Light Cone Dynamics of the QCD String with Quarks

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Abstract

The light–cone Hamiltonian of the effective QCD string with quarks is derived from Lorentz and gauge invariant Green function of $q\bar{q}$ system in confining gluonic fields. To incorporate confinement we start first with the Euclidean dynamics of the system and make use of the minimal area law asymptotis of averaged Wilson loop. It leads to the effective action of valence quarks connected by the frozen string. Minkowski dynamics of this effective model is investigated and different asymptotical regimes are found.

1. Introduction

The question how nonperturbative phenomena like confinement and chiral symmetry breaking arise in light cone (l.c.) frame is of primary importance to take the real advantage of light cone quantization [1] of QCD. In order to make profit of the results elaborated in Euclidean formulation of the theory we start with the consideration of Euclidean Green function of $q\bar{q}$ system where the interaction is described by the averaged Wilson loop. Confinement is included via the minimal area law asymptotics of Wilson loop which results (after continuation in Minkowski space) in the Hamiltonian of valence quarks connected by the frozen string.

As well as in our rest frame analysis [2], [3] the Hamiltonian contains explicitly the density of total momentum fraction carried by the string that eventually leads to existence of different dynamical regimes of the system.

2. Light cone Hamiltonian of the effective QCD string with quarks

We begin as in [2], [3] with quenched approximation for Green function of spinless quark and antiquark [4] interacting in Euclidean space

$$G(x\bar{x}, y\bar{y}) = \int_0^{+\infty} ds \int_0^{+\infty} d\bar{s}\, e^{-K-\bar{K}} Dz D\bar{z} < W(C) >_A \tag{1}$$

where $< W(C) >_A$ is the usual averaged Wilson loop operator along the contour consisting of z_μ, $\bar{z}_\mu$, trajectories and K, $\bar{K}$ are the kinetic terms of quarks

$$K = \frac{1}{2}\int_0^1 d\gamma \left(m_1^2 s + \frac{1}{s}\dot{z}_\mu^2 \right), \quad \bar{K} = \frac{1}{2}\int_0^1 d\bar{\gamma} \left(m_2^2 \bar{s} + \frac{1}{\bar{s}}\dot{\bar{z}}_\mu^2 \right) \tag{2}$$

The confinement enters the dynamics via the assumption of the area law for the asymptotics of the Wilson loop at large distances in Euclidean space (for recent numerical calculations see [5]) as it follows from the cluster expansion arguments [6]

$$< W(C) >_A \approx \exp[-\sigma S] \tag{3}$$

where S is the minimal surface for a given contour C.

Our aim is to continue eq. (1) into Minkowski space with light cone variables

$$z_\pm = \frac{z_3 \pm z_0}{\sqrt{2}}, \quad z_4 = -iz_0, \quad \vec{z}_\perp = (z_1, z_2) \tag{4}$$

and define the Hamiltonian through the equation $\partial G/\partial T = -iHT$ where the evolution parameter T is chosen as $T = z_+ = \bar{z}_+$.

To take advantage of this frame one is to exploit the fact that creation of massive states from the vacuum is suppressed on light cone. For the interaction (2) such creation is originated by the backward motion (in l.c. time) of quarks. Consequently one can separate in a selfconsistent way the domain of $q(\bar{q})$ trajectories with no backtracking in l.c. time and reparametrize eqs. (2), (3) from proper times γ, $\bar{\gamma}$ to the physical l.c. times z_+, $\bar{z}_+$ which is equivalent [7] to the following substitution in (2), (3) respectively

$$K \to K' = \frac{1}{2}\int d\tau \left(\mu_1(\dot{z}_\perp^2 + 2\dot{z}_-) - \frac{m_1^2}{\mu_1} \right), \quad \bar{K} = \frac{1}{2}\int d\tau \left(\mu_2(\dot{\bar{z}}_\perp^2 + 2\dot{\bar{z}}_-) - \frac{m_2^2}{\mu_2} \right) \tag{5}$$

