Low-x Physics in Deep Inelastic Scattering and the Regge Limit in QCD

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Abstract

A review is given over some recent theoretical developments in low-x physics and the Regge limit in QCD.

1. Introduction

The low-x region in deep-inelastic scattering (DIS) has attracted much interest recently. Not only have we obtained from HERA new and exciting data for the proton structure function in this region [1, 2], but there is also a genuine theoretical interest in this kinematic limit. The low-x region in DIS lies at the interface between perturbative and nonperturbative QCD: it can be viewed as a transition from one asymptotic limit to another, from the Bjorken limit ($Q^2 \to \infty$, x_B fixed and not too small) to the Regge limit ($x_B \to 0$, Q^2 fixed and of the order of a typical hadronic scale). For the first of these limits, QCD provides reliable perturbative predictions, whereas the latter one cannot be reached whithin perturbation theory. However, in between the two, there is new limit, a "modified Regge limit": $x_B \to 0$ at Q^2 fixed and large. Since α_s is small (at least for not too small x_B), one can start from perturbative QCD and then follow how perturbation theory requires more and more corrections and eventually needs nonperturbative contributions. Usually (and for good reasons) DIS is discussed in terms of the operator product expansion and, for the leading-twist term, the linear evolution equations of Gribov, Lipatov, Altarelli, and Parisi. In this talk I will take a more unconventual route: I start from the "modified Regge limit", for which the leading-logarithmic approximation in QCD is given by the BFKL-Pomeron [3], and then "return" to the DIS limit.

One of the motivations for proceeding in this way comes from the present experimental situation. Let me illustrate this in a few remarks:
(1) The energy dependence of the photoproduction cross section seems to be quite consistent with the (nonperturative) Regge-description of hadronic scattering at high energies. The simplest way [4] to describe the data seems to be the Pomeron pole with intercept at 1.08; unfortunately, we still do not know how to derive this simple description from QCD.
(2) The rise of $F_2(x, Q^2)$ at low x and $Q^2 > 8GeV^2$ is not simply a straightforward ex-

trapolation of the *nonperturbative* behaviour seen at $Q^2 = 0$ to larger Q^2-values. On the other hand, this rise seems to be compatible with the prediction of *perturbative* QCD; this I find extremely encouraging, since it provides us with a much more solid start for the use of perturbative QCD than expected.

(3) At present, data are not yet precise enough to allow a detailed investigation of the evolution in Q^2. So time has not yet come to investigate whether there is space for those higher twist terms in the Q^2-evolution of the structure function, which are expected at small x. This, in my opinion, seems to put more emphasis on the dependence upon x_B rather then Q^2.

(4) In the region of $Q^2 > 8GeV^2$, there are more events with a rapidity gap between the outgoing proton and the diffractive excitation of the photon than expected on the basis of standard DIS Monte Carlos. Their contribution to the total cross section is of the same order as expected from a conventional hadronic triple-Regge analysis. Furthermore, there is no strong Q^2-variation of these events. Since standard DIS Monte Carlo routines seem not to be capable to account for these events, something must be missing: what is it? In particular, how is it related to higher order corrections to the leading-logarithmic QCD predictions?

In respond to these observations, it seems fair to put more emphasis on the Regge aspects of the low-x region than on the Q^2-evolution. As mentioned in (3), the time for a detailed study of the variation in Q^2 has not yet come: the most acute problem, therefore, seems to be the x-dependence of F_2 and the implications of the diffractive events. Apart from these experimental observations, there are also theoretical reasons to believe that the calculational techniques which underly the derivation of the BFKL Pomeron may be better suited for studying the small-x region of DIS scattering than the conventional hard scattering approach: whereas it sacrifices accuracy in $1/x_B$ (by retaining only the leading logarithms), it gains in $\ln(Q^2/k^2)$ (by keeping all orders in this variable). This allows to treat the region of small x but finite transverse momentum more accurately.

