Quantum Schwarzschild spacetime

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Pasteura 5, Warsaw

Based on:

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“Ascribing quantum system to Schwarzschild spacetime with naked singularity”

arXiv:2110.08503 v2 [gr-qc]

OUTLINE

Introduction

- The Schwarzschild spacetime is one of the simplest vacuum solutions to Einstein's equations (Karl Schwarzschild, Johannes Droste, 1916)
- Quantization of gravitational system based only on explicit form of spacetime metric
- Novelty: quantization of spatial and temporal coordinates
- Rationale: distinction between space and time violates 4d diffeomorphism invariance of GR
- Aim:
  - testing simple but powerful quantization method
  - presenting the idea of time quantization
  - showing that our quantization scheme may resolve the singularity problem of Schwarzschild's spacetime

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The Schwarzschild metric

The Schwarzschild metric in the so-called Schwarzschild coordinates $(t, r, \theta, \phi) \in \mathbb{R} \times (0, \infty) \times S^2$ reads

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \, , \tag{1}$$

where

t, time coordinate;

r, radial coordinate measured as the circumference (divided by 2π) of sphere centered around isolated object;

M, mass parameter of the isolated object;

θ and ϕ are angle coordinates of the sphere S^2;

$M \to 0$ leads to the Minkowski metric (in spherical coordinates);

$r \to \infty$ leads to the Minkowski metric (Schwarzschild spacetime is asymptotically flat)

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Singularities

- at $r = 2M$ there isn't gravitational, but coordinate singularity called the event horizon
- curvature invariants for Schwarzschild's metric
  - $R = 0 = R_{\mu\nu} R^{\mu\nu}$, Ricci
  - K = $R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \neq 0$, Kretschmann

$$K = \frac{48M^2}{r^6}, \tag{2}$$

so that as $r \to 0$ the Kretschmann scalar blows up!

- for $M > 0$ we have the horizon so that the model with covered singularity, i.e. BH
- for $M < 0$ there is no event horizon so we have the model of spacetime with naked singularity

Remark: If isolated objects with naked singularities do occur in real world, observational data may bring highly valuable information to be used in the construction of quantum gravity (horizon may screen some essential quantum gravity details).

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Configuration space

The extended configuration space T of the system

$$T = \{(t, r) \mid (t, r) \in \mathbb{R} \times \mathbb{R}_+\}, \quad \mathbb{R}_+ = (0, +\infty)\,, \tag{3}$$

where t and r are time and radial coordinates, respectively, which occur in the line element (1).
The other space variables θ and ϕ of (1), do not enter the definition of T as the main observable to be quantized, the Kretschmann scalar, does not depend on these variables.

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Quantization

Roughly speaking, by quantization of system, represented by observables defined on our configuration space, I mean:

- ascribing to observables self-adjoint operators acting in Hilbert space
- calculating expectation values of quantum observables
- calculating variances of quantum observables

Considered observables:

- elementary observables: time and radial coordinates
- for $M < 0$, the coordinate r is spacelike and the coordinate t is timelike
- we quantize both temporal and spatial coordinates on the same footing
- composite observable: Kretschmann scalar

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Ascribing group structure

The configuration space T (see Eq. (3)) is a half-plane.
It can be identified with the affine group $\mathrm{Aff}(\mathbb{R}) =: G$

$$T \ni (t, r) \longrightarrow h(t, r) \in G\,, \tag{4}$$

by defining the group structure on T.
This identification is not unique:

- depends on the group parametrization, i.e. depends on the way we define group multiplication. For instance

$$h(t_1, r_1) \cdot h(t_2, r_2) := h(t_1 + t_2 r_1, r_1 r_2) \tag{5}$$

$$h(t_1, r_1) \cdot h(t_2, r_2) := h(t_1 + t_2/r_1, r_1 r_2) \tag{6}$$

  define two different parametrizations
- different parameterizations lead to unitarily inequivalent quantum theories

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Defining Hilbert space

The affine group Aff($\mathbb{R}$) has UIR realized in the Hilbert space $L^2(\mathbb{R}_+, d\nu(x))$, where $d\nu(x) = dx/x$, defined by

$$U(t,r)\psi(x) = e^{itx}\psi(rx)\,,$$

in the parametrization (5).

This enables defining the continuous family of affine coherent states $|t,r\rangle \in L^2(\mathbb{R}_+, d\nu(x))$ as follows

$$|t,r\rangle = U(t,r)|\phi\rangle\,,$$

where $|\phi\rangle \in L^2(\mathbb{R}_+, d\nu(x))$, is the so-called fiducial vector, which is a free "parameter" of this quantization scheme.

