

Self-Energies and Rotational Symmetry

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Abstract

The rotational invariance is violated in the lightcone quantization method when the Fock space is truncated for practical calculations. To what extent the rotation symmetry is broken in the lightcone quantization approach can be quantified by calculating the explicit rotation dependence of the two-body scattering phase shifts. We analyze the scattering phase shifts in a simple scalar field model, extending the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation in which the self-energy corrections are incorporated. We find that the self-energy effects significantly restore the rotation symmetry. Also, these effects make the phase shifts stabilize as the coupling constant grows. This is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects.

1. Introduction

For an explicit illustration of the rotation dependence in the two-body scattering phase shifts¹, let's consider a scalar field model^{2,3} which describes the interaction between two scalar particles $\phi, \bar{\phi}$ with equal mass m exchanging a scalar particle χ with mass λ . This model with $\lambda = 0$ is known as the Wick- Cutkosky model⁴ and the interaction Lagrangian is given by

$$\mathcal{L} = g\phi^2\chi. \quad (1)$$

Because the transverse components of the angular momentum (J_x and J_y) in the lightcone Poincare algebra⁵ contain interactions changing particle numbers in equal τ , the calculated scattering amplitude in the truncated Fock space is not rotationally invariant. The degree of the rotation symmetry breaking was quantified by our recent work of calculating the two-body scattering phase shifts¹. The numerical results showed that the rotation symmetry is broken more severely as the coupling constant of the model gets larger. More recently, we extended the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation in the same model⁶ and investigated the effects of the self-energy corrections and counter-terms to the rotation problem⁷. We found astonishingly a significant restoration of the rotation symmetry by this extension. Also, we observed that the self-energy effects stabilized the phase

shifts as the coupling constant grows. Even though the rotation problem is unavoidable in the lightcone quantization method with the Fock-space truncation, this calculation indicates that the rotation symmetry can be dynamically restored by adding the interactions which were neglected before. In this talk, we summarize our scattering formulation and discuss implications of our computation in the lightcone quantization scheme.

2. Formalism

In order to discuss the lightcone quantization more physically, we consider the c.m. system of two particles where the initial and final momenta of the first (second) particle are $\mathbf{k}(-\mathbf{k})$ and $\mathbf{l}(-\mathbf{l})$, respectively, and define the lightcone time τ as $\tau = t + \hat{\mathbf{n}} \cdot \mathbf{r}/c$ by introducing a unit vector $\hat{\mathbf{n}}$ on the lightcone surface (i.e. if $\tau = t + z/c$, then $\hat{\mathbf{n}} = \hat{z}$). In this reference frame, the lightcone two-body wavefunction, $\Psi(\mathbf{k}, \hat{\mathbf{n}})$ satisfies the following equation in the lowest order Tamm-Dancoff approximation including the self-energy corrections and counter-terms⁸ under the truncation of the Fock-space up to the three-body⁶:

$$(\mathbf{k}^2 - \mathbf{q}_{\text{in}}^2)\Psi(\mathbf{k}, \hat{\mathbf{n}}) = - \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{m^2}{\epsilon(\mathbf{l})} V(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}})\Psi(\mathbf{l}, \hat{\mathbf{n}}), \quad (2)$$

where $\epsilon(\mathbf{l}) = \sqrt{\mathbf{l}^2 + m^2}$, $\mathbf{q}_{\text{in}}^2 = s/4 - m^2$ (s is the square of the total c.m. energy), and

$$V(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) = \frac{4\pi\alpha V_{LA}(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}})}{1 + (\alpha m^2/4\pi)g(\mathbf{l}^2, \hat{\mathbf{n}} \cdot \mathbf{l})}.$$