The talk will be organized in three parts. First I will comment on the status of the BFKL Pomeron. Then I discuss what we presently know about corrections beyond the BFKL Pomeron. In the third part I will try relate these theoretical developments to experimental signals. Finally I will say a few words about future developments, in particular about prospects of deriving an effective field theory.

2. Status of the BFKL-Pomeron

As it is well-known, the Balitsky-Fadin-Kuraev-Lipatov (BFKL) ladder- approximation for the Pomeron constitutes the leading-logarithmic approximation of QCD in the Regge-limit: in order to justify the use of perturbation theory it is necessary to consider the "modified" Regge limit, as it was defined above. Since the properties of this approximation have been described in many review talks, I will list only very few of the main features: the power behavior of the scattering amplitude

$$T(s,0) \sim const \cdot s^{1+\omega_{BFKL}}, \;\; \omega_{BFKL} = \frac{4N_c \ln 2\, \alpha_s}{\pi} \approx 0.5 \tag{1}$$

or, in the context of deep inelastic scattering,

$$F_2(x,Q^2) \sim const \cdot x^{-\omega_{BFKL}} \tag{2}$$

This behavior is obtained from the integral equation

$$f(x,k) = f_0(x,k) + \int_x^1 \frac{dx'}{x'} \int d^2k' K(k,k') f(x',k'). \tag{3}$$

where the BFKL-kernel $K(k,k')$ is proportional to $\alpha_s(Q^2)$, and the momentum integration runs over the full two-dimensional space. In many applications, the fixed coupling $\alpha_s(Q^2$ is replaced by $\alpha_s(k^2)$ or $\alpha_s(k'^2)$: strictly speaking, this goes beyond the leading-logarithmic approximation. Since at this level many other contributions have to be taken into acount (see below), this procedure alone is not very satisfactory. If the inhomogeneous term in (3) does not depend upon x, the BFKL equation is equivalent to the evolution equation

$$\frac{\partial f}{\partial \ln \frac{1}{x}} = \int d^2k' K(k,k') f(x,k'). \tag{4}$$

If one applies this equation to deep inelastic scattering, the structure function F_2 is predicted to grow at small x (cf.(2)). For sufficiently large Q^2, one may think of expanding the solution in inverse powers of Q^2 and retaining only the leading term: its x-dependence is given by

$$f \sim \exp\left(\sqrt{\frac{48}{\beta_0} \ln(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}) \ln(\frac{1}{x})}\right) \tag{5}$$

This growth in $1/x$ is slightly weaker than the power law (2); for an intermediate region, however, the resulting x-distributions may be rather similar. In such a case, only the evolution in Q^2 will be able to distinguish between the two cases. As to the question which paramtrizations fits the HERA data best, it seems that those fits which are based on a power-law type behavior (eq.(2)) are generally doing better than those which are flat at small x; this applies to both the Durham group and the CTEQ group. The fact that the Dortmund curve is also in good agreement with the data may be explained by the long Q^2-evolution: the resulting x-dependence is of the type (5) which, simultaneously, represents the small-x tail of the GLAP evolution scheme and the leading-Q^2 term of the BFKL-Pomeron. In any case, it seems fair to say that the observed rise is “close to the predictions of perturbative QCD“.

Keeping in mind that the QCD-prediction for the small-x-behavior (to leading order) is given by the BFKL-Pomeron, it seems natural to ask whether the rise can be taken already as evidence for *having seen the BFKL Pomeron.* I would like to express several *caveats*. First of all, we do not yet know in what region the BFKL-approximation is valid. That is to say, the criterion

$$\frac{\alpha_s(Q^2)}{\pi} \ln \frac{1}{x} \leq 1 \qquad (6)$$

which is usually quoted is too crude and needs to be refined. The most natural way to proceed seems to calculate the first corrections (see below) and then to estimate at what values of x and Q^2 these corrections become as large as the leading-logarithmic approximation. The breakdown of the leading-logarithmic approximation can be seen in several places. First of all, unitarity corrections (which will be discussed further below) are expected to lead to terms which grow faster in $1/x$ than the BFKL-Pomeron itself. Since they come with an overall coefficient of the order $\alpha_s^2(\bar{Q}^2)$ (where $\bar{Q}^2$ lies somewhere between the hadronic scale and Q^2) they may be small as long as $1/x$ is not too large. At sufficiently small x, however, the smallness of α_s^2 will be compensated by the stronger growth in $1/x$. Since the sign of these unitarity corrections is opposite to that of the leading BFKL-Pomeron, they will tend to lower the increase in $1/x$ of F_2. At very small x, they eventually become larger than the BFKL approximation, and we need to sum *all* corrections.