The space of coherent states is highly entangled as we have

$$\langle t,r|t',r'\rangle \neq 0 \quad \text{if} \quad t \neq t' \;\text{ or }\; r \neq r'\,, \tag{7}$$

$$\langle t,r|t,r\rangle = 1 \quad \text{if} \quad \langle\phi|\phi\rangle = 1 \tag{8}$$

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Quantum operators

The irreducibility of the representation leads (due to Schur' lemma) to the resolution of the unity in $L^2(\mathbb{R}_+, d\nu(x)) =: \mathcal{H}_x$ as follows

$$\int_T d\mu(t,r)|t,r\rangle\langle t,r| = A_\phi \, \mathbb{I} \,, \tag{9}$$

where $d\mu(t,r) := dt\, dr/r^2$ is the left invariant measure on T, and where $A_\phi := \int_0^\infty |\phi(x)|^2 \frac{dx}{x^2} < \infty$ is a constant.

Using (9), enables quantization of any observable $f : T \rightarrow \mathbb{R}$ as follows

$$f \longrightarrow \hat{f} = \frac{1}{A_\phi} \int_T d\mu(t,r)|t,r\rangle f(t,r) \langle t,r| \,. \tag{10}$$

The operator $\hat{f} : \mathcal{H}_x \rightarrow \mathcal{H}_x$ is symmetric by construction.
No ordering ambiguity occurs (disaster of canonical quantization).

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Variance of quantum observable

Variance is a stochastic deviation from expectation value of quantum observable; it determines the value of smearing of quantum observable[1].

The variance is the average of the squared differences from the mean. In the quantum state labelled by ψ, the variance is defined to be

$$var(\hat{A};\psi) := \langle(\hat{A}-\langle\hat{A};\psi\rangle)^2;\psi\rangle = \langle\hat{A}^2;\psi\rangle - \langle\hat{A};\psi\rangle^2\,, \tag{11}$$

where $\langle\hat{B};\psi\rangle := \langle\psi|\hat{B}|\psi\rangle$.

If $\hat{A}$ is a self-adjoint operator, we have the important statement:

$$\Big(var(\hat{A};\psi) = 0\Big) \Longleftrightarrow \Big(\hat{A}\psi = \lambda\psi, \quad \lambda\in\mathbb{R}\Big)\,, \tag{12}$$

i.e., the variance of the operator $\hat{A}$ equals zero, if and only if, the quantum system is in an eigenstate of the operator $\hat{A}$.

[1] Standard deviation is calculated as the square root of variance.

Variance of quantum observable

Variance is a stochastic deviation from expectation value of quantum observable; it determines the value of smearing of quantum observable[1].

The variance is the average of the squared differences from the mean. In the quantum state labelled by ψ, the variance is defined to be

$$var(\hat{A};\psi) := \langle(\hat{A} - \langle\hat{A};\psi\rangle)^2;\psi\rangle = \langle\hat{A}^2;\psi\rangle - \langle\hat{A};\psi\rangle^2 , \qquad (11)$$

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Eigenproblem for the Kretschmann operator

Using our quantization rules (10), we get the quantum Kretschmann observable in the form

$$\hat{K} = 48M^2 \frac{1}{A_{\Phi_0}} \int_T d\mu(t,r) \langle t,r| \frac{1}{r^6} |t,r\rangle \,. \tag{13}$$

The eigenproblem reads

$$\int_{\mathbb{R}_+} d\nu(y)\, K_{\mathcal{K}}(x,y)\, \psi_k^{(\mathcal{K})}(y) = k\, \psi_k^{(\mathcal{K})}(x) \,, \tag{14}$$

where

$$K_{\mathcal{K}}(x,y) := \langle x|\hat{K}|y\rangle = \ldots = \mathcal{A}\, \delta(x-y) x^7 \,, \tag{15}$$

and where $\mathcal{A} = \frac{48M^2}{A_{\Phi_0}} \left[\int_{\mathbb{R}_+} \frac{dq}{q^8} |\Phi_0(q)|^2\right]$.

Direct calculations lead to the following generalized eigenfunctions

$$\psi_k^{(\mathcal{K})}(x) = \delta\left(x^6 - \frac{k}{\mathcal{A}}\right) \,, \tag{16}$$

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Expectation value and variance for $\hat{K}$ operator

In what follows we present calculations of expectation value of the operator $\hat{K}$ and the corresponding variance.