Here, the dimensionless coupling constant, α , is given by $\alpha = g^2/16\pi m^2$, the kernel in the ladder approximation, $V_{LA}(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}})$, is given by

$$V_{LA} = - \left[(\mathbf{k} - \mathbf{l})^2 + \lambda^2 - (\hat{\mathbf{n}} \cdot \mathbf{k})(\hat{\mathbf{n}} \cdot \mathbf{l}) \frac{(\epsilon(\mathbf{k}) - \epsilon(\mathbf{l}))^2}{\epsilon(\mathbf{k})\epsilon(\mathbf{l})} + \left(\epsilon^2(\mathbf{k}) + \epsilon^2(\mathbf{l}) - \frac{s}{2} \right) \left| \frac{\hat{\mathbf{n}} \cdot \mathbf{k}}{\epsilon(\mathbf{k})} - \frac{\hat{\mathbf{n}} \cdot \mathbf{l}}{\epsilon(\mathbf{l})} \right| \right]^{-1}, \quad (3)$$

and the self-energy corrections and counter-terms are summarized by

$$\begin{aligned} g(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k}) &= \frac{4}{a(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k})} \int_0^1 dz \log \left(1 + \frac{a(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k})(z - z^2)}{\lambda^2 z + m^2(1 - z)^2} \right) \\ &+ \frac{4}{b(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k})} \int_0^1 dz \log \left(1 + \frac{b(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k})(z - z^2)}{\lambda^2 z + m^2(1 - z)^2} \right) \end{aligned} \quad (4)$$

with

$$\begin{aligned} a(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k}) &= 2(\mathbf{k}^2 - \mathbf{q}_{\text{in}}^2) \left(1 + \frac{\hat{\mathbf{n}} \cdot \mathbf{k}}{\epsilon(\mathbf{k})} \right), \\ b(\mathbf{k}^2, \hat{\mathbf{n}} \cdot \mathbf{k}) &= 2(\mathbf{k}^2 - \mathbf{q}_{\text{in}}^2) \left(1 - \frac{\hat{\mathbf{n}} \cdot \mathbf{k}}{\epsilon(\mathbf{k})} \right). \end{aligned}$$

The conventional method to solve Eq.(2) is to set up an equivalent Lippman-Schwinger equation⁹ which is given by

$$T(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) = V(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) - \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{m^2}{\epsilon(\mathbf{q})} \frac{V(\mathbf{k}, \mathbf{q}, \hat{\mathbf{n}})T(\mathbf{q}, \mathbf{l}, \hat{\mathbf{n}})}{\mathbf{q}^2 - \mathbf{q}_{\text{in}}^2 + i\epsilon}. \quad (5)$$

Using the partial wave expansion of the scattering amplitude $T(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}})$ given by

$$T_j(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) = \int d\Omega T(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) P_j(\cos\theta), \quad (6)$$

where θ is the angle between \mathbf{k} and \mathbf{l} , and similarly defining $V_j(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}})$ as

$$V_j(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) = \int d\Omega V(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) P_j(\cos\theta), \quad (7)$$

we obtain

$$\begin{aligned} T_j(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) &= V_j(\mathbf{k}, \mathbf{l}, \hat{\mathbf{n}}) \\ &- \int \frac{|\mathbf{q}|^2 d|\mathbf{q}|}{(2\pi)^3} \frac{m^2}{\epsilon(\mathbf{q})} \frac{V_j(\mathbf{k}, \mathbf{q}, \hat{\mathbf{n}})T_j(\mathbf{q}, \mathbf{l}, \hat{\mathbf{n}})}{\mathbf{q}^2 - \mathbf{q}_{\text{in}}^2 + i\epsilon}. \end{aligned} \quad (8)$$

