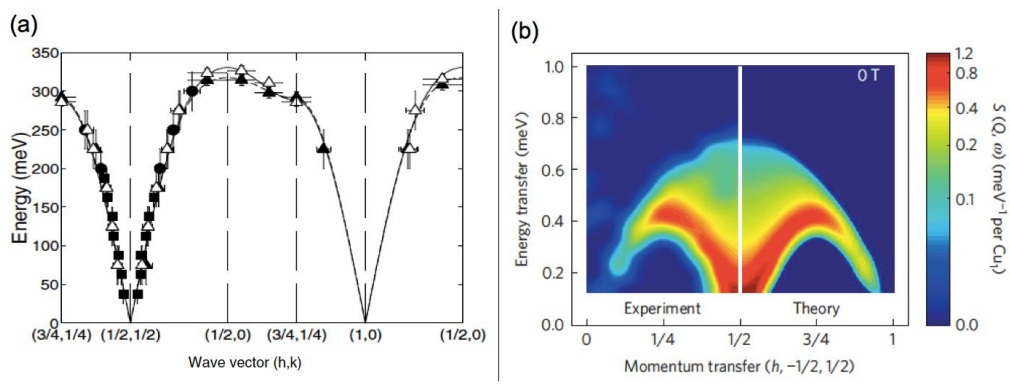

Figure 1: Collective spin excitations in the two- and one-dimensional antiferromagnets: (a) A well-defined spin wave dispersion can be nicely seen in the inelastic neutron scattering experiment on the two-dimensional antiferromagnet (La_2CuO_4; symbols) *and* in the linear spin wave approximation calculations for the Heisenberg model (solid line); figure adopted from R. Coldea *et al.*, Phys. Rev. Lett. **86**, 5377 (2001). (b) A two- and higher-order spinon continuum is seen in the the inelastic neutron scattering experiment on the one-dimensional antiferromagnet ($CuSO_4{\cdot}5D_2O$, left panel) which is confirmed by the theoretical calculations for the one-dimensional model using the Bethe-Ansatz technique (right panel); figure adopted from M. Mourigal *et al.*, Nature Physics **9**, 435 (2013). These two examples nicely show the differences in magnetism in one and two dimensions – in particular the more quantum nature of the one-dimensional magnetism.

The course is a brief introduction to the basic notions of the quantum theory of magnetism. During the course a particular emphasis will be devoted to: (i) the differences between quantum and classical effects, and (ii) the connections between the real materials and the here discussed effective spin models and its solutions. It is desired that after the exercise classes, the student should be able to understand the quantum nature of magnetism present in a number of solids and calculate the basic magnetic properties of quantum matter – with a particular emphasis on the 'very quantum' behavior of magnetism of the quasi-low-dimensional crystals, cf. Fig. 1.

The plan of the *exercise classes* is as follows:

Classes 1-2: Magnetic properties of matter

- Spin and orbital angular momentum and its connection to the magnetic

moment – revisited.

- Hund's exchange and Hund's rule in atoms – calculating magnetic ground states of various atoms.
- Stoner criterion – calculating (ferro)magnetic ground state of metals. Magnetic properties of matter versus spin and orbital quantum numbers of electrons.
- Overview of problems to be solved in the exercise classes.

Classses 3-4: Magnetism with localized magnetic moment – modelling La_2CuO_4

- Obtaining the tigh binding Hamiltonian from the LDA–derived bands for La_2CuO_4.
- Schrieffer-Wolff transformation: from Hubbard model to Heisenberg model.

Classes 3-8: Quantum magnetism – ground state and excitations of 2D Heisenberg model

(a) Classical (Ising) limit of the 2D Heisenberg model:

- Finding the ground state of the model.
- Calculating excited states using Green's function formalism.

(b) Ground state and excitations:

- Formal justificiation of the linear spin wave theory.
- Linear spin wave expansion and the Holstein-Primakoff transformation.
- Introduction to Bogoliubov and Fourier transformation.
- Calculating the ground and excited states of the model.

(c) Spontaneous symmetry breaking:

- Introducing the Lieb-Mattis (LM) toy-model.

- Ground state of the LM model.
- Different orders in taking the two limits of the LM model and the concept of the spontaneous symmetry breaking.
- Calculating the reduction of order parameter of the Hesenberg model by quantum fluctuations and quantum disorder.

(d) What is quantum and what is classical?

- Ground state of the ferromagnetic Heisenberg model.
- Holstein-Primakoff transformation for the ferromagnetic Heisnberg model and the excited states.
- Calculating the order parameter of the ferromagnetic Heisenberg model. (ground state, excitations, order parameter).

Classes 9-10: Revisiting La_2CuO_4: comparing the above discussed theoretical results for the 2D Heisenberg model with experiments

- Understading in detail one experimental technique: resonant inelastic x-ray scattering (RIXS).
- RIXS cross section and the Kramers-Heisenberg formula.
- Fast collision approximation for RIXS: justification, derivation and application.
- Calculating RIXS cross section for Cu L edge using fast collision approximation.

Classes 11-14: Properties of other spin models – ground state and collective excitations with a short discussion on its origin, application to materials, and experiments:

(a) One-dimensional Heisenberg model

- Calculating the destruciton of the order parameter of the one-dimensional Hesenberg model by quantum fluctuations using linear spin wave theory.

- Mean-field approach to the excitations of the model using the constrained fermion picture.
- Jordan-Wigner transformation and its utlity in solving this model (and other models) in one dimension.
- Overview of the Bethe Ansatz technique.

(b) Frustrated spin models

- Showing frusration of interactions on an example of a triangular Heisenberg model: calculating ground and excited states.
- Ground and excited states of the Majumdar-Ghosh model.
- Ground and excited states of the bilinear-biquadratic models for S=1.

(c) Adding orbital degrees of freedom (Kugel-Khomskii models)

- Derivation of the spin and orbital exchange interactions for an idealized two-orbital Hubbard model with two equivalent orbitals: introducing the multiplet structure of the virtual states and the SU(4) symmetric model in the limit of small Hund's exchange.
- Derivation of the spin and orbital exchange interactions for the Hubbard model with degenerate e_g orbitals.
- Finding the ground state of the spin-orbital models using mean-field decoupling.
- Failure of the 'standard' mean-field decoupling in one dimension and succes of the the mean-field decoupling using the constrained-fermion picture.
- Calculating spin-orbital entanglement.

Class 15: Summary of open problems in the field

- Derivation of the t-J mode using Schrieffer-Wolff transformation.
- Calculating motion of a single hole in the two-dimensional antiferromagnet using the retraceable path approximation and the self-consistent Born approximation: comparison between classical and quantum cases.

Literature:

1. "Interacting Electrons and Quantum Magnetism", Assa Auerbach, Springer-Verlag (New York 1994).

2. "Lecture Notes on Electron Correlation and Magnetism", Patrik Fazekas, World Scientific (Singapore 1999).

3. "Magnetism in Condensed Matter", Stephen Blundell, Oxford University Press (Oxford 2001).

4. "Introduction to Frustrated Magnetism: Materials, Experiments, Theory", Editors: C. Lacroix, P. Mendels, and F. Mila, Springer-Verlag (Heidelberg 2011).