Choosing our group parametrization (5) gives

$$\langle \hat{t}; g(t,r)\rangle = t, \qquad \langle \hat{r}; g(t,r)\rangle = r\,. \tag{17}$$

Using the second parametrization (6) would not lead to such a nice result.

For the Kretschmann operator we obtain

$$\langle \hat{K}; g(t,r)\rangle = 48M^2 \frac{A}{r^6}\,, \tag{18}$$

where A is a constant. Therefore, the mean value $\langle \hat{K}; g(t,r)\rangle$ has the singularity at $r = 0$, as in the classical case.

For the corresponding variance one gets

$$var(\hat{K}; g(t,r)) = (48M^2)^2\ \frac{B}{r^{12}}\,, \tag{19}$$

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Results of quantization

The operator $\hat{K}$ represents a well behaving smeared observable which is completely undetermined at the classical singularity $r = 0$

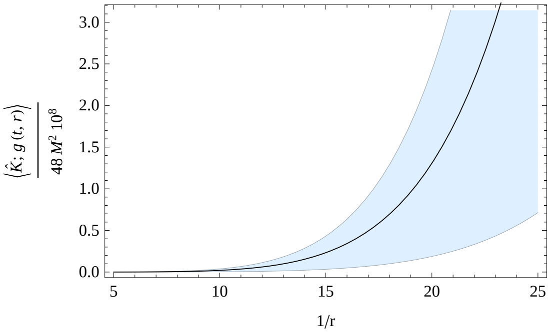

Figure: The $1/r$ dependence of expectation value of Kretschmann operator $\langle \hat{K}; g(t,r) \rangle$ defined by (18). The blue area defines the points for which distance from the expectation value is smaller than $\sqrt{var\langle \hat{K}; g(t,r) \rangle}$.

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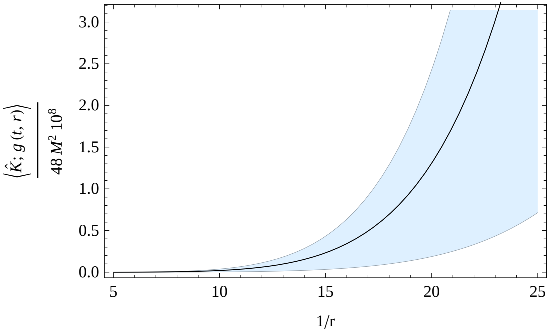

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Generalization

The above results have been obtained within the family of coherent states $|g(t,r)\rangle \in L^2(\mathbb{R}_+, d\nu(x))$ satisfying reasonable conditions (17).

We have verified these results by making use of the basis in the Hilbert space $L^2(\mathbb{R}_+, d\nu(x))$ defined as follows

$$\Psi_n(x) = Nx^n \exp\left[i\tau_0 x - \frac{\gamma^2 x^2}{2}\right], \tag{20}$$

where $n = 1, 2, \ldots$, and where $N^2 = 2\gamma^n/(n-1)!$

The expectation values of $\hat{t}$ and $\hat{r}$ in the states Ψ_n read

$$\langle \hat{t}; \Psi_n \rangle = \tau_0, \qquad \langle \hat{r}; \Psi_n \rangle = \frac{1}{A_{\Phi_0}} \frac{\Gamma\left(n - \frac{1}{2}\right)}{(n-1)!}\, \gamma\,. \tag{21}$$

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Generalization (cont)

The expectation value of $\hat{K}$ is the following

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Conclusions

- Both temporal and spatial variables have been treated on the same footing at quantum level.
- We have no problem with the choice of time at quantum level quite common to all other quantization schemes!
- Quantization of time variable has enabled avoiding gravitational singularity.
- The state corresponding to gravitational singularity cannot be the eigenstate of the the Kretschmann operator. The probability of finding considered black hole in the singular state equals zero.
- Our approach relies on using only metric tensor so that it can be applied to other isolated systems with known metrics.

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Prospects: quantization of isolated objects

- spherically symmetric isolated objects with covered or naked singularities
  - Schwarzschild's BH, in progress
  - shell model: Minkowski+shell+Sch, in progress
  - FRW+Sch (Oppenheimer-Snyder model), done
  - Lemaître-Tolman-Bondi (naked, covered), in progress
- anisotropic BHs: Bianchi type, planning
- rotating BHs: Kerr type, dreaming!

Remark: QG may be used to get insight into the dynamics of observed compact objects. And vice versa, observational data coming from strong gravitational fields may help to fix parameters of constructed QG models.

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