The $\hat{\mathbf{n}}$ -dependence in Eqs.(2)-(8) indicates the violation of the rotation invariance. However, we made a connection between our analysis and the bipolar harmonics formalism presented by Fuda¹⁰ and found that the physical amplitude suggested by Fuda is the rotational average of our $T_j(\mathbf{q}_{\text{in}}, \mathbf{q}_{\text{in}}, \hat{\mathbf{n}})$ over $\hat{\mathbf{n}}$ -direction and does not carry $\hat{\mathbf{n}}$ -dependence. Such procedure of integrating out the quantization axis dependence to obtain a physical amplitude is not possible in the ordinary equal t -quantization because the space for the boost operation is not compact. Integrating out the quantization axis dependence in equal t -quantization would necessarily require to include the lightcone surface. Therefore, the lightcone quantization method appears to be the most efficient way of solving quantum field theories. In order to quantify the dependence of the phase shift on the direction $\hat{\mathbf{n}}$, we fix the scattering plane as the plane made by $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ and the direction of initial momentum \mathbf{k} as $\hat{\mathbf{z}}$ and then vary the direction $\hat{\mathbf{n}}$. The effect of rotating the direction $\hat{\mathbf{n}}$ in a given scattering plane defined by its perpendicular direction $\mathbf{k} \times \mathbf{l}$ is equivalent to the effect of rotating $\mathbf{k} \times \mathbf{l}$ in a given direction of the lightcone time evolution, e.g., $\tau = t + z/c$. In any case, the focus of study is the dynamics dependent on the relative angle between $\hat{\mathbf{n}}$ and $\mathbf{k} \times \mathbf{l}$.

3. Numerical Results

We calculated both S-wave ($j = 0$) and P-wave ($j = 1$) phase shifts for various coupling constants ($\beta = \frac{\alpha}{\pi}$) and c.m. momenta. Since the detailed numerical results were presented in our recent papers^{1,7}, we discuss only the main features of the numerical results. For the small

β (e.g., $\beta = 0.1$), the light-cone results for $\hat{n} = \hat{x}, \hat{y}$ and \hat{z} are almost same whether the self energy corrections are included or not. As β gets larger (e.g. $\beta > 0.3$), however, one can see that the three lightcone results for $\hat{n} = \hat{x}, \hat{y}$, and \hat{z} deviate. The results including the self-energy corrections are consistently lower than the ones without them, indicating that the self-energy effects are repulsive. Also, we observe that the phase shifts with the self-energy correction do not change much as the coupling constant grows. This is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects, and make the binding energy be frozen as the coupling constant increases⁶. The similar results were obtained in the generalized theory of the Wick-Cutkosky model using discretized lightcone quantization¹¹ and in the Yukawa model¹². Furthermore, as we can observe from the numerical results, the deviations among $\hat{n} = \hat{x}, \hat{y}$, and \hat{z} are smaller after the self-energy corrections are included. Such reduction of the \hat{n} -dependence is more dramatic in the P-wave analyses. Especially, the dramatic falloff¹ of the phase shift with $\hat{n} = \hat{z}$ in the large c.m. momentum region ($k^2/m^2 > 1$) shown for $\beta = 20$ disappears completely by the self-energy corrections. This indicates a significant restoration of the rotational invariance in the scattering kernel by adding the self-energy interactions. It shows an example that the rotation symmetry in the lightcone quantization can be dynamically restored.

4. Summary and Conclusion

Practical computations using the light-cone quantization method require, in general, the truncation of the higher Fock states. As a consequence, the calculated scattering amplitude in the truncated Fock-space is not rotationally invariant because the transverse angular momentum operator whose direction is perpendicular to the direction of the quantization axis in the light-cone quantization method involves the interaction that changes the particle number. However, in view of the rotational compactness, the lightcone quantization appears to be most efficient in solving quantum field theories. The extent of the rotation symmetry breaking can be quantified by the explicit rotation dependence of the two-body scattering phase shifts. In a recent work⁷, we investigated the scattering problem in the light-cone formalism using a simple scalar field model by extending the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation which includes the self-energy corrections and counter-terms. We found that the self-energy interactions significantly restore the rotation symmetry and remove the dramatic falloff of the phase shifts observed¹ in the P-wave analysis with the large coupling and the large momentum. It shows an example that the rotation symmetry in the lightcone quantization can be dynamically restored. Also, we observe that the self-energy effects make the phase shifts frozen as the coupling constant is increased. This is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects.

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