Wave Function Properties in a High Energy Process

Arjun Berera
Department of Physics
Pennsylvania State University
University Park, Pennsylvania 16802, USA

Abstract

A model example is given of how properties of the hadronic light-cone wave function are revealed in a particular high energy process.

1. Introduction

The purpose of my talk is to give a pedagogical derivation of the light-cone wave function and demonstrate its use in a particular high energy process. Explicitly I will derive the meson wave function in scalar quark QCD. I will then apply it to recent work I have been doing with David Soper on diffractive hard scattering.

The Lagrangian for scalar quark QCD coupled to a meson field is,

$$\begin{aligned} \mathcal{L} &= (D_\mu q)^\dagger (D^\mu q) - m^2 q^\dagger q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a - \frac{1}{2\xi}(\partial_\mu A^\mu_a)(\partial_\nu A^\nu_a) \\ &\quad + \text{Faddeev-Popov terms} - \frac{g_4}{4}(q^\dagger q)^2. \end{aligned} \tag{1}$$

Here $A^\mu_a(x)$ as an SU(3) gauge field as in normal QCD. There is also a color triplet quark field $q_i(x)$ with quark mass m, but we take $q_i(x)$ to be a scalar field instead of a Dirac field. Since the theory includes a scalar quark field, a 4-quark coupling is necessary, but we have set the renormalized coupling constant g_4 to a negligible small value. This theory has the same behavior as spinor QCD for collinear and soft gluon emission from quarks. Its chief advantage is that it allows a perturbative model for a quark-antiquark bound state. We introduce a scalar, color singlet meson field $\phi(x)$ and couple it to the quarks using

$$\mathcal{L}_\phi = G\, \phi(x)\, q_i^\dagger(x) q_i(x). \tag{2}$$

We work to lowest nontrivial order in the $\phi q^\dagger q$ coupling G, letting the $\phi q^\dagger q$ vertex play the role that is played by the (amputated) Bethe-Salpeter wave function of a meson in spinor QCD. We denote the mass of the meson by M and take M to be smaller than $2m$, so that the meson cannot decay into a quark and an antiquark.

2. Light-Cone Wave Function

For a meson moving in the minus direction, the light-cone wave function $\psi(x, \mathbf{k})_{ij}$ is the amplitude to find that the meson with momentum $P^\mu = (M^2/(2P^-), P^-, \mathbf{0})$ consists of a quark and an antiquark of colors i and anti-j respectively, with the quark having minus-momentum $k^- = xP^-$ and transverse momentum $\mathbf{k}$. The wave function is measured by operators defined on the null-plane $y^- = 0$. The precise definition, following the formalism of [1,2,3] is,

$$\psi(x, \mathbf{k})_{ij} = 2x(1-x)P^- \int d^4y \, e^{ik\cdot y} \, \delta(y^-) \, \langle 0|q_i(y) \, q_j^\dagger(0)|P\rangle \, . \tag{3}$$

Here we have chosen the normalization

$$(2\pi)^{-3} \int_0^1 \frac{dx}{2x(1-x)} \int d\mathbf{k} \sum_{ij} |\psi(x, \mathbf{k})_{ij}|^2 = P_2 \, , \tag{4}$$

where P_2 is the probability, which is of order G^2, that the meson state consists of a $(q, q^\dagger)$ pair. In terms of the covariant $\phi \, q \, q^\dagger$ Green function amputated on the ϕ-leg, the definition (**??**) can be written as

$$\psi(x, \mathbf{k})_{ij} = 2x(1-x)P^- \int \frac{dk^+}{2\pi} \, \mathcal{G}(k^\alpha, P^\beta)_{ij} \, . \tag{5}$$

At lowest order in α_s and G one has,

$$\mathcal{G}(k^\alpha, P^\beta)_{ij} = iG \, \delta_{ij} \, \frac{i}{k^2 - m^2 + i\epsilon} \, \frac{i}{(P-k)^2 - m^2 + i\epsilon} \, . \tag{6}$$

By integrating according to Eq. (**??**), we find

$$\psi(x, \mathbf{k})_{ij} = \frac{Gx(1-x)}{\mathbf{k}^2 + m^2 - x(1-x)M^2} \, \delta_{ij} \, . \tag{7}$$

Notice that $|\psi|^2 \propto 1/\mathbf{k}^4$ for large $\mathbf{k}^2$. This good behavior in the ultraviolet, which arises from the fact that G has dimensions of mass, is the essential reason for the usefulness of this model.