Secondly, with growing $1/x$ the high energy behaviour of the BFKL-ladders becomes increasingly sensitive to the infrared region where perturbation theory becomes unreliable. Formally the BFKL-approximation is infrared finite (that is why, in (3) the momentum integration extends down to $k' = 0$). In a given application, for example the structure function F_2 at $Q^2 = Q_0^2$, the dominant region of integration inside the ladder diagrams is centered around the external momentum scale, Q_0^2, but as the evolution in $1/x$ gets longer and longer, it diffuses more and more into both the infrared and ultraviolet regions. Obviously, even in a truly deep inelastic situation with a large Q_0^2, the calculation of the BFKL-ladders will eventually reach the "dangerous" infrared region, and that is where the application becomes unreliable. Obviously, the smaller Q_0^2 is, the worse the situation becomes. A numerical calculation of the k'^2 distribution inside F_2 has recently been performed [5] (similar to the one presented in [6]), and the contribution of momenta below, say, 1 GeV^2 is not small: the theoretical prediction, therefore, will depend on how we are dealing with the "dangerous region". The customary way to improve on the leading-logarithmic BFKL-approximation is the use of the running coupling constant inside the ladders rather than the fixed one. Although this leaves the leading-logarithmic approximation (see above), it is felt that this modification will at least lead closer to the "true" QCD prediction. As a result of this, the k'-integral has to stop at some minimal value $k_0^2 > \Lambda_{QCD}^2$, and all numerical results will be sensitive to the choice of this parameter. This has been clearly demonstrated in [7]. As a further improvement, one can define some sort of continuation below this infrared cutoff; as shown in [8], this makes the resulting power of $1/x$ less dependent on k_0^2, but the overall strength still varies with k_0^2. In brief, we are not really testing any more the BFKL-Pomeron but a phenomenological modification of it.

Before concluding this section it should be mentioned that there are other reactions where the BFKL-Pomeron can be tested more safely, for example the Mueller-Navelet [9] jets in hadron-hadron collisions or the associated jet production ("Hot Spots") at HERA [10]. The latter process has been studied by several groups [10]. The cross section reads:

$$\frac{xk^2 d^4\sigma}{dxdk^2dx_Bdy} = \frac{4\pi\alpha^2}{2Q^2}\sum e^2 \left(y\Phi_1(Q^2,k^2,\frac{x}{x_B}) + \frac{1-y}{y}\frac{pq}{M^2 x_B}\Phi_2(Q^2,k^2,\frac{x}{x_B})\right)$$
$$\frac{12\alpha_s(k^2)}{4\pi}\left(xG(x,k^2) + \frac{4}{9}\sum q(x,k^2)\right) \quad (7)$$

Calculations of the cross section have been presented in [10, 11], and a numerical computation of the k-distribution inside the ladders is contained in [6]. The latter one shows that the infrared region plays an unimportant role. As a consequence, there is obviously no need to use a running coupling constant inside the BFKL evolution equation, and this measurement looks like an excellent tool for "seeing" the BFKL-Pomeron. First experimental results have been reported in [12], and the event rates are in qualitative agreement with the theoretical BFKL-predicitions.