$$z_+(\tau) = \bar{z}_+(\tau) = \tau \tag{6}$$

As a result one arrives [7] to the following local in l.c. time $3D$ effective action

$$G = \int D\mu_1 D\mu_2 Dz_\perp D\bar{z}_\perp Dz_- D\bar{z}_- \exp[iA] \tag{7}$$

$$A = K + \bar{K} - \sigma \int d\tau\, d\beta \sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2} \tag{8}$$

where the straight line approximation [2,3,7] for the surface $S_{\min}$ is used

$$w_\mu(\tau,\beta) = z_\mu(\tau)\beta + \bar{z}_\mu(\tau)(1-\beta), \quad \dot{w}_\mu = \frac{\partial w_\mu}{\partial \tau}, \quad w'_\mu = \frac{\partial w_\mu}{\partial \beta} \tag{9}$$

and one is to perform path integration over $\mu_1(\tau)$, $\mu_2(\tau)$ in the same way as over $z_\perp$, $\bar{z}_\perp$ and z_-, $\bar{z}_-$.

To obtain the Hamiltonian we note first that action (8) contains time derivatives of w_μ in the interaction term and therefore the string carries a finite fraction of the total and relative momentum of the system. To incorporate it explicitly let us introduce in the same way as in ref.[3] the total R_μ and relative r_μ coordinates

$$R_\mu = x(\tau) z_\mu(\tau) + (1 - x(\tau))\bar{z}_\mu(\tau) \; , \qquad r_\mu = z_\mu(\tau) - \bar{z}_\mu(\tau) \tag{10}$$

where parameter $x(\tau)$ will be determined from the condition [3] that $\dot{R}_\mu$ is decoupled in the total action from $\dot{r}_\mu$.

We use the formalism of auxiliary fields [8] in order to write the effective action (8) in a gaussian form which is trasformed in the following Hamiltonian [7]

$$H = \frac{1}{2}\left\{ \frac{m_1^2}{\mu_1} + \frac{m_2^2}{\mu_2} + \frac{\vec{p}_\perp^{\,2} - \frac{(\vec{p}_\perp \vec{r}_\perp)}{r_\perp^2}}{a_3} + \frac{((\vec{p}_\perp \vec{r}_\perp) + \lambda(P_+ r_-))^2}{\tilde{\mu} r_\perp^2} + \int \frac{\sigma^2}{\nu} d\beta \, r_\perp^2 + \right. \tag{11}$$

$$\left. + \frac{\int \nu d\beta \cdot (P_+ r_-)^2}{P_+(\mu_1 + \mu_2) r_\perp^2} \right\}$$

where $\nu(\tau, \beta)$ is the auxiliary field which should be integrated out in the full path integral representation for the Green function and

$$\begin{aligned} a_3 &= \mu_1(1-x)^2 + \mu_2 x^2 + \textstyle\int (\beta - x)^2 \nu \, d\beta \\ \tilde{\mu} &= \mu_1\mu_2/(\mu_1 + \mu_2), \quad \lambda = x - \mu_1/(\mu_1 + \mu_2) \end{aligned} \tag{12}$$

Here we have introduced the total momentum

$$P_+ = p_{1+} + p_{2+} = \mu_1 + \mu_2 + \int \nu \, d\beta \; , \qquad \vec{P}_\perp = 0 \tag{13}$$

and Feynman–Bjorken variable x

$$x = \frac{p_{1+}}{p_{1+} + p_{2+}} = \frac{\mu_1 + \int \beta \nu \, d\beta}{\mu_1 + \mu_2 + \int \nu \, d\beta} \tag{14}$$

which coincides [7] with $x(\tau)$ entering eq. (10). From eqs. (13), (14) it follows that $\nu(\tau, \beta)$ is the density of fraction of total momentum carried by the string while $\mu_i(\tau)$ are that of quarks.

