# Analysis and simulation of the "Pi of the Sky" detector response

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#### Introduction

- 2 Shape of the star image
- 3 Laboratory measurements
- Parametrization of the detector's response
- 5 Frame simulation and model applications

The "Pi of the Sky" experiment

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- Independent detection of GRB optical counterparts
- Detection of other optical transients with help of parallax
- Analysis of other phenomena variable on timescales of tens of seconds
- "Pi of the Sky" currently consists of:
  - two-camera prototype (Chile): since 2011 in SPdA, in years 2004-2009 in LCO
  - first detector of the final system operational from October 2010 at INTA, Spain (Bootes-1 site)



#### The "Pi of the Sky" experiment (2)

- Constant high-time resolution sky monitoring
- Target FoV: 1.5 sr
- Single camera FoV:  $20^{\circ} \times 20^{\circ}$



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#### Very large field of view

 $\Rightarrow$ 

#### Very large deformations of image near frame's edges

### Results of the image deformation

For stars with deformed images:

 large uncertainty of star position measurement – astrometry



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How to improve measurements:

- discard deformed stars (applicable to objects with big statistics only)
- parametrize deformations

Two approaches to parametrization considered:

- diffractive approach
  - light propagation through lenses
- effective approach
  - direct image shape parametrization



#### PSF – point spread function

Image of the point object on the screen/frame - point spread function (PSF)

- Image formation (in general) light diffraction theory
- Simplifications for PSF analysis:
  - vector diffraction theory
  - scalar diffraction theory
    - $\rightarrow$  Rayleigh-Sommerfeld's formula



$$PSF(x_0, y_0, z) = \left| \frac{1}{i\lambda} \iint_{-\infty}^{\infty} U(x, y, 0) \frac{ze^{ikr}}{r^2} dx dy \right|^2$$

U(x, y, 0) - wave amplitude on the aperture, x, y - coordinates on the aperture, x<sub>0</sub>, y<sub>0</sub> - coordinates on the screen, z - aperture-screen distance, r - distance between (x, y, 0) and (x<sub>0</sub>, y<sub>0</sub>, z),  $\lambda$  - wavelength

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$$|z| \gg L_1 + L_2$$
  
 $L_1$  – aperture size,  $L_2$  – screen size

$$PSF(x_0, y_0, z) = \left| \frac{e^{ikz}}{i\lambda z} \iint_A U(x, y, 0) e^{\frac{i\pi}{\lambda z} ((x_0 - x)^2 + (y_0 - y)^2)} dx dy \right|^2$$

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$$PSF(x_0, y_0, z) = \left| \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2x}(x_0^2 + y_0^2)} \iint\limits_A U(x, y, 0) e^{\frac{i2\pi}{\lambda x}(x_0 x + y_0 y)} dx dy \right|^2$$

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 $z\sim L_1,L_2$ in "Pi of the Sky"

### PSF – point spread function (2)

The real optical system - optical aberrations of the wavefront

$$PSF_L(x_0, y_0, z) = \left| \frac{1}{i\lambda} \iint_A U(x, y, 0) \frac{z e^{ikr} e^{W(x, y)}}{r^2} dx dy \right|^2$$

where W(x,y) – aberrations function – is the deviation of the wavefront from sphericity.

Seidel parametrization:

defocus



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Seidel parametrization:

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$$W_C(\rho,\phi) = C\rho^3\cos(\theta)$$

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Seidel parametrization:

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- astigmatism



$$W_A(\rho,\phi) = A\rho^2\cos(2\theta)$$

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- spherical aberration (higher order defocus)



$$W_{S}(\rho,\phi) = S\rho^{4}$$

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- 🕨 coma
- astigmatism
- spherical aberration (higher order defocus)
- higher order and different aberrations, superposition

Often parametrized with similar (but orthogonal) Zernike polynomials.



 $W = W_C + W_A + W_S + \dots$ 

#### PSF parametrization method

Standard method of the PSF shape determination:

- reconstruction of a high resolution profile

   superposition of multiple star(s) images
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Drawbacks in the "Pi of the Sky" case:

- we search for a shape, not fit parameters of a known shape
- poor stars superposition difficult centre determination for deformed profiles
- stars colour influence
- image blur mount vibrations, fluctuations, etc.