3. First Corrections

From what has been said above it follows that we have to go beyond the BFKL-approximation in order to know where it is applicable. Generally speaking, one may think of two different strategies for approaching the full QCD theory in the Regge limit. First, one may start by finding corrections to the BFKL-kernel: this implies that one stays in the framework of ladder diagrams and computes contributions which are down by one power of $\ln 1/x$ compared with the leading logarithmic aproximation. This involves diagrams which belong to vertex corrections, self energies, and the production of two-gluon states with a small rapidity gap. At this stage also fermions will come in: the fermion box diagrams which are known from the tower diagrams in QED [13, 14]. One consequence of all these contribution will be that they provide the first logarithmic correction to the fixed coupling constant. One also expects that the power of s (or $1/x$) which governs the high energy behaviour will receive corrections of the order α_s^2. This line of calculations is being persued by Fadin and Lipatov, and recent results have been published in [15]. Another important benefit from this line of attacking non-leading contributions is the possibility of obtaining higher order contributions to the gluon anomalous dimension. In analogy to [16] where the eigenvalues of the BFKL kernel have been used to calculate all terms of the form $(\frac{\alpha_s}{n-1})^k$ in the gluon anomalous dimension, one expects to obtain, from

[15], the contributions of the form $\alpha_s(\frac{\alpha_s}{n-1})^k$. It has been argued [17] that these terms are extremely important in the GLAP evolution of flat (in x_B) input distributions.

It is, however, clear that these corrections to the BFKL-kernel are not enough. In particular, they will not help to cure the violation of unitarity which manifests itself in the power of s (eq.(1)). The restauration of unitarity requires contributions which go beyond the one-ladder structure: diagrams with more than two (reggeized) gluons in the t-channel. In a naive picture, one may think of these diagrams as resulting from a s-channel iteration of the exchange of BFKL-ladders. However, from general considerations (gauge invariance, cancellations of infrared divergencies) it immediately follows that, for example, two BFKL-ladders in the t-channel would represent a too crude approximation: instead of two isolated ladders one needs all pairwise interactions between all four gluon lines. More precisely, such contributions are obtained from multiple energy discontinuities of inelastic multiparticle amplitudes: the theoretical background has been laid down in [18], and a systematic way how to obtain these amplitudes in practice has been defined in [19]. In the present context, the first "unitarity" corrections are those with two or four gluons in the t-channel, and their analytic form is given in [20]. The numerical evaluation has turned out to be rather involved; in particular, the most interesting numbers (in particular ω_4, the location of the leading angular momentum plane singularity) have not been calculated yet. Attempts to compute the energy spectrum of the n-gluon state in the large-N_c limit have been reported in [21, 22]. The short distance limit of these QCD-diagrams has been investigated in [20], and, as a quantitative result, the anomalous dimension of the four-gluon operator has been obtained [20, 23]. For the remainder of this section we shall limit ourselves to the short distance aspects of the unitarity corrections.

So far all the discussion has been about the "modified" Regge limit $1/x_B \to \infty$ at fixed (but not too small) Q^2. The aspect to be discussed in the following is the connection with the DIS limit $Q^2 \to \infty$ at fixed (not too small) x_B: when taking the large-Q^2 limit (also referred to as the small-distance limit) of the BFKL-Pomeron and the unitarity corrections to it, one recovers the usual twist-expansion (to be more precise: the leading-log 1/x approximation to it). Let us discuss this term by term.

To begin with the BFKL-approximation and F_2, it has the following expansion in powers of k^2/Q^2:

$$\Phi(\omega, Q^2, k^2) = Q^2 \sum_{l,n} C_{(l,n)} e^{in\theta} \left(\frac{k^2}{Q^2}\right)^{l+1-\gamma_{(ln)}} \tag{8}$$

where $\omega = n - 1$ is the variable conjugate to $\ln 1/x_B$, and the anomalous dimensions are of the form

$$\gamma_{(ln)} = \frac{N_c \alpha_s}{\omega \pi} + O(\frac{{\alpha_s}^2}{\omega}) \tag{9}$$

The leading term ($l = 0$) coincides with the two-gluon operator, whereas the higher order terms are related to derivatives (trace terms) of the two gluon operator. So the BFKL-amplitude provides, not only for the leading-twist but also for a subset of nonleading-twist contributions, the most singular (in ω) part of the anomalous dimension. At large Q^2, the BFKL-approximation coincides with the small-x_B limit of the GLAP gluon structure function, but, in addition to that, it contains nonleading powers of k^2/Q^2.