One is to express [7] μ_i with the help of eqs. (13), (14) through x, P_+, ν and then substitute [2] $\nu(\tau, \beta)$ by its extremal value ν_{ext} defined by equation

$$\left.\frac{\delta H}{\delta \nu}\right|_{\nu_{ext}} = 0 \tag{15}$$

It results in Hamiltonian depending only on canonically conjugated pairs $\{x,(P_+r_-)\}$, $\{\vec{p}_\perp, \vec{r}_\perp\}$ and Weyl ordering enables to construct the operator of H which will be used to calculate wave functions and formfactors [9].

3. Dynamical regimes of the system on light cone

To illuminate the main features of Hamiltonian (11) let us consider different limiting dynamical regimes originated by the interplay between the quarks and the string degrees of freedom.

The first and the simplest one is the case of heavy quarks with masses $m_1,\ m_2 \gg \sqrt{\sigma}$ where we have [7] for $\vec{p}^2 \ll (m_1+m_2)^2$

$$H = \frac{M^2}{2P_+}, \quad M \approx m_1 + m_2 + \frac{\vec{p}^2}{2\tilde{m}} + \sigma|\vec{r}| \tag{16}$$

and the following canonically conjugated p_z, r_z naturally arise

$$p_z = (m_1+m_2)\left(x - m_1/(m_1+m_2)\right), \quad r_z = \frac{(P_+r_-)}{m_1+m_2} \tag{17}$$

In the case of light quarks pure dynamical regimes appear [7] either for stretched along z axis or for squeezed in perpendicular plane configurations.

The latter type of configurations corresponding to $L - |L_z| \ll L$ appear in regimes which are the direct counterparts of the rest frame ones [2,3]. The transverse linear potential describes [7] excitations of radial quantum number n_r

$$H \to \frac{1}{2P_+}\left(2|\vec{p}_\perp| + \sigma|\vec{r}_\perp|\right)^2, \quad n_r \gg L \approx |L_z| \gg 1 \tag{18}$$

so that $M^2 \to 2\pi\sigma(2n_r + \Delta_1)$ and in the opposite case one obtains [7] in the leading order the Hamiltonian of transverse rotating string [2,3] with

$$\nu(\beta) = \frac{P_+}{M}\left(\frac{8\sigma L}{\pi}\right)^{1/2} \frac{1}{\sqrt{1-4(\beta-1/2)^2}}, \quad L \approx |L_z| \gg n_r \tag{19}$$

and $M^2 \to 2\pi\sigma(L+\Delta_2)$.

The regime of stretched configurations is a specifically l.c. one and we arrive [7] in this limit $L \gg |L_z|$ at well known t'Hooft $1+1$ QCD Hamiltonian

$$H \to \frac{1}{2P_+}(2\sigma|P_+r_-|), \quad n_r \gg L \gg |L_z| \gg 1 \tag{20}$$

with $M^2 \to 2\pi\sigma[(2n_r) + \Delta_3]$.

Summarizing the spectrum at least asymptotically has the following simple pattern

$$M^2 = 2\pi\sigma(2n_r + L + \Delta) \tag{21}$$

which coincides with that of the rest frame Hamiltonian [2,3].

4. Discussion

To conclude we derive l.c. Hamiltonian of the effective QCD string with quarks wave functions of which simply connected [9] to form factors and structure functions. Due to the noninert nature of the interaction properly defined Feynman–Bjorken variable x and total momentum P_+ involve the contribution from the string. In is important to note that total momentum P_+ and relative distance r_- enter the mass squared operator M^2 only via combination (P_+r_-) which is canonically conjugated to x. This property violates usual assumption that P_+ is decoupled from M^2. Also we stress that M^2 can not be decomposed into a pure kinetic (usually refered to quarks) and pure potential (string) parts. It brings about in particular the existence of different dynamical regimes of the system. The second important consequence of the interplay between the string and the quarks is that properly defined l.c. orbital momentum will be dependent of the interaction.

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