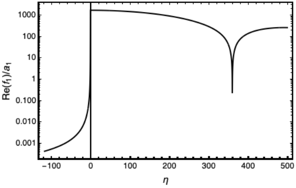


Biverse:

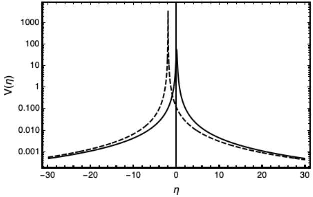


We pick these two solutions and determine respective Born-Oppenheimer solutions:





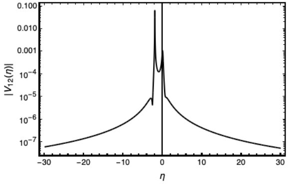
The amplification due to $\langle\psi_0|\hat{\mathcal{V}}|\psi_0\rangle_{bg}$ and $\langle\psi_1|\hat{\mathcal{V}}|\psi_1\rangle_{bg}$:



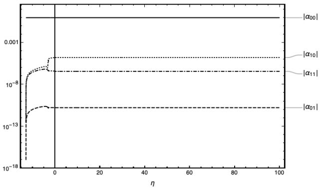
We go beyond Born-Oppenheimer solution:

$$|\Psi\rangle = \alpha_{00}|\psi_0\rangle_{bg}\cdot|\psi_0\rangle_{pert}+\alpha_{11}|\psi_1\rangle_{bg}\cdot|\psi_1\rangle_{pert}+\alpha_{01}|\psi_0\rangle_{bg}\cdot|\psi_1\rangle_{pert}+\alpha_{10}|\psi_1\rangle_{bg}\cdot|\psi_0\rangle_{pert}$$

The entanglement due to $\langle\psi_0|\hat{\mathcal{V}}|\psi_1\rangle_{bg}$:



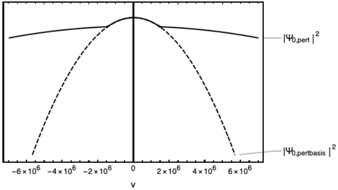
Starting from $|\Psi_0\rangle = |\psi_0\rangle_{bg}\cdot|\psi_0\rangle_{pert}$ we obtain the result:



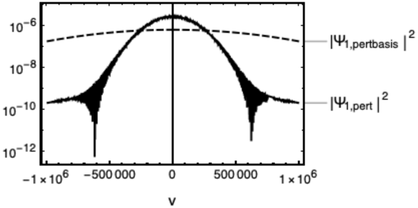
We rewrite $|\Psi\rangle$:

$$|\Psi\rangle = |\psi_0\rangle_{bg}\left[\alpha_{00}|\psi_0\rangle + \alpha_{01}|\psi_1\rangle\right]_{pert} + |\psi_1\rangle_{bg}\left[\alpha_{11}|\psi_1\rangle + \alpha_{10}|\psi_0\rangle\right]_{pert}$$

Non-gaussian probability density for primordial amplitude in $|\psi_0\rangle_{bg}$:



Non-gaussian probability density for primordial amplitude in $|\psi_1\rangle_{bg}$:



Conclusions:

Models of a trajectory universe furnished with matter fluctuations do not yield distinct observational quantum gravity signatures.

Born-Oppenheimer approximation has to be dropped for genuine quantum gravity effects to appear.

The entanglement between the background geometry and inhomogeneity is a truly quantum gravity effect and it generically alters the primordial perturbations.

The presented study only proves the existence of new effects and a more detailed study is needed:

One may expect that inclusion of more than one perturbation modes may result in entanglement between those modes.

More quantum gravity signatures such as primordial phase shift can appear.

The assumption of the B-O universe as the initial condition can be challenged.

The inflation could start in an entangled state and remain so during the exponential expansion.