The first unitarity correction are analysed most easily if we cut the diagrams with 4 t-channel gluons across the four-gluon state: the upper part then defines a four-gluon amplitude, and one has to study the limit $k_i^2/Q^2 \to 0$ (k_i denote the momenta of the gluons). As a result of the reggeization of the gluon, there is a leading term which contributes to twist two [29]. The next-to-leading term, twist four, has been investigated in [24, 23], and the anomalous dimension of the four-gluon operator has been calculated:

$$\gamma_4 = 4\frac{N_c\alpha_s(1+\delta_1)}{\omega\pi} \; , \; \delta_1 = 0.0123 \tag{10}$$

Recently it has been found [25] that this twist-four term of the four gluon amplitude has a very rich structure, which can be interpreted most easily in terms of operator product expansions. There is also mixing between the twist-four piece of the BFKL-Pomeron and the twist-four four gluon operator [20]. In analogy with the BFKL-Pomeron, the four-gluon amplitude allows for expansions in powers of k_i^2/Q^2, and there is a correspondence between the nonleading terms and higher order derivatives (trace terms) of the four-gluon operator.

Within the usual approach towards DIS, the twist expansion is usually truncated, and only the first term is kept. The reason why at small x_B this neglect of higher-twist terms is no longer justified can be seen very easily. The standard operator product expansion leads to the following expansion of the moments of the DIS structure function (for simplicity, we restrict ourselves to gluons only):

$$\begin{aligned} \mu(n,Q^2) &= \int_0^1 dx x^{n-1} F(x,Q^2) \\ &= C_2(Q^2/\mu^2;g)a_2 + \frac{M^2}{Q^2} C_4(Q^2/\mu^2;g)a_4 + ..., \end{aligned} \tag{11}$$

and the renormalization group analysis of the coefficient functions implies that

$$\begin{aligned} C_2(Q^2/\mu^2;g) &\sim \exp\int dt'\gamma_2(g(t')) \; , \; \gamma_2 = \frac{N_c\alpha_s}{(n-1)\pi} \\ C_4(Q^2/\mu^2;g) &\sim \exp\int dt'\gamma_4(g(t')) \; , \; \gamma_4 = 4\frac{N_c\alpha_s(1+\delta_1)}{(n-1)\pi} \end{aligned} \tag{12}$$

The small-x_B limit probes the moments near $n = 1$, where the anomalous dimensions become singular. Since the strength of this singularity grows quadratically with the number of gluons, the higher-twist terms become as important as (or even more important than) the leading term. This is why we are forced to investigate the singular parts of the nonleading-twist anomalous dimensions and coefficient functions.

Higher order unitarity corrections which lead to amplitudes with 2k gluon lines have not been constructed or analysed yet. Nevertheless, a first attempt has been made for calculating the anomalous dimension of the 2k-gluon operator [26]. Making certain simplifications and using a Bethe ansatz it has been found that

$$\gamma_{(2k)} = k^2 \frac{N_c \alpha_s (1+\delta_k)}{\omega \pi} \;, \delta_k = (k^2-1) \frac{1}{3(N_c^2-1)^2}. \tag{13}$$

There is no doubt that besides the leading anomalous dimensions there will be an extremely rich spectrum; an attractive way to attack this difficult problem seems to be the use the $1/N_c$- expansion. The first step in this direction has been done in [25], and further work is in progress [27].

It would certainly be extremely important if one could find a way to obtain the sum of these nonleading twist contributions, at least their most singular parts. Several years ago it has been suggested [28] that - under assumptions which are quite analogous to those underlying the eikonal approximation in high-energy hadron-hadron scattering - a nonlinear term, added to the rhs of the linear GLAP evolution equation, would effectively take into account not only all these higher twist contributions but also screening corrections to the leading twist. Unfortunately, the results on the anomalous dimension (10), (13), are in conflict with this equation, and a recent numerical estimate [25] indicates that that the disagreemet is not a small effect. The problem of finding a way to sum the nonleading twist contributions, therefore, remains one of the important tasks in low-x physics.