The wave function can be used to calculate, to order zero in α_s, the probability for finding a quark in a meson *i.e.* the parton distribution function:

$$\begin{aligned} f_{q/\phi}(x) &= (2\pi)^{-3} \frac{1}{2x(1-x)} \int d\mathbf{k} \sum_{ij} |\psi(x, \mathbf{k})_{ij}|^2 \\ &= \frac{3G^2}{16\pi^2} \frac{x(1-x)}{m^2 - x(1-x)M^2} \, . \end{aligned} \tag{8}$$

As a more realistic example, one can derive the light-cone wavefunction for quarks in a photon. For transverse polarization, we find

$$\psi(x, \mathbf{k}) = -\frac{e\mathcal{Q}}{2P^-} \, \frac{\overline{U} \; [x \, \epsilon \cdot \gamma \, k_T \cdot \gamma - (1-x) \, k_T \cdot \gamma \, \epsilon \cdot \gamma] \, \gamma^- V}{\mathbf{k}^2 + x(1-x)Q^2}, \tag{9}$$

where $k_T^\mu = (0, 0, \mathbf{k})$. This is quite similar to the scalar wave function, Eq. (**??**). The chief difference is the factor of $\mathbf{k}$ in the numerator, which leads to a logarithmic divergence in the normalization integral for ψ. (The spinors U and V depend on $\mathbf{k}$, but this dependence is eliminated when the spinors stand next to a γ^-.) For longitudinal polarization, we obtain

$$\psi(x, \mathbf{k}) = \frac{e\mathcal{Q}}{P^-}\, x(1-x)Q\, \frac{\overline{U}\gamma^- V}{\mathbf{k}^2 + x(1-x)Q^2}\,. \tag{10}$$

Again, $\overline{U}\gamma^- V$ is independent of $\mathbf{k}$. This wave function is small compared to that for transverse polarization when $Q^2 \ll \mathbf{k}^2$ but becomes comparable when $Q^2 \sim \mathbf{k}^2$.

3. Diffractive Hard Scattering

In 1985 Ingelmann and Schlein[4] predicted that events of the type

$$A + B \to A + \mathrm{jets} + X, \tag{11}$$

where hadron A is diffractively scattered, should occur with a small but not tiny probability. Here by "diffractively scattered," we mean that A emerges with a fraction $(1-z) > 0.9$ of its original longitudinal momentum and with a small transverse momentum $|\mathbf{P}_A'| \le 1\ \mathrm{GeV}$. The transverse momentum transfer can also be characterized using the invariant momentum transfer t from the hadron: $t = (P_A - P_A')^2 = -(\mathbf{P}_A'^2 + z^2 M_A^2)/(1-z) \approx -\mathbf{P}_A'^2$.

The picture for such diffractive hard scattering proposed by Ingelman and Schlein is that hadron A exchanges a pomeron with the rest of the system, where "pomeron" means whatever is exchanged in elastic scattering at large s, small t. Thus the cross section is proportional to the pomeron coupling to hadron A as measured in elastic scattering. The pomeron carries transverse momentum $-\mathbf{P}_A'$ and a fraction z of the hadron's longitudinal momentum. Here we do not need to know what a pomeron is, only that its momentum is carried by quarks and gluons. One of these collides with a parton from hadron B to produce the jets. Let the parton that participates in the hard scattering carry a fraction x of the longitudinal momentum of the incoming hadron A, and thus a fraction x/z of the longitudinal momentum transferred by the pomeron. Then the cross section in this model is proportional to a function $f_{a/P}(x/z, t; \mu)$, where $f_{a/P}(\xi, t; \mu)\, d\xi$ is interpreted as the probability to find a parton of kind a in a pomeron, where the parton carries a fraction ξ of the pomeron's longitudinal momentum.

The reaction (**??**) anticipated by Ingelman and Schlein has been seen at the CERN collider by the UA8 experiment[5]. However, the experiment suggests a feature not anticipated in [4,6,7]. It was expected that the functions $f_{a/P}(x/z)$ would have support only for $x < z$. That is, some of the momentum fraction x transferred from hadron A would be lost, appearing in low P_T particles rather than in the jets. Instead, the experiment suggests that a fraction of the events are lossless in the sense that $x = z$. It is as if the formula for the cross section contained a term proportional to $\delta(1 - x/z)$. A similar such distributional form was predicted in [8].

We will consider the cross section for lossless jet production in diffractive hard scattering. The details of the calculation are given in [9]. Our purpose here is to examine the role played by the light-cone meson wavefunction. The cross section is,

$$\left[\frac{d\sigma^{\rm diff}(A+B\to A+{\rm jets}+X)}{dE_T\,dX_A\,dX_B\,dz\,dt}\right]_0 \;\sim\; \delta\left(1-X_A/z\right)\int d\mathbf{r}\,\frac{|\psi(X_B,\mathbf{r})|^2}{2X_B(1-X_B)}$$

$$\times\sum_{j,k=1}^{2}\sum_{a,b=1}^{8}{\rm Tr}\Big\{\Big[G_a^j(-\mathbf{r};t,z)-G_a^j(\mathbf{0};t,z)\Big]^\dagger H_{ab}^{jk}\left(\hat{s},E_T\right)$$

$$\times\Big[G_b^k(-\mathbf{r};t,z)-G_b^k(\mathbf{0};t,z)\Big]\Big\}. \qquad (12)$$