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#### Solution: laboratory measurements



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#### Laboratory setup

#### Star

as seen from Earth - a point source

#### A point source

image of the source much smaller than the angular resolution given by a pixel size

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Light source,  $\phi=0.4 \text{ mm}$ 

#### A point source

image of the source much smaller than the angular resolution given by a pixel size

▶ light source: LED diode (colour or white)

- covered by a pinhole of 0.4 mm diameter
- powered by a pulse generator
- placed on a movable stand

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### Spatial pixel response function (PRF)

Source signal measured in a specific pixel vs. a spot position in respect to the pixel edge (measurement with reduced lenses opening – PSF close to pointlike).

An ideal case: constant inside the pixel, equal to zero outside the pixel.

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The real case:

- nonuniform inside the pixel pixel sensitivity depends on it's structure (electrodes, light penetration depths)
- sloping near the pixel edges spot's finite size
- non-zero outside the pixel finite PSF size or charge diffusion between pixels

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#### **PSF** reconstruction



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# Diffraction model

Fit of the parameters to the Rayleigh-Sommerfeld's formula with aberrations.

- The fit has been performed with 11 free parameters:
  - aberrations: coma, coma', astigmatism, spher. ab., spher ab.', trefoil
  - ▶ image coordinates: (x₀, y₀, z)
  - background and a scaling coefficient





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Drawbacks:

- very time-consuming computation
- simplification of the real case
- not successful at multi-wavelength fits
- not successful for other distances from the frame centre





### Polynomial model – formulation

Effective model focusing on the PSF shape on the frame, not on the "physics" of the shape's creation.

Parametrization in the image plane (not aperture):

- Simplified formulation
- Much shorter computing time

Model basis – modified Zernike polynomials:

 $\blacktriangleright$  finite aperture diameter  $\rightarrow$  infinite image size



$$Z(r,\phi) \rightarrow Z(u,\phi), \ u = 1 - e^{\frac{r}{\lambda}}$$

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$$PSF_L(r,\phi) = e^{-\frac{\mathbf{0.5}}{\lambda} \cdot Z(u,\phi) \cdot r^{\mathbf{p}}}$$

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- only symmetric terms allowed (symmetric PSF)
- + convolution with the pixel response function

$$PSF(r,\phi) = \iint_{CCD} PRF(r',\phi') \cdot PSF_L(r',\phi')dr'd\phi'$$





### Polynomial model – the real stars case

Profiles of the real stars:

- slightly different than diode's profiles (different focusing, etc.)
- depend on the azimuthal coordinate on the frame (imperfect camera assembly – optical axis ∦ CCD axis)



Model recalculation procedure:

reconstruction with the diode PSF

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- 2 refit of the polynomial coefficients

#### Possible model applications

The obtained polynomial model can be applied, among others, to:

- profile photometry and astrometry
- search for sources on the border of the detectors range
- more precise limits determination
- simulation of the "Pi of the Sky" frame

#### Profile photometry with the polynomial model

Comparison with the ASAS (aperture) photometry:

- similar results and behaviour
- bigger instability (expected)
- slightly better results for dark stars



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The lack of general improvement may be due to:

- different stars influencing fit
- real brightness fluctuations domination (not PSF shape)
- versatility of a circular aperture (highly improbable)



#### Profile astrometry with the polynomial model

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# The polynomial astrometry is a preferred choice for objects of special interest – which allow for a longer computing time

### "The naked-eye burst" precursor search

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The polynomial model has been used to:

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New limit: 12.25<sup>m</sup> compared to previous 11.5<sup>m</sup>. 0.75<sup>m</sup> limit improvement.

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 generates frame with catalogue stars position and brightness



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  - properly reproduces  $\frac{\Delta I}{I}(M)$



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The simulator can be used to:

- analyse photometric uncertainties
- test parallax determination
- test algorithms on crowded fields



etc.

# Summary

- A well working polynomial model of PSF has been created
- > Basic model-based photometry and astrometry has been tested
  - photometry may improve on new hardware
  - astrometry already performs much better
- > Additionally models can be used for:
  - search for weak signals
  - precise limits determination
- A realistic simulator of a frame has been created, useful for testing algorithms and hardware of the "Pi of the Sky"
- > Developed model could be used in future experiments with very large FoV

More details on the covered topics are available in my PhD thesis: http://www.fuw.edu.pl/~lewhoo/phd\_thesis.pdf.