4. What can be seen in Deep Inelastic Scattering at small x?

After this slightly theoretical discussion let me adress the more “pragmatic“ question what are the prospects of seeing experimental signals of these nonleading corrections. To start with the rise in F_2: the unitarity corrections to the BFKL-ladders which we have discussed before are expected to contain t-channel singularities to the right of ω_{BFKL}. For example, there is the contribution of the state consisting of two BFKL-Pomerons which is located at $\omega = 2\omega_{BFKL}$. There are, however, reasons to expect there will be a new singularity even further to the right at $\omega = \omega_4 > 2\omega_{BFKL}$. Together with the overall coefficient of the order α_s^2, the small-x behavior takes the form:

$$F_2 \sim (\frac{1}{x})^{\omega_{BFKL}} \left(1 - O(\alpha_s^2)(\frac{1}{x})^{\omega_4 - \omega_{BFKL}} \right) \tag{14}$$

A very crude estimate shows that the correction term does not have to be small in the HERA region [1]: for a precise estimate we need the number ω_4 as well as the coefficient in front (the latter contains both a perturbative and a nonperturbative part).

[1] As an example: taking $\omega_4 = 3/2$ and making the assumption that at $x_0 = 10^{-2}$ the suppression factor is $O(\alpha_s^2) = (\frac{0.2}{\pi})^2$, at $x = 10^{-4}$ the second term in (14) has reached already 40%.

However, even if these corrections to the BFKL-approximation are non-negligeable in the HERA region, the measurement of F_2 alone will, most likely, not be sufficient to establish their existence. All the uncertainties mentioned in connection with the BFKL-approximation become even bigger if we start to consider nonleading corrections. Hence one is lead to look into final state, in particular into those configurations which cannot be produced within the standard one-ladder picture. Most promising candidates are the events with large rapidity gaps which have been observed by both Zeus [34] and H1 [35]: within the standard one-ladder picture which underlies both the GLAP-evolution equations and the BFKL- approximation the final states are produced via color exchange, and large rapidity gaps are suppressed by color correlations. Therefore, if one tries to explain the HERA rapidity gap events within perturbative QCD one is lead to generalize the one-ladder picture in such a way that final states can be produced with color singlet exchange. This requires two-gluon pairs on both sides of the ladder, and we arrive at QCD diagrams with (at least) four gluons in the t-channel. A sytematic study of these (perturbative) contributions leads exactly to what I have discussed in the previous section. Therefore one is lead to the conclusion that, if the rapidity gap events (or part of them) can be explained within perturbative QCD, they provide evidence for the presence of nonleading corrections to the one-ladder picture (i.e.the BFKL Pomeron).

Reality is, of course, much more complicated [36], and a careful study of the rapidity gap events has to decide which part of them can be explained within perturbative QCD and which part requires the non-perturbative (soft) Pomeron. As a first example, let us look at the diffractive production of $q\bar{q}$ pairs with invariant mass M (the proton remains intact). The Pomeron which is exchanged between the quark pair and the proton can be soft or hard (e.g.BFKL), depending upon whether the quarks coming from the virtual photon are hard or soft. In the first case one expects, for fixed M^2, the W^2 dependence ($1/x \approx W^2/Q^2$):

$$\frac{d^2\sigma}{dtdM^2} \sim (\frac{1}{x})^{0.16}, \tag{15}$$

in the latter case

$$\frac{d^2\sigma}{dtdM^2} \sim (\frac{1}{x})^{2\omega_{BFKL}}. \tag{16}$$

It is therefore important to separate the "soft" from the "hard" contribution: a possible handle might be the transverse momentum of the quark which is closer to the rapidity gap. The observed W^2 behaviour [34] of the ratio $r = \sigma_{DD}/\sigma_{tot}(\gamma^* p)$ indicates that not all events can be of the type (15).