Despite its rather complicated structure, the interpretation of Eq. (**??**) is straightforward. In the model, meson B consists of a quark and an antiquark. With probability $\propto|\psi(X_B,\mathbf{r})|^2$, they are separated by a transverse distance $\mathbf{r}$. In order to restore the color of hadron A, we must absorb a gluon on either the antiquark (at position $-\mathbf{r}$) or the quark (at position $\mathbf{0}$). Since the quark and antiquark have opposite color charges, the absorption amplitude is proportional to the difference $G_a^j(-\mathbf{r};t,z)-G_a^j(\mathbf{0};t,z)$. Here $G_a^j(\mathbf{b};t,z)$ is the amplitude to absorb a color field quantum at transverse position $\mathbf{b}$ when the "active" gluon is annihilated at the origin of space-time and hadron A is diffractively scattered. Thus G describes the color field associated with the pomeron when one gluon from the pomeron has been annihilated at the origin.

Here we meet an interesting experimental possibility. The $\mathbf{b}$ dependence of $G_a^j(\mathbf{b};t,z)$ reflects the transverse structure of the pomeron. It has significant structure on some distance scale $R_{\rm P}$ characteristic of the pomeron. In the present model, $1/R_{\rm P}$ is of order of the quark mass m. Thus $G_a^j(-\mathbf{r};t,z)-G_a^j(\mathbf{0};t,z)$ is small when $|\mathbf{r}|\ll R_{\rm P}$. On the other hand, $|\psi(X_B,\mathbf{r})|^2$ is small when $|\mathbf{r}|\gg R_B$, where R_B is a characteristic size of hadron B. This size is also of order $1/m$ in the model. However, suppose that we generalize the model so that R_B can be separately adjusted. Then when $R_B\sim R_{\rm P}$, there will be a substantial contribution to the cross section proportional to $\delta\left(1-X_A/z\right)$. But when $R_B\ll R_{\rm P}$, this contribution will vanish.

So far, we have worked only with a simple model. But the model suggests a plausible conjecture. First, there can be a sizable contribution to diffractive jet production proportional to $\delta\left(1-X_A/z\right)$, arising from using one gluon from the pomeron to make the jets and absorbing on the partons of hadron B the rest of the color field needed to make hadron A back into a color singlet. Second, when the size R_B of hadron B is small compared to the transverse size $R_{\rm P}$ associated with the color field in pomeron exchange, then hadron B should act as a color singlet and this contribution should disappear.

In order to test this conjecture, and probe the transverse structure of the pomeron, one needs to use hadrons of adjustable size. At HERA, one manufactures bremsstrahlung photons from the electron beam. The virtuality $Q=[-P_B^\mu P_{B\mu}]^{1/2}$ of the photon is measured by the deflection of the electron, and can be anything from nearly zero to many GeV. The photon can collide with a proton (hadron A) to make jets with $E_T\gg Q$. The cross section for this process can be (roughly) divided into two parts. In one part, the photon acts as a parton and scatters directly with a parton from hadron A to make the jets. In the other part, the photon acts as a hadron, made of constituent partons. For $Q \approx 0$, this hadron is essentially a ρ-meson, with a size $R_B \approx 1\ \mathrm{fm}$. For $Q \gg 1\ \mathrm{fm}^{-1}$, the "hadron" consists of a quark-antiquark pair, with wave functions given in Eq. (**??**) for transverse polarization and Eq. (**??**) for longitudinal polarization. These wave functions are characterized by a size $R_B \approx [X_B(1 - X_B)Q^2]^{-1/2}$. Since Q^2 and X_B are measurable, this size is adjustable.

We must emphasize that the proposal given above is a conjecture based on a simple model, not a proven consequence of QCD. It should be a challenge to investigate the structure of diffractive hard scattering further and to discover what features of the model survive a higher order analysis.

Acknowledgment

I thank R. Perry and S. Glazek for inviting me to this meeting. Partial funding was provided by the U. S. Department of Energy grant DE-FG03-91ER40674.

References

1. J. B. Kogut and D. E. Soper, Phys. Rev. **D1**, 2901 (1970).
2. J. D. Bjorken, J. B. Kogut and D. E. Soper, Phys. Rev. **D3**, 1382 (1971).
3. S. J. Brodsky and G. P. Lepage, Phys. Rev. **D22**, 2157 (1980).
4. G. Ingelman and P. Schlein, Phys. Lett. **B152**, 256 (1985).
5. A. Brandt, et. al. , Phys. Lett. **B297**, 417 (1992).
6. H. Fritzsch and K. H. Streng, Phys. Lett. **B164**, 391 (1985).
7. E. L. Berger, J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B286**, 704 (1987).
8. L. Frankfurt and M. Strikman, Phys. Rev. Lett. **64**, 1914 (1989).
9. A. Berera and D. E. Soper, Phys. Rev. **D50**, in press 1994.