Renormalized Light-front Tamm-Dancoff Approach to QED and Compton Scattering

Edsel A. Ammons
Department of Physics
Center for Nonlinear and Nonequilibrium Aeroscience
Florida A&M University
Tallahassee, Florida 32307

Abstract

We will do Compton scattering within the context of the Light-front Tamm-Dancoff approximation. We have chosen quantum electrodynamics to illustrate further features of Wilson's renormalization group with a gauge theory but without the complications of quantum chromodynamics. Wilson's renormalization group is an approach to the solution of the renormalization problem for Hamiltonian methods in relativistic quantum field theory.

1. Introduction

We will do Compton scattering within the context of Light-front Tamm- Dancoff quantum electrodynamics (LFTD QED). We will assume a Tamm- Dancoff truncation at the very beginning of the analysis. This differs from Perry's approach[1] in which a Tamm-Dancoff truncation is never used. This fact, plus the correctness of his approach, allows Perry to obtain results consistent with ordinarily renormalized field theory. However, any purely numerical Hamiltonian approach will have to institute a Tamm-Dancoff truncation at the very beginning. Our preliminary results indicate that even though we institute an invariant mass cut-off, we nevertheless find divergences, associated with small longitudinal momenta.

A philosophical comment is that Hamiltonian methods perhaps promise to be more powerful than Lagrangian methods when working with field theories, such as QCD, that have composite particles, especially when these composite particles are relativistic, non-perturbative bound states. This may be so if only because Hamiltonian methods promise to give, unambiguously, the wave functions of the composites.

As has been alluded to previously at this conference, the simplified vacuum structure of the front-form formulation may allow a simplified Fock space structure of the excited states, the particles. Hence, this may allow a successful constituent picture of hadrons to emerge from QCD, and it may allow Tamm-Dancoff truncations to be justified.

2. Methods

Zhang and Harindranath[2] formulate light-front QCD. We took their results, in their equations (3.1) to (3.8), and obtained the front-form of QED in the $A^+ = 0$ gauge by setting the structure constants, f_{ijk} equal to zero. We chose an extreme truncation, keeping only the bare electron and bare electron-bare photon sectors of Fock space. This resulted in an interaction Hamiltonian with ten terms in it. The fermion mass terms have also been included in this list of ten as interactions. We number the couplings for these interactions from 1 to 10, where canonically, $m^2 = \mu_1\Lambda_0^2$, $m^2 = \mu_2\Lambda_0^2$, $e = g_3$, $e = g_5$, $em = \mu_7\Lambda_0$, $e^2 = g_9$, and $e^2 = g_{10}$. These couplings are numbered as such, because under renomalization, they will run differently. Λ_0 is the invariant mass cut-off in the problem.

Since g_9, g_{10}, and μ_7 are higher order in smallness, we begin with a renormalization group analysis[3] that is second order in the remaining couplings. Such an analysis requires only four Hamiltonian diagrams[1] if all irrelevant operators are dropped, as is done throughout. (These diagrams look like the fermion self-energy diagrams of standard perturbation theory.) The result is that only the coupling, μ_1 runs. In a third order analysis, 12 more diagrams are needed, and the additional couplings g_3, g_5, and μ_7 will run.

The renormalization group analysis provides a difference equation for how μ_1 runs. The idea of coupling coherence[1] is used to solve this equation, and this fixes the strength of μ_1 at

$$\mu_1 = \frac{g_3^2}{16\pi^2} f(\epsilon)$$

where $f(\epsilon) = -3/2 + 2\epsilon - \epsilon^2 - ln\epsilon$. The limit ϵ approaching zero must be taken. This then is evidence of the small longitudinal momentum divergence, since ϵ is the longitudinal momentum fraction of the electron in the electron-photon Fock state being integrated out in the renormalization group analysis. This is a problem. How do we fix it? One idea is to use massive quanta as Fock space constituents instead of massless quanta, as we have done. It turns out that use of massive electrons does regulate this divergence. The immediate implication of this is that as the ultraviolet cut-off, which initially is very large, is lowered, the constituent masses, initially very small, grow. As the cut-off becomes comparable to these masses, the original classification, with massless constituents, of the operators of the theory, in terms of relevant, marginal, and irrelevant, completely breaks down. The similarity renormalization scheme of Głazek and Wilson[4,5] promises to overcome this problem of small longitudinal momentum divergences by consistently renormalizing these divergences as well as the large transverse momentum divergences. We will press forward with an implementation of this scheme in an LFTD QED calculation.

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