Most interesting, at least from the theoretical viewpoint, are events where the diffractively excited photon contains, in addition to the $q\bar{q}$-pair, also gluons. Such events are expected to contribute to the region of large M. Again, we have to distinguish between a "hard" and a "soft" Pomeron across the rapidity gap. The W^2-dependence (at fixed M) is again given by (15) and (16). The dependence on M^2, on the other hand, depends upon the internal structure of the $q\bar{q} - gluon$ system: if the exchanged Pomerons are "hard", i.e. of the BFKL-type, the perturbative triple-Pomeron vertex appears which has been derived recently [29, 30], and the cross section is predicted to have the form:

$$\frac{d^2\sigma}{dtdMm^2} \sim (\frac{1}{x})^{2\omega_{BFKL}} \left(\frac{M^2 + Q^2}{Q^2} \right)^{\omega_4 - 2\omega_{BFKL} - 1} \tag{17}$$

where ω_4 is the new singularity of the four gluon state mentioned before. Also the observation has been made that the behavior near $t = 0$ may be quite interesting [30, 31]: the conformal invariance of the BFKL-Pomeron leads to a decoupling at $t = 0$ of coupling between three BFKL-singularities.

As another example of diffraction scattering in DIS, one might consider [32] the diffractive production of vector mesons at large momentum transfer (the proton can either scattered elastically or, alternatively, produce a diffractive final state): here the momentum transfer across the Pomeron provides the large momentum scale which allows the use of the BFKL Pomeron. In [33] it has been shown that in DIS also the cross section of the forward diffractive production of vector mesons is calculable within perturbative QCD.

5. Future prospects: towards an Effective Field Theory

Finally a few words should be said about future developments in this field, in particular about theoretical ideas on the Regge limit. Presently, the most attractive program consists of the derivation and solution of an effective 2+1 dimensional field theory. The underlying idea looks simple and attractive: in a high energy scattering process with zero or small momentum transfer the two transverse degrees of freedom (two-dimensional impact parameter or its conjugate, the transverse momentum) and the longitudinal degree of freedom (rapidity or angular momentum of the cross channel) play completely different roles. Therefore it appears to be an attractive idea to formulate an effective field theory which lives in the two-dimensional transverse space (with rapidity as the time variable). Several attempts in this direction have already been made before [37]. But as it has been said before, the low-x limit of DIS provides a new direction of attacking the Regge limit; it is therefore necessary to review the idea of such a lower-dimensional field theory in this new context.

Starting point for the formulation of such a field theory are the Green's functions $G_{n \to m}(\rho_1, ..., \rho_n; \rho'_1, ..\rho'_m)$ which describe the transition: n reggeized t-channel gluons $\to$ m reggeized t-channel gluons. They live in the two-dimensional impact parameter space and also depend upon angular momentum ω. The simplest example for such a Green's function is $G_{2\to 2}$, the BFKL-Pomeron. The first nontrivial generalization to this is the function $G_{2\to4}$ which has been derived in [20] and contains, as a new kernel, a $2 \to 4$ gluon transition vertex [29]. For the former case it has been shown in [38] that it is invariant under Moebius transformations; an analogous investigation of the latter one is in progress [39]. The (perturbative) dynamics of these Green's functions can be used in order to study their short distance behavior (anomalous dimensions, fusion coefficients): examples are again the two Green's functions $G_{2\to2}$ and $G_{2\to4}$. As a result of the investigations in [38] and [20] it seems possible to define short distance expansions which allows to identify the Green's functions as vacuum expectation values of some (possibly: conformal) field theory:

$$G_{n\to m}(\rho_1, ..., \rho_n; \rho'_1, ..., \rho'_m) \sim < \phi(\rho_1)...\phi(\rho_n)\phi(\rho'_1)...\phi(\rho'_m > \tag{18}$$

(note that these fields cannot be identified with the QCD-gluon field operators; the property of being "reggeized" makes the gluons already somewhat "composite"). The short distance behavior of this field theory has to be derived by extending the perturbative analysis of QCD which has been scetched in the second part of this talk.

Clearly, presently we are still at a very early stage of this program. It will be very interesting to see how far this approach will take us.

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